A consistent three-flavour approach to possible evidence of neutrino oscillations

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Abstract

We thoroughly analyse the interplay of the two possible indications of neutrino masses and mixing — the solar ν -deficit and the atmospheric ν -anomaly — and their compatibility with the negative results of oscillation searches at accelerators and reactors. First, world neutrino data — with the exception of upward-going muons and multi-GeV atmospheric ν 's — are analysed in a three-flavour oscillation framework, characterized by an extremized form of the natural neutrino mass hierarchy. A set of solutions is found in the neutrino mass/mixing parameter space. Then these solutions are shown to be stable by means of a perturbative-like approach, leading to a definitely reasonable and consistent hierarchical scenario. Solar and Earth matter effects on neutrino propagation are accurately taken into account in both steps of the analysis. Finally, the recent Kamiokande results on the atmospheric ν_μ/ν_e ratio in the multi-GeV energy range are compared with our fits of non-solar ν data. The impressive agreement reinforces our oscillation scenario.

1 Introduction

The discovery of neutrino flavour oscillations represents one of the greatest challenges of elementary particle physics. Its implications in terms of neutrino masses and mixings would be of paramount importance for our understanding of physics beyond the Standard Model of electroweak interactions.

A variety of neutrino beams have been used to test the oscillation hypothesis. According to the location of the beam source, the relevant experiments carried out so far can be broadly divided into two classes:

1. Solar neutrino experiments;
2. Terrestrial neutrino experiments, including here both those detecting atmospheric neutrinos and those using laboratory-produced neutrinos (accelerator and reactor experiments).

In spite of the negative indications coming from direct searches at accelerators and reactors, the well-known solar neutrino deficit and the atmospheric neutrino anomaly can be regarded as possible evidences of neutrino oscillations. In the absence of conclusive confirmations, the experimental results are generally interpreted in the simplest (two-flavour) oscillation scenario, characterized by one squared mass difference δm^2 and one mixing angle θ . However, it is well known that solar and terrestrial neutrinos give rather different information on δm^2 : $\delta m_{\nu}^2(\text{solar}) < \delta m_{\nu}^2(\text{terrestrial})$. In particular, the sensitivity ranges are approximately: $\delta m_{\nu}^2(\text{solar}) \lesssim 10^{-4} \text{ eV}^2$, $\delta m_{\nu}^2(\text{atmos.}) \gtrsim 10^{-4} \text{ eV}^2$, $\delta m_{\nu}^2(\text{reac.}) \gtrsim 10^{-2} \text{ eV}^2$, and $\delta m_{\nu}^2(\text{accel.}) \gtrsim 10^{-1} \text{ eV}^2$. It follows that one squared mass difference is not sufficient to describe all the data. Moreover, one cannot describe the mixing in the different oscillation channels with one and the same angle θ .

We are thus naturally led towards a three-generation scenario, in which solar and terrestrial neutrino oscillations are respectively related to “light” and “heavy” neutrino mass squared difference. We will show in the following that a three-flavour oscillation scenario compatible with world neutrino data does exist. A family of solutions will be determined, characterized by the “natural” mass hierarchy: $m_1 < m_2 < m_3$.

We have already discussed this set of solutions in a previous paper [1]. In this work, we will not only update the results of [1] in the light of the most recent experimental data, but we will also drop some theoretical assumptions and approximations used there to simplify the calculations. The resulting analysis is considerably more complicated; however its self-consistency can now be explicitly proved.

Using the same notation as in [1], we assume a real 3×3 neutrino mixing matrix, parametrized according to the same prescription as for quarks in the *Review of Particle Properties* [2], with vacuum mixing angles $\theta_{12} = \omega$, $\theta_{13} = \phi$, and $\theta_{23} = \psi$. The mass m_1 is conventionally set to zero, so that the total parameter space is five-dimensional: $m_2, m_3, \omega, \psi, \phi$. In the assumed hierarchy $m_3 > m_2 > m_1 = 0$, solar neutrinos will be sensitive mainly to the light mass m_2 and secondarily to the average effect of the short-wavelength m_3 -driven oscillations. Conversely, terrestrial neutrinos will probe the heavy mass m_3 , the long-wavelength m_2 -driven oscillation being almost “frozen” at the length scale of the Earth for atmospheric neutrinos and of the laboratory for accelerator/reactor neutrino beams.

The plan of the paper is as follows. After a review of the experimental data in Section 2, we present in Section 3 our theoretical approach to solar and terrestrial neutrino oscillations. A global analysis of all data is then performed in a two-step perturbative-like approach. In the first step, covered in Section 4, an extremized form of mass hierarchy is used ($m_2/m_3 \rightarrow 0$), in order to find a preliminary set of approximate confidence regions in the space $(m_2, m_3, \omega, \psi, \phi)$. In the second step, described in Section 5, both m_2 and m_3 are taken as finite and an improved set of solutions is found. Slight differences with respect to the first step will be found: their smallness will demonstrate the stability of our approach. Finally, in Section 6 the mass-mixing regions allowed by the terrestrial neutrino data are compared with the very interesting recent results from Kamiokande, concerning a possible evidence for oscillations of multi-GeV atmospheric neutrinos. Section 7 summarizes our results and collects our conclusions.

