A NOTE ON SOME PHYSICAL AND COSMOLOGICAL IMPLICATIONS OF
CONFORMAL METRIC FLUCTUATIONS

J.L. Rosales and J.L. Sánchez-Gómez

Departamento de Física Teórica, Universidad Autónoma de Madrid,
Cantoblanco, 28049-Madrid, Spain.

See 8449

CERN LIBRARIES, GENEVA

SCAN-9412052

Abstract

We show that stochastic fluctuations of the space-time metric -which various authors invoke as a possible physical agent of decoherence in macroscopic systems- must be conformal in order to avoid some bizarre physical and cosmological implications.

Gravitation has been -in various ways- recently invoked as a possible objective agent of the quantum wave-packet reduction for macroscopic systems [1-6]. In this regard, the model of Károlyházy et al. [1] (K-model hereafter) would provide us, in principle, of an example of a rather elaborated model in which stochastic fluctuations of the space-time metric can give rise to a breakdown of the quantum superposition principle in the case of macroscopic systems, without affecting in a noticeable manner the strict quantum behaviour of isolated microscopic systems (see [1] for further details).

Unfortunately, as pointed out very recently by Diósi and Lukács [7], the K-model is little plausible as it predicts some bizarre physical and cosmological facts, such as deadly X-ray emission from -apparently- quite harmless solid matter, and/or vacuum densities comparable to or higher than those in neutron stars [8]. Now one might, perhaps, be tempted to believe that all models of stochastic metric fluctuations are unrealistic, and that, therefore, this kind of approach to the problem of wave-packet reduction for macroscopic systems is actually ruled out. This is not necessarily so, however, as we hopefully will show in this note.

The key point is, in our opinion, the nature of the said fluctuations. We have already stressed [9] the importance that they be *conformal* in order to maintain, maximally, the symmetry of the vacuum. In this spirit a model of stochastic (conformal) metric fluctuations has been produced [10] in which decoherence has been achieved -at least approximately- in the case of macro-systems without affecting the coherent quantum behaviour of isolated microsystems (particles, atoms, etc.). Furthermore, we will show now that *conformality* prevents such fluctuations from oddly behaving, both "physically" and "cosmologically", so that Diósi and Lukács' acute criticism to the K-model does not apply to ours.

Let us, first of all, briefly summarize the main features of the model (see [10] for further details).

The fundamental assumption is that the spacetime metric is not strictly defined since it undergoes stochastic fluctuations of a conformal nature, i.e.

$$g_{\mu\nu}(x) = e^{\phi(x)} \langle g_{\mu\nu}(x) \rangle , \quad (1)$$

where ϕ is a (classical) stochastic field and $\langle . \rangle$ stands for stochastic mean -the background metric is taken as minkowskian, for the sake of simplicity.

From (1), and by assuming the probability distribution to be Riemannian, the following equations are easily derived [10]:

$$\begin{aligned} \langle e^{\phi(x)} \rangle &= 1 , \quad \langle e^{\phi(x)} \phi_{\mu}(x) \rangle = 0 , \quad \langle \phi_{\mu}(x) \rangle = 0 , \\ \langle \phi_{\mu}(x) \phi(x) \rangle &= 0 ; \end{aligned} \quad (2)$$

thus enabling us to consider ϕ as a gaussian stochastic process with variance (the mean, by symmetry, is taken to be zero)

$$\langle \phi^2(x) \rangle = \sigma^2 . \quad (3)$$

Also, we assume the following correlation function

$$\langle \phi(\mathbf{x}, t) \phi(\mathbf{x}', t') \rangle = \sigma^2 e^{-(t-t')^2/\tau^2} e^{-(\mathbf{x}-\mathbf{x}')^2/L^2} , \quad (4)$$

where $\tau = L/c$, and L is a (by now) free parameter representing the correlation length. The parameters σ and L can be related to each other by means of simple dimensional arguments [10]:

$$\sigma^2 \sim \hbar G / (L^2 c^3) = \Lambda_p^2 / L^2 \quad (5)$$

where Λ_p is the Planck length . We are then left with only one free parameter -the correlation length L . It can be easily seen that, for all reasonable values of L , one has $\sigma^2 \ll 1$; hence we are in the weak field case, and then all gravitational effects can be accounted for by a stochastic "newtonian" potential

$$V(\mathbf{x}, t) = 1/2 \, mc^2 \phi(\mathbf{x}, t) . \quad (6)$$

By using the preceding stochastic potential, and with the help of the Feynman-Vernon's formalism [11], one is able to get decoherence in the case of quantum states representing "macro-systems" while quantum "microsystems" (not interacting with macroscopic ones) practically follow a unitary time evolution, as it must be (further details can be found in [10]).

Let us go now to the cosmological problems put forward by Diósi and Lukács [7] regarding the K-model. Here, we shall show that such problems do not concern our model and that, moreover, cosmology is in fact useful in order to obtain an approximate lower bound to the correlation length, L , introduced above.

As already commented, it has been pointed out [8] that the K-model predicts -for almost all sensible values of the cutoff parameter of the model- a vacuum gravitational energy larger than that of a neutron star. Let us see now what happens in our model.

