Nuclear Many-Body Theory for the Study
of QCD Sum Rules in Matter

L.S. Celenza, C.M. Shakin, Wei-Dong Sun, and J. Szweda
Department of Physics and Center for Nuclear Theory
Brooklyn College of the City University of New York
Brooklyn, N.Y. 11210

(November, 1994)

SCW9450

Submitted to Physical Review C
(Subnucleon Aspects of Nuclei/Physics of Hadrons)

Abstract

We discuss the calculation of QCD sum rules for hadrons in matter from the point of view of nuclear many-body theory and treat the sum rule for the rho meson as an example. We describe the factorization approximation that is used to calculate four-quark condensates in vacuum and in matter and discuss the corrections to that approximation in some detail. In the case of the rho meson, we find that corrections to the factorization approximation in matter can vary from about 10 to 40 percent depending upon the model used to describe the nucleon. If the nucleon is considered to be composed of a quark coupled to a scalar ($T = 0$) diquark, the corrections are significant, while, if the quark is coupled to a axial-vector ($T = 1$) diquark, the corrections are small. The physical nucleon may be taken to have equal scalar and axial-vector diquark components. In that case, corrections of about 20 percent are found for the density-dependent part of the vector-isovector polarization tensor. More precise estimates require a more detailed picture of nucleon structure than that considered here. In a previous work we discussed the corrections to the factorization approximate for the nucleon-nucleon correlator evaluated in nuclear matter. The corrections we calculated served to eliminate a problematic density dependence of the four-quark condensates that arose when using the factorization approximation in matter. However, our study provided a successful analysis only when the interpolating field advocated by Ioffe was used. The situation in the case of sum rules for the vector-isovector polarization tensor is subject to fewer uncertainties, since the choice of the current is unique. We find that, while the corrections we calculate for the density dependence of the four-quark condensates are not small, they lead to only very small modifications in the predicted density dependence of the rho mass.

I. Introduction

In recent years we have seen the development of novel techniques for the study of hadron properties in matter based upon the use of QCD sum rules [1-11]. A formalism that makes use of condensates of lowest dimension has been shown to give a good account of the nucleon self-energy in matter [1,11]. It was found that the nucleon experienced a strong (repulsive) Lorentz-vector interaction and a strong (attractive) Lorentz-scalar interaction, in accord with Dirac phenomenology [12]. The picture is more obscure if one studies the role of condensates up to dimension six. In particular, the results are sensitive to the treatment of the four-quark condensates. The factorization approximation for the evaluation of the four-quark condensates in vacuum reads

$$\langle \bar{q}_{\alpha i}^A q_{\beta j}^B \bar{q}_{\gamma k}^C q_{\delta l}^D \rangle_0 = \langle \bar{q}_{\alpha i}^A q_{\beta j}^B \rangle_0 \langle \bar{q}_{\gamma k}^C q_{\delta l}^D \rangle_0 - \langle \bar{q}_{\alpha i}^A q_{\delta l}^D \rangle_0 \langle \bar{q}_{\gamma k}^C q_{\beta j}^B \rangle_0 . \quad (1.1)$$

In vacuum, the condensate $\langle \bar{q}q \rangle_0$ is finite. In nuclear matter, the factorization approximation reads

$$\langle \bar{q}_{\alpha i}^A q_{\beta j}^B \bar{q}_{\gamma k}^C q_{\delta l}^D \rangle_{\rho_B} = \langle \bar{q}_{\alpha i}^A q_{\beta j}^B \rangle_{\rho_B} \langle \bar{q}_{\gamma k}^C q_{\delta l}^D \rangle_{\rho_B} - \langle \bar{q}_{\alpha i}^A q_{\delta l}^D \rangle_{\rho_B} \langle \bar{q}_{\gamma k}^C q_{\beta j}^B \rangle_{\rho_B} , \quad (1.2)$$

where the subscripts denotes an evaluation made for nuclear matter of density ρ_B . In matter, the condensates $\langle \bar{q}q \rangle_{\rho_B}$ and $\langle q^\dagger q \rangle_{\rho_B}$ are finite. The use of Eq. (1.2), with the in-medium value of $\langle \bar{q}q \rangle_{\rho_B}$ replacing $\langle \bar{q}q \rangle_0$ in the calculation of the four-quark condensates, leads to a result where the large scalar field found previously in the nucleon self-energy is much reduced [5]. This problem raises the question of how one should calculate the four-quark condensate terms, if one wishes to go beyond the factorization approximation of Eq. (1.2).

The proper treatment of four-quark condensates is also quite important when one studies the properties of the rho meson in matter [8-10]. We will study the vector-isovector correlator in this work and will calculate various corrections in the evaluation of the four-quark condensates that go beyond the result of the factorization approximation. Indeed, it can be shown that the factorization approximation is essentially a mean-field approximation and the corrections to that mean-field approximation can be large. (In Ref. [13] we demonstrated the relation between a QCD sum rule calculation for the nucleon in matter that considered only the lowest-dimension quark condensates and a mean-field description based upon a bosonization of the Nambu–Jona-Lasinio model [14,15].) The mean-field characterization of the QCD analysis was made explicit in our work.)

In this work we will consider the vector-isovector quark current correlation function which allows us to obtain the properties of the ρ meson in vacuum and in matter. As is standard in this area of study, we only consider modifications of the vacuum results that are linear in the density of nuclear matter. The plan of our work is as follows. In Section II we review the calculation for the vacuum of the polarization tensor for the vector-isovector current and show how the calculation is generalized if one calculates the polarization tensor in the presence of matter. We exhibit the result obtained for the terms linear in the density, if one makes the factorization approximation. We then go on to show how the term proportional to $\hat{m}_q \langle \bar{q}q \rangle_\rho$ in the polarization tensor may be calculated in a diagrammatic analysis based upon the Nambu–Jona-Lasinio model [14,15]. (Here $\hat{m}_q$ is the average current quark mass.)

Our motivation here is to show the relation between a many-body theory that allows us to calculate various nonperturbative matrix elements of quark operators and the QCD analysis

made using the operator-product expansion. In Section III we discuss the calculation of four-quark condensates for the vector-isovector polarization tensor in the presence of nuclear matter. We consider two models of the nucleon. One has a quark coupled to a scalar ($T = 0$) diquark and the other has a quark coupled to an axial-vector ($T = 1$) diquark [16]. (The physical nucleon may be described as having about equal magnitudes for the scalar diquark and axial-vector diquark components.) In Section IV we go on to consider additional four-quark condensate terms that appear in the formalism. The corrections to the factorization approximation considered in this section are found to be small relative to those discussed in Section III.

Unless otherwise stated, we will use the notation $\bar{q}q = \bar{u}u + \bar{d}d$. Thus, $\langle \bar{q}q \rangle_0 = \langle \bar{u}u + \bar{d}d \rangle_0 = 2 \langle \bar{u}u \rangle_0$, since we put $\langle \bar{u}u \rangle_0 = \langle \bar{d}d \rangle_0$. In the literature, one often finds that $\langle \bar{q}q \rangle_0 = \langle \bar{u}u \rangle_0 = \langle \bar{d}d \rangle_0$, so that one must be careful in evaluating various expressions containing $\langle \bar{q}q \rangle_\rho$ or $\langle \bar{q}q \rangle_0$. (We will note which convention we are using at various places in this work.)

In our work we will use some results given in Ref. [17]. For ease of comparison, we use the notation adopted there, where $A, B, \dots$ are flavor indices, $\alpha, \beta, \gamma, \dots$ are color indices and $a, b, \dots$ are Dirac matrix indices. Greek letters such as $\mu, \eta, \dots$ will denote Lorentz indices. As we will see, our procedures include forming an approximation to a four-quark nucleon density matrix of the form $\langle N | \bar{q}_{\alpha a}^A \bar{q}_{\beta b}^B q_{\gamma c}^C q_{\delta d}^D | N \rangle$, where all the quark operators are evaluated at the origin. The complete specification of such a density matrix requires a large number of terms. In this work we consider only four terms that are expected to be important. Two of these terms arise from the meson "cloud" of the nucleon and may be said to represent

a sigma meson or pion "condensate". The other two terms have their origin in the constituent quarks of the nucleon and may be considered to represent a "scalar ($T = 0$) diquark condensate" and a "axial-vector ($T = 1$) condensate". The relevance of these designations will become clear in Sections III and IV.

II. Calculation of the Vector-Isovector Polarization Tensor in Vacuum and in Nuclear Matter

We consider the calculation of the current-current correlator of the vector-isovector current

$$J^\mu(x) = \frac{1}{2} [\bar{u}(x) \gamma^\mu u(x) - \bar{d}(x) \gamma^\mu d(x)] \quad (2.1)$$

$$= \bar{q}(x) \gamma^\mu \frac{\tau_3}{2} q(x) \quad . \quad (2.2)$$

We define the vacuum polarization tensor,

$$\Pi_{(\rho)}^{\mu\nu}(q) = i \int d^4x e^{iq \cdot x} \langle 0 | T(J^\mu(x) J^\nu(0)) | 0 \rangle \quad , \quad (2.3)$$

and put

$$\Pi_{(\rho)}^{\mu\nu}(q) = (q^\mu q^\nu - q^2 g^{\mu\nu}) \Pi_{(\rho)}(q^2) \quad . \quad (2.4)$$

Thus, we may write

$$\Pi_{(\rho)}(q^2) = -\frac{i}{3q^2} \int d^4x e^{iq \cdot x} \langle 0 | T(J^\mu(x) J_\mu(0)) | 0 \rangle \quad . \quad (2.5)$$

One can also define these quantities in the presence of matter. [See Appendix A.] In that case there is another four vector, u^μ , to be considered. That vector describes the flow of the nuclear matter. In the rest frame, $u^\mu = [1, 0, 0, 0]$. Thus, there is a second invariant, $q \cdot u$, in addition to q^2 to be considered. In general, we should write $\Pi_{(\rho)}(q^2, q \cdot u)$. We also define contributions to $\Pi_{(\rho)}(q^2, q \cdot u)$ that are even and odd in $q \cdot u$:

$$\tilde{\Pi}_{(\rho)}(q^2, q \cdot u) = \tilde{\Pi}_{(\rho)}^{(e)}(q^2, (q \cdot u)^2) + q \cdot u \tilde{\Pi}_{(\rho)}^{(o)}(q^2, (q \cdot u)^2) \quad . \quad (2.6)$$

Since $\tilde{\Pi}_{(\rho)}^{(e)}(q^2, q \cdot u)$ and $\tilde{\Pi}_{(\rho)}^{(o)}(q^2, q \cdot u)$ satisfy separate dispersion relations, we will

concentrate on the calculation of $\tilde{\Pi}_{(\rho)}^{(e)}(q^2)$ in the following and will drop the superscript for convenience.