2 Survey of experimental data

2.1 Solar neutrinos

Concerning solar neutrino experiments, the recent results from the chlorine (Cl) detector at the Homestake mine [3], the Kamiokande (Kam) water-Cherenkov detector [4], and the gallium (Ga) detectors GALLEX [5] and SAGE [6], all confirm the long-standing deficit of the measured neutrino rates (see Table 1), when compared to the corresponding theoretical expectations of standard solar models. Here, in order to estimate the theoretical rates and their uncertainties we use the detailed solar model of Bahcall and Pinsonneault [7], according to the analytical prescriptions reported in Ref. [8].

A further experimental information can be added, i.e. the asymmetry of the night (N) and day (D) solar neutrino rates as measured at Kamiokande [9], at present compatible with zero: $\frac{N-D}{N+D} = 0.07 \pm 0.08$.

Table 1: Measured solar neutrino rates and 1σ total experimental errors for the chlorine (Cl), water-Cherenkov (Kam) and the combined gallium (Ga) experiments. The Kamiokande rate is normalized to the Bahcall and Pinsonneault standard solar model prediction (BPSSM) [7].

Experiment	Ref.	Rate	Error	Units
Homestake (Cl)	[3]	2.55	0.25	SNU
Kamiokande (Kam)	[4]	0.51	0.07	Kam/BPSSM
GALLEX+SAGE (Ga)	[5, 6]	74.3	8.5	SNU

A night-day asymmetry value significantly different from zero would represent unmistakable evidence of Earth matter effects on neutrino propagation, and thus of the Mikheyev–Smirnov–Wolfenstein (MSW) mechanism of matter-enhanced neutrino oscillations [10]. The MSW effect is at present the most economical and robust solution to the solar neutrino deficit, and we will adopt it later in the interpretation of solar ν data.

2.2 Atmospheric neutrinos

As far as atmospheric neutrinos are concerned, the current water-Cherenkov experiments IMB [11] and Kamiokande [12] report an experimental ratio $R_{\mu/e}$ of muon-to-electron neutrino-induced contained events of about half the expected theoretical rate $R_{\mu/e}^0$. This anomaly in the flavour composition of the detected events may be interpreted again as a signal of neutrino oscillations. The statistical significance of the anomaly is not spoiled by the opposite results of the two (concluded) lower-statistics iron-detector experiments Fréjus [13] and NUSEX [14]. The experimental situation is shown in Table 2.

Table 2: Measured-to-predicted μ/e double ratio $R_{\mu/e}/R_{\mu/e}^0$ and 1σ total experimental error of the four atmospheric neutrino experiments considered in our analysis. For Fréjus and NUSEX, the slight difference of the double ratio with respect to the published value is due to our cut of 2.2 GeV in energy (see Subsection 4.1). Moreover, the NUSEX error has been symmetrized. Other values and cuts are as published.

Experiment	Ref.	Exposure (kton·yr)	$R_{\mu/e}/R_{\mu/e}^0$ $\pm 1\sigma$	Cuts in ν_e events (GeV)	Cuts in ν_μ events (GeV)
IMB	[11]	7.70	0.55 ± 0.10	$0.1 \leq p_e \leq 1.5$	$0.3 \leq p_\mu \leq 1.5$
Kamiokande	[12]	7.70	0.60 ± 0.07	$0.1 \leq p_e \leq 1.33$	$0.2 \leq p_\mu \leq 1.5$
Fréjus	[13]	1.56	1.02 ± 0.23	$0.2 \leq E_{\nu_e} \leq 2.2$	$0.2 \leq E_{\nu_\mu} \leq 2.2$
NUSEX	[14]	0.74	1.2 ± 0.4	$0.2 \leq E_e \leq 2.2$	$0.2 \leq E_\mu \leq 2.2$

Here and in the following the Bartol neutrino and antineutrino fluxes [15] are used to calculate the theoretical rates at each detector location. A further 5% flux theoretical uncertainty should be added as a common systematic error on $R_{\mu/e}/R_{\mu/e}^0$. We do not include in Table 2 the Soudan 2 data [16], which are still preliminary and not sufficiently documented. We also exclude from our analysis the so-called upward-going muon data, whose theoretical interpretation is at present affected by the very large ($\sim 30\%$) uncertainties on the *absolute* ν -fluxes [17].

