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OSCILLATOR AND ITS COHERENT STATES

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GENERALIZED DEFORMED PARA-BOSE OSCILLATOR AND ITS COHERENT STATES

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ABSTRACT

Generalized deformed commutation relations for a single mode para-Bose oscillator are constructed. The connection of generalized deformed para-Bose oscillators with para-Bose oscillators is determined. From these, the energy spectrum of generalized deformed para-Bose oscillators and the coherent states of the annihilation operators are discussed.

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In the last few years there has been increasing interest in particles obeying statistics different from Bose or Fermi. These generalized statistics are called para-Bose and para-Fermi statistics [1-4]. Since the advent of the theory of parastatistics there have been many attempts to generalize the canonical Commutation Relations (CR). In particular, quantum deformations of the Heisenberg algebra and their possible physical applications have been extensively investigated [4-25]. Chaturvedi and Srinivasan [9] have shown that a single para-Bose oscillator may be regarded as a deformed Bose oscillator. In Ref.[10] the CR for a single mode of the harmonic oscillator which contains para-Bose and q-oscillator CR have been constructed. The connection of q-deformed para-Bose oscillators with para-Bose oscillators has been determined in Ref.[18]. Chakrabarti and Jagannathan [20] have considered two-parameter deformation of a parabosonic algebra underlying the two-particle Calogero model and have discussed corresponding coherent states. It is noteworthy that starting from an arbitrary deformation of the oscillator Daskaloyannis [21] has constructed all the general deformed oscillator algebra and has studied its properties. Next, the generalized deformed oscillators corresponding to the energy spectrum of the Pöschl-Teller potential [22] or satisfying the same CR as a system of fermion pairs of zero angular momentum in a single-j shell [23] have been discussed. Furthermore, in Ref.[25] the analogues of the single-photon and multiphoton coherent states for the generalized deformed bosonic oscillator system have been constructed in a unified way.

The following questions can be raised: Is it possible to construct generalized deformed CR for a single mode para-Bose oscillator? Is there a connection between generalized deformed para-Bose oscillators and para-Bose oscillators? The main purpose of this work is contributed to these questions. In addition, we will discuss about the energy spectrum and the coherent states corresponding to generalized deformed para-Bose oscillators.

As is well known, a single mode para-Bose system is characterized by the CR [9,10]

$$[a, \mathcal{N}] = a, \quad [a^\dagger, \mathcal{N}] = -a^\dagger \quad (1)$$

where

$$\mathcal{N} = \frac{1}{2}(aa^\dagger + a^\dagger a) - \frac{p}{2} \quad (2)$$

and p is the order of the para-Bose system.

Also

$$aa^\dagger = \tilde{f}(\mathcal{N} + 1), \quad a^\dagger a = \tilde{f}(\mathcal{N}) \quad (3)$$

with

$$\tilde{f}(n) = n + \frac{1}{2} \{1 + (-1)^n\} (p - 1). \quad (4)$$

Hence

$$[a, a^\dagger] = \tilde{f}(\mathcal{N} + 1) - \tilde{f}(\mathcal{N}) = 1 + (-1)^{\mathcal{N}} (p - 1). \quad (5)$$

From these relations, an operator A^+ was constructed so that [9,10]

$$A^+ = a^\dagger \frac{\mathcal{N} + 1}{\tilde{f}(\mathcal{N} + 1)} \quad (6)$$

$$[a, A^+] = 1, \quad [A^+, \mathcal{N}] = -A^+ \quad (7)$$

By the results just mentioned above the number operator $\mathcal{N}$ can be written as follows

$$\mathcal{N} = A^+ a. \quad (8)$$

Let us now turn to the question of generalized deformed para-Bose oscillator corresponding to the annihilation and creation operators $\tilde{a}$ and $\tilde{a}^+$ respectively. According to the method of Daskaloyannis [21, 22] we begin by establishing an operator $\tilde{A}^+$ and consider the generalized deformed CR

$$\tilde{a} \tilde{A}^+ = g(\tilde{A}^+ \tilde{a}) \quad (9)$$

where $\tilde{A}^+$ is defined by [9, 10]

$$\tilde{A}^+ = \frac{\tilde{a}^+ (N + 1)}{f(N + 1)}. \quad (10)$$

In the q-deformed oscillator algebra the function $g(x)$ takes the form

$$g(x) = 1 + qx. \quad (11)$$

The number operator N satisfies, by definition, the CR

$$[\tilde{a}, N] = \tilde{a}, \quad [\tilde{A}^+, N] = -\tilde{A}^+. \quad (12)$$

It can be shown that N is given by the relation [21, 22]

$$N = f(\tilde{A}^+ \tilde{a}) \quad (13)$$

where the function f and the inverse function $F = f^{-1}$ are related to the function g as follows [21, 22]

$$g(x) = F\{1 + f(x)\}. \quad (14)$$

If $|\alpha\rangle$ is a base of eigenvectors of the number operators N ,

$$N|\alpha\rangle = \alpha|\alpha\rangle \quad (15)$$

then from Eq.(12) one has

$$\tilde{a}|\alpha\rangle = \sqrt{[\alpha]}|\alpha-1\rangle \quad (16)$$

$$\tilde{A}^+|\alpha\rangle = \sqrt{[\alpha+1]}|\alpha+1\rangle \quad (17)$$

where [21, 22]

$$[\alpha] = F(\alpha). \quad (18)$$

Substituting (10) into (17) we find

$$\tilde{a}^+|\alpha\rangle = \frac{\tilde{f}(N)}{N}\sqrt{[\alpha+1]}|\alpha+1\rangle. \quad (19)$$