Thus, we will define

$$\tilde{\Pi}_{(\rho)}(q^2, (q \cdot u)^2) = -\frac{i}{3q^2} \int d^4x e^{iq \cdot x} \left\langle T(J^\mu(x) J_\mu(0)) \right\rangle_{\rho_B}, \quad (2.7)$$

with the understanding that we will discard terms odd in $(q \cdot u)$. In Eq. (2.7), the subscript ρ_B again denotes a matrix element evaluated in the presence of nuclear matter of baryon density ρ_B .

If we disregard the condensate $\langle \bar{q} \sigma_{\mu\nu} G^{\mu\nu} q \rangle$, we have for the vacuum value of $\Pi_{(\rho)}(Q^2)$ [17],

$$\begin{aligned} \Pi_{(\rho)}(Q^2) = & -\frac{1}{8\pi^2} \left[1 + \frac{\alpha_S}{\pi} \right] \ln Q^2 + \frac{1}{2} \hat{m}_q \langle \bar{q} q \rangle_0 \frac{1}{Q^4} \\ & + \frac{1}{24} \left\langle \frac{\alpha_S}{\pi} G_{\mu\nu} G^{\mu\nu} \right\rangle_0 - \frac{112}{81} \pi \alpha_S \langle \bar{u} u \rangle_0^2 \frac{1}{Q^6} + \dots, \end{aligned} \quad (2.8)$$

where $Q^2 = -q^2$. Here, we use the notation $\bar{q}q = \bar{u}u + \bar{d}d$ and have put $\langle \bar{u}u \rangle_0 = \langle \bar{d}d \rangle_0$. Further, we use the same value for the current quark masses of the up and down quarks, $\hat{m}_q = \hat{m}_u = \hat{m}_d$. The last term in Eq. (2.8) arises from the use of the factorization assumption for the four-quark condensates [4]. We will have more to say about that approximation in the next section.

In the presence of matter, we may use the factorization approximation to write

$$\begin{aligned}\tilde{\Pi}_{(\rho)}(Q^2) = & -\frac{1}{8\pi^2} \left[1 + \frac{\alpha_S}{\pi} \right] \ln Q^2 + \frac{1}{2} \frac{\hat{m}_q}{Q^4} \langle \bar{q}q \rangle_{\rho_B} \\ & + \frac{1}{24} \frac{1}{Q^4} \left\langle \frac{\alpha_S}{\pi} G_{\mu\nu} G^{\mu\nu} \right\rangle_{\rho_B} - \frac{112}{81} \frac{\pi\alpha_S}{Q^6} \langle \bar{u}u \rangle_{\rho_B}^2, \end{aligned} \quad (2.9)$$

where we have replaced $\langle \bar{q}q \rangle_0$ by $\langle \bar{q}q \rangle_{\rho_B}$ and $\langle \bar{u}u \rangle_0^2$ by $\langle \bar{u}u \rangle_{\rho_B}^2$.

In general, for an operator O , we have

$$\langle O \rangle_{\rho_B} = \langle O \rangle_0 + \langle N | O | N \rangle_{\rho_B} + \dots \quad (2.10)$$

to lowest order in ρ_B . Using that expansion we may write the density-dependent part of

$\tilde{\Pi}_{(\rho)}(Q^2)$ as

$$\Delta \Pi_{(\rho)}(Q^2) \equiv \tilde{\Pi}_{(\rho)}(Q^2) - \Pi_{(\rho)}(Q^2, \rho_B = 0) \quad (2.11a)$$

$$= \frac{1}{2} \frac{\hat{m}_q}{Q^4} \langle N | \bar{q}q | N \rangle_{\rho_B} + \frac{1}{24} \left\langle N \left| \frac{\alpha_S}{\pi} G_{\mu\nu} G^{\mu\nu} \right| N \right\rangle_{\rho_B} - \frac{112}{81} \frac{\pi\alpha_S}{Q^6} (2) \langle \bar{u}u \rangle_0 \langle N | \bar{u}u | N \rangle_{\rho_B} .$$

$$(2.11b)$$

There are two additional condensates of dimension four and dimension six that may be expressed in terms of moments of the quark distribution functions for the nucleon. (These will not concern here and we do not include them in Eq. (2.11).) The density-dependent contribution of these terms to $\tilde{\Pi}_{(\rho)}(Q^2)$ is

$$\frac{m_N}{4Q^4} A_2^{u+d} \rho_B - \frac{5m_N^3}{24Q^6} A_4^{u+d} \rho_B ,$$

where m_N is the nucleon mass and $A_n^{u+d} \equiv A_n^u + A_n^d$. For example,

$$A_n^u(\mu^2) = 2 \int_0^1 dx x^{n-1} \left[u(x, \mu^2) + (-1)^n \bar{u}(x, \mu^2) \right] , \quad (2.12)$$

where $u(x, \mu^2)$ and $\bar{u}(x, \mu^2)$ are the scale-dependent distribution function for up quarks and up antiquarks in the nucleon, etc.

One of our goals is to understand the origin of Eq. (2.11) and to assess its validity. In this section we will concentrate on the first term of Eq. (2.11) and consider four-quark condensates at a later stage. One way to proceed is to write

$$\langle \bar{q}q \rangle_{\rho_B} = \langle \bar{q}q \rangle_0 + \langle N | \bar{q}q | N \rangle_{\rho_B} + \dots , \quad (2.13)$$

where $\langle N | \bar{q}q | N \rangle$ is the nucleon matrix element of $\bar{q}q$ (averaged over spin and isospin). It is convenient to write Eq. (2.13) in terms of the pion-nucleon sigma term [18]

$$\sigma_N = \hat{m}_q \langle N | \bar{q}q | N \rangle , \quad (2.14)$$

so that

$$\langle \bar{q}q \rangle_{\rho_B} = \langle \bar{q}q \rangle_0 + \frac{\sigma_N}{\hat{m}_q} \rho_B . \quad (2.15)$$

(Here, we again use the notation $\bar{q}q = \bar{u}u + \bar{d}d$.) We note that the density-dependent correction of Eq. (2.15) may be considered to be model-independent [19].

To gain some familiarity with our procedures, we provide a derivation of Eq. (2.13) making use of a diagrammatic analysis. It is useful to note that [17]

$$\langle : \bar{q}_{\alpha i}^A(x) q_{\beta j}^B(0) : \rangle_0 = \frac{1}{12} \delta_{\alpha\beta} \delta_{AB} \langle : \bar{q}^A q^B : \rangle_0 \left[\delta_{ij} + \frac{i}{4} \hat{m}_q x_\mu (\gamma^\mu)_{ji} + \dots \right] . \quad (2.16)$$

Here, A, B are flavor indices, α, β are color indices and i, j are Dirac matrix indices. (The

$\hat{m}_q = \hat{m}_u = \hat{m}_d$. The last term in Eq. (2.8) arises from the use of the factorization assumption for the four-quark condensates [4]. We will have more to say about that approximation in the

colons on either side of the bracketed operators denote a normal product formed with respect to the perturbative vacuum. In most cases we will not write these colons.)

One contribution to the polarization in vacuum is given in terms of up and down quark condensates [17],

$$\Pi_{(\phi)}(q^2) = \frac{1}{12q^2} \int d^4x \int \frac{d^4p}{(2\pi)^4} e^{-i(p-q) \cdot x} \left\{ 2 \langle 0 | : \bar{u}_{\alpha i}(x) u_{\alpha j}(0) : | 0 \rangle \left[\gamma^\mu S(p) \gamma_\mu \right]_{ij} + u \rightarrow d \right\} . \quad (2.17)$$

Here, $S(p)$ denotes the quark propagator. The factor of 2 in the bracket appears because of the presence of two diagrams in Fig. 1(a). One makes use of Eq. (2.16)-(2.17) to obtain the second term in Eq. (2.8). We now consider a similar calculation made in the presence of matter. The analog of Eq. (2.16) is

$$\langle N | \bar{q}_{\alpha i}^A(x) q_{\beta j}^B(0) | N \rangle_{\rho B} = \frac{1}{12} \delta_{\alpha\beta} \delta_{AB} \langle N | \bar{q}^A q^B | N \rangle_{\rho B} \left[\delta_{ij} + \frac{i}{4} \hat{m}_q x^\mu (\gamma_\mu)_{ji} + \dots \right] . \quad (2.18)$$

Actually, it is useful to be more precise. For example, note that nucleons in a noninteracting Fermi gas have a four-momentum P^μ , with $P^0 = E_N(P)$ being the on-mass-shell energy. Averaging over spin and isospin, we may put

$$\langle \bar{P} | \bar{q}_{\alpha i}^A(0) q_{\beta j}^B(0) | \bar{P} \rangle = \delta_{ij} \delta_{\alpha\beta} \frac{\delta_{AB}}{24} \langle \bar{P} | \bar{q}(0) q(0) | \bar{P} \rangle + \delta_{\alpha\beta} \delta_{AB} \frac{1}{96} (\gamma_\mu)_{ji} \left[\frac{P_\mu}{m_N} \right] \left[\frac{m_N}{E_N(\bar{P})} \right] . \quad (2.19)$$

This result should then be summed over the nucleons in the Fermi sea with $|\bar{P}| \leq k_F$. The second term in Eq. (2.19) ultimately gives rise to a term proportional to $q \cdot u$ (or q^0 in the rest frame), and therefore contributes to the second term in Eq. (2.6). Thus, Eq. (2.19) actually

represents a more complex calculation, than that implied by Eq. (2.18). However, we will use the simpler notation here.

It is useful to understand the origin of the first term in Eq. (2.11b) using diagrammatic techniques in the context of the Nambu–Jona-Lasinio model. In that model we can consider the nucleon as composed of three constituent (or valence) quarks and a "meson cloud" [15]. The contribution to the density matrix due to the valence quarks is indicated schematically in Fig. 1(b). The corresponding nucleon matrix element will be denoted as $\langle N | \bar{q}q | N \rangle_{val}$.

Now consider Figs. 1(c) and (d). There the solid dot denotes a factor of iG_S , where G_S is the coupling constant of the NJL model, whose Lagrangian is

$$\mathcal{L}(x) = \bar{q}(x)[i\partial - \hat{m}_q]q(x) + \frac{G_S}{2} [(\bar{q}q)^2 + (\bar{q}i\gamma_5 \vec{\tau}q)^2] \quad (2.20)$$

in the case of $SU(2)$ - flavor symmetry.

