A fresh look into the heavy quark-mass values

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Abstract

Using the recent *world average* $\alpha_s(M_Z^2) = 0.118 \pm 0.006$, we give the *first direct extraction* from the Ψ and Υ data of the values of the *running heavy quark masses* within QCD spectral sum rules to two-loops in the $\overline{MS}$ -scheme: $\overline{m}_b(M_b^{PT2}) = (4.23^{+0.03}_{-0.04} \pm 0.02)$ GeV and $\overline{m}_c(M_c^{PT2}) = (1.23^{+0.02}_{-0.04} \pm 0.03)$ GeV, (the errors are respectively due to α_s and to the gluon condensate), and the corresponding value of the *short-distance perturbative pole masses to two-loops*: $M_b^{PT2} = (4.62 \pm 0.02)$ GeV, $M_c^{PT2} = (1.41 \pm 0.03)$ GeV, which we compare with the updated values of the *non-relativistic pole masses* re-extracted *directly* from the two-loop non-relativistic sum rules: $M_b^{NR} = (4.69^{+0.01}_{-0.02} \pm 0.02)$ GeV and $M_c^{NR} = (1.44 \pm 0.02 \pm 0.03)$ GeV. It is also informative to compare the *three-loop* values of the short-distance pole masses: $M_b^{PT3} = (4.87 \pm 0.05 \pm 0.02)$ GeV and $M_c^{PT3} = (1.62 \pm 0.07 \pm 0.03)$ GeV, with the *dressed mass* $M_b^{nr} = (4.94 \pm 0.10 \pm 0.03)$ GeV, entering into the *non-relativistic Balmer formula* including higher order α_s corrections. The *small* mass-differences $M_b^{NR} - M_b^{PT2} \simeq M_b^{nr} - M_b^{PT3} \simeq 70$ MeV and $M_c^{NR} - M_c^{PT2} \simeq (30 \pm 20)$ MeV *can measure the size* of the non-perturbative effect induced by *renormalon* type-singularities. An analogous analysis is pursued for the heavy-light mesons, where a simultaneous *re-fit* of the B and B^* masses from relativistic sum rules leads to: $M_b^{PT2} = (4.63 \pm 0.08)$ GeV, while the full-QCD and HQET sum rules in the large mass limit give the *meson-quark mass difference to two-loops*: $(M_B - M_b^{NR})_\infty \simeq (0.6^{+0.20}_{-0.10})$ GeV. A comparison of these *new* and *accurate* results with the existing ones in the literature is done. As a consequence, the *updated* values of the *pseudoscalar decay constants to two-loops* are: $f_D = (1.37 \pm 0.04 \pm 0.06)f_\pi$ and $f_B = (1.49 \pm 0.06 \pm 0.05)f_\pi$, which lead to $f_B\sqrt{B_B} = (1.49 \pm 0.14)f_\pi$.

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1 Introduction

Present accurate measurements of the QCD coupling α_s , mainly from Z-physics and tau-decay data, motivate a careful reconsideration of the existing estimates of the standard model parameters. Among others, of a *prime importance* is the one of the quark masses. However, unlike the electron mass, the definition of the quark masses needs a theoretical framework due to the complication implied by confinement. In particular, the notion of the pole mass from perturbation theory, though gauge independent, appears to be ambiguous, as the summation of large order perturbation theory induces a non-perturbative term due to renormalon-type singularities. However, one can still provide a *good definition* of this pole mass for a truncated QCD series, which can be used *consistently* in a given observable known at the same order of perturbation theory. Unlike the *pole* mass, the *running* mass of the $\overline{MS}$ -scheme does not suffer from this ambiguity, such that its *direct estimate* from the data without passing through the pole mass should be very useful.

In this paper, we shall use QCD spectral sum rules (QSSR) in order *to extract, directly* from the data, and *for the first time*, the value of the *running* heavy quark mass $\overline{m}_Q^1$, which is a useful (and well-defined) quantity for GUTS and some other phenomenology. Indeed, many papers have been devoted in the past, either to the *direct* estimate of the *Euclidian* mass $m_Q(p^2 = -m_Q^2)$ or/and of the *pole* mass $M_Q(p^2 = M_Q^2)$ [?]-[?] while the running mass has been mainly deduced [?, ?] from its perturbative relation with the pole mass [?]²:

$$M_Q(M_Q^2) = \overline{m}_Q(M_Q^2) \left\{ 1 + \frac{4}{3} \left(\frac{\alpha_s(M_Q^2)}{\pi} \right) + K_Q \left(\frac{\alpha_s(M_Q^2)}{\pi} \right)^2 \right\}, \quad (1)$$

where $K_b \simeq 12.4$, $K_c \simeq 13.3$ [?]. Instead, once, we obtain the running mass from the data, we shall use the previous relation in order to deduce the short-distance *perturbative* pole mass M_Q^{PT} , which we shall compare with the *non-relativistic* pole mass M_Q^{NR} extracted directly from the data, using non-relativistic sum rules, and with the mass M_Q^{nr} entering into the non-relativistic Balmer formula. We shall interpret the difference between these two values as a measure of the size of the renormalon effect into the pole mass definition.

Finally, in comparing our results for the pole masses with the existing ones in the literature, we shall test the reliability of these previous results and look for the possible sources of some eventual differences between these and the ones in this paper.

2 Input values of α_s and matching conditions

We use the value of $\alpha_s(M_Z)$ in the range given in Table 1, where the new world average is 0.118 ± 0.006 [?], but we have also considered a slightly higher value of 0.127 in order to be more conservative. Then, we run this value until $M_b = 4.6 - 4.7$ GeV, using the

¹However, an extraction of the running mass using local duality FESR has been done in [?].

²For consistency with the two-loop expression of the two-point correlator, we shall not use the three-loop relation in the sum rule analysis.