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INSTANTANEOUS CHIRAL QUARK MODEL
FOR RELATIVISTIC MESONS
IN A HOT AND DENSE MEDIUM¹

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1 Introduction

One of the most interesting fields of current research in nuclear and particle physics is the search for effects occurring in hot and dense hadronic matter [1]. This research is becoming more topical in the light of the construction of new accelerators such as the LHC at CERN and the RHIC at Brookhaven, which are planned to provide temperatures and particle densities where the quark (and gluon) substructure of the hadrons are to become evident not only by changes in the hadronic properties but, even more interestingly, by the emergence of a new phase of matter characterized by chiral symmetry and quark deconfinement. This therefore, makes an accurate simultaneous description of all the modifications of the properties of hadrons and their interactions at finite temperatures and densities necessary. In addition it is not yet fully understood which observables are suitable for testing the state of matter produced during the course of the envisaged heavy ion collision.

A modern theory for these problems should be based on quark and gluon degrees of freedom and should obey the symmetries of Quantum Chromodynamics. However, up to now, most of the investigations at finite temperature and density have been performed within the framework of QCD motivated models of the Nambu- and Jona-Lasinio (NJL) type [2] – [7], where the quark interactions are approximated by contact forces and gluonic degrees of freedom are not considered. This local theory is valid only for low energy interactions of light quarkonia and omits very interesting phenomena such as the behaviour of formfactors, the heavy quarkonium spectroscopy and hadron – hadron interactions.

On the other hand, the heavy quarkonia are very well described within the approximation of a Schrödinger equation and phenomenological confining potentials [11]. There is a difficulty in generalizing this approximation to describe heavy-light quark mesons, i.e. open charm and open beauty, which are interesting particles for the above mentioned task of hot hadronic matter spectroscopy. Thus, both the nonrelativistic potential models and the relativistic NJL model have only a limited region of validity.

Recently, a QCD inspired approach to the description of the spectrum of mesons and their interactions was formulated in Refs. [8, 9, 10] as covariant generalization of nonrelativistic potential models [11], containing, as limiting cases, the above mentioned descriptions of both heavy as well as light quarkonia. This instantaneous approach to mesons is based on the following three assumptions:

- The Markov – Yukawa definition bilocal fields as irreducible representation of the Poincare group [12, 13].
- The dominant role of equal – time interactions in the formation of these bound states [8, 9, 10].
- The method of the gaugeless reduced phase space quantization of chromodynamics, which is particularly suitable for the description of bound states. This method leads to a unique separation of equal time and retardation parts in gluon exchange interaction with respect to the eigen-vector of the bound state total momentum operator [14, 15].

With regards to the problem of the description of quark and meson mass spectra in the meson rest frame, the bilocal instantaneous model [8, 9, 10, 16] completely reproduces the QCD motivated potential models for spontaneous chiral symmetry breaking [17] – [23] while, in relation to the description of light meson interactions, it generalizes the successful NJL – type [6, 7] and chiral Lagrangian [24, 25] approaches.

In the present paper we present a chiral quark model with covariant instantaneous interactions at finite temperature and chemical potential, thus generalizing the previous bilocal field theoretical approaches to apply to the mesonic bound states. Moreover, after having shown the covariance of bound states in the vacuum, we demonstrate how covariance is broken in the medium by examining the case of mesons which are Lorentz-boosted relative to the surrounding thermal medium.

At first we consider the common spontaneous symmetry breaking mechanism, leading to the creation of massive quarks. As is well known [17] – [23] the corresponding Schwinger–Dyson equation for the mass operator has the same structure as the famous Bogoliubov gap equation of BCS-theory. This analogy makes it clear that heating the system reduces the quark mass and forces it to disappear at some critical temperature T_c , signaling the thermal restoration of chiral symmetry. Similarly, a finite baryon number density of the surrounding matter leads to the occupation of lowest positive energy states, causing a decrease of the quark mass quite similar to the Pauli – blocking in odd – mass atomic nuclei acting to diminish the gap parameter of the neighboring even mass system in the BCS-theory of pair correlations.

Our main interest is, however, in a rather complete formulation of temperature and baryon number dependent effects in meson structure, described by correlated quark – antiquark pairs above the chiral symmetry violating vacuum. In the main, the solving of this task is known from the pioneer work [26] but, contrary to the situation with the Schwinger – Dyson equation [21, 22], a lot of principal aspects of the theory of bound states, such as the influence of nonlocality in coordinate space, general structure of temperature and density dependent effects, flavour dependence, the influence of the total momentum of moving mesons, decay properties, the baryon structure, so have so far not been addressed in considerable detail in a thermodynamic approach.

We will fill this gap by generalizing our $T = 0, \mu = 0$ results to obtain justified meson masses and wave functions, depending on temperature, chemical potential and finite meson velocity relative to the medium as a first step in this direction.

The paper is organized as follows. In section 2 we shortly review the path integral bosonization for an instantaneous four-point interaction kernel which allows for the introduction of covariant bilocal meson fields. We derive the Schwinger–Dyson as well as the Bethe–Salpeter equation from the effective chiral quark model Lagrangian by functional integral techniques. In section 2 we consider quark pair correlations at zero temperature. We show that, for the instantaneous interaction model, a manifestly covariant description of bound states can be given. We concentrate on a discussion of pseudoscalar mesons (pion, kaon). In section 4 we generalize the covariant description given in the previous section to the case of finite temperatures and densities, by applying the Matsubara technique of finite temperature Green functions. We show that, in this case, the covariance of the bound states is broken by the medium. The quark and the meson mass spectra are dependent on the velocity relative to the surrounding medium. We show that a fast moving bound state is protected against chiral symmetry restoration, in other words, a finite boost of a quark pair relative to the medium acts towards chiral symmetry breaking. In section 5, we present the numerical evaluation of the presented approach for the case of a separable quark antiquark interaction. Finally, section 6 contains a summary of the results obtained and the conclusions.

2 Covariant bound states in effective QCD

The covariant formulation of hadronic bound states on the basis of a QCD- motivated theory has been worked out over the past few years [8]. In this section we present the principal notations and results of this approach.

In the effective chiral quark models, the gluonic degrees of freedom are not treated explicitly but are incorporated in the effective quark-quark interaction. The quark action functional can be represented in the form

$$S_{\text{eff}}[q, \bar{q}] = \int dx_1 dx_2 \bar{q}_\alpha(x_1) [G_0^{-1}(x_1, x_2)]_{\alpha\beta} q_\beta(x_2) \\ - \frac{1}{2} \int dx_1 dx_2 dy_1 dy_2 q_\delta(x_2) \bar{q}_\alpha(x_1) [\mathcal{K}^\eta(x_1, x_2; y_1, y_2)]_{\alpha\beta; \gamma\delta} q_\beta(y_1) \bar{q}_\gamma(y_2). \quad (1)$$

Here, the first term is the free quark action, where $[G_0^{-1}(x_1, x_2)]_{\alpha\beta} = (i\hat{\partial} - \hat{m}_0) \delta_{\alpha\beta} \delta(x_1 - x_2)$ is the free quark Green function and $\hat{m}_0 = \text{diag}(m_0^a)$, $a = 1, \dots, N_f$ is the current quark mass matrix. The greek indices $\alpha \dots \delta$ are a compact notation for Dirac as well as flavour and color indices. Flavour and colour numbers are denoted by N_f and N_c , respectively.

The instantaneous interaction kernel is assumed to obey the channel decomposition

$$[\mathcal{K}^\eta(x_1, x_2; y_1, y_2)]_{\alpha\beta; \gamma\delta} = \sum_H C_{\alpha\beta}^H \mathcal{K}^\eta(x, y|X, Y) C_{\gamma\delta}^H, \quad (2)$$

where $C_{\alpha\beta}^H$ are the flavor and color wave functions in the hadronic channel H (see ref.[27]). The orbital part of the interaction

$$\mathcal{K}^\eta(x, y|X, Y) = \not{W}(x^\perp, y^\perp) \delta^4(X - Y) \delta(x \cdot \eta) \delta(y \cdot \eta) \not{\eta} \quad (3)$$

has a relativistic covariant form [9], with $x = x_1 - x_2$, $X = (x_1 + x_2)/2$ being the four-vectors of the relative and the center of mass coordinates of the incoming quark-antiquark pair; $y = y_1 - y_2$, $Y = (y_1 + y_2)/2$ are those of the outgoing one, respectively. The four-dimensional δ -functions guarantee the condition of the center of mass conservation, which is a consequence of the homogeneity of the space-time continuum.

The instantaneous interaction kernel is further assumed to neglect retardation effects in the s -channel so that it depends via $W(x^\perp, y^\perp)$ only on the transverse components of the four-distances x_μ and y_μ with respect to the (conserved) four-vector P_μ of the center of mass momentum:

$$x_\mu^\perp = x_\mu - x_\mu^\parallel, \quad x_\mu^\parallel = \eta_\mu(x \cdot \eta), \quad \eta_\mu = \frac{P_\mu}{\sqrt{P^2}}, \quad (4)$$

$y_\mu^\perp$ and $y_\mu^\parallel$ are introduced accordingly. The neglect of retardation effects in the s -channel has been discussed with regard to formulating a wave equation. This ansatz is justified by the assumption of the dominance of instantaneous interactions between the meson constituents as expressed by the condition $\delta(x \cdot \eta)$. This potential of the effective quark interaction can be constructed in a gauge invariant way within the reduced phase space quantization scheme [14, 15].

This general form is related to the bilocal theory where we assume that

$$W(x^\perp, y^\perp) = \delta^3(x^\perp - y^\perp) V(x^\perp), \quad (5)$$

and the potential $V(x^\perp)$ depends on the transversal component of the four - momentum x_μ . The general form of $W(x^\perp, y^\perp)$ also includes separable potentials

$$W(x^\perp, y^\perp) = V_0 g(x^\perp) g(y^\perp) \quad (6)$$

Within the standard functional integral approach to bilocal field theory [3, 28] the action (1) can be transformed by introducing bilocal bosonic fields $\mathcal{M}^H = \mathcal{M}^H(x_1, x_2)$. After integration over the quark fields, the effective bosonized action takes the form

$$S_{\text{eff}}[\mathcal{M}] = -N_c \left\{ \frac{1}{2} (\mathcal{M}(\mathcal{K}^\eta)^{-1} \mathcal{M}) + i \text{Tr} \ln(-G_0^{-1} + \mathcal{M}) \right\} \quad (7)$$

In this equation the trace Tr acts on the four-momenta as well as the flavor - and Dirac indices. We confine ourselves to the case of color singlet states such that the trace over color indices is performed yielding the factor N_c .