For the moment, we will work with the current quark mass, $\hat{m}_q$, rather than the constituent mass m_q . We denote the quark propagator as $S(x, y)$ in coordinate space and $S(p)$ in momentum space. For example, consider a term representing the evaluation of the diagram of Fig. 1(c),

$$I_1(q^2) = 2n_c n_f \langle N | \bar{q}q | N \rangle_{val} \rho_B(iG_S) \int d^4x \int d^4y [S(0, y) S(y, x)]_{ab} [\gamma_\mu S(x, 0) \gamma^\mu]_{ba} e^{iq \cdot x} . \quad (2.21)$$

Here 2 is a statistical factor. Further, $n_c = 3$ and $n_f = 2$ are the number of colors and flavors, respectively. Now, consider putting $x = 0$ in the first bracket of Eq. (2.21), and note that

$$\int d^4y [S(0, y) S(y, 0)]_{ab} = \int \frac{d^4k}{(2\pi)^4} [S(k) S(k)]_{ab} . \quad (2.22)$$

It is useful to define

$$\hat{J}_S(r) = n_f n_c i \int \frac{d^4 k}{(2\pi)^4} \text{Tr}[S(k+r)S(k)] , \quad (2.23)$$

so that

$$\hat{J}_S(0) = n_c n_f i \int \frac{d^4 k}{(2\pi)^4} \text{Tr}[S(k)S(k)] . \quad (2.24)$$

Now we may expand the matrix on the right-hand side of Eq. (2.22) in a complete set of Dirac matrices. Only the unit matrix survives upon performing the four-dimensional integral over k^μ , so that we may put

$$\int \frac{d^4 k}{(2\pi)^4} [S(k)S(k)]_{ab} = \frac{\delta_{ab}}{4} \int \frac{d^4 k}{(2\pi)^4} \text{Tr}[S(k)S(k)] . \quad (2.25)$$

In this manner, upon using

$$\text{Tr}[\gamma_\mu S(q) \gamma^\mu] = \frac{16 \hat{m}_q}{q^2} , \quad (2.26)$$

we find a density-dependent term,

$$\tilde{\Pi}_{(\omega)}(q^2) = \frac{1}{6q^2} \left\{ \frac{1}{4} \langle N | \bar{q} q | N \rangle_{val} \rho_B \frac{16 \hat{m}_q}{q^2} G_S \hat{J}_S(0) + \dots \right\} \quad (2.27)$$

to add to the term arising from Fig. 1(b):

$$\tilde{\Pi}_{(\omega)}(q^2) = \frac{1}{6q^2} \left\{ \frac{1}{4} \langle N | \bar{q} q | N \rangle_{val} \rho_B \frac{16 \hat{m}_q}{q^2} \right\} . \quad (2.28)$$

Combining these, we have the (partial) contribution to $\tilde{\Pi}_{(\omega)}(q^2)$:

$$\tilde{\Pi}_{(\varphi)}(q^2) = \left[\frac{2}{3q^4} \right] \hat{m}_q \frac{\langle N | \bar{q} q | N \rangle_{val}}{1 - G_S \hat{J}_S(0)} \rho_B + \dots \quad (2.29)$$

$$= \frac{2}{3q^2} \hat{m}_q \langle N | \bar{q} q | N \rangle \rho_B , \quad (2.30)$$

where we have defined

$$\langle N | \bar{q} q | N \rangle = \frac{\langle N | \bar{q} q | N \rangle_{val}}{1 - G_S \hat{J}_S(0)} . \quad (2.31)$$

The additional terms that appear when expanding the denominator of Eqs. (2.29) and (2.31) are generated by diagrams such as that of Fig. 1(d), for example.

The term obtained by putting $x = 0$ in $S(y, x)$ of Eq. (2.19) does not give the entire answer. A comparable term is obtained if we expand the first bracket in Eq. (2.19) to first order in x . Thus, we consider the matrix

$$(I_0)_{ij} = \int d^4 y [S(0, y) x^\lambda \partial_\lambda S(y, 0)]_{ij} . \quad (2.32)$$

Now

$$\int d^4 y [S(0, y) \partial_\lambda S(y, 0)]_{ij} = \frac{D}{4} (\gamma_\lambda)_{ij} , \quad (2.33)$$

where D is a constant. We have

$$4D = Tr \int d^4 y [S(0, y) \partial_\lambda S(y, 0) \gamma^\lambda] , \quad (2.34)$$

$$= Tr \int \frac{d^4 k}{(2\pi)^4} [S(k) i \not{k} S(k)] , \quad (2.35)$$

$$= i \hat{m}_q \int \frac{d^4 k}{(2\pi)^4} Tr [S(k) S(k)] . \quad (2.36)$$

Thus,

$$I_0 = ix^\lambda \gamma_\lambda \frac{\hat{m}_q}{4} \int \frac{d^4 k}{(2\pi)^4} \text{Tr}[S(k)S(k)]/4 . \quad (2.37)$$

The contribution to $\Pi_{(\varphi)}(q^2)$ from I_0 is readily obtained following the procedure outlined in Ref. [17].

The combined result of the terms considered thus far is a contribution to $\tilde{\Pi}_{(\varphi)}(Q^2)$ of the form

$$\tilde{\Pi}_{(\varphi)}(Q^2) = \frac{\hat{m}_q}{2Q^4} \frac{\langle N | \bar{q}q | N \rangle_{val}}{1 - G_S \hat{J}_S(0)} \rho_B , \quad (2.38)$$

where $\bar{q}q = \bar{u}u + \bar{d}d$. The result presented in Eq. (2.38) is given in terms of $\hat{J}_S(0)$. That quantity is given in Eq. (2.24) in terms of the propagators $S(k)$, which are expressed in terms of the current quark mass. In the NJL model, an integral similar to $\hat{J}_S(0)$ is defined in terms of quarks with a constituent mass m_q , rather than the current mass, $\hat{m}_q$. We may denote a propagator of a constituent quark as $S_c(k) = [k - m_q + i\epsilon]^{-1}$. The origin of the constituent mass may be understood by considering the effects due to the vacuum condensate of the NJL model. [See Fig. 2(a).] The sum of the insertions on the lines of momentum k serves to replace $\hat{J}_S(0)$ by

$$J_S(0) = n_c n_f i \int \frac{d^4 k}{(2\pi)^4} \text{Tr}[S_c(k)S_c(k)] . \quad (2.39)$$

We note that this integral is divergent and requires a cutoff of the type used in the NJL model.

That cutoff may be made in Euclidean space such that $k_E^2 = k_0^2 + \vec{k}^2 \leq \Lambda^2$.

At this point we may replace Eq. (2.38) by

$$\tilde{\Pi}_{(\omega)}(Q^2) = \frac{\bar{m}_q}{2Q^4} \langle N | \bar{q}q | N \rangle_{\rho_B} , \quad (2.40)$$

where, now, instead of Eq. (2.31), we have defined

$$\langle N | \bar{q}q | N \rangle = \frac{\langle N | \bar{q}q | N \rangle_{val}}{1 - G_S J_S(0)} . \quad (2.41)$$

Equation (2.40) is the same as the result obtained when replacing $\langle \bar{q}q \rangle_0$ by $\langle \bar{q}q \rangle_0 + \langle N | \bar{q}q | N \rangle_{\rho_B}$ in the presence of matter. In this discussion we have shown how that result may be derived using a specific many-body theory. In the next section we turn to a discussion of the corrections to the factorization approximation that arise when calculating the value of four-quark condensates in nuclear matter. These corrections may be thought of as dynamical corrections that go beyond a mean-field approximation. Alternatively, we may view nuclear matter as providing various mesonic and diquark condensates when evaluating the four-quark density matrix in nuclear matter. The basis for this last observation will be given in Sections III and IV.

III. Corrections to the Factorization Approximation for Four-Quark Condensates

A rather complete discussion of the calculation of the vector-isovector polarization tensor in vacuum is given in Ref. [17]. We consider the various processes considered in that work and use the same diagrammatic representation. We begin by considering the diagrams of Figs. 3(a) and 3(b). The calculation represented by these diagrams requires the specification of a four-quark density matrix. For completeness, we record the expression given in Ref. [17] for the contribution of the diagrams in Figs. 3(a) and 3(b) to $\Pi_{(\omega)}(q^2)$. (Here $\rho_B = 0$.) These contributions (plus those arising from diagrams with the direction of the fermion lines reversed) are given by

$$\begin{aligned}
\Pi_{(\omega)}^{(1)}(q^2) = & \frac{g^2}{24q^2} \int d^4x \int d^4y \int d^4z \int \frac{d^4k}{(2\pi)^4} \int \frac{d^4p_1}{(2\pi)^4} \int \frac{d^4p_2}{(2\pi)^4} \\
& \times \sum_{AB} (\tau_3)_{AA} (\tau_3)_{BB} \frac{1}{k^2} e^{i(q+p_1) \cdot x} e^{-i(k+p_1) \cdot y} e^{i(k-p_2) \cdot z} \\
& \times \left\{ \left[\gamma^\lambda S(p_1) \gamma^\mu \right]_{mj} \left[\gamma_\lambda S(p_2) \gamma_\mu \right]_{se} \right. \\
& \times \left[\left\langle 0 \left| \bar{q}_{\beta m}^A(y) \bar{q}_{\alpha s}^B(z) q_{\alpha j}^A(x) q_{\beta e}^B(0) \right| 0 \right\rangle - \frac{1}{3} \left\langle 0 \left| \bar{q}_{\alpha m}^A(y) \bar{q}_{\beta s}^B(z) q_{\alpha j}^A(x) q_{\beta e}^B(0) \right| 0 \right\rangle \right] \\
& - \left[\gamma_\mu S(p_2) \gamma_\lambda \right]_{kt} \left[\gamma^\lambda S(p_1) \gamma^\mu \right]_{mj} e^{i(q+p_1) \cdot x} e^{-i(k+p_2) \cdot y} e^{i(k+p_2) \cdot z} \\
& \left. \left[\left\langle 0 \left| \bar{q}_{\alpha k}^B(0) \bar{q}_{\gamma m}^A(y) q_{\alpha j}^A(x) q_{\gamma e}^B(z) \right| 0 \right\rangle - \frac{1}{3} \left\langle 0 \left| \bar{q}_{\beta k}^B(0) \bar{q}_{\alpha m}^A(y) q_{\alpha j}^A(x) q_{\beta e}^B(z) \right| 0 \right\rangle \right] \right\} \\
& + \text{inverted diagrams}
\end{aligned} \tag{3.1}$$

where we have used the notation of Ref. [17].