Recently, the Kamiokande collaboration has reported new and interesting data on events with visible energy above the cuts of Table 2 (the so-called multi-GeV data) [12]. The double ratio for this sample is estimated to be $R_{\mu/e}/R_{\mu/e}^0 = 0.57 \pm 0.10$, thus reinforcing the evidence of an anomaly in the flavour distribution of atmospheric ν events. However, an accurate treatment of these data implies a much more detailed simulation of the geometry and of the response function of the detector than required for low-energy data¹. We plan to perform a fit including multi-GeV events in a future work. In Subsection 4.2 we shall limit ourselves to an approximate and separate analysis of these data, in order to compare later their implications on neutrino masses and mixings with the bounds coming from all the other non-solar ν data.

2.3 Accelerator and reactor neutrinos

We conclude the survey of neutrino data with the results from accelerator and reactor experiments. At present they do not show any indication in favour of neutrino oscillations. The most stringent bounds in the $(\nu_e \rightarrow \nu_x)$ disappearance channel are provided by the reactor experiments at Gösgen [18] and Krasnoyarsk [19]. The accelerator experiments E531 [20] and E776 [21] at FNAL give the strongest limits to the two flavour transition probabilities $P(\nu_\mu \rightarrow \nu_\tau)$ and $P(\nu_\mu \rightarrow \nu_e)$, respectively. Finally, the disappearance channel $(\nu_\mu \rightarrow \nu_x)$ has been probed by the CDHSW experiment [22] at CERN.

3 Solar neutrino analysis

As already stated, we analyze solar ν data in the light of the MSW mechanism. In the simplest 2ν scenario, the basic quantity to compute is the survival probability $P_{2\nu}(\nu_e \rightarrow \nu_e)$, which depends only on m_2 and ω .

¹This is necessary, for instance, in order to link the incoming neutrino energy to the visible energy of partially contained events.

In a 3ν scenario with $m_3 \rightarrow \infty$, this probability is replaced by $P_{3\nu}$, which, however, is strictly related to $P_{2\nu}$:

$$P_{3\nu}(\nu_e \rightarrow \nu_e) = c_\phi^4 P_{2\nu}(\nu_e \rightarrow \nu_e) + s_\phi^4 \quad . \quad (1)$$

In this work we will also take into account the *leading* correction induced by a third “heavy” generation, which implies the following expression [23] for $P_{3\nu}$:

$$P_{3\nu}(\nu_e \rightarrow \nu_e) = c_\phi^2 c_{\phi_m}^2 P_{2\nu}(\nu_e \rightarrow \nu_e) + s_\phi^2 s_{\phi_m}^2 \quad , \quad (2)$$

where ϕ_m , which represents the ϕ mixing angle in matter at the production point in the solar core, is given by

$$s_{2\phi_m}^2 = \frac{s_{2\phi}^2}{(c_{2\phi} + 2\sqrt{2}G_F N_e E_\nu / m_3^2)^2 + s_{2\phi}^2} \quad . \quad (3)$$

Notice that the survival probability for the electron flavour now depends on m_2 , m_3 , ω and ϕ , but is always independent of the mixing angle ψ [24]. Concerning the “non-oscillatory” ingredients of our computation of the solar neutrino rates (neutrino fluxes, distributions, capture cross sections, etc.) we refer the reader to [1].

4 Terrestrial neutrino analysis

4.1 Atmospheric neutrinos

In Ref. [1] the atmospheric neutrino phenomenology was analysed in considerable detail, and for the first time an accurate global fit of the IMB, Kamiokande and Fréjus data was performed. However, we made use of two simplificative assumptions: 1) Oscillations were considered to occur only in vacuum, implying a restriction of the neutrino event sample to zenith angles above the horizon; 2) Only the effects of the heaviest mass m_3 were taken into account, m_2 being set to zero.

Both approximations can be relaxed by using a more refined approach to atmospheric neutrino oscillations. Technically, we are now able to propagate numerically the 3-component (anti)neutrino amplitudes from the production point in the atmosphere to the detector, for any given choice of m_2 , m_3 , ω , ψ , ϕ .

Earth density effects are taken into account along trajectories below the horizon. To speed up calculations we assume (as for solar ν ’s) a simplified step-like profile for the Earth electron density N_e (five shells are considered along the Earth radius), fitting the seismological data of Ref. [25]. By using constant densities, the equations of neutrino

propagation in the Earth can be solved after diagonalization of the (constant) mass matrices in each shell (with flavour-conserving matching conditions of the amplitudes at the interface), thus avoiding lengthy Runge-Kutta methods². The Earth effect is expected to play a rôle in the range $10^{-4} \lesssim m_3^2 \lesssim 10^{-3} \text{ eV}^2$, i.e. for m_3^2 of the order of the matter-induced squared mass term $A = 2\sqrt{2}G_F N_e E_\nu$.

The squared final amplitudes are then integrated, together with the detector cross-section and efficiency functions, over the zenith angle, the neutrino energy and the momentum of the final-state lepton (see also [1] for details).

As a consequence, not only can we correctly analyse the full 4π data sample of the IMB, Kamiokande and Fréjus experiments, but also include the NUSEX data, in which upwards and downwards neutrino-induced events are folded. We cut only a few high-energy iron-detector events [those having E_ν (E_ℓ) $> 2.2 \text{ GeV}$ for Fréjus (NUSEX)], thus avoiding to consider the onset of deep inelastic scattering of neutrinos off the target nuclei. The modest loss in statistics is responsible for the slight difference between the published iron-detector data and those reported in Table 2.