Due to the particular choice of the instantaneous interaction kernel (3), the fields $\mathcal{M}_{\alpha\beta}^H(x_1, x_2) = C_{\alpha\beta}^H \mathcal{M}^H(x|X)$ can be introduced in such a way that they satisfy the Markov - Yukawa condition [12, 13] for instantaneous bilocal meson fields

$$x_\mu \frac{\partial}{\partial X_\mu} \mathcal{M}^H(x|X) = 0. \quad (8)$$

These fields are irreducible representations of the Poincare group with definite mass $P^2 = M_H^2$, spin and orbital part which can be expressed as

$$\begin{aligned} \mathcal{M}^H(x|X) = & \int \frac{d\mathbf{P}}{(2\pi)^{3/2} \sqrt{2\omega_H(\mathbf{P})}} \{ e^{-iP \cdot X} \Gamma_H(x^\perp|P) a_H^+(P) + \\ & + e^{iP \cdot X} \bar{\Gamma}_H(x^\perp|P) a_H^-(P) \} \delta(x \cdot \eta^H), \end{aligned} \quad (9)$$

where $a_H^+(P), a_H^-(P)$ are creation and annihilation operators of a bound state. The index H denotes the set of hadron quantum numbers and P_0 is fixed as $P_0 = \omega_H = \sqrt{\mathbf{P}^2 + M_H^2}$. $\Gamma_H(x^\perp|P)$ and $\bar{\Gamma}_H(x^\perp|P)$ are the quark-meson vertex amplitudes which are functions of the transverse component of the four-distance $x^\perp$ with respect to the four-vector P_μ of the center of mass momentum.

The extremum condition for the action (7)

$$\left. \frac{\delta S_{\text{eff}}[\mathcal{M}(x_1, x_2)]}{\delta[\mathcal{M}(x_1, x_2)]} \right|_{\mathcal{M}(x_1, x_2) = \Sigma(x_1, x_2) - \hat{m}_0 \delta(x_1 - x_2)} = 0 \quad (10)$$

coincides with the Schwinger - Dyson (gap) equation for the quark mass operator $\Sigma(x_1, x_2)$

$$\Sigma(x_1, x_2) = \hat{m}_0 \delta(x_1 - x_2) + i N_c N_f \int dy_1 dy_2 \text{tr} [\mathcal{K}^\eta(x_1, x_2, y_1, y_2) G(y_1, y_2)] , \quad (11)$$

where $G^{-1}(y_1, y_2) = \delta(y_1 - y_2) i \not{\partial} - \Sigma(y_1, y_2)$ denotes the quark Green function. This equation defines the dynamical quark mass spectrum which is spontaneously generated by the mechanism of chiral symmetry breaking. Expansion of action (7) around the classical solution (11) over fluctuations, $\mathcal{M} = \mathcal{M}' + (\Sigma - \hat{m}_0)$, gives

$$S_{\text{eff}}[\mathcal{M}] = S_{\text{eff}}^{(0)}[\Sigma - \hat{m}_0] + S_{\text{eff}}^{(2)}[\mathcal{M}'] + S_{\text{eff}}^{\text{int}}[\mathcal{M}'], \quad (12)$$

where

$$S_{\text{eff}}^{(2)}[\mathcal{M}] = -\frac{N_c}{2} \left\{ [\mathcal{M}'(\mathcal{K}^\eta)^{-1} \mathcal{M}'] + i \text{Tr}(G \mathcal{M}')^2 \right\} \quad (13)$$

and

$$S_{\text{eff}}^{\text{int}}[\mathcal{M}] = i N_c \sum_{n=3}^{\infty} \frac{1}{n} \text{Tr}(G \mathcal{M}')^n. \quad (14)$$

The second variation of $S_{\text{eff}}[\mathcal{M}]$ with respect to $\mathcal{M}'(x_1, x_2)$

$$\frac{\delta^2 S_{\text{eff}}[\mathcal{M}'(x_1, x_2)]}{\delta[\mathcal{M}'(x_1, x_2)]^2} \Big|_{\mathcal{M}'(x_1, x_2)=0} \Gamma(x_1, x_2) = 0 \quad (15)$$

leads to the homogeneous Bethe–Salpeter equation in the ladder approximation for the vertex function $\Gamma(x_1, x_2)$ of the $[q(x_1)q(x_2)]$ state

$$\Gamma(x_1, x_2) = i N_c \int dy_1 dy_2 \mathcal{K}^\eta(x_1, x_2; y_1, y_2) [G(x_1, y_1) \Gamma(y_1, y_2) G(y_2, x_2)] \quad (16)$$

Eq. (16) has to be solved together with equation (11) and describes bound states as well as scattering states of the quark–antiquark system. It has been shown in refs. [8, 17] that the equations (11) and (16) describe in the limit $m_0 \rightarrow 0$ both the chiral symmetry breaking as well as the bilocal version of the Goldstone theorem, so that the action (13) can be considered as the generalization of chiral Lagrangians. In the limit $m_0 \rightarrow \infty$, equations (11) and (16) reduce to the Schrödinger equation.

3 Quark pair correlations at zero temperature

3.1 Gap equation

In momentum space, the gap equation (11) for the self energy (mass operator) with the covariant notation (3) for the interaction kernel is given by [17]

$$\begin{aligned} \Sigma^a(q^\perp) &= m_0^a + i N_c N_f \int \frac{dp}{(2\pi)^4} W(q^\perp, p^\perp) \not{h} G^a(p) \not{h} \\ &= m_0^a + N_c N_f \int \frac{dp^\perp}{(2\pi)^3} W(q^\perp, p^\perp) \left[i \int \frac{dp^\parallel}{(2\pi)} \not{h} G^a(p^\parallel, p^\perp) \not{h} \right], \end{aligned} \quad (17)$$

where the index a denotes the quark flavor. The quark four–momentum is decomposed into components parallel $p_\mu^\parallel = \eta_\mu(p \cdot \eta)$ and perpendicular $p_\mu^\perp = p_\mu - p_\mu^\parallel$ to the center of mass four–momentum P_μ of the bilocal field $\mathcal{M}(p, P)$, $dp = dp^\perp dp^\parallel$. In order to solve the equation (17), we introduce the following representation of the self energy

$$\Sigma^a(p^\perp) = E^a(p^\perp) [S^a(p^\perp)]^2 + \not{p}^\perp \quad (18)$$

The matrix $S^a(p^\perp)$ is the generalized Foldy–Wouthuysen matrix

$$[S^a(p^\perp)]^{\pm 2} = \cos[2\vartheta^a(p^\perp)] \mp \frac{\not{p}^\perp}{|p^\perp|} \sin[2\vartheta^a(p^\perp)], \quad (19)$$

that allows the defining of the projection operators on positive and negative energy states

$$\Lambda_{\pm}^a(p^{\perp}) = [S^a(p^{\perp})]^{-1} \left(\frac{1 \pm \not{n}}{2} \right) [S^a(p^{\perp})] = \frac{1}{2} \left[1 \pm \not{n} [S^a(p^{\perp})]^2 \right], \quad (20)$$

Using Eq. (18), the propagator of a massive quark can be represented as [8, 16]

$$G^a(p^{\parallel}, p^{\perp}) = \frac{1}{\not{p} - \Sigma^a(p)} = \left\{ \frac{\Lambda_+^a(p^{\perp})}{p^{\parallel} - E^a(p^{\perp}) + i\epsilon} + \frac{\Lambda_-^a(p^{\perp})}{p^{\parallel} + E^a(p^{\perp}) - i\epsilon} \right\} \not{n}, \quad (21)$$

in such a way that the dependence on the parallel component is separated. The poles of the propagator (21) at positive and negative energies correspond to quarks and antiquarks, respectively.

The integration over $p^{\parallel}$ in (17) with the representation of Green function (21) facilitates the presenting of the gap equation (17) as a set of two coupled integral equations

$$E^a(p^{\perp}) \cos[2\vartheta^a(p^{\perp})] = m_0^a + \frac{1}{2} N_c N_f \int \frac{dq^{\perp}}{(2\pi)^3} W(q^{\perp}, p^{\perp}) \cos[2\vartheta^a(q^{\perp})], \quad (22)$$

$$E^a(p^{\perp}) \sin[2\vartheta^a(p^{\perp})] = |p^{\perp}| + \frac{1}{2} N_c N_f \int \frac{dq^{\perp}}{(2\pi)^3} W(q^{\perp}, p^{\perp}) \frac{p^{\perp} \cdot q^{\perp}}{|p^{\perp}| |q^{\perp}|} \sin[2\vartheta^a(q^{\perp})], \quad (23)$$

for the two unknown functions $E^a(p^{\perp})$ and $\vartheta^a(p^{\perp})$. The system (22) – (23) describes parametrically the quark dispersion relation for a nonlocal interaction in a relativistic covariant form. For a given form of the (attractive) quark–antiquark interaction potential $V(q^{\perp}, p^{\perp})$, there exists a critical coupling strength to the extent that the (approximative) chiral symmetry with $m_0 \sim 0$ is broken in the vacuum and a constituent quark mass is generated dynamically.

3.2 Salpeter equation

In terms of vertex functions the Salpeter equation (16) with the kernel (3) in momentum space has the form

$$\begin{aligned} \Gamma^{ab}(q^{\perp}|P) &= iN_c \int \frac{dp}{(2\pi)^4} W(q^{\perp}, p^{\perp}) \not{n} G^a(p + \frac{P}{2}) \Gamma^{ab}(p^{\perp}|P) G^b(p - \frac{P}{2}) \not{n} \\ &= N_c \int \frac{dp^{\perp}}{(2\pi)^3} W(q^{\perp}, p^{\perp}) \left[i \int \frac{dp^{\parallel}}{2\pi} \not{n} G^a(p^{\parallel} + \frac{P}{2} | p^{\perp}) \Gamma^{ab}(p^{\perp}|P) G^b(p^{\parallel} - \frac{P}{2} | p^{\perp}) \right]. \end{aligned} \quad (24)$$

In this formula, P is the total 4 - momentum of the bound state. Due to the choice of the interaction kernel (3), the vertex function $\Gamma^{ab}(p^{\perp}|P)$ has no dependence on the parallel component $p^{\parallel}$. The factor $\not{n}$ appearing in eq. (24) for the effective interaction in the case of the moving bound states $\vec{P} \neq 0$ was evaluated by taking η in the direction of P so that $\eta_{\mu} = P_{\mu}/\sqrt{P^2}$.

Integrating (24) over $p^{\parallel}$, we obtain the the Salpeter equation in the form

$$\Gamma^{ab}(q^{\perp}|P) = N_c \int \frac{dp^{\perp}}{(2\pi)^3} W(q^{\perp}, p^{\perp}) \cdot \quad (25)$$

$$\not{n} \left\{ \frac{\Lambda_+^a(p^{\perp}) (\not{n} \Gamma^{ab}(p^{\perp}|P) \not{n}) \Lambda_-^b(-p^{\perp})}{[E^a(p^{\perp}) + E^b(p^{\perp})] - \sqrt{P^2} - i\epsilon} + \frac{\Lambda_-^a(p^{\perp}) (\not{n} \Gamma^{ab}(p^{\perp}|P) \not{n}) \Lambda_+^b(-p^{\perp})}{[E^a(p^{\perp}) + E^b(p^{\perp})] + \sqrt{P^2} + i\epsilon} \right\} \not{n}.$$

The equation (25) has the relativistic covariant form since it depends on the center of mass momentum of the quark pair in an invariant way via $\sqrt{P^2}$.