We observe, further, that

$$\tilde{a}^+\tilde{a} = \frac{\tilde{f}(N)}{N}[N], \tilde{a}\tilde{a}^+ = [N+1]\frac{\tilde{f}(N+1)}{N+1}. \quad (20)$$

With the help of (20) we obtain

$$[\tilde{a}, \tilde{a}^+] = \tilde{f}(N+1)\frac{[N+1]}{N+1} - \tilde{f}(N)\frac{[N]}{N}. \quad (21)$$

So, we have just constructed the most general CR (21) for generalized deformed para-Bose oscillators. It includes the various forms of the CR defined in the literature [10, 18, 20] as special cases. For example, the q -deformed para-Bose CR [10, 18] can be reproduced with the following choice

of the structure functions:

$$F(x) = \frac{q^x - 1}{q - 1}, f(x) = \frac{\ln\{1 + (q-1)x\}}{\ln q}.$$

Let us now turn to the problem of the connection between generalized deformed para-Bose oscillators and para-Bose oscillators. The boson realization of the algebra (12), following from (15)-(17) is given by

$$\tilde{a} = a\sqrt{\frac{[N]}{N}} \quad (22)$$

$$\tilde{A}^+ = \sqrt{\frac{[N]}{N}}A^+ \quad (23)$$

$$N = A^+a = \mathcal{N}. \quad (24)$$

Combining (6), (10) and (23) yields

$$\tilde{a}^+ = \sqrt{\frac{[N]}{N}}a^+. \quad (25)$$

So, owing to the relations (22) and (25) we have just expressed the operators $\tilde{a}$ and $\tilde{a}^+$ of generalized deformed para-Bose oscillator in terms of the operators a and a^+ of single mode para-Bose oscillator. This result is general and can be applied for any deformed para-Bose oscillators, including the q -deformed ones [18].

As an important immediate consequence of (25) we have

$$[\tilde{a}^+, N] = -\tilde{a}^+. \quad (26)$$

This formula will be used later.

Now, by the above relations we can also obtain the energy spectrum corresponding to the energy operator $H = \frac{\lambda}{2}(\tilde{a}^+ \tilde{a} + \tilde{a} \tilde{a}^+ + p - 1)$. This energy is

$$E_n = \frac{\lambda}{2} \left\{ \tilde{f}(n) \frac{[n]}{n} + \tilde{f}(n+1) \frac{[n+1]}{n+1} + p - 1 \right\}. \quad (27)$$

As $p \rightarrow 1$ this becomes

$$E_n = \frac{\lambda}{2} \{ [n] + [n+1] \}. \quad (28)$$

Therefore the expression (26) reduces to the energy spectrum of the generalized deformed ordinary oscillator [21, 22]. Next, letting $[n] \rightarrow n$ into (27) we get also the energy spectrum of para-Bose oscillator [3].

In the last part of the paper we consider the construction of coherent states of the annihilation $\tilde{a}$, that is

$$\tilde{a}|z\rangle = z|z\rangle \quad (29)$$

where z is a complex number.

The construction of these coherent states is most easily done following a simple technique [8, 9] applicable to any generalized boson oscillator. Let

$$C = \tilde{a} \frac{N}{[N]}, C^+ = \frac{N}{[N]} \tilde{A}^+ \quad (30)$$

such that

$$[\tilde{a}, C^+] = 1, [C, \tilde{a}^+] = 1, [N, C] = -C, [N, C^+] = C^+. \quad (31)$$

The coherent states will have the form

$$|z\rangle \sim \exp(zC^+) |0\rangle. \quad (32)$$

By using (10), (26), (30) finally the normalized coherent state is

$$|z\rangle = \left\{ \sum_{n=0}^{\infty} \frac{|z|^{2n}}{[n]!} \right\}^{-\frac{1}{2}} \exp \left\{ z \frac{N^2}{[N]\tilde{f}(N)} \tilde{a}^+ \right\} |0\rangle \quad (33)$$

with

$$[n]! = F(n)F(n-1)\dots F(1), [0]! = 1. \quad (34)$$

In the special case of $p = 1$, one obtains an expression

$$|z\rangle = \left\{ \sum_{n=0}^{\infty} \frac{|z|^{2n}}{[n]!} \right\}^{-\frac{1}{2}} \exp \left\{ z \frac{N}{[N]} \tilde{a}^+ \right\} |0\rangle \quad (35)$$

known in the generalized bosonic oscillator literature [25]. Therefore it also contains the coherent state of the annihilation operator of a q oscillator [8] in the $p = 1$, $[x] = \frac{q^x - 1}{q - 1}$ case. These coherent states (33) for the cases of q-deformed and (p,q)-deformed para-Bose oscillators coincide with the known relations of the coherent states of Refs.[10, 20]. Besides, this state includes the various forms of the coherent states defined in Ref.[24]. It has already been shown that our expression (33) extends and generalizes some of the corresponding ones in several special cases [8, 10, 20, 24, 25].

We conclude this paper with the following comments. The eigenvalues of generalized deformed para-Bose oscillator (27) are of great importance since we can use them for the calculation of the masses of several particles. Our results may also lead to constructing the analogues of the single-photon

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and multiphoton coherent states for generalized deformed parabosonic oscillator systems. Another interesting problem is the study of the solution of the general equation (27), which determines the structure function $F(x)$ from a given energy spectrum E_n and deriving the equivalent deformed paraoscillator. We will study these topics elsewhere.

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