The value in vacuum of $\Pi_{(\omega)}^{(1)}(q^2)$ is obtained by use of the factorization approximation.

One finds [17]

$$\Pi_{(\omega)}^{(1)}(q^2) = \frac{16}{9} \left[\frac{\pi \alpha_s}{q^6} \right] \langle \bar{u}u \rangle_0^2 \quad (3.2)$$

as the result of evaluating the diagrams of Figs. 3(a) and 3(b) and the inverted forms. Density-dependent corrections to that result are of two types, those that involve the meson cloud of the nucleon and those arising from the presence of diquarks in the nucleon. For example, a density-dependent correction to Fig. 3(a) due to a meson (double line) originating in a nucleon (heavy line) is shown in Fig. 3(c).

In our calculations, we again encounter a four-quark operator of the form $\bar{q}_{\alpha a}^A \bar{q}_{\beta b}^B q_{\gamma c}^C q_{\delta d}^D$ and its matrix element taken between nucleon states. (That matrix element is averaged over spin and isospin projections of the nucleon.) The four-quark operator may be organized in several ways. For example, we may couple the first operator with the fourth and the second with the third. That coupling is related by a Fierz rearrangement to the case where the first and third (and second and fourth) are coupled together. One or another coupling scheme is most useful for the calculation of a particular dynamical processes. However, one must proceed in such a manner as to avoid double counting.

In this work we will consider contributions of the sigmas and pions that are present in the nucleon's meson cloud. First, we consider the corrections to the vacuum values calculated for Figs. 3(a) and 3(b) that are due to the sigma meson. That result may be obtained by considering only a single term in the complete expansion for the density matrix

$$\bar{q}_{\alpha a}^A \bar{q}_{\beta b}^B q_{\gamma c}^C q_{\delta d}^D = \frac{1}{(4)(144)} \delta_{AD} \delta_{BC} \delta_{\alpha\delta} \delta_{\beta\gamma} \delta_{ad} \delta_{bc} (\bar{q}q)(\bar{q}q) + \dots \quad (3.3)$$

Here all the q 's are at the origin, i.e. $\bar{q}_{\alpha a}^A = \bar{q}_{\alpha a}^A(0)$, etc., and we again use $\bar{q}q = \bar{u}u + \bar{d}d$.

Using Eq. (3.3), we found

$$\Pi_{(\rho)}^{(1)}(q^2) = \frac{16\pi}{9(4)} \alpha_S \langle (\bar{q}q)(\bar{q}q) \rangle_{\rho} \frac{1}{q^6} \quad (3.4)$$

in an earlier work [20]. We then wrote the matrix element in Eq. (3.4) as

$$\langle \bar{q}q\bar{q}q \rangle_{\rho} = \langle \bar{q}q\bar{q}q \rangle_0 + 2\langle N | \bar{q}q | N \rangle \langle \bar{q}q \rangle_0 \rho_B + \langle N | (\bar{q}q)(\bar{q}q) | N \rangle_C \rho_B \quad (3.5)$$

where the last term represents a correction to the factorization approximation. Thus, one contribution to $\tilde{\Pi}_{(\rho)}^{(1)}(q^2)$ is

$$\tilde{\Pi}_{(\rho)}^{(1)}(q^2) = \frac{16}{9} \frac{\pi\alpha_S}{q^6} \left(\langle \bar{u}u \rangle_0^2 + 2\langle \bar{u}u \rangle_0 \langle N | \bar{u}u | N \rangle \rho_B + \frac{1}{4} \langle N | (\bar{q}q)(\bar{q}q) | N \rangle_C \rho_B \right) \quad (3.6)$$

Here $\langle N | (\bar{q}q)(\bar{q}q) | N \rangle_C$ is defined such that it does not contain a factor of $\langle \bar{q}q \rangle_0$. The origin of the first two terms on the right-hand side of Eq. (3.6) may be understood by inspection of Fig. 4. There the double line denotes a sigma-like field that has its origin in the nucleons of nuclear matter and which is proportional to $\langle N | \bar{q}q | N \rangle \rho_B$. The contribution of the "sigma condensate" is represented by the last term in Eq. (3.6). For that term, we have

$$\tilde{\Pi}_{(\rho)}^{(1)}(q^2) = \frac{4}{9} \left[\frac{\pi\alpha_S}{Q^6} \right] A_{\sigma} \rho_B \quad (3.7)$$

where $A_{\sigma} = \langle N | (\bar{q}q)(\bar{q}q) | N \rangle = 0.0317 \text{ GeV}^3$ was calculated in an earlier work [20].

Now consider the contribution of the term in the four-quark operator $\bar{q}_{\alpha a}^A \bar{q}_{\rho b}^B q_{\gamma c}^C q_{\delta d}^D$ of the form

$$\bar{q}_{\alpha a}^A \bar{q}_{\beta b}^B q_{\gamma c}^C q_{\delta d}^D = \frac{1}{4(144)} (i\gamma_5)_{da} (i\gamma_5)_{cb} (\tau_j)_{DA} (\tau_k)_{CB} \delta_{\alpha\delta} \delta_{\beta\gamma} (\bar{q} i\gamma_5 \tau_j q) (\bar{q} i\gamma_5 \tau_k q) + \dots \quad (3.8)$$

We found the contribution of the pion to the diagram in Fig. 3(c), and the corresponding correction originating from Fig. 3(b), to be [20]

$$\Pi_{(\phi)}^{(1)}(q^2) = -\frac{4}{27} \left[\frac{\pi\alpha_s}{q^6} \right] C_{\pi} \rho_B \quad , \quad (3.9)$$

where C_{π} is the value of the direct term arising in the evaluation of $\langle N | (\bar{q} i\gamma_5 \bar{\tau} q) \cdot (\bar{q} i\gamma_5 \bar{\tau} q) | N \rangle_C$. We found $C_{\pi} = 0.108 \text{ GeV}^3$, so that this contribution to $\Pi_{(\phi)}^{(1)}(q^2)$ is small and may be neglected [20]. (See Table 1.)

Much larger corrections are obtained if we consider diquark processes such as that shown in Fig. 3(d). [Figure 3(d) is based upon Fig. 3(a). There is another diquark process associated with Fig. 3(b) that we do not show.] The diquark processes may be calculated if we organize the nucleon density in another fashion. We put

$$\begin{aligned} \langle N | \bar{q}_{\alpha m}^A \bar{q}_{\beta s}^B q_{\alpha' j}^{A'} q_{\beta' e}^{B'} | N \rangle &= \frac{1}{(24)(24)} \sum_{cc'} (C\gamma_5)_{sm} (t_c)_{\beta\alpha} (\tau_2)_{BA} (\tau_2)_{B'A'} (\gamma_5 C^{-1})_{ej} (t_{c'})_{\beta'\alpha'} \\ &\times \langle N | (\bar{q} \gamma_5 C t_c \tau_2 \bar{q}^T) (q^T C^{-1} \gamma_5 \tau_2 t_{c'} q) | N \rangle + \dots \quad , \end{aligned} \quad (3.10)$$

and define $C_{SD} = \langle N | (\bar{q} \gamma_5 C t_c \tau_2 \bar{q}^T) (q^T C^{-1} \gamma_5 \tau_2 t_{c'} q) | N \rangle$. Again, all the operators are evaluated at the origin. Equation (3.10) may be used in Eq. (3.1) to calculate the contribution to $\Pi_{(\phi)}^{(1)}$ due to the presence of a scalar-diquark condensate. Carrying out the various trace operations, we find the contribution of a scalar-diquark condensate to be

$$\Delta \Pi_{(\rho)}^{(1)}(q^2) = \frac{2}{9} \left[\frac{\pi \alpha_S}{q^6} \right] C_{SD} \rho_B \quad , \quad (3.11)$$

where C_{SD} was defined after Eq. (3.10). In our previous work [20], we found $C_{SD} = 0.660 \text{ GeV}^3$ for a nucleon composed of a quark and a scalar diquark. Thus, we obtain a contribution to $\Delta \Pi_{(\rho)}^{(1)}(q^2)$:

$$\Delta \Pi_{(\rho)}^{(1)}(q^2) = 1.85 \times 10^{-4} \left[\frac{\pi \alpha_S}{q^6} \right] \text{ GeV}^6 \quad . \quad (3.12)$$

where we have put $\rho_B = (0.108 \text{ GeV})^3$. However, the vacuum value is

$$\Pi_{(\rho)}^{(1)}(q^2) = \frac{16}{9} \frac{\pi \alpha_S}{q^6} \langle \bar{u} u \rangle_0^2 \quad (3.13)$$

$$\simeq 4.34 \times 10^{-4} \left[\frac{\pi \alpha_S}{q^6} \right] \text{ GeV}^6 \quad . \quad (3.14)$$

Therefore we see that the correction given in Eq. (3.11) is about 40 percent of the vacuum result and has the same sign. This is by far the largest correction that we find. [See Tables 1 and 2.]