Using the explicit form of the projectors $\Lambda_{\pm}(p^{\perp})$, it is convenient to introduce the "undressed" vertex functions $\tilde{\Gamma}^{ab}(p^{\perp})$:

$$\tilde{\Gamma}^{ab}(p^{\perp}) = S_a(p^{\perp})\Gamma^{ab}(p^{\perp})S_b(p^{\perp}) . \quad (26)$$

The vertex functions (26) satisfy the equation

$$\begin{aligned} \tilde{\Gamma}^{ab}(q^{\perp}|P) = & -N_c \int \frac{dp^{\perp}}{(2\pi)^3} W(q^{\perp}, p^{\perp}) [S_a(q^{\perp})S_a^{-1}(p^{\perp})] \\ & \left\{ \frac{\Lambda_+^0}{[E^a(p^{\perp}) + E^b(p^{\perp})] - \sqrt{P^2} - i\epsilon} + \frac{\Lambda_-^0}{[E^a(p^{\perp}) + E^b(p^{\perp})] + \sqrt{P^2} + i\epsilon} \right\} \\ & \tilde{\Gamma}^{ab}(p^{\perp}|P) [S_b^{-1}(p^{\perp})S_b(q^{\perp})] , \end{aligned} \quad (27)$$

where $\Lambda_{\pm}^0 = (1 \pm \not{h})/2$.

We can see that the vertex function $\tilde{\Gamma}^{ab}$ can be represented as the sum of two functions $\tilde{\Gamma}_1^{ab}$ and $\tilde{\Gamma}_2^{ab}$ expanding over the full system of Dirac matrices [18, 19, 20]

$$\tilde{\Gamma}^{ab}(p^{\perp}|P) = \Gamma_1^{ab}(p^{\perp}|P) + \not{h}\Gamma_2^{ab}(p^{\perp}|P) , \quad (28)$$

with

$$\Gamma_l^{ab} = 1 \cdot \mathcal{S}_l^{ab} + \gamma_5 \cdot \mathcal{P}_l^{ab} + (\gamma_{\mu} - \eta_{\mu}\not{h}) \cdot (\mathcal{V}^{ab})_l^{\mu} + \gamma_5(\gamma_{\mu} - \eta_{\mu}\not{h}) \cdot (\mathcal{A}^{ab})_l^{\mu} , \quad (29)$$

$l = 1, 2$; the quantities $\mathcal{S}_l^{ab}, \mathcal{P}_l^{ab}, (\mathcal{V}_l^{ab})^{\mu}, (\mathcal{A}_l^{ab})^{\mu}$ are vertex functions of scalar, pseudoscalar, vector, axial- vector mesons, respectively. The investigation of the equation (27) in the rest frame for the zero temperature case was recently carried out in Ref. [7, 29] in the rest frame for the zero temperature case. The aim of the present work is to generalize this result to apply to the finite temperature case as it will be shown in sec. 4.

3.3 Decay constant f_P

In order to calculate the pseudoscalar meson decay constant f_P , we consider the inclusion of weak interactions in the effective action (1). The weak interactions change the quark flavour indices. We must therefore restore the flavour indices in the effective action. The quadratic part of the effective action (13) in momentum space has the form

$$S_{\text{eff}}^{(2)} = i \frac{N_c}{2} \text{tr} \int \frac{d^4 p}{(2\pi)^4} \left(\tilde{\Gamma}^{ab}(p^{\perp}|P) G^b(p^{\parallel}) + \frac{P}{2} |p^{\perp}\rangle \Gamma^{ba}(p^{\perp}|P) G^a(p^{\parallel}) - \frac{P}{2} |p^{\perp}\rangle \right) . \quad (30)$$

Now we consider the inclusion of weak interactions in the action (30). The Lagrangian of the weak interactions has the form [33]

$$\mathcal{L}_w = \frac{G}{\sqrt{2}} \{ V_{ab}(\bar{Q}_a O_{\mu} q_b)(\bar{l} O_{\mu} \nu_l) + h.c. \} = \frac{G}{\sqrt{2}} \{ V_{ab}(\bar{Q}_a O_{\mu} q_b) l_{\mu} + h.c. \} , \quad (31)$$

with the elements V_{ab} of the Kobayashi- Maskawa matrix, and the Fermi constant $G = 10^{-5}/m_p^2$. Q denotes the column of (u, c, t) quark and q stands for the column of (d, s, b)

quark fields. The leptons $l = e, \mu, \tau$ and their neutrinos $\nu_l = \nu_e, \nu_\mu, \nu_\tau$ couple to the quark - antiquark pairs as a pair field $l_\mu = \bar{l} O_\mu \nu_l$, where $O_\mu = \gamma_\mu(1 + \gamma_5)$. Thus the weak interaction Lagrangian (31) is very similar in its structure to the quark - antiquark effective action (30).

For the definition of the meson decay constants we will introduce in the nonlocal effective action the local leptonic weak current $\hat{L}_{ab}$ by the substitution

$$\Gamma^{ab} \rightarrow \Gamma^{ab} + \hat{L}^{ab}, \quad (32)$$

where

$$\hat{L}^{ab} = \frac{G}{\sqrt{2}} V_{ab} \hat{l}(P_L), \quad (33)$$

$\hat{l} = O_\mu l_\mu$, and P_L is the total momentum of the leptonic pair.

In the result we obtain the following form for effective action, which corresponds to weak interaction

$$\text{Tr}(\bar{\Gamma}^{ab} G^b \Gamma_{ba} G^a) \rightarrow 2 \text{Tr}(G^a \bar{\Gamma}^{ab} G_b \hat{L}_{ba}) = 2 \frac{G}{\sqrt{2}} V_{ab} O_\mu (G \bar{\Gamma} G)^{ab} l_\mu. \quad (34)$$

and the matrix element for the decay of a pseudoscalar meson H into a leptonic pair ($l\nu$)

$$\langle l\nu | S_{\text{eff}}^{(2)} | H \rangle = (2\pi)^4 \delta(P_H - P_L) \frac{1}{\sqrt{(2\pi)^3 2\omega_H}} \frac{G}{\sqrt{2}} \langle l\nu | l_\mu | 0 \rangle V_{ab} f_H^{ab} P_\mu \quad (35)$$

where

$$f_H^{ab} P_\mu = i N_c \int \frac{dp^\perp dp^\parallel}{(2\pi)^4} \text{tr} \left\{ O_\mu G_a(p^\parallel - \frac{P}{2} | p^\perp) \bar{\Gamma}_H^{ab} G_b(p^\parallel + \frac{P}{2} | p^\perp) \right\} \quad (36)$$

After integration over $p^\parallel$ and performing the trace operation, we obtain for the decay constant f_P^{ab} of pseudoscalar mesons

$$f_P^{ab} = \frac{4 N_c}{M_P^{ab}} P \int \frac{dq^\perp}{(2\pi)^3} \left[\mathcal{P}_2^{ab}(p^\perp) g_+(p^\perp) + \mathcal{P}_1^{ab}(p^\perp) g_-(p^\perp) \right] \cos(\vartheta^a + \vartheta^b) \quad (37)$$

where $M_P^{ab} = \sqrt{P^2}$ (see (27)) is the mass of pseudoscalar meson and

$$g_\pm(p^\perp) = \left(\frac{1}{[E^a(p^\perp) + E^b(p^\perp)] - M_P^{ab}} \pm \frac{1}{[E^a(p^\perp) + E^b(p^\perp)] + M_P^{ab}} \right) \quad (38)$$

denotes the symmetric (+) and antisymmetric (-) modes of the particle - antiparticle propagator.

4 Pair correlations in the medium at finite density and temperature

4.1 Gap equation for quark pairs moving in the medium

Let us consider a Lorentz boosted quark pair in the medium. The Lorentz boost is defined by the momentum of center of mass $P_\mu = (P_0, |\mathbf{P}|, 0, 0)$ the quark - antiquark

pair, where we chose the direction of the three - momentum of the meson parallel to the p_1 - axis. Accordingly, the separation of parallel and perpendicular components of the relative momentum p_μ is realized using the 4 - vector η_μ in the form

$$\eta_\mu = \frac{P_\mu}{\sqrt{P^2}} = \left(\frac{1}{\sqrt{1-v^2}}, \frac{v}{\sqrt{1-v^2}}, 0, 0 \right), \quad (39)$$

where the velocity of the $(q\bar{q})$ - pair is defined by

$$v = \frac{|\mathbf{P}|}{P_0}. \quad (40)$$

The representations of $\not{p}, p^\parallel, p^\perp$ for this choice are

$$\begin{aligned} \not{p} &= \gamma_\mu \eta_\mu = \frac{\gamma_0 - v\gamma_1}{\sqrt{1-v^2}} = \gamma_0^{(v)} \cdot \gamma_1^{(v)} = \frac{\gamma_1 - v\gamma_0}{\sqrt{1-v^2}} \\ p_\mu^\parallel &= (p_0^{(v)}, \mathbf{0}) \cdot p_\mu^\perp = \left(\frac{v}{\sqrt{1-v^2}} p_1^{(v)}, \frac{1}{\sqrt{1-v^2}} p_1^{(v)}, p_2, p_3 \right) = p_\mu^{\perp(v)}, \\ p_0^{(v)} &= \frac{p_0 - vp_1}{\sqrt{1-v^2}}, \quad p_1^{(v)} = \frac{p_1 - vp_0}{\sqrt{1-v^2}}. \end{aligned} \quad (41)$$

For the solution of the Schwinger - Dyson equation, we consider the representation of the (mass operator) self energy

$$\begin{aligned} \Sigma^a(p^{\perp(v)}|v) &= \gamma_0^{(v)} \Delta E^a(p^{\perp(v)}) + E^a(p^{\perp(v)}) [S^a(p^{\perp(v)})]^2 - (\gamma^{(v)} \cdot p^{\perp(v)}) \\ &= \gamma_0^{(v)} \Delta E^a(p^{\perp(v)}) + E^a(p^{\perp(v)}) \cos[2\vartheta^a(p^{\perp(v)})] + \\ &+ E^a(p^{\perp(v)}) \frac{(\gamma^{(v)} \cdot p^{\perp(v)})}{|p^{\perp(v)}|} \{ \sin[2\vartheta^a(p^{\perp(v)})] - |p^{\perp(v)}| \}. \end{aligned} \quad (42)$$

Now, the Green function (21) can be represented as

$$G^a(p_0^{(v)}, p^{\perp(v)}) = \left\{ \frac{\Lambda_+^a(p^{\perp(v)})}{p_0^{(v)} - E_+^a(p^{\perp(v)}) + i\epsilon} + \frac{\Lambda_-^a(p^{\perp(v)})}{p_0^{(v)} + E_-^a(p^{\perp(v)}) - i\epsilon} \right\} \gamma_0^{(v)}, \quad (43)$$

where

$$\Lambda_\pm^a(p^{\perp(v)}) = \frac{1}{2} \left[1 \pm \gamma_0^{(v)} [S^a(p^{\perp(v)})]^2 \right], \quad (44)$$

$$[S(p^{\perp(v)})]^{\pm 2} = \cos[2\vartheta^a(p^{\perp(v)})] \mp \frac{\not{p}^{\perp(v)}}{|p^{\perp(v)}|} \sin[2\vartheta^a(p^{\perp(v)})], \quad (45)$$

and

$$E_\pm^a(p^{\perp(v)}) = E^a(p^{\perp(v)}) \pm \Delta E^a(p^{\perp(v)}) \quad (46)$$

In order to introduce temperature and chemical potential of the surrounding medium, we have to replace the zero component of the relative four - momentum in the rest frame of the medium (p_0) by Matsubara frequencies ($\omega_n^a = p_0^a + \mu^a$) [30, 31].

