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RENORMALIZATION GROUP ANALYSES OF PARTICLE MASSES IN FULL FOUR-GENERATION FERMION CONDENSATE SCHEME

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ABSTRACT

The renormalization group analyses of the masses of the heavy fermions and the Higgs boson, including the discussions of possible fixed points and applicability of perturbation theory, are expounded in the full four-generation quark-lepton condensate scheme of the minimal electroweak symmetry breaking. The results also indicate that the model could accommodate itself to the top-quark mass $m_t = 160 - 180 GeV$. The predicted Higgs boson mass is in fact not too sensitive to the above change of m_t . It is also found that for a model without the fourth generation of leptons, the limitation to Λ imposed by the requirement that the masses of the fourth generation of extra quarks are bigger than $m_t = 174 GeV$ could be greatly relaxed up to $\leq 10^{11} GeV$.

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1 Introduction

The top-quark condensate scheme [1,2] of electroweak symmetry breaking of Nambu-Jona-Lasinio (NJL)-form [3] provides a dynamical explanation of the origin of the Higgs boson and the large top-quark mass. The extensions of this scheme to many fermion generations have been also investigated by some authors in different scenarios [4-9]. In fact, the minimal (one Higgs-doublet) dynamical breaking of the electroweak gauge group could be realized by means of the condensates of many generations of fermions. This idea, partly motivated by solving the fine-tuning problem of the coupling constant [7], has been proven to be valid through the calculations in the bubble approximation [8]. An important feature of its results is that the top-quark together with the other heavy fermions (for example, the fourth generation of quarks and leptons) will simultaneously enter the vacuum condensates and the components of the Higgs boson, i.e. both the vacuum condensates and the Higgs boson are now some linear combinations of the bilinear quantities of the heavy fermions, where the lighter fermions (e.g. the first two generations of quarks and leptons as well as the bottom-quark) can be omitted because the weights of the fermionic bilinears in the combinations are proportional to the masses of corresponding fermions [9]. Therefore, in a full four-generation fermion condensate scheme, the top-quark and the fourth generation of quarks and leptons must be treated equally. For instance, when we intend to generalize the results obtained in the bubble approximation [7-9] to the case including the full gauge interactions and the dynamics of the composite Higgs boson based on the technique proposed in Ref.[2], we must include simultaneously the evolution of the coupling constants of all the heavy fermions in the renormalization group (RG) equations.

In view of this point, the scheme discussed in Ref.[5], although it emphasizes more the right-handed neutrino condensates, will not be able to be called the full four-generation condensate one, because the evolution of the heavy top-quark-Higgs Yukawa coupling constant has been neglected from all the RG equations. A true full four-generation quark-lepton condensate scheme based on the technique in Ref.[2] has been outlined and some most interesting results have been reported in Ref.[10]. In this paper, we will make a

systematic and complete description of the renormalization group analyses in the general four-generation fermion condensate scheme where the fourth generation will not necessarily be the normal quark-leptons.

The basic idea of the technique in Ref.[2] lies in that the results obtained in the bubble approximation of the effective four-fermion interactions responsible to spontaneous electroweak symmetry breaking can be equivalent to an effective Lagrangian at the scales below the momentum cut-off Λ . This low-energy effective Lagrangian contains an induced gauge-invariant kinetic term and a quartic self-interaction term of the Higgs boson and will have the similar form to the Lagrangian of the standard model. The full dynamics may be considered by means of the RG evolution of the coupling constants. However, since the Higgs boson is now composite particle, all the Higgs-dependent coupling constants must satisfy the compositeness boundary conditions at the momentum cut-off (or compositeness scale) Λ .