An important check of our modellization of the various detectors is the comparison of the published Monte Carlo simulations [13, 14, 26] with our results in the no-oscillation case. In particular, in Fig. 1 we compare our calculations of the spectral distributions of R_μ and R_e (separately) with the published estimates for each detector. The agreement is impressive, taking also into account that the comparison is absolute, i.e. no normalization of the curves has been used.

4.2 Approximate analysis of Kamiokande multi-GeV data

The anomaly in the flavour distribution of multi-GeV Kamiokande ν data, as the corresponding anomaly at lower energies, is suggestive of neutrino flavour oscillations. Within this hypothesis, the Kamiokande collaboration itself presents an analysis [12], considering two possible oscillation channels: $\nu_e \leftrightarrow \nu_\mu$ and $\nu_\mu \leftrightarrow \nu_\tau$. The first is not relevant to our hierarchical scenario; the second corresponds to assume, in our notation, both m_2 and ϕ equal to zero, the results being presented in the plane $(s_{2\psi}^2, m_3^2)$. In particular, the Kamiokande collaboration finds that $2 \times 10^{-3} \lesssim m_3^2 \lesssim 9 \times 10^{-2} \text{ eV}^2$ and $s_{2\psi}^2 \gtrsim 0.6$.

In order to estimate *precisely* the corresponding bounds when m_2 and/or ϕ are different from zero, it would be unavoidable to reanalyse the published data by means of a very detailed simulation of the Kamiokande detector response to partially contained events. However, at least in the case $m_2 = 0$ and $s_\phi^2 > 0$ (but not too large), we will *approximately*

²The final integration over neutrino energies and trajectories smears out differences with respect to the true density profile case, as we have explicitly checked.

draw ϕ -dependent bounds by developing the following simplifications, first proposed in Ref. [27].

Earth matter effects are neglected, so that the transition probabilities are the same for ν 's and $\bar{\nu}$'s. Dropping antineutrino labels for the sake of brevity, it is understood that for each flavour α ($\alpha = e, \mu$) one should add ν_α and $\bar{\nu}_\alpha$ events. Let us define:

$$\begin{cases} N_{\alpha\beta} &= \int \phi_\alpha \sigma_\beta \epsilon_\beta \\ \overline{P}_{\alpha\beta} &= N_{\alpha\beta}^{-1} \int \phi_\alpha P_{\alpha\beta} \sigma_\beta \epsilon_\beta \end{cases} \quad (\alpha, \beta = e, \mu) \quad , \quad (4)$$

where ϕ_α and σ_α are the (differential) neutrino fluxes and interaction cross sections, ϵ_α the detector efficiencies, and $P_{\alpha\beta}$ the flavour transition probabilities for $\nu_\alpha \rightarrow \nu_\beta$. An integration over the initial neutrino energy, the final lepton momentum and the zenith angle is understood.

In the multi-GeV ν energy range, e and μ neutrino fluxes have similar shapes, apart from an overall scale factor: $\phi_e \propto \phi_\mu$. Moreover, $\sigma_e \simeq \sigma_\mu$ and $\epsilon_e \simeq \epsilon_\mu$ [12]. These approximations imply

$$\frac{N_{\mu e}}{N_{ee}} \simeq \frac{N_{\mu\mu}}{N_{e\mu}} \equiv r \quad , \quad (5)$$

so that the double μ/e ratio can be expressed as [27]

$$\frac{R_{\mu/e}}{R_{\mu/e}^0} \simeq \frac{\overline{P}_{\mu\mu} + \overline{P}_{e\mu}/r}{\overline{P}_{ee} + \overline{P}_{\mu e} \cdot r} \quad . \quad (6)$$

We can now link the above expression for $R_{\mu/e}/R_{\mu/e}^0$ with a continuous transformation from the pure $\nu_\mu \leftrightarrow \nu_\tau$ oscillation case to a more general situation in which ϕ is variable (but not too large) and $m_3 > m_2 = m_1 = 0$ (one dominant mass scale). The two following cases may occur:

Pure $\nu_\mu \leftrightarrow \nu_\tau$ oscillations. In this case, using the usual 2ν mass/mixing parameters δm^2 and θ , the average probabilities $\overline{P}_{\alpha\beta}$ are given by

$$\begin{cases} \overline{P}_{\mu\mu} &= 1 - s_{2\theta}^2 \overline{S} \\ \overline{P}_{e\mu} &= \overline{P}_{\mu e} = 0 \\ \overline{P}_{ee} &= 1 \end{cases} \quad , \quad (7)$$

where $\overline{S}$ is the average value of the oscillation factor $S = \sin^2(\delta m^2 L / 4E_\nu)$ (L = path length), so that:

$$\frac{R_{\mu/e}}{R_{\mu/e}^0} = 1 - s_{2\theta}^2 \overline{S} \quad . \quad (8)$$