Therefore, we have to apply an inverse Lorentz - transformation in order to express p_0 by the independent variables ($p_0^{(v)}, p_1^{(v)}$) of the boosted system, i.e.

$$p_0 = \frac{p_0^{(v)} + vp_1^{(v)}}{\sqrt{1-v^2}} \longrightarrow p_0'' + \mu'' \quad (47)$$

Inserting (47) in the expression in (43), we obtain

$$G^a(p_0^n, p^\perp(v)) = \left\{ \frac{\Lambda_+^a(p^\perp(v))}{(p_0^n + \mu^a) - E_+^a(p^\perp(v)|v) + i\epsilon} + \frac{\Lambda_-^a(p^\perp(v))}{(p_0^n + \mu^a) + E_-^a(p^\perp(v)|v) + i\epsilon} \right\} \frac{\gamma_0^{(v)}}{\sqrt{1-v^2}}, \quad (48)$$

where

$$E_\pm^a(p^\perp(v)|v) = \frac{E_\pm^a(p^\perp(v)) \pm v p_1^{(v)}}{\sqrt{1-v^2}}. \quad (49)$$

The spectral function is found to be

$$2\text{Im}G^a(p_0, p^\perp(v)) = 2\pi \left\{ \Lambda_+^a(p^\perp(v)) \delta(p_0 + \mu^a - E_+^a(p^\perp(v)|v)) + \Lambda_-^a(p^\perp(v)) \delta(p_0 + \mu^a + E_-^a(p^\perp(v)|v)) \right\} \frac{\gamma_0^{(v)}}{\sqrt{1-v^2}}. \quad (50)$$

Inserting the spectral density (50) in the Landau spectral representation [31] for the Green function, we obtain the finite temperature propagator of a moving quark

$$\mathcal{G}(p_0, p^\perp(v)) = \left\{ \frac{\Lambda_+^a(p^\perp(v)) [f_+^a(p^\perp(v)|v)]}{p_0 + \mu^a - E_+^a(p^\perp(v)|v) - i\epsilon} + \frac{\Lambda_+^a(p^\perp(v)) [1 - f_+^a(p^\perp(v)|v)]}{p_0 + \mu^a - E_+^a(p^\perp(v)|v) + i\epsilon} + \frac{\Lambda_-^a(p^\perp(v)) [1 - f_-^a(p^\perp(v)|v)]}{p_0 + \mu^a + E_-^a(p^\perp(v)|v) - i\epsilon} + \frac{\Lambda_-^a(p^\perp(v)) [f_-^a(p^\perp(v)|v)]}{p_0 + \mu^a + E_-^a(p^\perp(v)|v) + i\epsilon} \right\} \frac{\gamma_0^{(v)}}{\sqrt{1-v^2}}, \quad (51)$$

where we have used the notation

$$f_\pm^a(p^\perp(v)|v) = \left\{ \exp \left[\frac{E_\pm^a(p^\perp(v)|v) \mp \mu^a}{T} \right] + 1 \right\}^{-1} \quad (52)$$

for the Fermi distribution function of the boosted argument.

The Schwinger – Dyson equation for the self energy takes the form

$$\Sigma^a(p^\perp(v)|v) = m_0^a + N_c N_f \int \frac{dq^\perp(v)}{(2\pi)^3} W(p^\perp(v), q^\perp(v)) \gamma_0^{(v)} \left[i \int \frac{dq_0}{2\pi} \sqrt{1-v^2} \mathcal{G}(q_0, q^\perp(v)) \right] \gamma_0^{(v)}. \quad (53)$$

After performing the q_0 -integration and taking into account the properties of the projection operator $\gamma_0^{(v)} \gamma_0^{(v)} = 1$, we compare the result with the representation (42) of the self

energy $\Sigma^a(p^\perp(v)|v)$. We can then rewrite (53) as a coupled system of equations

$$E^a(p^\perp(v))\cos[2\vartheta^a(p^\perp(v))] = m_0^a + \frac{1}{2}N_c N_f \int \frac{dq^\perp(v)}{(2\pi)^3} W(p^\perp(v), q^\perp(v)) \times \\ \times \cos[2\vartheta^a(q^\perp(v))] [1 - f_+^a(q^\perp(v)|v) - f_-^a(q^\perp(v)|v)], \quad (54)$$

$$E^a(p^\perp(v))\sin[2\vartheta^a(p^\perp(v))] = |p^\perp(v)| + \frac{1}{2}N_c N_f \int \frac{dq^\perp(v)}{(2\pi)^3} W(p^\perp(v), q^\perp(v)) \times \\ \times \xi \sin[2\vartheta^a(p^\perp(v))] [1 - f_+^a(q^\perp(v)|v) - f_-^a(q^\perp(v)|v)], \quad (55)$$

$$\Delta E^a(p^\perp(v)) = -\frac{1}{2}N_c N_f \int \frac{dq^\perp(v)}{(2\pi)^3} W(p^\perp(v), q^\perp(v)) [f_+^a(q^\perp(v)|v) - f_-^a(q^\perp(v)|v)]. \quad (56)$$

Here $\xi = p^\perp(v) \cdot q^\perp(v)/|p^\perp(v)||q^\perp(v)|$. The system of equations (54) – (56) is the new result of the approach presented. It generalizes the result obtained in the zero temperature case [7, 8, 32] in twofold way:

1. Medium effects are included,
2. Lorentz boost with respect to the medium is performed.

The effect of the boost relative to the rest system of the medium can be reformulated as a change in the distribution functions of the surrounding matter. The result (54) we would have obtained using the covariant distribution function

$$f_\pm^a(p^\perp(v)|v) \longrightarrow f_\pm^a(p) = \left\{ \exp \left[\frac{p_\mu v^\mu \mp \mu^a}{T} \right] + 1 \right\}^{-1}, \quad (57)$$

in performing the Matsubara summation.

Let us comment on the case where the quark pair is at rest in the medium. This case corresponds to the choice $\not{h} = \gamma_0$ for the projector operator. The four - momentum vector p_μ is decomposed into $p_\mu = p_\mu^\perp + p_\mu^\parallel$ with $p_\mu^\parallel = (p_0, \mathbf{0})$ and $p_\mu^\perp = (0, \mathbf{p})$ and $dp = dp_0 dp$. In the Schwinger – Dyson equations (54) – (56) we have to replace $f_\pm^a(p^\perp(v)|v)$ with the usual Fermi – function

$$f_\pm^a(\mathbf{p}|0) = \left\{ \exp \left[\frac{E_\pm^a(\mathbf{p}) \mp \mu^a}{T} \right] + 1 \right\}^{-1} = f_\pm^a(\mathbf{p}) \quad (58)$$

and obtain the result which is form equivalent to (22) – (23).

4.2 Salpeter equation for moving bound states in the medium

The Salpeter equation (16) with the kernel (3) in medium has the form

$$\Gamma_H^{ab}(q^\perp(v)|P) = N_c \int \frac{dp^\perp(v) dp^\parallel}{(2\pi)^4} W(q^\perp(v), p^\perp(v)). \quad (59)$$

$$\not{h} \left[\mathcal{G}^a(p_0, p^\perp(v)|\frac{P}{2}) \Gamma^{ab}(p^\perp(v)|P) \mathcal{G}^b(p_0, p^\perp(v)|-\frac{P}{2}) \right] \not{h}$$

Integration (59) over $p^\parallel$ leads to

$$\begin{aligned}
\Gamma_H^{ab}(q^\perp(v)|P) = & -N_c P \int \frac{dp^\perp(v)}{(2\pi)^3} W(q^\perp(v), p^\perp(v)) \gamma_0^{(v)} \cdot \\
& \left\{ \frac{\Lambda_+^a(p^\perp(v)) \not{n} \Gamma^{ab}(p^\perp(v)|P) \not{n} \Lambda_-^b(-p^\perp(v)) [1 - f_+^a(p^\perp(v)|v) - f_-^b(p^\perp(v)|v)]}{[E_+^a(p^\perp(v)) + E_-^b(p^\perp(v))] - M_H^{ab}(v)} + \right. \\
& + \frac{\Lambda_-^a(p^\perp(v)) \not{n} \Gamma^{ab}(p^\perp(v)|P) \not{n} \Lambda_+^b(-p^\perp(v)) [1 - f_-^a(p^\perp(v)|v) - f_+^b(p^\perp(v)|v)]}{[E_-^a(p^\perp(v)) + E_+^b(p^\perp(v))] + M_H^{ab}(v)} + \\
& + \frac{\Lambda_+^a(p^\perp(v)) \not{n} \Gamma^{ab}(p^\perp(v)|P) \not{n} \Lambda_+^b(-p^\perp(v)) [f_+^b(p^\perp(v)|v) - f_+^a(p^\perp(v)|v)]}{[E_+^a(p^\perp(v)) - E_+^b(p^\perp(v))] + M_H^{ab}(v)} + \\
& \left. + \frac{\Lambda_-^a(p^\perp(v)) \not{n} \Gamma^{ab}(p^\perp(v)|P) \not{n} \Lambda_-^b(-p^\perp(v)) [f_-^b(p^\perp(v)|v) - f_-^a(p^\perp(v)|v)]}{[E_-^a(p^\perp(v)) - E_-^b(p^\perp(v))] - M_H^{ab}(v)} \right\} \gamma_0^{(v)}, \quad (60)
\end{aligned}$$

where

$$M_H^{ab}(v) = \sqrt{P^2} + (\mu_a - \mu_b) \sqrt{1 - v^2}. \quad (61)$$

Using the explicit form of the projectors $\Lambda_\pm(p^\perp(v))$, it is convenient to introduce the "undressed" vertex functions $\hat{\Gamma}_H^{ab}(p^\perp(v)|P)$:

$$\hat{\Gamma}_H^{ab}(p^\perp(v)|P) = S_a(p^\perp(v)) \Gamma_H^{ab}(p^\perp(v)|P) S_b(p^\perp(v)). \quad (62)$$