In order to generalize the above technique to the case with full four-generation of fermions, we must put down the low-energy effective Lagrangian at the scale μ below the momentum cut-off Λ . It has now the following form:

$$\mathcal{L} = \mathcal{L}_{\text{kinetic}} - (H^\dagger C + C^\dagger H) + \Delta L_{\text{gauge}} + Z_H |D_\mu H|^2 - m_H^2 H^\dagger H - \frac{\lambda_0}{2} (H^\dagger H)^2 \quad (1.1)$$

$$C = \sum_{\alpha=3,4} \left[g_{U_\alpha}^0 (-i\sigma_2) (\bar{Q}_{\alpha L}^T U_{\alpha R}) + g_{D_\alpha}^0 (\bar{D}_{\alpha L} Q_{\alpha R}) \right] \quad (1.2)$$

where all the loops are defined with respect to the low-energy scale μ .

$$H = \begin{pmatrix} H^+ \\ H^0 \end{pmatrix} \quad (1.3)$$

is the Higgs-field doublet and

$$Q_{\alpha L} = \begin{pmatrix} U_\alpha \\ D_\alpha \end{pmatrix}_L, Q_{\alpha R} = U_{\alpha R}, D_{\alpha R} \quad (\alpha = 3, 4) \quad (1.4)$$

is the fermion doublets containing the heavy fermions, i.e. $\alpha = 3$ corresponds to the third generation of quarks (t, b) and $\alpha = 4$ to the fourth generation of fermions. The Q -fields are also assigned in the $SU_c(3)$ representation R with dimension $d_Q(R)$ and have anomaly-free $U_Y(1)$ -charge assignments. They are all assumed to be in their mass eigenstates [8].

$\mathcal{L}_{\text{kinetic}}$ represents the $SU_c(3) \times SU_L(2) \times U_Y(1)$ gauge-invariant kinetic terms of the gauge and the Q -fermion fields. The $g_Q^0 (Q = U_\alpha, D_\alpha)$ are the bare Q -fermion-Higgs Yukawa coupling constants and σ_2 is the Pauli matrix. The ΔL_{gauge} represents the fermion loop contributions after deducting the low-energy loop terms to the gauge coupling constants in the bubble approximation of the effective four-fermion interactions and D_μ is the gauge covariant derivative. The constants Z_H in the induced kinetic term, m_H^2 in the mass term and λ_0 in the induced quartic Higgs self-interaction term take the following forms:

$$\begin{aligned} Z_H &= \sum_Q d_Q(R) (g_Q^0)^2 (4\pi)^{-2} \ln \frac{\Lambda^2}{\mu^2}, \\ m_H^2 &= m_0^2 - \sum_Q 2d_Q(R) (g_Q^0)^2 (4\pi)^{-2} (\Lambda^2 - \mu^2), \\ \lambda_0 &= \sum_Q 2d_Q(R) (g_Q^0)^4 (4\pi)^{-2} \ln \frac{\Lambda^2}{\mu^2} \end{aligned} \quad (1.5)$$

where the Q in the sums run over all the heavy fermions in the third and the fourth generation except the light bottom quark (i.e. assuming $g_b^0 = 0$). The m_0 is the bare mass of the Higgs field when it appears as a static field in the Lagrangian of Yukawa-form equivalent to the original effective four-fermion Lagrangian [2]. For enough high Λ we always have $m_H^2 < 0$ and this means spontaneous breaking of the electroweak $SU_L(2) \times U_Y(1)$ gauge group. We may obtain the low-energy physical tree spectrum from the Lagrangian (1.1), as has been made in Ref.[10]. It is seen from Eq.(1.5) that we will have the following limits at the compositeness scale Λ that

$$Z_H \rightarrow 0, m_H^2 \rightarrow m_0^2, \lambda_0 \rightarrow 0, g_Q^0 \rightarrow \text{constant, when } \mu \rightarrow \Lambda \quad (1.6)$$

They imply that when the low-energy scale μ approaches the momentum cut-off Λ , the Higgs field H will lose its kinetic term and return to the static one. Therefore, Eq.(1.6) simply represents the compositeness boundary conditions in tree (or bubble) approximation. These conditions will play a key role in the total renormalization group analyses.