One dominant mass scale oscillations. In this case, the transition probabilities are given by

$$\begin{cases} \overline{P}_{\mu\mu} &= 1 - 4c_\phi^2 s_\psi^2 (1 - c_\phi^2 s_\psi^2) \overline{S} \\ \overline{P}_{e\mu} &= \overline{P}_{\mu e} = 4s_\phi^2 c_\phi^2 s_\psi^2 \overline{S} \\ \overline{P}_{ee} &= 1 - 4s_\phi^2 c_\phi^2 \overline{S} \end{cases}, \quad (9)$$

where now $\overline{S}$ is the average value of $S = \sin^2(m_3^2 L / 4E_\nu)$, so that:

$$\frac{R_{\mu/e}}{R_{\mu/e}^0} = \frac{1 - 4s_\psi^2 c_\phi^2 \overline{S} (1 - c_\phi^2 s_\psi^2 - s_\phi^2 / r)}{1 - 4s_\phi^2 c_\phi^2 \overline{S} (1 - s_\psi^2 \cdot r)} \quad (10)$$

We will see in Section 5 that the combined analysis of terrestrial neutrino data (multi-GeV events excluded) limits s_ϕ^2 to relatively small values: $s_\phi^2 \lesssim 0.25$. If the denominator in Eq. (10) is expanded at first order in s_ϕ^2 , one obtains:

$$\frac{R_{\mu/e}}{R_{\mu/e}^0} \simeq 1 - 4c_\phi^2 \overline{S} \left[s_\psi^2 (1 - c_\phi^2 s_\psi^2) + s_\phi^2 (2s_\psi^2 - 1) \right] \quad (11)$$

where use has been made of the numerical estimate $(r - 1/r) \simeq 2$.

Notice that there is a one-to-one correspondence between Eqs. (8) and (11), provided one makes the substitutions:

$$\begin{cases} \delta m^2 &\longleftrightarrow m_3^2 \\ s_{2\theta}^2 &\longleftrightarrow 4c_\phi^2 \left[s_\psi^2 (1 - c_\phi^2 s_\psi^2) + s_\phi^2 (2s_\psi^2 - 1) \right] \end{cases}, \quad (12)$$

which allow the published Kamiokande-allowed region to be mapped for multi-GeV ν oscillation from their plane $(s_{2\theta}^2, \delta m^2)$ to our plane $(s_{2\psi}^2, m_3^2)$ at fixed s_ϕ^2 .

In principle, a similar treatment could be applied to low-energy atmospheric neutrino data for which, however, we have made use of the refined analysis described in the previous subsection. In order to estimate the degree of approximation of the approach, we have compared for these data the exact results with those obtained by using the above simplifications, and found that, for $m_2^2 = 0$ and $s_\phi^2 \leq 0.25$, they differ by less than $\sim 10\%$ in the phenomenologically interesting region where $R_{\mu/e} / R_{\mu/e}^0 \lesssim 0.8$. Since the various simplifications used work better at higher ν energies, it can be reasonably guessed that, at least for $R_{\mu/e} / R_{\mu/e}^0 \lesssim 0.8$ and $0 \leq s_\phi^2 \leq 0.25$, also our approximate approach to multi-GeV ν data is affected by uncertainties of $\mathcal{O}(10\%)$.

4.3 Accelerator and reactor neutrinos

In Ref. [1] we have shown that the published 2ν bounds from accelerator and reactor experiments can be used to get limits on 3ν scenarios, provided that no neutrino but the third is massive. In particular, we have explicitly shown how to map exclusion plots from the familiar 2ν plane $(s_{2\theta}^2, \delta m^2)$ to the 3ν subspace $(s_{2\psi}^2, m_3^2)$ at any given ϕ , for the different oscillation channels probed by appearance or disappearance experiments.

The inclusion of the subleading effects induced by a finite value of m_2 would spoil this 2ν – 3ν correspondence, and would require a complete reanalysis of the raw data in each experiment. However, as we will see, the value of m_2^2 is always bound by solar ν data to be less than $\sim 10^{-4} \text{ eV}^2$. This value is so low with respect to the sensitivity range of the present accelerator and reactor experiments, that the subleading oscillation driven by m_2 can be safely neglected in all cases we consider. Thus the approach developed in [1] can be adopted without further corrections.

5 First step: $m_2/m_3 \rightarrow 0$

5.1 Approach

In the first step of our global analysis of solar, atmospheric, accelerator and reactor neutrino data, the rôle of the leading oscillation terms is purposely maximized by taking the limit $m_2/m_3 \rightarrow 0$. More precisely, for solar ν 's m_2 is taken to be finite and $m_3 = \infty$, whilst for terrestrial ν 's it is assumed that m_3 is finite and $m_2 = 0$.