These vertex functions allow the rewriting the Salpeter equation with the help of $\Lambda_\pm^0$:

$$\begin{aligned}
\Lambda_\pm(p^\perp(v)) \not{n} \Gamma^{ab}(p^\perp(v)|P) \not{n} \Lambda_\mp(-p^\perp(v)) &= -\Lambda_\pm^0 \hat{\Gamma}^{ab}(p^\perp(v)|P) \Lambda_\mp^0 \\
\Lambda_\pm(p^\perp(v)) \not{n} \Gamma^{ab}(p^\perp(v)|P) \not{n} \Lambda_\pm(-p^\perp(v)) &= \Lambda_\pm^0 \hat{\Gamma}^{ab}(p^\perp(v)|P) \Lambda_\pm^0
\end{aligned} \quad (63)$$

From (63) we can see that the vertex function $\hat{\Gamma}^{ab}$ can be represented as the sum of two parts: $\hat{\Gamma}^{ab} = \{\Gamma\}^{ab} + [\Gamma]^{ab}$ according to the commutation properties with matrix $\not{n}$

$$\begin{aligned}
\Lambda_\pm^0 \hat{\Gamma}^{ab}(p^\perp(v)|P) \Lambda_\mp^0 &= \Lambda_\pm^0 \{\Gamma\}^{ab}(p^\perp(v)|P) \\
\Lambda_\pm^0 \hat{\Gamma}^{ab}(p^\perp(v)|P) \Lambda_\pm^0 &= \Lambda_\pm^0 [\Gamma]^{ab}(p^\perp(v)|P)
\end{aligned}$$

We can therefore decompose these parts with respect to Dirac - matrices

$$\begin{aligned}
\{\Gamma\}(p^\perp(v)|P) &= [\mathcal{P}_1(p^\perp(v)|P) + \not{n} \mathcal{P}_2(p^\perp(v)|P)] \gamma_5 \\
&+ [\mathcal{V}_1^i(p^\perp(v)|P) + \not{n} \mathcal{V}_2^i(p^\perp(v)|P)] \gamma_i
\end{aligned} \quad (64)$$

$$\begin{aligned}
[\Gamma](p^\perp(v)|P) &= [\mathcal{S}_1(p^\perp(v)|P) + \not{n} \mathcal{S}_2(p^\perp(v)|P)] \cdot 1 \\
&+ [\mathcal{A}_1^i(p^\perp(v)|P) + \not{n} \mathcal{A}_2^i(p^\perp(v)|P)] \gamma_5 \gamma_i
\end{aligned} \quad (65)$$

The matrices $\vec{\gamma} = \{\gamma_i\}$ can be decomposed into a sum of the longitudinal and transversal components with respect to the three momentum $\vec{p}$.

In particular, the pseudoscalar mesons are described by the following components of the vertex function

$$\{\Gamma\}(p^{\perp(v)}|P) = [\mathcal{P}_1(p^{\perp(v)}|P) + \not{p}\mathcal{P}_2(p^{\perp(v)}|P)]\gamma_5, \quad (66)$$

$$\{\Gamma\}(p^{\perp(v)}|P) = [\mathcal{A}_1(p^{\perp(v)}|P) + \not{p}\mathcal{A}_2(p^{\perp(v)}|P)]\hat{p}^{\perp(v)}\gamma_5$$

With respect to this decomposition, in terms of these vertex functions, the Salpeter equation has a more explicit form. We obtained the Salpeter equation as the coupled system

$$\begin{aligned} \mathcal{P}_1(p^{\perp(v)}|P) &= N_c \int \frac{d^3 p^{\perp(v)}}{(2\pi)^3} W(q^{\perp(v)}, p^{\perp(v)}) \left[\mathcal{P}_1(q^{\perp(v)}, p^{\perp(v)}|P) \{g\}_+(p^{\perp(v)}) + \right. \\ &\quad \left. + \mathcal{P}_2(q^{\perp(v)}, p^{\perp(v)}|P) \{g\}_-(p^{\perp(v)}) \right], \end{aligned} \quad (67)$$

$$\begin{aligned} \mathcal{P}_2(p^{\perp(v)}|P) &= N_c \int \frac{d^3 p^{\perp(v)}}{(2\pi)^3} W(q^{\perp(v)}, p^{\perp(v)}) \left[\mathcal{P}_2(q^{\perp(v)}, p^{\perp(v)}|P) \{g\}_+(p^{\perp(v)}) + \right. \\ &\quad \left. + \mathcal{P}_1(q^{\perp(v)}, p^{\perp(v)}|P) \{g\}_-(p^{\perp(v)}) \right], \end{aligned} \quad (68)$$

$$\begin{aligned} \mathcal{A}_1(p^{\perp(v)}|P) &= -N_c \int \frac{d^3 p^{\perp(v)}}{(2\pi)^3} W(q^{\perp(v)}, p^{\perp(v)}) \left[\mathcal{A}_1(q^{\perp(v)}, p^{\perp(v)}|P) [g]_+(p^{\perp(v)}) + \right. \\ &\quad \left. + \mathcal{A}_2(q^{\perp(v)}, p^{\perp(v)}|P) [g]_-(p^{\perp(v)}) \right], \end{aligned} \quad (69)$$

$$\begin{aligned} \mathcal{A}_2(p^{\perp(v)}|P) &= -N_c \int \frac{d^3 p^{\perp(v)}}{(2\pi)^3} W(q^{\perp(v)}, p^{\perp(v)}) \left[\mathcal{A}_2(q^{\perp(v)}, p^{\perp(v)}|P) [g]_+(p^{\perp(v)}) \right. \\ &\quad \left. + \mathcal{A}_1(q^{\perp(v)}, p^{\perp(v)}|P) [g]_-(p^{\perp(v)}) \right], \end{aligned} \quad (70)$$

where the particle – antiparticle propagator is given as

$$\begin{aligned} \{g\}_{\pm}(p^{\perp(v)}) &= \left(\frac{1 - f_+^a(p^{\perp(v)}) - f_-^b(p^{\perp(v)})}{[E_+^a(p^{\perp(v)}) + E_-^b(p^{\perp(v)})] - M^{ab}(P_0)} \right. \\ &\quad \left. \pm \frac{1 - f_-^a(p^{\perp(v)}) - f_+^b(p^{\perp(v)})}{[E_-^a(p^{\perp(v)}) + E_+^b(p^{\perp(v)})] + M^{ab}(P_0)} \right), \end{aligned} \quad (71)$$

which can be considered as the normal mode (c.f. eq.(38)) and the particle – particle (antiparticle – antiparticle) propagator as

$$\begin{aligned} [g]_{\pm}(p^{\perp(v)}) &= \left(\frac{f_+^b(p^{\perp(v)}) - f_+^a(p^{\perp(v)})}{[E_+^a(p^{\perp(v)}) - E_+^b(p^{\perp(v)})] + M^{ab}(P_0)} \right. \\ &\quad \left. \pm \frac{f_-^b(p^{\perp(v)}) - f_-^a(p^{\perp(v)})}{[E_-^a(p^{\perp(v)}) - E_-^b(p^{\perp(v)})] - M^{ab}(P_0)} \right), \end{aligned} \quad (72)$$

In eqs. (67) – (70), we have used the following abbreviation

$$\begin{aligned}\mathcal{P}_1(q^\perp(v), p^\perp(v)) &= c^-(q^\perp(v)) \left[c^-(p^\perp(v)) \mathcal{P}_1(p^\perp(v)) - s^-(p^\perp(v)) \mathcal{A}_1(p^\perp(v)) \right] + \\ &+ \xi s^-(q^\perp(v)) \left[c^-(p^\perp(v)) \mathcal{A}_1(p^\perp(v)) + s^-(p^\perp(v)) \mathcal{P}_1(p^\perp(v)) \right],\end{aligned}\quad (73)$$

$$\begin{aligned}\mathcal{P}_2(q^\perp(v), p^\perp(v)) &= c^+(q^\perp(v)) \left[c^+(p^\perp(v)) \mathcal{P}_2(p^\perp(v)) + s^+(p^\perp(v)) \mathcal{A}_2(p^\perp(v)) \right] - \\ &- \xi s^+(q^\perp(v)) \left[c^+(p^\perp(v)) \mathcal{A}_2(p^\perp(v)) - s^+(p^\perp(v)) \mathcal{P}_2(p^\perp(v)) \right],\end{aligned}\quad (74)$$

$$\begin{aligned}\mathcal{A}_1(q^\perp(v), p^\perp(v)) &= \xi c^-(q^\perp(v)) \left[c^-(p^\perp(v)) \mathcal{A}_1(p^\perp(v)) + s^-(p^\perp(v)) \mathcal{P}_1(p^\perp(v)) \right] - \\ &- s^-(q^\perp(v)) \left[c^-(p^\perp(v)) \mathcal{P}_1(p^\perp(v)) + s^-(p^\perp(v)) \mathcal{A}_1(p^\perp(v)) \right],\end{aligned}\quad (75)$$

$$\begin{aligned}\mathcal{A}_2(q^\perp(v), p^\perp(v)) &= \xi c^+(q^\perp(v)) \left[c^+(p^\perp(v)) \mathcal{A}_2(p^\perp(v)) - s^+(p^\perp(v)) \mathcal{P}_2(p^\perp(v)) \right] + \\ &+ s^+(q^\perp(v)) \left[c^+(p^\perp(v)) \mathcal{P}_2(p^\perp(v)) - s^+(p^\perp(v)) \mathcal{A}_2(p^\perp(v)) \right],\end{aligned}\quad (76)$$

and

$$s^\pm(p^\perp(v)) = \sin(\vartheta_a(p^\perp(v)) \pm \vartheta_b(p^\perp(v))),$$

$$c^\pm(p^\perp(v)) = \cos(\vartheta_a(p^\perp(v)) \mp \vartheta_b(p^\perp(v))).$$

Solution of the equations (67) – (70) facilitate the obtaining of the meson mass spectrum. The numerical calculations will give in the next section.