In Sect.2 we will consider Eq.(1.1) as a starting point and give the RG equations of the heavy-fermion-Higgs Yukawa coupling constants and the Higgs-quartic coupling constant. In particular, we will expound how to transfer the compositeness boundary

conditions (1.6) to the case with many running couplings. In Sect.3 the RG equations given in Sect.2 will be solved numerically and the masses of the heavy fermions and the Higgs boson will be obtained for the model with four generations of quarks and leptons. In the meantime, we will also discuss the behavior of the solutions at possible fixed points and demonstrate the applicability of perturbation theory in the lower momentum cut-off case. In Sect.4 the RG equations will be solved for a model with the fourth generation of quarks but without corresponding leptons and a comparison between the models in Sect.3 and Sect.4 will be made. Finally in Sect.5 we come to our conclusions.

2 Renormalization group equations and compositeness boundary conditions

The precise predictions about the masses of the heavy fermions and the Higgs boson depend on the complete dynamics contained in the low-energy effective Lagrangian (1.1). Such dynamics is embodied in the RG evolutions of the gauge coupling constants and the fermion-Higgs Yukawa and the Higgs-quartic coupling constants. Since Eq.(1.1) has the same form as the Lagrangian of the standard model, we can use the RG evolutions of these couplings in the standard model. However, the Higgs fields are now composite and not fundamental ones, the relevant running couplings must obey some boundary conditions consistent with Eq.(1.6) at the compositeness scale Λ . When the color and the electroweak gauge interactions are opened and the dynamics of the Higgs fields is considered, the effective low-energy Lagrangian (1.1) will take the following form:

$$\begin{aligned} \mathcal{L} = & \mathcal{L}_{\text{kinetic}}^{\text{gauge}} + \sum_Q [\bar{Q}_L i \gamma^\mu D_\mu Q_L + \bar{Q}_R i \gamma^\mu D_\mu Q_R] \\ & - \left\{ H^\dagger \sum_{\alpha=3,4} [\bar{g}_{U_\alpha} (-i\sigma_2) (\bar{Q}_{\alpha L}^T U_{\alpha R} + \bar{g}_{D_\alpha} (\bar{D}_{\alpha R} Q_{\alpha L})] + h.c. \right\} \\ & + |D_\mu H|^2 - \frac{m_H^2}{Z_H} H^\dagger H - \frac{\bar{\lambda}}{2} (H^\dagger H)^2 \end{aligned} \quad (2.1)$$

where $\mathcal{L}_{\text{kinetic}}^{\text{gauge}}$ represents the kinetic terms of the gauge fields, and in Eq.(2.1) the Higgs field and the fermion fields have been rescaled by the replacements

$$H \rightarrow H/\sqrt{Z_H}, \quad Q_L \rightarrow Q_L/\sqrt{Z_{Q_L}}, \quad Q_R \rightarrow Q_R/\sqrt{Z_{Q_R}} \quad (2.2)$$

where Z_H, Z_{Q_L} and Z_{Q_R} are respectively the divergent wave function normalization constants of the Higgs fields, the left-handed and the right-handed Q -fermion fields. Eq.(2.2) leads to the conventional normalization convention i.e. the kinetic terms have the forms of the free field theory, as can be seen from(2.1). In this normalization convention, the physical coupling constants $\bar{g}_Q$ and $\bar{\lambda}$ are defined by

$$\begin{aligned} \bar{g}_Q &= \frac{Z_{HQ}}{\sqrt{Z_H Z_{Q_L} Z_{Q_R}}} g_Q^0, \\ \bar{\lambda} &= \frac{Z_{4H}}{Z_H^2} \lambda_0 \end{aligned} \quad (2.3)$$

where Z_{HQ} and Z_{4H} are the proper vertex renormalization constants for the Higgs-Yukawa and the Higgs-quartic interactions separately.