By using the "newtonian" potential V given in (6), one can compute -by means of standard techniques- the mean value of the spacetime curvature ($c=1$):

$$\tilde{R} = -\frac{1}{2} \langle \exp(-2\phi) \{ 6\Box\phi + 3[(\partial\phi/\partial t)^2 - (\nabla\phi)^2] \} \rangle. \quad (7)$$

Conformality -through Eqs.(2)- and the correlation function (4) imply, as one easily checks (note also (5)):

$$\langle \Box\phi \rangle = 0 ; \quad \langle (\partial\phi/\partial t)^2 \rangle = \langle (\nabla\phi)^2 \rangle = 4\sigma^2/L^2 = \frac{4}{L^2} (\Lambda_p/L)^2. \quad (8)$$

Then $\tilde{R} = 0$ up to order ϕ^4 (recall that the background metric is minkowskian). Therefore our model has no troubles as far as the mean curvature of the universe is concerned. Now one must also

compute the $_{00}$ -component of the energy-momentum tensor, which, of course, is not a Minkowski scalar. Note, however, that the relative *dynamic* velocity (i.e. cosmic expansion not included) between any pair of *physically relevant* systems seems to be, as a matter of fact, much smaller than the velocity of light, and hence one is (probably) allowed to assume that $\langle T_{00} \rangle$ -computed in the reference frame (Ether?) wherein Eq.(4) holds- gives a reliable estimate of the mean energy density of the universe stored in the fluctuating vacuum. By using standard techniques one gets

$$\langle T_{00}/c^2 \rangle = c^2/(32\pi GL^2) (\Lambda_p/L)^2 \quad (9)$$

If one now assumes $\langle T_{00} \rangle/c^2$ to be less than 10^{-29} g/cm³, since it seems reasonable to believe that vacuum fluctuations are not "closing" the universe, then one can obtain a lower bound to L , $L \geq 10^{-3}$ cm, which, in spite of being really a very rough bound, gives us -at least- a hint about the order of magnitude of this parameter.

Now one might be tempted to speculate about the nature of the hypothetical metric fluctuations discussed here, relating them, for instance, to primordial gravity waves, the wavelength of which, being at present of the order of L , has "inflated" starting from a length of the order of the Planck length at the epoch in which gravitation decoupled from the other fundamental interactions. (A different - albeit in some way related also to gravitational waves- possible origin of the metric fluctuations is discussed in [12]).

Let us go now to the other point risen by Diósi and Lukács.

In [7] they convincingly show that the K-model predicts that a macroscopic piece (a gram, say) of condensed matter, by virtue of the oscillations of charged particles induced by the metric fluctuations, would spontaneously radiate a considerable amount of electromagnetic energy, mainly in the form of hard X-rays, what obviously -and fortunately- does not happen. Now we shall show here that such a "terrible" fact is absent in our model.

We start from the well-known dipole radiation formula [13], which in the present case can be written down as

$$\langle W \rangle = \frac{2e^2}{3c^3} \langle a^2 \rangle, \quad (10)$$

where a is the stochastic acceleration of the particle (e being its charge) produced by the stochastic potential V . So that, from (6)

$$\langle W \rangle = \frac{2e^2}{3c^3} \frac{1}{m^2} \frac{m^2 c^4}{4} \langle (\nabla \phi)^2 \rangle, \quad (11)$$

and, finally, from (8)

$$\langle W \rangle = \frac{2e^2}{3c^3} \left(\frac{c^2}{L} \right)^2 \left(\frac{\Lambda_p}{L} \right)^2. \quad (12)$$

If we introduce in this expression the corresponding numerical values (with $L = 10^{-3}$ cm), we obtain $\langle W \rangle \approx 5 \times 10^{-51}$ eV/s. Then for a macroscopic system ($\sim 10^{23}$ particles) $\langle W \rangle \approx 10^{-46}$ w, which is absolutely negligible.

In conclusion, we have shown that neither cosmological nor "radiative" problems appear in the model of metric fluctuations discussed here -this good feature being (mainly) due to their assumed conformal nature. We thus think that being conformal is

indeed a property stochastic metric fluctuations have to have in order to be realistic and consistent.

This work has been partially supported by CICYT (Spain) , under contract No. PB 91-0124.

References

- [1] F. Károlyházy, "The Breakdown of the Superposition Principle", in *Sixty-two Years of Uncertainty*, edited by A.I. Miller, Plenum Press (New York, 1991) and references therein.
- [2] R. Penrose, "Gravity and State Vector Reduction", in *Quantum Concepts in Space and Time*, edited by R. Penrose and C. Isham, Clarendon Press (Oxford, 1989).
- [3] L. Diosi, Phys. Rev. **A40**, (1989) 1165.
- [4] G.C. Ghirardi, R. Grassi, and A. Rimini, Phys. Rev. **A43** (1991) 9.
- [5] J.L. Rosales and J.L. Sánchez-Gómez, Phys.Lett. **A166** (1992) 111.
- [6] J. Unturbe and J.L. Sánchez-Gómez, Il Nuovo Cimento **107B** (1992) 211; Physics Essays **6** (1993) 580.
- [7] L. Diósi and B. Lukács, Phys. Lett. **A181** (1993) 366.
- [8] L. Diósi and B. Lukács, preprint KFKI-1993-05/AB (1993).
- [9] J.L. Rosales and J.L. Sánchez-Gómez, Phys. Lett. **A142** (1989) 207 and **A156** (1991) 207.
- [10] J.L. Sánchez-Gómez, "Decoherence through Stochastic Fluctuations of the Gravitational Field", in *Stochastic Evolution of Quantum States*, eds. L. Diósi and B. Lukács, World Scientific (Singapore, 1994); J.L. Sánchez-Gómez and J.L. Rosales, Proc. New York Acad. Sc. (in press).

- [11] R.P. Feynman and F. Vernon, *Ann. Phys.(N.Y.)* **24** (1963) 118.
- [12] J.P. Vigiér, *Astron. Nach.* **303** (1982) 55; N. Cufaro-Petroni and J.P. Vigiér, *J. Phys. A: Math. Gen.* **17** (1984) 599.
- [13] L.D. Landau and E.M. Lifchitz, *The Classical Theory of Fields*, Pergamon Press (Oxford, 1962).