We can rewrite the Salpeter equation in terms of meson wave function

$$\Gamma_H^{ab}(q^\perp(v)) = N_c \int \frac{dp^\perp(v)}{(2\pi)^3} W(q^\perp(v), p^\perp(v)) \gamma_0^{(v)} \psi_H^{ab}(p^\perp(v)|P) \gamma_0^{(v)}, \quad (77)$$

where wave function $\psi_H^{ab}(p^\perp(v)|P)$ has the form

$$\begin{aligned}\psi_H^{ab}(p^\perp(v)|P) &= [\psi_{+-}^{ab}(p^\perp(v)|P) + \psi_{-+}^{ab}(p^\perp(v)|P)] \\ &+ [\psi_{++}^{ab}(p^\perp(v)|P) + \psi_{--}^{ab}(p^\perp(v)|P)],\end{aligned}\quad (78)$$

with

$$\begin{aligned}\psi_{\pm\mp}^{ab}(p^\perp(v)|P) &= \Gamma_{\pm\mp}^{ab}(p^\perp(v)) \left\{ \frac{(1 - f_\pm^a(p^\perp(v)|v)) (1 - f_\mp^b(p^\perp(v)|v))}{E_{\pm\mp}^{ab}(p^\perp(v)) \mp M^{ab}(v) - i\epsilon} \right. \\ &- \left. \frac{f_\pm^a(p^\perp(v)|v) f_\mp^b(p^\perp(v)|v)}{E_{\pm\mp}^{ab}(p^\perp(v)) \mp M^{ab}(v) + i\epsilon} \right\},\end{aligned}\quad (79)$$

$$\begin{aligned}\psi_{\pm\pm}^{ab}(p^\perp(v)|P) &= \Gamma_{\pm\pm}^{ab}(p^\perp(v)) \left\{ \frac{(1 - f_\pm^a(p^\perp(v)|v)) f_\pm^b(p^\perp(v)|v)}{E_{\pm\pm}^{ab}(p^\perp(v)) \mp M^{ab}(v) - i\epsilon} \right. \\ &\quad \left. - \frac{f_\pm^a(p^\perp(v)|v) (1 - f_\pm^b(p^\perp(v)|v))}{E_{\pm\pm}^{ab}(p^\perp(v)) \mp M^{ab}(v) + i\epsilon} \right\}.\end{aligned}$$

We used the following notations

$$\Gamma_{\pm\mp}^{ab}(p^\perp(v)) = \Lambda_\pm^a(p^\perp(v)) \not{n} \Gamma^{ab}(p^\perp(v)|P) \not{n} \Lambda_\mp^b(-p^\perp(v)) \quad (80)$$

$$\Gamma_{\pm\pm}^{ab}(p^\perp(v)) = \Lambda_\pm^a(p^\perp(v)) \not{n} \Gamma^{ab}(p^\perp(v)|P) \not{n} \Lambda_\pm^b(-p^\perp(v))$$

and

$$E_{\pm\mp}^{ab}(p^\perp(v)) = E_\pm^a(p^\perp(v)) + E_\mp^b(p^\perp(v)) \quad (81)$$

$$E_{\pm\pm}^{ab}(p^\perp(v)) = E_\pm^a(p^\perp(v)) - E_\mp^b(p^\perp(v))$$

Finally, we obtain the Salpeter equation in following form

$$\begin{aligned}[E_{\pm\mp}^{ab}(q^\perp(v)) \mp M^{ab}(v)] \tilde{\psi}_{\pm\mp}^{ab}(q^\perp(v)) &= \Lambda_\pm^0 \left[\int \frac{dp^\perp(v)}{(2\pi)^3} W(q^\perp(v), p^\perp(v)) \right. \\ &\quad \left. \tilde{\psi}^{ab}(p^\perp(v)) (1 - f_\pm^a(p^\perp(v)|v) - f_\mp^b(p^\perp(v)|v)) \right] \Lambda_\mp^0\end{aligned} \quad (82)$$

$$\begin{aligned}[E_{\pm\pm}^{ab}(q^\perp(v)) \mp M^{ab}(v)] \tilde{\psi}_{\pm\pm}^{ab}(q^\perp(v)) &= \Lambda_\pm^0 \left[\int \frac{dp^\perp(v)}{(2\pi)^3} W(q^\perp(v), p^\perp(v)) \right. \\ &\quad \left. \tilde{\psi}^{ab}(q^\perp(v), p^\perp(v)) (f_\pm^b(p^\perp(v)|v) - f_\pm^a(p^\perp(v)|v)) \right] \Lambda_\pm^0\end{aligned} \quad (83)$$

where the abbreviations

$$\dot{\psi}_{\pm\mp}^{ab}(q^\perp(v)) = S_a(q^\perp(v)) \psi_{\pm\mp}^{ab}(q^\perp(v)) S_b(q^\perp(v)), \quad (84)$$

$$\dot{\psi}_{\pm\pm}^{ab}(q^\perp(v)) = S_a(q^\perp(v)) \psi_{\pm\pm}^{ab}(q^\perp(v)) S_b(q^\perp(v)),$$

and

$$\begin{aligned}\tilde{\psi}^{ab}(q^\perp(v), p^\perp(v)) &= [S_a(q^\perp(v)) S_a^{-1}(p^\perp(v))] \left[\tilde{\psi}_{+-}^{ab}(p^\perp(v)) + \tilde{\psi}_{-+}^{ab}(p^\perp(v)) + \right. \\ &\quad \left. + \tilde{\psi}_{++}^{ab}(p^\perp(v)) + \tilde{\psi}_{--}^{ab}(p^\perp(v)) \right] [S_b^{-1}(p^\perp(v)) S_b(q^\perp(v))]\end{aligned} \quad (85)$$

are used.

4.3 Quark – meson coupling constant

To calculate the decay constant, we need the normalization condition for the meson vertex function.

For the normalization of vertex functions we can use the quadratic part of the effective action (13) for the quantum bilocal meson with commutation relations

$$\left[a_{H'}^-(\vec{P}'), a_H^+(\vec{P}) \right] = \delta_{HH'} \delta^3(\vec{P}' - \vec{P}). \quad (86)$$

The substitution of the off – shell $\sqrt{P^2} \neq M_H$ decomposition (9) into the free part of the effective action defines the Green function of the bilocal field $\mathcal{G}_H(\sqrt{P^2}|v)$

$$S_{\text{eff}}^{(2)}[\mathcal{M}] = 2\pi\delta(P_0 - P'_0) \sum_H \int \frac{d\vec{P}}{\sqrt{2\omega_H}} a_H^+(\vec{P}) a_H^-(\vec{P}) \mathcal{G}_H^{-1}(\sqrt{P^2}|v), \quad (87)$$

where $\mathcal{G}_H^{-1}(\sqrt{P^2}|v)$ is the difference of two terms

$$\mathcal{G}_H^{-1}(\sqrt{P^2}|v) = I(\sqrt{P^2}|v) - I(M_H^{ab}(\sqrt{P^2}|v)). \quad (88)$$

Here $M^{ab}(P_0)$ is the eigenvalue of the Salpeter equation, and

$$I(\sqrt{P^2}|v) = N_c \int \frac{d^3 q^\perp(v)}{(2\pi)^3} \left\{ \frac{i}{2\pi} \int dq^\parallel \cdot \right. \\ \left. \text{tr} \left[G^b(q^\parallel - \frac{P}{2} | q^\perp(v)) \Gamma^{ba}(q^\perp(v) | -P) G^a(q^\parallel + \frac{P}{2} | q^\perp(v)) \Gamma^{ab}(q^\perp(v) | P) \right] \right\}. \quad (89)$$

According to quantum field theory, the normalization condition is defined by formula

$$2\omega_H(\vec{P}) = \frac{\partial \mathcal{G}_H^{-1}(\sqrt{P^2}|v)}{\partial P_0} \Big|_{P_0=\omega_H(\vec{P})} = \frac{dM^{ab}(P_0)}{dP_0} \frac{dI(M^{ab})}{dM^{ab}} \Big|_{P_0=\omega_H(\vec{P})} \\ = (M^{ab}(\omega_H))' \frac{dI(M^{ab})}{dM^{ab}}. \quad (90)$$

After the integration over q_0 we obtain the normalization condition for the vertex functions $\Gamma(p^\perp(v)|P)$

$$\frac{2\omega_H(\vec{P})}{(M^{ab}(\omega_H))'} = P \int \frac{dp^\perp(v)}{(2\pi)^3} \cdot \\ \left\{ \frac{|\Lambda_+^a(p^\perp(v)) \Gamma^{ab}(p^\perp(v) | P) \Lambda_-^b(-p^\perp(v))|^2 [1 - f_+^a(p^\perp(v)) - f_-^b(p^\perp(v))]}{([E_+^a(p^\perp(v)) + E_-^b(p^\perp(v))] - M^{ab}(\omega_H))^2} - \right. \\ \left. - \frac{|\Lambda_-^a(p^\perp(v)) \Gamma^{ab}(p^\perp(v) | P) \Lambda_+^b(-p^\perp(v))|^2 [1 - f_-^a(p^\perp(v)) - f_+^b(p^\perp(v))]}{([E_-^a(p^\perp(v)) + E_+^b(p^\perp(v))] + M^{ab}(\omega_H))^2} \right\} \quad (91)$$

$$\begin{aligned}
& - \frac{|\Lambda_+^a(p^\perp(v))\Gamma^{ab}(p^\perp(v)|P)\Lambda_+^b(-p^\perp(v))|^2 [f_+^b(p^\perp(v)) - f_+^a(p^\perp(v))]}{([E_+^a(p^\perp(v)) - E_+^b(p^\perp(v))] + M^{ab}(\omega_H))^2} + \\
& + \frac{|\Lambda_-^a(p^\perp(v))\Gamma^{ab}(p^\perp(v)|P)\Lambda_-^b(-p^\perp(v))|^2 [f_-^b(p^\perp(v)) - f_-^a(p^\perp(v))]}{([E_-^a(p^\perp(v)) - E_-^b(p^\perp(v))] - M^{ab}(\omega_H))^2} \Big\} ,
\end{aligned}$$

and for the pseudoscalar mesons

$$\begin{aligned}
\frac{2\omega_H(\vec{P})}{(M^{ab}(\omega_H))'} &= \mathcal{P} \int \frac{dp^\perp(v)}{(2\pi)^3} \cdot \\
& \left\{ \frac{|\mathcal{P}_1(p^\perp(v)) + \mathcal{P}_2(p^\perp(v))|^2 [1 - f_+^a(p^\perp(v)) - f_-^b(p^\perp(v))]}{([E_+^a(p^\perp(v)) + E_-^b(p^\perp(v))] - M^{ab}(\omega_H))^2} - \right. \\
& - \frac{|\mathcal{P}_1(p^\perp(v)) - \mathcal{P}_2(p^\perp(v))|^2 [1 - f_-^a(p^\perp(v)) - f_+^b(p^\perp(v))]}{([E_-^a(p^\perp(v)) + E_+^b(p^\perp(v))] + M^{ab}(\omega_H))^2} - \\
& - \frac{|\mathcal{A}_1(p^\perp(v)) + \mathcal{A}_2(p^\perp(v))|^2 [f_+^b(p^\perp(v)) - f_+^a(p^\perp(v))]}{([E_+^a(p^\perp(v)) - E_+^b(p^\perp(v))] + M^{ab}(\omega_H))^2} + \\
& + \left. \frac{|\mathcal{A}_1(p^\perp(v)) - \mathcal{A}_2(p^\perp(v))|^2 [f_-^b(p^\perp(v)) - f_-^a(p^\perp(v))]}{([E_-^a(p^\perp(v)) - E_-^b(p^\perp(v))] - M^{ab}(\omega_H))^2} \right\} .
\end{aligned} \tag{92}$$

and

$$[M^{ab}(\omega_H)]' = \frac{\omega_H}{M^{ab}(\omega_H)} + (\mu_a - \mu_b) \frac{\omega_H^2 - [M^{ab}(\omega_H)]^2}{M^{ab}(\omega_H)\omega_H^2} . \tag{93}$$