The RG equations satisfied by $\bar{g}_Q(\mu)$ and $\bar{\lambda}(\mu)$ in the standard model have appeared in references [11,5]. However, for convenience of our use we will give their following forms for n fermion-generation case by an independent derivation.

$$\begin{aligned} (4\pi)^2 \mu \frac{d\bar{g}_Q}{d\mu} = & \left\{ \left[\frac{3}{2} + d_Q(R) \right] \bar{g}_Q^2 + \sum_{Q' \neq Q} d_{Q'}(R) \bar{g}_{Q'}^2 - \frac{3}{2} (\delta_{QU_\alpha} \bar{g}_{D_\alpha}^2 + \delta_{QD_\alpha} \bar{g}_{U_\alpha}^2) \right. \\ & \left. - 3(\epsilon_Q^2 + \frac{1}{4} Y_{Q_L}^2) \bar{g}_1^2 - \frac{9}{4} \bar{g}_2^2 - 6C_2(R_Q) \bar{g}_3^2 \right\} \bar{g}_Q. \end{aligned} \quad (2.4)$$

and

$$\begin{aligned} (4\pi)^2 \mu \frac{d\bar{\lambda}}{d\mu} = & 12 \left\{ \bar{\lambda}^2 + \left[\frac{1}{3} \sum_Q d_Q(R) \bar{g}_Q^2 - A \right] \bar{\lambda} + B - \frac{1}{3} \sum_Q d_Q(R) \bar{g}_Q^4 \right\} \\ A = & \frac{1}{4} \bar{g}_1^2 + \frac{3}{4} \bar{g}_2^2, \quad B = \frac{1}{16} (\bar{g}_1^4 + 2\bar{g}_1^2 \bar{g}_2^2 + 3\bar{g}_2^4) \end{aligned} \quad (2.5)$$

where the definitions

$$\left. \begin{matrix} \delta_{QU_\alpha} \\ \delta_{QD_\alpha} \end{matrix} \right\} = \frac{1}{2} (1 \pm \delta_Q) \quad (2.6)$$

with

$$\delta_Q = \begin{cases} 1 & \text{for } Q = U_\alpha \\ -1 & \text{for } Q = D_\alpha, \end{cases} \quad (2.7)$$

have been used. The ϵ_Q is the value of the electric charge of the Q -fermions defined by

$$\epsilon_Q = \frac{1}{2} (\delta_Q + Y_{Q_L}) \quad (2.8)$$

and Y_{Q_L} is the $U_Y(1)$ charge of the left-handed Q -fermions. $C_2(R_Q)$ is the eigenvalue of the $SU_c(3)$ quadratic Casimir operator in the representation R of the Q -fermions. Eqs.(2.4) and (2.5) are applicable to the model with n generations of fermions if Q runs over all of the n generations. However, we will use them still only for the models with two heavy fermion generations and omit the coupling constant $\bar{g}_b$ of the light bottom-quark from the sums of Q and the equations. It is also easy to verify that when only the top-quarks are included in the condensate scheme, i.e. when $\bar{g}_Q = 0$ for all $Q \neq t$, Eqs.(2.4) and (2.5) will go back to Eqs.(4.6) and (4.12) in Ref.[2] respectively. The running coupling constants $\bar{g}_1^2(\mu)$, $\bar{g}_2^2(\mu)$ and $\bar{g}_3^2(\mu)$ of the $U_Y(1)$, $SU_L(2)$ and $SU_c(3)$ gauge group will be determined by the RG evolution formulas in the standard model [12]

$$\bar{g}_i^2(\mu) = \bar{g}_i^2(M_Z) / \left[1 + \bar{g}_i^2(M_Z) \frac{\beta_0^{(i)}}{8\pi^2} \ln \frac{\mu}{M_Z} \right], \quad i = 1, 2, 3 \quad (2.9)$$