We may now consider an alternative model where the nucleon is composed of a quark coupled to an axial-vector ($T = 1$) diquark. Instead of Eq. (3.10), we now consider a term in the expansion of the density matrix of the form

$$\begin{aligned} \langle N | \bar{q}_{\alpha m}^A \bar{q}_{\beta s}^B q_{\alpha' j}^{A'} q_{\beta' e}^{B'} | N \rangle &= \frac{1}{(96)(96)} \sum (C \gamma^\mu)_{sm} (i t_c)_{\beta\alpha} (\tau_j \tau_2)_{BA} (\gamma^\nu C^{-1})_{ej} (i t_{c'})_{\beta' \alpha'} (\tau_2 \tau_k)_{B' A'} \\ &\times \langle N | (\bar{q}(i t_c) (\tau_j \tau_2) \gamma_\mu C \bar{q}^T) (q^T C^{-1} \gamma_\nu (\tau_2 \tau_k) (i t_{c'}) | N \rangle + \dots \end{aligned} \quad (3.15)$$

We also use the relation

$$\begin{aligned} &\langle N | (\bar{q}(i t_c) (\tau_j \tau_2) \gamma_\mu C \bar{q}^T) \cdot (q^T C^{-1} \gamma_\nu (\tau_2 \tau_k) (i t_c) q) | N \rangle \\ &= \frac{g_{\mu\nu}}{4} \langle N | (\bar{q}(i t_c) (\tau_j \tau_2) \gamma^\rho C^{-1} q) \cdot (q^T C^{-1} \gamma_\rho (\tau_2 \tau_k) (i t_c) q) | N \rangle . \end{aligned} \quad (3.16)$$

The result then depends upon the matrix element of a hermitian (scalar) operator:

$$C_{VD} = \langle N | (\bar{q} i t_c (\bar{\tau} \tau_2) \gamma^\rho C \bar{q}^T) \cdot (q^T C^{-1} \gamma_\rho (\tau_2 \bar{\tau}) i t_c q) | N \rangle . \quad (3.17)$$

The calculation of this quantity is discussed in Appendix B. [See Fig. 5.] We find $C_{VD} = 0.380 \text{ GeV}^3$. Forming the appropriate traces in the evaluation of Eq. (3.1), we have for the contribution of Figs. 3(a) and 3(b) (and the inverted diagrams)

$$\Delta \Pi_{(\rho)}^{(1)}(q^2) = - \frac{1}{27} \left[\frac{\pi \alpha_S}{q^6} \right] C_{VD} \rho_B \quad (3.18)$$

as the axial-vector diquark contribution to $\Delta \tilde{\Pi}_{(\rho)}^{(1)}$. Inserting our value for C_{VD} , we have

$$\Delta \Pi_{(\rho)}^{(1)}(q^2) = - 1.77 \times 10^{-5} \left[\frac{\pi \alpha_S}{q^6} \right] \text{ GeV}^6 \quad (3.19)$$

which is an order of magnitude smaller than the scalar-diquark contribution given in Eq. (3.12). Comparing to the vacuum value given in Eq. (3.14), we see that Eq. (3.19) represents only a 4 percent correction. Therefore, we see that the model of the nucleon that consists of a quark coupled to an axial-vector diquark yields much smaller corrections than the model where a quark

coupled to a scalar diquark. As we believe that the physical nucleon has approximately equal components of scalar and axial-vector diquarks, we estimate corrections due to diquark processes to be about 15 to 20 percent of the vacuum value of $\Pi_{(\phi)}^{(1)}(q^2)$.

IV. Corrections to the Factorization Approximation: Mixed Condensates

The diagrams of Fig. 6 contribute to a mixed condensate of dimension six, $\hat{m}_q \langle \bar{q}^A \sigma_{\mu\nu} G^{\mu\nu} q^A \rangle_0$. We will not attempt to obtain corrections to this matrix element that go beyond the approximation

$$\langle \bar{q}^A \sigma_{\mu\nu} G^{\mu\nu} q^A \rangle_\rho = \langle \bar{q}^A \sigma_{\mu\nu} G^{\mu\nu} q^A \rangle_0 + \langle N | \bar{q}^A \sigma_{\mu\nu} G^{\mu\nu} q^A | N \rangle_\rho + \dots \quad (4.1)$$

However, we will consider the contribution of the diagrams in Fig. 7 in some detail. These diagrams contribute to a quantity denoted as $\Pi_{(\phi)}^{(2)}(q^2)$ in Ref. [17]. The contribution of the diagram in Fig. 7(a) is obtained in terms of the value of the matrix element $\langle \bar{q}^A \gamma^\tau \frac{\lambda}{2} q^A \sum_C \bar{q}^C \gamma_\tau \frac{\lambda^a}{2} q^C \rangle_0$ [17]. With SU(2)-flavor symmetry, this matrix element is equal to $(1/2) \langle \bar{q} \gamma^\tau (\lambda^a/2) q \bar{q} \gamma_\tau (\lambda^a/2) q \rangle_0$, in our notation. For the vacuum, we calculate that

$$\Pi_{(\phi)}^{(2)}(q^2) = \frac{4\pi}{9} \frac{\alpha_S}{q^6} \left\langle \left(\bar{q}^A \gamma^\tau \frac{\lambda}{2} q^A \right) \sum_C \bar{q}^C \gamma_\tau \frac{\lambda^a}{2} q^C \right\rangle_0, \quad (4.2)$$

$$= \frac{4\pi}{9} \frac{\alpha_S}{q^6} \left[-\frac{8}{9} \langle \bar{u}u \rangle_0^2 \right], \quad (4.3)$$

$$= -\frac{32}{81} \frac{\pi\alpha_S}{q^6} \langle \bar{u}u \rangle_0^2, \quad (4.4)$$

where we have evaluated the matrix element in the factorization approximation. (The result given in Eq. (4.4) agrees with that of Ref. [17] for $\Pi_{(\phi)}^{(2)}(q^2)$.)

In matter, the factorization approximation would give

$$\Pi_{(\rho)}^{(2)}(q^2) = -\frac{32}{81} \frac{\pi\alpha_S}{q^6} \langle \bar{u}u \rangle_\rho^2, \quad (4.5)$$

where we have replaced $\langle \bar{u}u \rangle_0$ by $\langle \bar{u}u \rangle_\rho$. However, in our formulation we have a correction term that does not depend upon $\langle \bar{u}u \rangle_0$. We write

$$\tilde{\Pi}_{(\rho)}^{(2)}(q^2) = \frac{2}{9} \left[\frac{\pi\alpha_S}{q^6} \right] \left\langle \left[\bar{q} \gamma^\tau \frac{\lambda^a}{2} q \right] \cdot \left[\bar{q} \gamma_\tau \frac{\lambda^a}{2} q \right] \right\rangle_\rho, \quad (4.6)$$

$$= \frac{2}{9} \left[\frac{\pi\alpha_S}{q^6} \right] \left[\left\langle \left[\bar{q} \gamma^\tau \frac{\lambda^a}{2} q \right] \left[\bar{q} \gamma_\tau \frac{\lambda^a}{2} q \right] \right\rangle_\rho + \langle N | \left[\bar{q} \gamma^\tau \frac{\lambda^a}{2} q \right] \left[\bar{q} \gamma_\tau \frac{\lambda^a}{2} q \right] | N \rangle_C \rho_B \right] \quad (4.7)$$

$$= \frac{4}{9} \left[\frac{\pi\alpha_S}{q^6} \right] \left[-\frac{8}{9} \langle \bar{u}u \rangle_\rho^2 + \frac{1}{2} \langle N | (\bar{q} \gamma^\tau \frac{\lambda^a}{2} q) (\bar{q} \gamma_\tau \frac{\lambda^a}{2} q) | N \rangle_C \rho_B \right] \quad (4.8)$$

with $\langle \bar{u}u \rangle_\rho = \langle \bar{u}u \rangle_0 + 2 \langle \bar{u}u \rangle_0 \langle N | \bar{u}u | N \rangle_{\rho_B}$. (Recall that the matrix element $\langle N | (\bar{q} \gamma^\tau (\lambda^a/2) q) (\bar{q} \gamma_\tau (\lambda^a/2) q) | N \rangle_C$ is defined such that it does not have a factor of $\langle \bar{q}q \rangle_0$.)

Thus we need to calculate the nucleon matrix element appearing on the right of Eq. (4.8). Upon Fierz rearrangement we have

$$\frac{1}{2} \langle N | \left[\bar{q} \gamma^\tau \frac{\lambda^a}{2} q \right] \cdot \left[\bar{q} \gamma_\tau \frac{\lambda^a}{2} q \right] | N \rangle_{\rho_B} = -\frac{1}{18} \langle N | (\bar{q} \gamma_5 C \tau_2 t_c \bar{q}^T) (q^T C^{-1} \gamma_5 \tau_2 t_c q) | N \rangle_{\rho_B} \quad (4.9)$$

$$= -\frac{1}{18} C_{SD} \rho_B, \quad (4.10)$$

$$= -4.63 \times 10^{-5} \text{ GeV}^6, \quad (4.11)$$

where we have used $\rho_B = (0.108 \text{ GeV})^3$ and the result for the nucleon matrix element calculated in Ref. [20]. This may be compared to the leading

term in Eq. (4.7). We have $-(8/9)\langle\bar{u}u\rangle_0^2 = -2.17 \times 10^{-4} \text{ GeV}^6$ and $-2(8/9)\langle\bar{u}u\rangle_0\langle N|\bar{u}u|N\rangle\rho_B = 1.43 \times 10^{-4} \text{ GeV}^6$, where we have put $\langle N|\bar{u}u|N\rangle = 4.09$. Thus, Eq. (4.11) represents a correction of the order of 10 percent of the vacuum term $-(8/9)\langle\bar{u}u\rangle_0^2$. [See Tables 1 and 2.]

We may also consider the model in which the nucleon is considered to be composed of a quark coupled to an axial-vector ($T = 1$) diquark. To calculate the nucleon matrix element of the relevant operator, we again perform a Fierz rearrangement,

$$\begin{aligned} \frac{1}{2}\langle N|\left[\bar{q}\gamma^\mu\frac{\lambda^a}{2}q\right]\left[\bar{q}\gamma_\mu\frac{\lambda^a}{2}q\right]|N\rangle_{\rho_B} &\stackrel{F.R.}{=} \frac{1}{36}\langle N|\left[\bar{q}(it_c)(\bar{\tau}\tau_2)\gamma^\nu C\bar{q}^T\right] \\ &\times \left[\bar{q}^T C^{-1}\gamma_\nu(\tau_2\bar{\tau})(it_c)q\right]|N\rangle_{\rho_B} + \dots, \end{aligned} \quad (4.12)$$

$$= \frac{1}{36}C_{VD}\rho_B + \dots, \quad (4.13)$$

where $C_{VD} = 0.380 \text{ GeV}^3$. [See Tables 1 and 2.] In this case, the contribution to $\Pi_{(\omega)}^{(2)}(Q^2)$ is

$$\Pi_{(\omega)}^{(2)}(Q^2) = -\frac{4}{9}\left[\frac{\pi\alpha_S}{Q^6}\right]\frac{C_{VD}}{36}\rho_B. \quad (4.14)$$

This represents a very small correction. [See Table 1.]

We now turn to Fig. 7(b) and consider the corrections due to the non-zero values of the matrix elements $A_\sigma = \langle N|(\bar{q}q)(\bar{q}q)|N\rangle$ and $C_\pi = \langle N|(\bar{q}i\gamma_5\bar{\tau}q)(\bar{q}i\gamma_\tau\tau q)|N\rangle$. Upon Fierz rearrangement, into the sigma channel, we have [20]

$$\frac{1}{2} \langle N | \left[\bar{q} \gamma^\tau \frac{\lambda^a}{2} q \right] \left[\bar{q} \gamma_\tau \frac{\lambda^a}{2} q \right] | N \rangle \rho_B \stackrel{F.R.}{=} -\frac{1}{9} A_\sigma \rho_B , \quad (4.15)$$

$$= -4.4 \times 10^{-6} \text{ GeV}^6 , \quad (4.16)$$

upon using $A_\sigma = 0.0317 \text{ GeV}^3$. This is a quite small correction and may also be neglected.