The wave function $\psi^{ab}(p^\perp(v))$ satisfies the normalization condition

$$\begin{aligned}
\frac{2\omega_H(\vec{P})}{(M^{ab}(\omega_H))'} &= \int \frac{dp^\perp(v)}{(2\pi)^3} \text{tr} \left\{ \frac{|\psi_{+-}^{ab}(p^\perp(v))|^2}{1 - f_+^a(p^\perp(v)) - f_-^b(p^\perp(v))} \right. \\
& - \frac{|\psi_{-+}^{ab}(p^\perp(v))|^2}{1 - f_-^a(p^\perp(v)) - f_+^b(p^\perp(v))} - \frac{|\psi_{++}^{ab}(p^\perp(v))|^2}{f_+^b(p^\perp(v)) - f_+^a(p^\perp(v))} + \\
& + \left. \frac{|\psi_{--}^{ab}(p^\perp(v))|^2}{f_-^b(p^\perp(v)) - f_-^a(p^\perp(v))} \right\} .
\end{aligned} \tag{94}$$

4.4 Decay constant f_P for a meson moving in the medium

To investigate the decay constant as a function of temperature and velocity, we can consider the expression

$$f_H^{ab} P_\mu = iN_c \int \frac{dp^\perp(v) dp^\parallel}{(2\pi)^4} \text{tr} \left\{ O_\mu \mathcal{G}_a(p - \frac{P}{2}) \Gamma^{ab} \mathcal{G}_b(p + \frac{P}{2}) \right\} , \tag{95}$$

which is the straightforward generalization of the Eq.(36). After integration over $p^\parallel$ in (95) with temperature Green functions, we obtain the decay constant f_H as a function of T, μ, v

$$\begin{aligned}
f_{\mathbf{P}}^{ab} &= \frac{4N_c}{M_{\mathbf{P}}} \int \frac{dq^{\perp(v)}}{(2\pi)^3} \times \\
&\times \left\{ \left[\mathcal{P}_2^{ab}(p^{\perp(v)})[g]_{+}(p^{\perp(v)}) + \mathcal{P}_1^{ab}(p^{\perp(v)})[g]_{-}(p^{\perp(v)}) \right] \cos(\vartheta^a + \vartheta^b) \right. \\
&\left. - \left[\mathcal{A}_2^{ab}(p^{\perp(v)})\{g\}_{+}(p^{\perp(v)}) + \mathcal{A}_1^{ab}(p^{\perp(v)})\{g\}_{-}(p^{\perp(v)}) \right] \sin(\vartheta^a + \vartheta^b) \right\}.
\end{aligned} \tag{96}$$

The numerical investigation is presented in the following section for the pion case.

5 Numerical evaluation for a separable model

In order to give an example for the solution of the equations (54) – (56) we have to specify the interaction potential which can be given after Fourier transformation in momentum space

$$W(p^{\perp}, q^{\perp}) = \int dx dy e^{i(px+qy)} V(x^{\perp}, y^{\perp}) \delta(x \cdot \eta) \delta(y \cdot \eta). \tag{97}$$

If the potential depends only on the difference of the transverse momentum vectors,

$$W(p^{\perp}, q^{\perp}) = V(p^{\perp} - q^{\perp}), \tag{98}$$

one arrives at the bilocal relativistic theory of mesons [8, 7] which has recently been applied to the case of heavy mesons at zero temperature [29]. Here we give an evaluation for the case of a separable representation of the interaction, [32, 27]

$$W(p^{\perp}, q^{\perp}) = V_0 g(p^{\perp}) g(q^{\perp}), \tag{99}$$

where the integrals can be immediately performed, see also [32, 27]. A particular simple interaction form factor arises in the Nambu – Jona – Lasinio model, where $g(p^{\perp}) = \Theta(\Lambda - p^{\perp})$. It corresponds to a constant interaction in momentum space characterized by the coupling constant V_0 and the cut-off parameter Λ which regularizes the occurring integrals. Note that in the framework of a separable approach it is possible to use more realistic form factors such as Gaussian- or Lorentzian shapes. A systematic study of this issue was published recently [27].

For the sake of simplicity, we confine ourselves here to the simple NJL model. Furthermore, we consider the degenerate two-flavor case, where $m_0 = m_0^u = m_0^d$ which means that the flavor index can be dropped.

We have fixed the parameters of the model $\Lambda = 900 \text{ MeV}$, $V_0 = 1.92 \cdot 10^{-5} \text{ MeV}^{-2}$, $m_0 = 5.1 \text{ MeV}$ such a way that the pion mass ($M_{\pi} = 140 \text{ MeV}$), the pion decay constant ($f_{\pi} = 93 \text{ MeV}$) and the constituent quark mass ($m = 330 \text{ MeV}$) are recovered at temperature $T = 0$, chemical potential $\mu = 0$ and boost velocity $v = |\mathbf{P}|/P_0 = 0$.

Results are shown for the masses of the constituent quarks, the pion and kaon masses as well as the pion decay constant at finite values for temperature and boost velocity. Furthermore, results for the constituent quark mass at finite chemical potential are given.

5.1 Solution of the gap equation

In the case of a separable approach, the set of equations (54) – (56) simplifies to

$$m(T, \mu, v) = m_0 + \frac{V_0 N_c N_f}{2} \int^\Lambda \frac{dp^\perp(v)}{(2\pi)^3} \frac{m}{E} [1 - f_+(p^\perp(v)|v) - f_-(p^\perp(v)|v)], \quad (100)$$

$$\Delta E = -\frac{V_0 N_c N_f}{2} \int^\Lambda \frac{dp^\perp(v)}{(2\pi)^3} [f_+(p^\perp(v)|v) - f_-(p^\perp(v)|v)], \quad (101)$$

since the integral in equation (55) vanishes, such that

$$E_\pm^a(p^\perp(v)) \cos[2\vartheta^a(p^\perp(v))] = m^a(p^\perp(v)).$$

The occupation numbers $f_\pm(p^\perp(v)|v)$ as defined according to Eq. (52) depend on $E_\pm(p^\perp(v)|v)$.

For further numerical evaluation, we will replace $E_\pm$ with E thus neglecting the contribution of the energy shift ΔE , which is small in the region of interest.

The solution of Eq. (100) is shown in Figs. 1 - 3 for nonstrange quark matter, where $N_c = 3$ and $N_f = 2$.

In Fig. 1, the mass of the constituent quark is shown for the NJL model as a function of the temperature for zero chemical potential and zero velocity relative to the medium. The restoration of chiral symmetry which would occur at temperature $T_c = 220$ MeV in the chiral limit $m_0 = 0$ is washed out due to the explicit chiral symmetry breaking by the finite current quark mass. In Fig. 2, the dependence of the constituent quark mass is shown as a function of the chemical potential μ at fixed temperature $T = 10$ MeV and zero velocity relative to the medium. The critical value for the restoration of chiral symmetry would occur at $\mu_c = 370$ MeV, which corresponds to a critical quark density ($n_c = 3.5 \text{ fm}^{-3}$) of the many quark system.

Of special interest is the dependence of the quark mass on the boost velocity with respect to the medium. In solving the gap equation (100) for finite values of the boost velocity relative to the medium, we have to perform the integration over the angle θ between the boost velocity (chosen in the direction of p_1) and the momentum $p^\perp(v)$ which occurs in the Pauli - blocking factors (52) due to $E_\pm(p^\perp(v)|v) = [E(p^\perp(v))_\pm \pm vp^\perp(v) \cos \theta] / \sqrt{1-v^2}$, see Eq. (49). The integration can be performed analytically and leads to the following result for the Pauli-blocking factor

$$\begin{aligned} Q(p^\perp(v)|v) &= \frac{1}{4\pi} \int d\Omega_{p^\perp(v)} [1 - f_+(p^\perp(v)|v) - f_-(p^\perp(v)|v)] \\ &= \frac{2T\sqrt{1-v^2}}{p^\perp(v) v} \ln \left\{ \frac{\cosh(\bar{E}_+(p^\perp(v)|v)/T) + \cosh(\mu/T)}{\cosh(\bar{E}_-(p^\perp(v)|v)/T) + \cosh(\mu/T)} \right\}, \end{aligned} \quad (102)$$

where $\bar{E}_\pm(p^\perp(v)|v) = E_\pm(p^\perp(v)|v)|_{\theta=0}$. The results for the constituent quark mass as a function of the boost velocity are shown in Fig. 3 for zero chemical potential and different values of the medium temperature. The restoration of chiral symmetry occurs at parameter values of the boost velocity which are strongly temperature dependent. With regard to finite current quark masses, we observe smooth behaviour of the quark mass. As an important result, above $T = 100$ MeV shows a strong dependence on the boost velocity. This behaviour is caused by the ineffectiveness of the Pauli blocking for quarks with large momenta relative to the medium.

5.2 Solution of the Salpeter equation for pseudoscalar mesons

In order to calculate the meson mass spectrum, we have to perform the decomposition of the Salpeter equation into the different Dirac channels. In the frame of a separable approach, this decomposition leads to a simple matrix equation for the meson vertex functions. In particular, the pion mass M_π is defined as a solution of the equation

$$1 - Re J^\pi(\sqrt{P^2} = M_\pi|v) = 0, \quad (103)$$

if one neglects pseudoscalar-axial vector mixing. The polarization function $J^\pi(P|v)$ is given as

$$J^\pi(P|v) = \frac{2V_0 N_c N_f}{\pi^2} \int dp^\perp(v) p^\perp(v)^2 \frac{E(p^\perp(v))}{E^2(p^\perp(v)) - (P/2)^2} Q(p^\perp(v)|v). \quad (104)$$

We will now discuss the behaviour of the mass of pseudoscalar mesons as a function of temperature. First of all, we notice that the Goldstone theorem is also valid for $T, \mu \neq 0$ and arbitrary boost velocity v . The equation for the pion mass (103) and the gap equation for the constituent mass m coincide in the chiral limit $m_0 = 0$ if the pion appears as a massless Goldstone boson. The finite pion mass $M_\pi = 140$ MeV is a result of explicit chiral symmetry breaking due to finite current quark masses.