This extremized form of the natural mass hierarchy $m_1 < m_2 < m_3$ leads to welcome advantages in the formalism [1], useful in determining a sort of first-order fit to all the data:

1. For solar neutrinos, the $3\nu \rightarrow 2\nu$ reduction formula (2) for the ν_e survival probability reduces to Eq. (1), and consequently the parameter space is spanned by three variables only: m_2, ω, ϕ .
2. For terrestrial neutrinos, the degeneracy of m_2 and m_1 (both zeroed) allows the angle $\theta_{12} = \omega$ to be rotated away, so that the parameter space reduces to m_3, ψ, ϕ . The 3ν oscillations formalism simplifies considerably, even in the presence of Earth matter effects [28].

We choose to fix s_ϕ^2 at three representative values: $s_\phi^2 = 0$ (no 1–3 mixing); $s_\phi^2 = 0.1$ (“moderate” 1–3 mixing); $s_\phi^2 = 0.25$ (relatively “large” 1–3 mixing). At any given value

of ϕ , solar and terrestrial ν data will be analysed respectively in the mass/mixing planes $(s_{2\omega}^2/c_{2\omega}, m_2^2)$ and $(s_{2\psi}^2, m_3^2)$. Symbolically, in this first step the parameter space gets factorized:

$$(m_2, m_3, \omega, \phi, \psi) \longrightarrow (s_\phi^2) \otimes \underbrace{(s_{2\omega}^2/c_{2\omega}, m_2^2)}_{solar} \otimes \underbrace{(s_{2\psi}^2, m_3^2)}_{terrestrial} . \quad (13)$$

5.2 Results for $\sin^2 \phi = 0$

Solar neutrinos. At $\phi = 0$, solar ν oscillations are pure $\nu_e \leftrightarrow \nu_\mu$. The MSW solutions to the solar neutrino problem are reported in Fig. 2. In particular, the first four subfigures labelled ^{71}Ga , ^{37}Cl , Kam and Kam N–D/N+D correspond to GALLEX+SAGE, Homestake, Kamiokande (average rate) and Kamiokande (night/day rate asymmetry). Solid lines represent the theoretical iso-SNU or iso-rate curves, including also Earth matter effects on solar neutrino propagation. The grey areas represent the allowed solutions at 2σ , adding to the experimental errors also the theoretical uncertainties related to the solar model and to the neutrino capture cross section. In ALL, the solid lines represent iso-standard-deviation contours ($\sigma = 1, 2, 3, 4 \iff \Delta\chi^2 = 1, 4, 9, 16$) for the global fit, adding or not the theoretical uncertainties (theo) to the experimental errors (expt). It is seen that the inclusion of the theoretical errors enlarges considerably the two allowed regions at small and large values of ω .

Terrestrial neutrinos. At $\phi = 0$, terrestrial ν oscillations are pure $\nu_\mu \leftrightarrow \nu_\tau$: thus Earth matter effects are absent in the propagation of atmospheric neutrinos, and no bounds can be derived from reactor data. Separate and global fits to data are shown in Fig. 3. In the first three subfigures (Kamiokande, IMB and Fréjus+NUSEX) we show the ν -oscillation fit to the atmospheric neutrino data of the corresponding experiment. The solid lines represent the theoretical predictions for the double μ/e ratio, and the grey areas the 2σ -allowed regions³, including a 5% theoretical error on the flux. The atmospheric neutrino fit is clearly dominated by the high-statistics IMB and Kamiokande experiments, which provide lower bounds both on ψ and on m_3 . The fourth subfigure shows a superposition of the most stringent accelerator exclusion plots (at 90% C.L.), which bound m_3 from above. The combination of all terrestrial neutrino data is finally shown in the last two subfigures (ALL), with and without the theoretical errors associated to the atmospheric neutrino fluxes. In ALL, solid lines represent iso-standard-deviation contours and the grey areas are allowed at 2σ . Notice that accelerator limits are not strictly combined, but more simply superimposed, since their format is not χ^2 -like.

³In the subfigure labelled Fréjus+NUSEX the grey area corresponds to the combination of the two experiments; the solid lines refer to Fréjus, and are very similar to the NUSEX curves (not shown).

5.3 Results for $\sin^2 \phi = 0.1$

For $\phi \neq 0$, the oscillations of solar and terrestrial neutrinos cannot be reduced respectively to pure $\nu_e \leftrightarrow \nu_\mu$ and $\nu_\mu \leftrightarrow \nu_\tau$ oscillations, and new interesting effects emerge both in vacuum and in matter.

Solar neutrinos. Concerning solar ν oscillations, the factor c_ϕ^4 in Eq. (1) tends to lower the predicted rates, so that both the iso-SNU contours and the allowed regions are slightly shifted and modified in shape, as shown in Fig. 4. In particular, the large angle solution is now considerably reduced, being allowed at the 2σ level only when theoretical errors are included. Moreover, the preferred values of m_2^2 are slightly higher than those derived in the corresponding fit at $\phi = 0$ (Fig. 3).