[See Tables 1 and 2.]

Finally, we perform Fierz rearrangement to obtain a term proportional to C_π . We find

$$\frac{1}{2} \langle N | \left[\bar{q} \gamma^\tau \frac{\lambda^a}{2} q \right] \left[\bar{q} \gamma_\tau \frac{\lambda^a}{2} q \right] | N \rangle \rho_B = -\frac{1}{9} C_\pi \rho_B , \quad (4.17)$$

$$= -1.5 \times 10^{-5} \text{ GeV}^6 , \quad (4.18)$$

where we have used $C_\pi = 0.108 \text{ GeV}^3$. Again, this correction is quite small and may be neglected.

Putting our various results together, we have, in the case of a pure axial-vector diquark condensate,

$$\bar{\Pi}_{(\rho)}^{(2)}(Q^2) = -\frac{4}{9} \left[\frac{\pi \alpha_S}{Q^6} \right] \left[C_{VD} - \frac{1}{9} A_\sigma - \frac{1}{9} C_\pi \right] \rho_B . \quad (4.19)$$

[See Tables 1 and 2.] For a pure scalar-diquark condensate, C_{VD} is to be replaced by $-C_{SD}$ in Eq. (4.19). If the nucleon is a mixture of axial-vector and scalar diquark components, these results have to be modified accordingly.

V. Spectral Densities and Sum Rules for the Vector-Isovector Polarization Tensor in Nuclear Matter

Here we follow Ref. [10] and consider the case of $\vec{q} = 0$. In this case the longitudinal polarization tensor is related to the transverse tensor, $\tilde{\Pi}_l(q^2, q \cdot u) = q_0^2 \tilde{\Pi}_t(q^2, q \cdot u)$, where $q^2 = (q^0)^2$. Also, we have

$$\Pi_l(q, \rho_B) = -\frac{1}{3(q^0)^2} \Pi_\mu^\mu(q, \rho_B) \quad , \quad (5.1)$$

when $q^2 = (q^0)^2$. We may also write

$$\Pi_l(q_0^2, \rho_B) = \int_0^\infty ds \frac{\rho(s, \rho_B)}{s - q_0^2} \quad , \quad (5.2)$$

and, with $q_0^2 = -Q^2$, we define

$$\Delta\Pi_l(Q^2) = \Pi_l(q^2, \rho_B) - \Pi_l(q^2, \rho_B = 0) \quad , \quad (5.3)$$

$$= \int_0^\infty ds \frac{[\rho(s, \rho_B) - \rho(s)]}{s + Q^2} \quad . \quad (5.4)$$

The next step is represented by the following large $-Q^2$ expansion,

$$\Delta\Pi_l(Q^2) = \int_0^\infty [\rho(s, \rho_B) - \rho(0)] \left[\frac{1}{Q^2} - \frac{s}{Q^4} + \frac{s^2}{Q^6} + \dots \right] \quad . \quad (5.5)$$

Since $\Delta\Pi_l(Q^2)$ starts off as $1/Q^4$, we have the relation

$$\int_0^\infty ds [\rho(s, \rho_B) - \rho(s)] = 0 \quad . \quad (5.6)$$

Now, in the factorization approximation, $\Delta\Pi_l(Q^2)$ defined in Eq. (5.2) is [10]

$$\begin{aligned}
\Delta\Pi_l(Q^2) = & \frac{\hat{m}_q}{2Q^4}, \langle N | \bar{u}u + \bar{d}d | N \rangle \rho_B + \frac{1}{24Q^4} \langle N | (\alpha_S/\pi) G_{\mu\nu} G^{\mu\nu} | N \rangle \rho_B \\
& + \frac{m_N}{4Q^4} A_2^{u+d} \rho_B - \frac{224}{81} \left[\frac{\pi\alpha_S}{Q^6} \right] \langle \bar{u}u \rangle_0 \langle N | \bar{u}u | N \rangle \rho_B \\
& - \frac{5m_N^3}{24Q^4} A_4^{u+d} \rho_B - \frac{112}{81} \left[\frac{\pi\alpha_S}{Q^6} \right] \langle \bar{u}u \rangle_0^2 \left[\frac{\rho_B}{\rho_{NM}} \right] .
\end{aligned} \tag{5.7}$$

In the pole-plus-continuum approximation, we have in the case of the vacuum,

$$\rho(s) = F\delta(s - m_\rho^2) + \frac{1}{8\pi^2} \left[1 + \frac{\alpha_S}{\pi} \right] \theta(s - s_0) , \tag{5.8}$$

where s_0 is the continuum threshold. Further, we note that $F = f_\rho^2$, where f_ρ is the pion-decay constant. We have $f_\rho \simeq (0.152 \pm 0.015)$ GeV, so that the value of F is determined to about 20 percent. In the study of nuclear matter one writes [10]

$$\rho(s, \rho_B) = \rho_{sc}\delta(s) + F^* \delta(s - m_\rho^{*2}) + \frac{1}{8\pi^2} \left[1 + \frac{\alpha_S}{\pi} \right] \theta(s - s_0^*) , \tag{5.9}$$

where the asterisk denotes the in-medium values. In our work we can drop the "scattering term" ρ_{sc} , since we are making an analysis for the zero-temperature case.

One finds, upon using Eq. (5.4), that [10]

$$F^* - F + \frac{1}{8\pi^2} \left[1 + \frac{\alpha_S}{\pi} \right] (s_0 - s_0^*) = 0 \tag{5.10}$$

$$\begin{aligned}
& F^* m_\rho^{*2} - F m_\rho^2 + \frac{1}{16\pi^2} \left[1 + \frac{\alpha_S}{\pi} \right] (s_0^2 - s_0^{*2}) \\
& = -\hat{m}_q \langle N | \bar{u}u | N \rangle \rho_B - \frac{1}{24} \langle N | \frac{\alpha_S}{\pi} G_{\mu\nu} G^{\mu\nu} | N \rangle \rho_B - \frac{m_N}{4} A_2^{u+d} \rho_B ,
\end{aligned} \tag{5.11}$$

and

$$\begin{aligned}
& F^* m_\rho^{*4} - F m_\rho^4 + \frac{1}{24\pi^2} \left[1 + \frac{\alpha_S}{\pi} \right] (s_0^3 - (s_0^*)^3) \\
& = -\frac{224}{81} \pi \alpha_S \langle \bar{u}u \rangle_0 \langle N | \bar{u}u | N \rangle \rho_B - \frac{5m_N^3}{24} A_4^{u+d} \rho_B.
\end{aligned} \tag{5.12}$$

From Ref. [10], we have $\sigma_N \simeq 45$ MeV, $\hat{m}_q = 5.5$ MeV, $\langle N | (\alpha_S/\pi) G_{\mu\nu} G^{\mu\nu} | N \rangle \simeq -650$ MeV, $A_2^{u+d} \simeq 0.938$ and $A_4^{u+d} \simeq 0.121$, $\langle \bar{u}u \rangle_0 = -(0.245 \text{ GeV})^3$, $m_\rho = 770$ MeV, and $\alpha_S = 0.3$. All the terms of Eqs. (5.10-5.12) were taken into account in Ref. [10]. We now consider our corrections to the mean-field result for the four-quark condensate. The correction is here written as a factor κ times the vacuum value of the four-quark condensate (see Table 1). We put ρ_{NM} equal to the density of nuclear matter, $\rho_{NM} = (0.108 \text{ GeV})^3$, and ρ_B is a variable baryon density. Therefore, including our new term, Eq. (5.12) is modified to read

$$\begin{aligned}
& F^* m_\rho^{*4} - F m_\rho^4 + \frac{1}{24\pi^2} \left[1 + \frac{\alpha_S}{\pi} \right] (s_0^3 - (s_0^*)^3) \\
& = -\frac{224}{81} \pi \alpha_S \langle \bar{u}u \rangle_0 \langle N | \bar{u}u | N \rangle \rho_B - \frac{112}{81} \pi \alpha_S \langle \bar{u}u \rangle_0^2 \left[\frac{\rho_B}{\rho_{NM}} \right] \kappa,
\end{aligned} \tag{5.13}$$

where we can vary κ from about 0.1 to 0.4. (See Tables 1 and 2.) We take $\kappa = 0.25$ as a reasonable physical value based upon the suggestion of an equal fraction of scalar and axial-vector diquark condensate in the ground state of nuclear matter.

In our analysis we took $F = (f_\rho^{eff})^2$ with $f_\rho^{eff} = 0.145$ GeV and $s_0 = 1.0$ GeV. (That value of f_ρ^{eff} takes into account the finite width of the resonance term in $\rho(s)$. The correction for the finite width of the resonance represents only a small modification with respect to the results obtained when the delta function of Eq. (5.8) is used.) If we use Eqs. (5.9)-(5.11), we

find that

$$\frac{m_\rho^*}{m_\rho} = 1 - 0.148 \left[\frac{\rho_B}{\rho_{NM}} \right] . \quad (5.14)$$

We also find $f_\rho^* = 0.0932$ GeV and $s_0^* = 0.655$ GeV². However, if we use Eq. (5.13) (with $\kappa = 0.25$) instead of Eq. (5.11), we obtain the effect of our correction to the factorization approximation. In that case, we find

$$\frac{m_\rho^*}{m_\rho} = 1 - 0.118 \left[\frac{\rho_B}{\rho_{NM}} \right] . \quad (5.15)$$

We also find $f_\rho^* = 0.0993$ GeV and $s_0^* = 0.744$ GeV².

Thus, we see that even though our correction to the four-quark condensate term is about 25 percent of the vacuum value, we only find a quite small change in m_ρ^*/m_ρ . (We ascribe this behavior to the nonlinear nature of Eqs. (5.9)-(5.11).) We conclude that, in the example studied here, corrections to the factorization approximation are quite small. (In previous work, the size of such corrections was unknown [9].)

VI. Discussion

Various of our results are summarized in Tables 1 and 2. In the evaluation of the four-quark condensate terms we have seen the importance of matrix elements of the form $\langle N | (\bar{q} t_c \gamma_5 C \tau_2 \bar{q}^T) (q^T C^{-1} t_c \tau_2 \gamma_5 q) | N \rangle$. Indeed, we can interpret this matrix element (when multiplied by ρ_B) as providing a measure of a "scalar-diquark condensate" somewhat analogous to the term $\langle N | \bar{q} q | N \rangle \rho_B$ that appears when forming $\langle \bar{q} q \rangle_{\rho_B}$. However, while $\langle \bar{q} q \rangle_0$ is nonzero, there is no corresponding diquark condensate in the vacuum.