in which M_Z is the mass of the Z -boson and

$$\beta_0^{(1)} = -\frac{20}{9}n_g - n_q - \frac{1}{6}, \quad \beta_0^{(2)} = \frac{43}{6} - \frac{4}{3}n_g - n_q, \quad \beta_0^{(3)} = 11 - \frac{4}{3}(n_g + n_q) \quad (2.10)$$

where n_g is the number of the quark-lepton generation and n_q is the number of the exotic quark-like generation (assuming that they are in the $SU_c(3)$ -triplet representations). We will ignore the effect induced by the large mass difference $m_t - m_b$ on the conventional running of the gauge couplings $\bar{g}_1^2(\mu)$ and $\bar{g}_2^2(\mu)$ which was indicated in Ref.[9] and leave it to further more precise calculations. The above results in the conventional normalization convention are not completely appropriate to the case with composite Higgs boson, because the coefficient before the Higgs kinetic term in Eq.(2.1) can not keep to be unity at any scale μ owing to the compositeness boundary conditions (1.6). In addition, we must also change the compositeness boundary conditions (1.6) into the form appropriate to many running coupling constants. Therefore, following the method in Ref.[2], we will take the "unconventional" normalization convention which can be realized in Eq.(2.1) through the transformation

$$H \rightarrow H/\bar{g}_t^2(\mu) \quad (2.11)$$

where $\bar{g}_t(\mu)$ is the running top-quark-Higgs Yukawa coupling constant in the conventional normalization convention (in fact, the top-quark may also be replaced by any heavy Q -

fermion). We thus obtain the wave function normalization constant $\tilde{Z}_H$ of the Higgs kinetic term and the Higgs quartic self-coupling constant $\tilde{\lambda}$ in the unconventional normalization convention as follows.

$$\tilde{Z}_H(\mu) = \frac{1}{\bar{g}_t^2(\mu)} \quad (2.12)$$

$$\tilde{\lambda}(\mu) = \frac{\bar{\lambda}(\mu)}{\bar{g}_t^4(\mu)} \quad (2.13)$$

In addition, we also define the unconventional normalization constants $\tilde{Z}_Q$ for the other heavy Q -fermions by

$$\tilde{Z}_Q(\mu) = \frac{1}{\bar{g}_Q^2(\mu)}, \quad \text{for } Q \neq t \quad (2.14)$$

It is easy to verify that owing to Eq.(1.6) we will have the following boundary conditions:

$$\tilde{Z}_H(\mu) \rightarrow 0, \quad \tilde{\lambda}(\mu) \rightarrow 0 \quad \text{and} \quad \tilde{Z}_Q(\mu) \rightarrow 0 \quad (Q \neq t), \quad \text{when } \mu \rightarrow \Lambda \quad (2.15)$$

However, the ratio between the Yukawa coupling constants $\bar{g}_Q^2$ and $\bar{g}_t^2$ will keep to be a constant at the scale Λ , i.e.

$$\frac{\tilde{Z}_H(\mu)}{\tilde{Z}_Q(\mu)} \equiv \frac{\bar{g}_Q^2(\mu)}{\bar{g}_t^2(\mu)} = \eta_{Qt} \left[\frac{Z_{H\bar{Q}}^{G+F}(\mu)}{Z_{Ht}^{G+F}(\mu)} \right]^2 \frac{Z_{tL}^{G+F}(\mu) Z_{tR}^{G+F}(\mu)}{Z_{QL}^{G+F}(\mu) Z_{QR}^{G+F}(\mu)} \rightarrow \text{constant, when } \mu \rightarrow \Lambda \quad (2.16)$$