Terrestrial neutrinos. Figure 5 shows the analysis of terrestrial ν data. In the first three subfigures, solid lines represent contours of the double ratio $R_{\mu/e}/R_{\mu/e}^0$ (which can now be greater than 1, due to ν_e 's oscillating into other flavours and increasing $R_{\mu/e}$). Earth matter effects are now active for atmospheric ν trajectories below the detector horizon, as can be seen by comparing the solid curves with the dashed curves, obtained with the Earth density set to zero (vacuum). As expected, the effect is maximized in the range $m_3^2 \simeq 10^{-4} - 10^{-3} \text{ eV}^2$, which happens to be below the region allowed at 2σ by all the atmospheric data. Thus the global fit of atmospheric data would be only moderately changed if a simplified pure vacuum oscillation description were used.

Concerning accelerator and reactor neutrinos, for $s_\phi^2 = 0.1$ the reactor bounds are probed, and provide the most stringent (and ψ -independent) upper limit on m_3 . In combination with atmospheric data (ALL) this limit reduces considerably the allowed region in the $(s_{2\psi}^2, m_3^2)$ plane with respect to the $\phi = 0$ case.

5.4 Results for $\sin^2 \phi = 0.25$

The trend emerging in the comparison of the two previous cases is enhanced for such a large value of ϕ .

Solar neutrinos. In Fig. 6 it can be seen that the maximum predicted rate for each experiment is much lower than for $\phi = 0$, and higher values of m_2^2 are now allowed at 2σ for the small - and large - angle solutions. Moreover, both solutions extend towards smaller values of the mixing angle ω with respect to Figs. 2 and 4.

Terrestrial neutrinos. In Fig. 7 the best fit to atmospheric ν data is pushed towards the high- ψ side, since the theoretical value of $R_{\mu/e}/R_{\mu/e}^0$ can be very large (~ 1.6) at the opposite side ($\psi = 0$). As in Fig. 5, the solid (dashed) iso-double-ratio lines are obtained including (excluding) Earth matter effects. The allowed region for atmospheric

ν 's, however, is only marginally compatible with the Krasnoyarsk bound, which would be even stronger for higher values of ϕ . The incompatibility between atmospheric and reactor ν data then provides an (approximate) upper bound on ϕ at 2σ : $s_\phi^2 \lesssim 0.25$.

6 Second step: m_2 and m_3 finite

6.1 Approach

In the first step of the analysis, we start by forcing the ratio m_2/m_3 to be *zero*, and derive both (squared) masses ranging in *finite* intervals, thus raising a problem of self-consistency. Since the preferred ranges for m_2 and m_3 are separated by an order of magnitude at least (see Figs. 2–7), we guess that the inclusion of finite mass effects should not change dramatically the previous analysis. However, it is desirable to quantify explicitly the maximum effect that would be induced on the fit results by the hitherto neglected subleading oscillation terms.

This can be accomplished by using the solutions found in Section 5 as “source terms” for a new determination of the allowed regions, obtained by “crossing” the information coming from solar and terrestrial neutrinos in a perturbative-like approach. More precisely:

1. For solar neutrinos, we fix m_3^2 at the value $m_3^2 = 10^{-3} \text{ eV}^2$, which is sufficiently low to enhance the subleading m_3 -dependent term in Eqs. (2) and (3), but not too low to be excluded at 2σ by the terrestrial ν data fit (see Figs. 1, 3 and 5). The value of ψ is, as said, totally irrelevant.
2. For atmospheric neutrinos, we fix m_2 and ω at the values $m_2^2 = 10^{-4} \text{ eV}^2$ and $s_\omega^2 = 0.25$, which are representative of the largest values allowed by solar ν data at the 2σ level⁴. Notice that the five parameters m_2 , m_3 , ω , ψ , ϕ are now used with finite values, so that the full 3-generation structure of the propagation in vacuum and matter is probed.
3. For accelerator and reactor neutrino data, subleading effects induced by a “light” mass m_2 as small as $m_2^2 = 10^{-4} \text{ eV}^2$ are bound to be safely negligible in the sensitivity range of present experiments, so that it is not worth refining the data analysis in this second step.

With the above choices, we have maximized at the 2σ level the perturbations induced by assuming for atmospheric ν 's a finite value of m_2 and ω , and for solar ν 's a finite value

⁴Lower values of m_2 and/or ω would induce smaller subleading effects on atmospheric neutrinos.

of m_3 . Should the new regions allowed by neutrino data prove to be stable against such perturbations, then it could be concluded that our approach is not only self-consistent, but also sufficiently accurate even in the first — simpler — step of the analysis.