In this work, corrections of about 20 percent were found if we consider the nucleon to have equal scalar and axial-vector diquark components. (Such a model of the nucleon is needed if one is to obtain reasonable values for the magnetic moments. For example, a pure scalar diquark model would significantly underestimate the nucleon magnetic moments, while a wave function that only contains axial-vector diquarks would overestimate the moments.) A model with equal scalar ($T = 0$) and axial-vector ($T = 1$) diquark components is also suggested by the SU(6) constituent-quark wave function of the nucleon.

As seen in Section V, the value of m_ρ^* is quite insensitive to modifications of the four-quark condensate term. Our result may lend some further confidence in the use of QCD sum rules to determine the properties of the rho meson in matter.

Appendix A

The polarization operator in matter takes a more complex form than that usually considered, since there is an additional function, other than $\Pi_{(\omega)}(q^2)$ of Eq. (2.5), to be specified. For example, let us use the vector u^μ , that describes the flow of nuclear matter, to define a vector $\hat{u}^\mu = u^\mu + (u \cdot q)q^\mu/q^2$. (Note that $q \cdot \hat{u} = 0$.) In matter, we may write [21]

$$\Pi_{(\omega)}^{\mu\nu}(q, u) = (q^\mu q^\nu - q^2 g^{\mu\nu}) \Pi_{(\omega)}(q^2, q \cdot u) + \frac{\hat{u}^\mu \hat{u}^\nu}{\hat{u}^2} \hat{\Pi}_{(\omega)}(q^2, q \cdot u) . \quad (\text{A1})$$

Thus, in the general case, there are two functions to determine. While the function $\hat{\Pi}_{(\omega)}(q^2, q \cdot u)$, defined in Eq. (A1), is not important for the work reported here, it is important if we study the excitation of nucleon particle-hole states in the calculation of $\Pi_{(\omega)}^{\mu\nu}$ [21]. That comment may be understood by noting that the propagator for nucleons in nuclear matter may be written as [21]

$$G(p, u) = G_0(p) + G_m(p, u) , \quad (\text{A2})$$

where $G_0(p) = i(\not{p} - m_N + i\epsilon)^{-1}$ and

$$G_m(p, u) = -2\pi(\not{p} + m_N)\delta(p^2 - m_N^2)\theta(p^0)\theta(E_F - p \cdot u) . \quad (\text{A3})$$

Here m_N is the nucleon mass and E_F is the Fermi energy. Note that in the frame where $u^\mu = [1, 0, 0, 0]$, the last theta function in Eq. (A3) is $\theta(E_F - p^0)$. The dependence of $G_m(p, u)$ on u leads to nonzero values the second term in Eq. (A1) [21].

Appendix B

In this appendix we wish to describe how the matrix element

$$C_{VD} = \left\langle N \left| \left[\bar{q}(it_c)(\bar{\tau}\tau_2)\gamma^\mu C\bar{q}^T \right] \cdot \left[q^T C^{-1}\gamma_\mu(\tau_2\bar{\tau})(it_c)q \right] \right| N \right\rangle \quad (\text{B1})$$

is calculated. (The calculation of C_{SD} , defined after Eq. (3.10), was described in an earlier work [20].) With reference to Fig. 8(d), we have

$$C_{VD} = -4g_{Dqq}^2 \frac{1}{2} \text{Tr} \int \frac{d\bar{k}}{(2\pi)^3} \Gamma_\beta \Lambda^{(+)}(\bar{P}) \frac{J_{(\rho)}^{\beta\mu}(P-Q)}{i} \frac{J_{(\rho)\mu\alpha}(P-Q)}{i} \\ \times \frac{m_q}{E(\bar{Q})} \Lambda^{(+)}(\bar{Q}) \Gamma^\alpha \Lambda^{(+)}(\bar{P}) \frac{1}{\left[(P-Q)^2 - m_{VD}^2 \right]^2} \quad (\text{B2})$$

where the factor of 4 is a statistical factor. Here, m_{VD} is the mass of the axial-vector diquark, Γ_β and Γ_α are vertex functions for the nucleon to go into a quark and an axial-vector ($T = 1$) diquark and g_{Dqq} is the coupling constant between the diquark and the constituent quarks of mass m_q . Isospin factors are here absorbed in the vertex functions, Γ_β and Γ_α . The tensor $J_{(\rho)}^{\beta\mu}(P-Q)$ is a basic quark-loop integral of the NJL model:

$$J_{(\rho)}^{\beta\mu}(P-Q) = n_c n_f i \text{Tr} \int \frac{d^4 k}{(2\pi)^4} S(P-Q+k) \gamma^\beta S(k) \gamma^\mu \quad (\text{B3})$$

The tensor defined in Eq. (B3) may be written as

$$J_{(\rho)}^{\beta\mu}(P-Q) = J_{(\rho)}((P-Q)^2) \bar{g}^{\beta\mu}(P-Q) \quad , \quad (\text{B4})$$

where

$$\bar{g}^{\beta\mu}(k) = g^{\beta\mu} - \frac{k^\beta k^\mu}{k^2} \quad . \quad (\text{B5})$$

Finally, $\Lambda^{(+)}(\bar{P})$ and $\Lambda^{(+)}(\bar{Q})$ are positive-energy projection operators. For example

$$\Lambda^{(+)}(\bar{Q}) = \frac{Q + m_q}{2m_q} \quad (\text{B6})$$

with $Q^0 = E_q(\bar{Q}) = [\bar{Q}^2 + m_q^2]^{1/2}$.

We find

$$C_{VD} = 4g_{Dqq}^2 \frac{1}{2} \text{Tr} \int \frac{d^3 Q}{(2\pi)^3} \Lambda^{(+)}(\bar{P}) \Gamma_\alpha \frac{m_q}{E_q(Q)} \times \Lambda^{(+)}(\bar{Q}) \Gamma^\alpha \Lambda^{(+)}(\bar{P}) J_{(\rho)}^2((P - Q)^2) \frac{1}{[(P - Q)^2 - m_{VD}^2]^2} . \quad (\text{B7})$$

It is useful to introduce a wave function $\psi^{(+)}(\bar{Q})$ that parameterizes the vertex. We write

$$\Gamma^\alpha \Lambda^{(+)}(\bar{P}) = \gamma_5 \frac{\hat{\gamma}^\alpha(P, Q)}{\sqrt{2}} \Lambda^{(+)}(\bar{Q}) \Lambda^{(+)}(\bar{P}) \psi^{(+)}(\bar{Q}) \left[\frac{2E_q(\bar{Q})}{E_q(\bar{Q}) + m_q} \right] [(P - Q)^2 - m_{VD}^2] . \quad (\text{B8})$$

Here

$$\hat{\gamma}^\alpha(P, Q) = \tilde{\gamma}^\alpha(P) - \frac{Q(P) \hat{Q}^\alpha(P)}{\hat{Q}^2(P)} , \quad (\text{B9})$$

where

$$\tilde{\gamma}^\alpha(P) \equiv \gamma^\alpha - \frac{P P^\alpha}{P^2} , \quad (\text{B10})$$

and

$$\hat{Q}^\alpha(P) \equiv Q^\alpha - \frac{(Q \cdot P) P^\alpha}{P^2} . \quad (\text{B11})$$

(Because of the form $\hat{\gamma}^\alpha(P, Q)$, we can use $g^{\beta\mu}$ rather than $\tilde{g}^{\beta\mu}$ and obtain the same results.)

In a study of a quark-diquark model of nucleon structure, we found that the most important part of the quark-diquark vertex could be written in the form given in Eq. (B8) [22].

Thus,

$$C_{VD} = 4g_{Dqq}^2 \frac{1}{2} \text{Tr} \int \frac{d^3Q}{(2\pi)^3} \frac{m_q}{E_q(\bar{Q})} \left[\frac{2E_q(\bar{Q})}{E_q(\bar{Q}) + m_q} \right] J_{(\omega)}^2 \langle (P - Q)^2 \rangle [\psi^{(+)}(\bar{Q})]^2 \quad (\text{B12})$$

$$\times \left[\Lambda^{(+)}(\bar{P}) \Lambda^{(+)}(\bar{Q}) \frac{\hat{\gamma}_\alpha(P, Q)}{\sqrt{2}} \gamma_5 \Lambda^{(+)}(\bar{Q}) \gamma_5 \hat{\gamma}^\alpha(P, Q) \Lambda^{(+)}(\bar{Q}) \Lambda^{(+)}(\bar{P}) \right] ,$$

$$= 4g_{Dqq}^2 \int \frac{d^3Q}{(2\pi)^3} \left[\frac{2m_q}{E_q(\bar{Q}) + m_q} \right] J_{(\omega)}^2 \langle (P - Q)^2 \rangle [\psi^{(+)}(\bar{Q})]^2 \frac{1}{2} \text{Tr} [\Lambda^{(+)}(\bar{P}) \Lambda^{(+)}(\bar{Q}) \Lambda^{(+)}(\bar{P})] , \quad (\text{B13})$$

$$= 4g_{Dqq}^2 \int \frac{d^3Q}{(2\pi)^3} J_{(\omega)}^2 \langle (P - Q)^2 \rangle [\psi^{(+)}(\bar{Q})]^2 . \quad (\text{B14})$$

We not put $g_{Dqq} = 3.3$ and $m_q = 0.30$ GeV. For small q^2 we note that $J_{(\omega)}(q^2) \simeq [0.083 + q^2 / g_{\rho qq}^2] \text{ GeV}^2$. Here the rho-quark coupling constant is $g_{\rho qq} = 3.8$ and q^2 is in units of GeV^2 . Finally, we find that $C_{VD} = 0.38 \text{ GeV}^3$, when we insert a wave function obtained in our study of a quark-diquark model of the nucleon [22]. For completeness, we note that the wave function satisfies the normalization condition

$$2 \int \frac{d^3Q}{(2\pi)^3} [m_N - E_q(\bar{Q})] [\psi^{(+)}(\bar{Q})]^2 = 1 . \quad (\text{B15})$$

[See Fig. 9.]

Acknowledgement

This work was supported in part by a grant from the National Science Foundation and by the PSC-CUNY Faculty Research Award program of the City University of New York.