where the parameter η_{Qt} is defined by

$$\eta_{Qt} = (g_Q^0/g_t^0)^2 \quad (2.17)$$

which is the ratio between the squared Yukawa coupling constants $(g_Q^0)^2$ and $(g_t^0)^2$, or say the ratio between the squared masses $(m_Q^0)^2$ and $(m_t^0)^2$ in the bubble approximation [8]. In Eq.(2.16) we have kept only the contributions $Z_{H\bar{Q}}^{G+F}$, Z_{QL}^{G+F} and Z_{QR}^{G+F} coming from the fermion and gauge-boson loops in the corresponding wave function normalization constants $Z_{H\bar{Q}}$, Z_{QL} and Z_{QR} separately, this is due the fact that when $\mu \rightarrow \Lambda$, $\tilde{Z}_H \rightarrow 0$, i.e. the kinetic term of the Higgs fields will disappear and the Higgs boson will cease to propagate; however, the gauge fields and the fermion fields are the fundamental ones, there is no reason to omit their loop contributions at the scale Λ . To one-loop order these constants can be calculated and the resulting expressions are that

$$Z_{H\bar{Q}}^{G+F}(\mu) = 1 + \left[2e_Q Y_{Q_L} \bar{g}_1^2(\mu) + 4C_2(R_Q) \bar{g}_3^2(\mu) \right] (4\pi)^{-2} \ln \frac{\mu^2}{M_Z^2},$$

$$Z_{Q_L}^{G+F}(\mu) = 1 + \left[\frac{1}{4} Y_{Q_L}^2 \bar{g}_1^2(\mu) + \frac{3}{4} \bar{g}_2^2(\mu) + C_2(R_Q) \bar{g}_3^2(\mu) \right] (4\pi)^{-2} \ln \frac{\mu^2}{M_Z^2},$$

$$Z_{Q_R}^{G+F}(\mu) = 1 + \left[e_Q^2 \bar{g}_1^2(\mu) + C_2(R_Q) \bar{g}_3^2(\mu) \right] (4\pi)^{-2} \ln \frac{\mu^2}{M_Z^2} \quad (2.18)$$

Here and in Eq.(2.9) we have taken the Z -boson mass M_Z to be the low-energy scale parameter and this amounts to specify a subtraction scheme of the renormalization. Changing the scale parameter will lead to only an additional finite renormalization and not bring about any essential influence on the results. By means of Eq.(2.9) we may introduce the denotations

$$b_i = \frac{\bar{g}_i^2(\Lambda)}{8\pi^2} \ln \frac{\Lambda}{M_Z} = \left[\beta_0^{(i)} + \frac{8\pi^2}{\bar{g}_i^2(M_Z) \ln \frac{\Lambda}{M_Z}} \right]^{-1}, i = 1, 2, 3 \quad (2.19)$$

hence the boundary condition (2.16) will become that

$$\frac{\dot{Z}_H(\Lambda)}{\dot{Z}_Q(\Lambda)} = \frac{\bar{g}_Q^2(\Lambda)}{\bar{g}_t^2(\Lambda)} = \eta_Q \left[\frac{1 + 4C_2(R_Q)b_3 + 2e_Q Y_{Q_L} b_1}{1 + 4C_2(R_t)b_3 + 2e_t Y_{t_L} b_1} \right]^2 \frac{1 + C_2(R_t)b_3 + \frac{3}{4}b_2 + \frac{1}{4}Y_{t_L}^2 b_1}{1 + C_2(R_Q)b_3 + \frac{3}{4}b_2 + \frac{1}{4}Y_{Q_L}^2 b_1}$$

$$\cdot \frac{1 + C_2(R_t)b_3 + e_t^2 b_1}{1 + C_2(R_Q)b_3 + e_Q^2 b_1} \quad (2.20)$$