In Fig. 4, we show the result of the numerical solution of Eq. (103) for the pion mass as a function of the temperature if the pion is at rest in the medium for zero chemical potential (vanishing baryon number): For comparison, the behaviour of the sum of the constituent quark masses is also shown in that figure. Note the fact that, regardless of the drastic changes in the constituent quark masses for temperatures above $T \simeq 100$ MeV, the mass of the pion remains fairly constant until the temperature of the Mott transition $T \simeq 200$ MeV is reached where the pion enters the continuum of unbound quark-antiquark states. This "protection" of the pion against medium effects is analogous to the well-known effect in nuclear physics, where self energy effects in the one-particle sector are partially compensated by Pauli-blocking for the two-particle states, see [34].

Let us now study the effects of a finite boost velocity relative to the medium on the behaviour of the pion mass. Inspection of the Schwinger-Dyson (100) and Salpeter equations (104) shows that the boost velocity influences on the quark and meson mass spectra via the Pauli blocking factor $Q(p^\perp(v)|v)$ only. For boost velocities near the velocity of light, $Q(p^\perp(v)|v)$ becomes nearly independent of the temperature and chemical potential of the medium. Accordingly, the chiral restoration (see Fig. 2) occurs at larger values of T and μ . This behaviour is shown in Fig. 6 for boost velocities $v/c = 0.1, 0.95, 0.99$. The arguments for the "protection" of the pionic bound state due to the compensation of the medium influence for the bound state also apply in this case. At velocity $v = 0.95 c$, e.g., the pion is a bound state up to temperatures $T \simeq 450$ MeV, see Fig. 6.

The formula for the electroweak decay constant of the pion has the following form in the simple case of the NJL interaction without mixing

$$f_\pi = \frac{2\mathcal{N}N_c}{\pi^2} \int^\Lambda dp^\perp(v) p^\perp(v)^2 \frac{1}{E(p^\perp(v))} \frac{m(p^\perp(v))}{E^2(p^\perp(v)) - (P/2)^2} Q(p^\perp(v)|v) \quad (105)$$

$$\mathcal{N}^{-2} = \frac{2N_c}{\pi^2} \int^\Lambda dp^\perp(v) p^\perp(v)^2 \frac{E(p^\perp(v))}{(E^2(p^\perp(v)) - (M_\pi/2)^2)^2} Q(p^\perp(v)|v)$$

with the normalization $\mathcal{N}$. The temperature dependence of f_π is plotted in Fig. 5. The decay constant is proportional to the quark mass and drops to zero when the chiral symmetry is restored also at finite boost velocities, Fig. 6.

5.3 Inclusion of strangeness degree of freedom

In the flavor SU(3) case we have to consider two different bare quark masses $m_0 = m_0^u = m_0^d$ and m_0^s in the result that the quark energies $E_{\pm}^a, E_{\pm}^b$ in the Salpeter equation (60) are no longer degenerate and the contributions of all four terms have to be considered in the numerical solution. In order to obtain the kaon mass $M_K = 490$ MeV at zero temperature we chose $m_0^s = 140$ MeV. The temperature dependence of M_K for zero chemical potential and vanishing boost velocity is shown in Fig. 5. The behaviour of the kaon as a pseudoscalar meson in the flavor SU(3) multiplet is similar to that of the pion in Fig. 4. Note that the continuum for the Mott transition of the kaon does not coincide with that of the pion since the constituent mass of strange quark differs from that of the light flavors.

6 Conclusions

We have considered here a covariant description of the relativistic mesons as quark - antiquark bound states in hot and dense quark matter.

The starting point was the relativistic theory of mesons (inspired by the Heisenberg - Pauli type minimal quantization of QCD [8]) which unifies the chiral Lagrangian for light quarkonia and the nonrelativistic potential model for heavy ones [29].

This approach makes it possible to apply the relativistic covariant generalization of the quantum statistical methods of the investigation of many particle systems for hadrons at finite temperature and density.

The final result of the paper is the derivation of relativistic covariant equations for the quark self- energy and the meson vertex functions describing the behaviour of the quark and meson spectra in a hot and dense medium.

Our main interest is concentrated on meson properties at finite temperature and density. As a qualitatively new aspect, we included in our considerations the influence of a nonvanishing three - momentum of mesons $\vec{P} = \vec{r}\omega_H$. Some general conclusions may be obtained just from the structure of the equations. One interesting property with possible experimental significance concerns the dependence of the spectrum on the total momentum of bound states. Looking at the asymptotics of fast particles with $\vec{P}^2 \gg \Sigma^2(0), M_H^2$, one finds a characteristic reduction of Pauli- blocking leading to a stabilization of the bound states against break up into continuum states. This behaviour is similar to that observed for clusters in nuclear matter [35].

A very interesting effect which has not yet been discussed as far as we know is the appearance of $q\bar{q}$ - and $q q$ - correlations for $T \neq 0$ which is a common situation from the point of view of many- body theory. As follows from the Bethe Salpeter equations, these correlations appear only for non- equal energies of the quarks in the pair. As a consequence, we anticipate characteristic temperature dependent mass difference in the spectrum, e.g., of $\pi^\pm$, and π^0 mesons or for mesons with strangeness or charm.

The behavior of the pion masses with increasing temperature consents with similar predictions made in other recent publications [36]. Consideration of a schematic model shows that the pion is a low - lying collective mode (Goldstone mode) in the spirit of many- body theory and the excitation is inside gap: $M_\pi \leq 2m$. With increasing temperature, the binding energy diminishes according to the dropping constituent quark mass such that at a critical temperature $T = T^{Mott}$ the pion is dissolved in the $q\bar{q}$ continuum when

$$M_\pi(T^{Mott}, P) = 2m(T^{Mott}, P).$$

We emphasize that part of the peculiarities observed in the low $\mathbf{p}$ momentum part of pion and kaon spectra from violent heavy ion collisions could be caused by the momentum dependence of the quark pair correlations in hot and dense matter.

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Fig. 1 The T -dependence of the constituent quark mass m_u for $\mu_u = 0$ shows the restoration of the chiral symmetry at $T_c = 220$ MeV. The vacuum value of $m_u = 330$ MeV is fixed by the parameters.

Fig. 2 The μ -dependence of the constituent quark mass m_u for $T = 0$ shows the restoration of the chiral symmetry at $\mu_c = 380$ MeV.

Fig. 3 The T -dependence of M_π and $m_u + m_d$ is displayed. The π -meson mass is fixed to $M_\pi = 140$ MeV at zero temperature and chemical potential by the model parameters. The π -mass does not change up to a temperature of 150 MeV. Starting at $T_{Mott} = 225$ MeV the π mass becomes bigger than the threshold $m_u + m_d$.

Fig. 4 The T -dependence of M_K and $m_u + m_s$ is shown. The K- meson mass is fixed to $M_K = 490$ MeV at zero temperature and chemical potential by the model parameters. Starting at $T_{Mott} = 250$ MeV the K- mass becomes bigger than the the threshold $m_u + m_s$.

Fig. 5 The T -dependence of pion decay constant f_π shows a strong decrease of f_π near T_c . The electro-weak decay of the π meson is strongly suppressed since the π - meson can decay into two quarks for temperatures higher than T_c .

Fig. 6 The dependence of the constituent quark mass on the boost velocity for different temperatures below, at and above T_c is displayed in the chiral limit and with a finite bare quark mass. The mechanism of the restoration of the chiral symmetry is suppressed at finite velocities and even above T_c in nonvanishing dynamical quark mass is generated due to the motion of the bound state.

Fig. 7 The T -dependence of the scalar- and pseudoscalar mesons, the pion decay constant as well as the double quark mass are plotted for two different center of mass momenta of the quark-antiquark pair.

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Установлена следующая стоимость подписки на 12 месяцев на издания ОИЯИ, включая пересылку, по отдельным тематическим категориям:

Индекс	Тематика	Цена подписки на год
1.	Экспериментальная физика высоких энергий	915 р.
2.	Теоретическая физика высоких энергий	2470 р.
3.	Экспериментальная нейтронная физика	365 р.
4.	Теоретическая физика низких энергий	735 р.
5.	Математика	460 р.
6.	Ядерная спектроскопия и радиохимия	275 р.
7.	Физика тяжелых ионов	185 р.
8.	Криогеника	185 р.
9.	Ускорители	460 р.
10.	Автоматизация обработки экспериментальных данных	560 р.
11.	Вычислительная математика и техника	560 р.
12.	Химия	90 р.
13.	Техника физического эксперимента	720 р.
14.	Исследования твердых тел и жидкостей ядерными методами	460 р.
15.	Экспериментальная физика ядерных реакций при низких энергиях	460 р.
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17.	Теория конденсированного состояния	365 р.
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По всем вопросам оформления подписки следует обращаться в издательский отдел ОИЯИ по адресу: 141980, г.Дубна, Московской области

Бляшке Д. и др.

Одновременная киральная кварковая модель
для релятивистских мезонов в плотной и горячей среде

E2-94-307

MPG-VT-UR 40/94

Сформулирована киральная кварковая модель с ковариантным мгновенным взаимодействием с помощью релятивистских термодинамических функций Грина. Этот подход используется для описания мезонов как связанных состояний в горячей и плотной кварковой среде. Уравнение Швингера — Дайсона для массового оператора кварка получено для ковариантного четырехточечного ядра взаимодействия. Уравнения Солпитера для кварк-антикварковых связанных состояний в двухкомпонентной релятивистской кварковой плазме даны в хорошо известных скаляр-псевдоскалярном и вектор-псевдовекторном каналах. Рассмотрен случай ненулевого общего импульса связанного состояния относительно среды. Результаты численных расчетов для спектра масс мезонов и константы распада пиона при конечной температуре представлены для специального случая сепарабельного взаимодействия. Эти вычисления могут быть обобщены на случай других релятивистских потенциалов.

Работа выполнена в Лаборатории теоретической физики им. Н.Н.Боголюбова ОИЯИ и в Университете Ростка (ФРГ).

Препринт Объединенного института ядерных исследований. Дубна, 1994

Blaschke D. et al.

Instantaneous Chiral Quark Model for Relativistic Mesons
in a Hot and Dense Medium

E2-94-307

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A chiral quark model with covariant instantaneous interactions is formulated using relativistic thermodynamic Green functions. The approach is applied to the description of mesons as relativistic bound state in hot and dense quark matter. The Schwinger — Dyson equation for the quark mass operator is obtained for a covariant four — point interaction kernel. The Salpeter equations for quark — antiquark bound states in a two — component relativistic quark plasma are given in the scalar — pseudoscalar as well as vector — axial-vector channels. The case of nonvanishing total momentum of bound state relative to the medium is considered. Numerical results for the meson mass spectrum and the pion decay constant at finite temperature are presented for the special case of a separable interaction which can be applied to the case of more realistic potentials.

The investigation has been performed at the Bogoliubov Laboratory of Theoretical Physics, JINR and at the University of Rostock (Germany).

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