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Table 1 Calculation of $\Pi_{(\rho)}(Q^2)$: Four-quark condensates in vacuum and in nuclear matter

Vacuum condensate [Eq.(2.8)]	$-\frac{112}{81} \frac{\pi\alpha_S}{Q^6} \langle \bar{u}u \rangle_0^2$	$-3.37 \times 10^{-4} \left[\frac{\pi\alpha_S}{Q^6} \right] \text{ GeV}^6$
Mean-field correction in matter [Eq.(2.11)]	$-\frac{112}{81} \frac{\pi\alpha_S}{Q^6} \left(2 \langle N \bar{u}u N \rangle \langle \bar{u}u \rangle_{0\rho_B} \right)$	$2.45 \times 10^{-4} \left[\frac{\pi\alpha_S}{Q^6} \right] \text{ GeV}^6$
Calculation of $\Pi_{(\rho)}^{(1)}(Q^2)$ [Section III]		
a) Sigma-meson condensate [Eq.(3.7)]	$-\frac{4}{9} \left[\frac{\pi\alpha_S}{Q^6} \right] A_\sigma \rho_B$	$-0.177 \times 10^{-4} \left[\frac{\pi\alpha_S}{Q^6} \right] \text{ GeV}^6$
b) Pion condensate [Eq.(3.9)]	$\frac{4}{27} \left[\frac{\pi\alpha_S}{Q^6} \right] C_\pi \rho_B$	$0.202 \times 10^{-4} \left[\frac{\pi\alpha_S}{Q^6} \right] \text{ GeV}^6$
c) Scalar-diquark condensate [Eq.(3.11)]	$-\frac{2}{9} \left[\frac{\pi\alpha_S}{Q^6} \right] C_{SD} \rho_B$	$-1.85 \times 10^{-4} \left[\frac{\pi\alpha_S}{Q^6} \right] \text{ GeV}^6$
d) Axial-vector diquark condensate [Eq.(3.18)]	$\frac{1}{27} \left[\frac{\pi\alpha_S}{Q^6} \right] C_{VD} \rho_B$	$0.177 \times 10^{-4} \left[\frac{\pi\alpha_S}{Q^6} \right] \text{ GeV}^6$
Calculation of $\Pi_{(\rho)}^{(2)}(Q^2)$ [Section IV]		
a) Sigma-meson condensate [Eq.(4.15)]	$\frac{4}{81} \left[\frac{\pi\alpha_S}{Q^6} \right] A_\sigma \rho_B$	$0.197 \times 10^{-6} \left[\frac{\pi\alpha_S}{Q^6} \right] \text{ GeV}^6$
b) Pion condensate [Eq.(4.17)]	$\frac{4}{81} \left[\frac{\pi\alpha_S}{Q^6} \right] C_\pi \rho_B$	$6.72 \times 10^{-6} \left[\frac{\pi\alpha_S}{Q^6} \right] \text{ GeV}^6$
c) Scalar-diquark condensate [Eq.(4.10)]	$\frac{4}{9} \left[\frac{\pi\alpha_S}{Q^6} \right] \frac{C_{SD}}{18} \rho_B$	$2.05 \times 10^{-5} \left[\frac{\pi\alpha_S}{Q^6} \right] \text{ GeV}^6$
d) Axial-vector diquark condensate [Eq.(4.13)]	$-\frac{4}{9} \left[\frac{\pi\alpha_S}{Q^6} \right] \frac{C_{VD}}{36} \rho_B$	$-1.556 \times 10^{-6} \left[\frac{\pi\alpha_S}{Q^6} \right] \text{ GeV}^6$

Table 1. (Continued)

<p>Vacuum value of</p> $\Pi_{(\phi)}^{(1)}(Q^2) = -\frac{16}{9} \left[\frac{\pi\alpha_S}{Q^6} \right] \langle \bar{u}u \rangle_0^2$	$-4.34 \times 10^{-4} \left[\frac{\pi\alpha_S}{Q^6} \right] \text{ GeV}^6$
<p>Vacuum value of</p> $\Pi_{(\phi)}^{(2)}(Q^2) = -\frac{32}{81} \left[\frac{\pi\alpha_S}{Q^6} \right] \langle \bar{u}u \rangle_0^2$	$0.964 \times 10^{-4} \left[\frac{\pi\alpha_S}{Q^6} \right] \text{ GeV}^6$

Table 2 Nucleon Matrix Elements: C_{SD} is the value for a quark plus scalar diquark model of the nucleon [20] and C_{VD} is the value for a quark and axial-vector diquark model of the nucleon. Calculations of A_σ , C_π and C_{SD} were presented in Ref. [20].

$A_\sigma = \langle N (\bar{q}q)(\bar{q}q) N \rangle$	0.0317 GeV^3
$C_\pi = \langle N (\bar{q}i\gamma_5 \bar{\tau}q) \cdot (\bar{q}i\gamma_5 \bar{\tau}q) N \rangle$	0.108 GeV^3
$C_{SD} = \langle N (\bar{q}t_c\tau_2\gamma_5 C\bar{q}^T)(q^T C^{-1}\gamma_5\tau_2 t_c q) N \rangle$	0.660 GeV^3
$C_{VD} = \langle N (\bar{q}it_c(\bar{\tau}\tau_2)\gamma^\mu C\bar{q}^T) \cdot (q^T C^{-1}\gamma_\mu(\tau_2 \bar{\tau})it_c q) N \rangle$	0.380 GeV^3

Figure Captions

- Fig. 1.
- (a) Two diagrams that arise in the calculation of $\Pi_{(\rho)}(q^2)$ in vacuum. The pairs of filled circles denote the vacuum quark condensate. The current which carries a momentum q is indicated by a dashed line. The thick quark line indicates a quark of high momentum.
 - (b) A diagram that arises when calculating $\Pi_{(\rho)}(q^2)$ in matter. Here N denotes a nucleon containing three valence quarks. The diagram exhibits an exchange process between quarks in the nucleon and those excited from the vacuum by the current. Evaluation of this process requires the nucleon density matrix for valence quarks. [See Section II.] Again, the thick line denotes the quark that carries the large momentum.
 - (c) Here the large filled circle denotes the quark-quark interaction of the NJL model and provides a factor of iG_s when evaluating the diagram. [See the text.]
 - (d) This diagram gives rise to two factors of iG_s .
 - (e) The operator product expansion serves to separate the short-distance and long-distance parts of the calculation. Here we expand the condensate about $x_\mu = 0$ and keep only the leading term. (See Fig. 1(c) above.)
 - (f) See caption (e) and Fig. 1(d).

Fig. 2. Various insertions into the light lines representing low-momentum quarks are shown for the NJL model. They give rise to the constituent quark mass, m_q , of that model. [See Section II.]

- Fig. 3. (a) Diagrammatic representation of a contribution to $\Pi_{(\omega)}^{(1)}(q^2)$ due to the four-quark condensate in the vacuum. The wavy line denotes a gluon and the heavy lines denote quarks that carry the large momentum, q .
- (b) Similar caption to (a) above.
- (c) A contribution to $\Pi_{(\omega)}^{(1)}(q^2)$ that is linear in the density ρ_B . Here the triple line denotes a nucleon of nuclear matter and the double line denotes a meson of the nucleon's meson "cloud". (Compare to Fig. 3(a).)
- (d) A contribution to $\Pi_{(\omega)}^{(1)}(q^2)$ due to diquark components (cross-hatched line) of the nucleon. Here the labelling of the points is taken to be the same as in (a).

- Fig. 4. (a) A diagram that contributes to $\Pi_{(\omega)}^{(1)}(q^2)$ in the vacuum [17]. Here the small filled circles denote the vacuum condensate and the wavy line is a gluon. This diagram yields a term proportional to $\langle \bar{q}q \rangle_0^2$ in $\Pi_{(\omega)}^{(1)}(q^2)$. [See Eq. (3.5.)]
- (b)-(c) The origin of the factorization approximation may be understood from these diagrams. In these diagrams the double line denotes the sigma-like field whose source is the nucleons of nuclear matter. Both (b) and (c) give equal contributions and they give rise to the term $2\langle \bar{q}q \rangle_0 \langle N | \bar{q}q | N \rangle \rho_B$ in Eq. (3.5).

Fig. 5. (a) Here Fig. 3(d) has been drawn so as to clarify the nature of the diquark contribution to $\Pi_{(\rho)}^{(1)}(q^2)$. Again the heavy lines denote quarks carrying the large momentum q .

(b) This diagram is similar to that shown in (a).

Fig. 6. Two diagrams evaluated in Ref. [17] in terms of the mixed condensate $\langle \bar{q} \sigma_{\mu\nu} G^{\mu\nu} q \rangle_0$. (Here the gluon carries zero momentum.) These diagrams give zero contribution to $\Pi_{(\rho)}(q^2)$.

Fig. 7. (a) A diagram that contributes to the value of $\Pi_{(\rho)}^{(2)}(q^2)$ in vacuum [17]. Here the gluon carries zero momentum.

(b) A correction to the diagram of (a) in the presence of matter. Here mesons of the nucleon's meson cloud are taken in account. [See Section IV.]

(c) A density-dependent correction to the diagram shown in (a) that has its origin in a quark-diquark model of the nucleon. Here the cross-hatched lines denote diquarks.

Fig. 8. (a) Schematic representation of a four-quark operator whose matrix element is taken between nucleon states. (See (3.10) and (3.15), for example.)

(b) The vertex for a diquark (cross-hatched double line) to go into two constituent quarks is shown.

(c) The vertex for a nucleon (heavy line) to go into a diquark and a quark is shown.

(d) Evaluation of the matrix element $C_{VD} = \langle N | \left(\bar{q} i t_c (\bar{\tau} \tau_2) \gamma^\mu C \bar{q}^T \right) \left(q^T C^{-1} \gamma_\mu (\tau_2 \bar{\tau}) i t_c q \right) | N \rangle$, defined in

Eq. (3.17), is indicated in a schematic fashion. We have only calculated the direct term shown in the figure. That term has a large statistical factor. [See Appendix B for a detailed discussion.]

Fig. 9. The wave function, $\psi^{(+)}(\bar{Q})$, of Eq. (B8) is shown. This wave function describes the relative motion of a quark and an axial-vector ($T = 1$) diquark [22]. The normalization condition for this wave function is given in Eq. (B15).

FIG. 1

FIG. 2

FIG. 3

(a)

(b)

(c)

FIG. 4

FIG. 5

(a)

(b)

FIG. 6

FIG. 7

(a)

(b)

(c)

(d)

Fig. 8

FIG. 9