6.2 Results

The results of the inclusion of the finite mass effects lead to new fits analogous to those presented in Figs. 2–7. For the sake of brevity, only the final results (combinations of all solar/terrestrial ν data with the inclusion of theoretical errors) will be shown. More precisely, in Fig. 8 all the solar neutrino data are combined in the three subfigures on the left, and all terrestrial neutrino data on the right, according to the values of s_ϕ^2 : $s_\phi^2 = 0, 0.1, 0.25$. Thick solid lines enclose the regions (in grey) allowed at 2σ in this second step of the analysis, to be compared with the results obtained in the first step (thin solid lines). Dash-dotted lines enclose the Kamiokande-allowed regions for the oscillation parameters of the multi-GeV neutrinos.

Differences in the solar neutrino solutions when passing from the first to the second step of the analysis are so small as to be graphically unobservable. Concerning terrestrial neutrinos, the allowed regions are instead slightly modified (and enlarged for $\phi = 0$).

Notice that we have chosen very unfavourable situations at the 2σ level to enhance the perturbations, which are nevertheless very small. This means that the gap between the preferred values of m_2^2 and m_3^2 is wide enough to suppress subleading effects. We have thus found a *stable* family of solutions to both the solar neutrino problem and the atmospheric neutrino deficit, compatible with the bounds coming from negative oscillation searches at accelerators and reactors.

Finally let us discuss the oscillation parameter regions allowed by Kamiokande multi-GeV data at 90% C.L. (dash-dotted lines, right side of Fig. 8). For $s_\phi^2 = 0$ the region is exactly as published in Ref. [12] for the pure $\nu_\mu \leftrightarrow \nu_\tau$ oscillation case, whilst for $s_\phi^2 = 0.1, 0.25$ it has been obtained by means of the approximate substitution rules in Eq. (12). It can be noticed that there is considerable overlap between this region and the grey area allowed by all other terrestrial neutrino data, thus corroborating the picture emerging from our analysis.

7 Summary and conclusions

We have combined the most recent experimental data on solar, atmospheric, accelerator and reactor neutrinos in a 3-flavour oscillation scheme characterized by a natural hierarchy

of masses. Solar and atmospheric neutrino data have been analysed in a very refined framework, which allows a study of the coupled effects of the low and high neutrino mass ranges. Published accelerator and reactor bounds have been properly transformed into a 2-dimensional subspace of the 3ν parameter space. As a result, solutions to the solar ν problem and the atmospheric ν deficit — compatible with the negative searches for oscillation of laboratory-produced ν 's — have been found. Bounds have been placed on all the five mass/mixing neutrino parameters m_2 , m_3 , ω , ψ , ϕ . By means of a perturbative-like approach, the solutions, initially found by using an extremized form of mass hierarchy, have been shown to be stable with respect to a further iteration of the analysis. Finally, the recent evidence for a flavour anomaly in the Kamiokande multi-GeV atmospheric ν 's, even if analysed in an only approximate way, has been shown to be highly consistent with the global terrestrial neutrino data fit.

Our results do not exclude the possibility of different theoretical approaches, e.g. those using an unnatural hierarchy of masses [29]. However, a *complete* exploration of all experimentally allowed scenarios in the parameter space (m_2 , m_3 , ω , ψ , ϕ) would be exceedingly complex. Analyses such as the one we have presented in this paper have the virtue of showing the subtle interplay among different experimental results, but their combination is still open to more than one solution. We hopefully wait for the next generation of solar and terrestrial neutrino experiments to give new, more significant and constraining evidence of neutrino flavour oscillations.

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Figure Captions

Fig. 1 : Comparison between our calculations (solid lines) and the published Monte-Carlo estimates (histograms) for ν_μ - and ν_e -induced events in the four atmospheric neutrino experiments Kamiokande, IMB, Fréjus and NUSEX. The labels SR and CC denote “Single (Cherenkov) Ring” and “Charged Current” events.

Fig. 2 : MSW solutions to the solar neutrino problem for $s_\phi^2 = 0$. The grey areas, allowed at the 2σ level, refer to each experiment separately in the upper four subfigures, and to their combination in the lower two (ALL).

Fig. 3 : Neutrino oscillation solutions to the atmospheric neutrino deficit for $s_\phi^2 = 0$, compared with the accelerator and reactor limits. The grey areas are allowed at the 2σ level (or 90%, where indicated). The combined bounds are shown in the last two subfigures.

Fig. 4 : As in Fig. 2, but for $s_\phi^2 = 0.1$.

Fig. 5 : As in Fig. 3, but for $s_\phi^2 = 0.1$. The dashed curves are obtained by excluding the Earth effect.

Fig. 6 : As in Fig. 2, but for $s_\phi^2 = 0.25$.

Fig. 7 : As in Figs. 3 and 5, but for $s_\phi^2 = 0.25$.

Fig. 8 : Comparison of the global solutions obtained in our analysis at the first step (thin solid lines) and at the second step (thick solid lines, grey areas). The difference is modest for atmospheric neutrinos, and graphically unobservable for solar neutrinos. This proves the stability and consistency of our hierarchical scenario. Also shown are the boundaries of the regions preferred by the multi-GeV Kamiokande data (dash-dotted lines). See the text for details.