Since the compositeness boundary conditions (2.16) or (2.20) have included the effects of the gauge interactions, it is expected that we will be able to obtain more reliable physical results. By means of Eqs.(2.12)-(2.14), Eqs.(2.4) and (2.5) can be transformed into an equations system about $\dot{Z}_H$ and $\dot{Z}_Q$ (here Q represents the heavy fermions other than the top-quark) and an equation about $\dot{\lambda}$. They have the following forms:

$$(4\pi)^2 \frac{d\dot{Z}_H}{dt} = -9 - 2 \sum_{Q \neq t} d_Q(R) \frac{\dot{Z}_H}{\dot{Z}_Q} + \left(\frac{17}{6} \bar{g}_1^2 + \frac{9}{2} \bar{g}_2^2 + 16 \bar{g}_3^2 \right) \dot{Z}_H \quad (2.21)$$

$$(4\pi)^2 \frac{d\dot{Z}_Q}{dt} = -(3 + 2d_Q(R)) - 6 \frac{\dot{Z}_Q}{\dot{Z}_H} - 2 \sum_{Q' \neq Q, t} d_{Q'}(R) \frac{\dot{Z}_Q}{\dot{Z}_{Q'}} + 3 \left(\frac{\dot{Z}_Q}{\dot{Z}_{D_o}} \delta_{Q U_o} + \frac{\dot{Z}_Q}{\dot{Z}_{U_o}} \delta_{Q D_o} \right)$$

$$+ \left[6 \left(e_Q^2 + \frac{1}{4} Y_{Q_L}^2 \right) \bar{g}_1^2 + \frac{9}{2} \bar{g}_2^2 + 12 C_2(R_Q) \bar{g}_3^2 \right] \dot{Z}_Q, (Q \neq t) \quad (2.22)$$

$$(4\pi)^2 \frac{d\dot{\lambda}}{dt} = 12 \left\{ \frac{\dot{\lambda}^2}{\dot{Z}_H^2} + \left[\frac{2}{9} \bar{g}_1^2 + \frac{8}{3} \bar{g}_3^2 - \frac{1}{2 \dot{Z}_H} \right] \dot{\lambda} + B \dot{Z}_H^2 - \frac{1}{3} \sum_{Q \neq t} d_Q(R) \frac{\dot{Z}_H^2}{\dot{Z}_Q^2} - 1 \right\} \quad (2.23)$$

where the variant t is defined by

$$t = \ln \frac{\mu}{M_Z} \quad (2.24)$$

These equations are non-linear, the first order differential ones and must be solved numerically. However, we notice that in the equation (2.23) of $\dot{\lambda}$, there are the terms like $\dot{\lambda}/\dot{Z}_H$ and $(\dot{\lambda}/\dot{Z}_H)^2$ whose boundary values at the scale Λ will be of the indefinite form of $0/0$. Therefore, we must specify the boundary condition satisfied by $\dot{\lambda}/\dot{Z}_H$ additionally so as to make the numerical solution of Eq.(2.23) become possible. Define the function

$$x = \dot{\lambda}/\dot{Z}_H \quad (2.25)$$

then it is easy to verify that x obeys the differential equation

$$(4\pi)^2 \frac{dx}{dt} = 12 \bar{g}_t^2 \left[x^2 + \left(\frac{1}{4} + E(t) \right) x + (1 + D(t)) \right] = 12 \bar{g}_t^2 [x - x_-(t)] [x - x_+(t)] \quad (2.26)$$

where the denotations

$$E(t) = \dot{Z}_H(t) \left[\frac{1}{6} \sum_{Q \neq t} \frac{d_Q(R)}{\dot{Z}_Q(t)} - \frac{1}{72} \bar{g}_1^2(t) - \frac{3}{8} \bar{g}_2^2(t) + \frac{4}{3} \bar{g}_3^2(t) \right],$$

$$D(t) = \dot{Z}_H^2(t) \left[\frac{1}{3} \sum_{Q \neq t} \frac{d_Q(R)}{\dot{Z}_Q^2(t)} - B(t) \right] \quad (2.27)$$

we may assume that $E(t) > 0$ and $D(t) > 0$ because of the weakness of the electroweak gauge couplings $\bar{g}_i^2 (i = 1, 2)$. The right-handed side of Eq.(2.26) has the zero-points

$$x_{\pm}(t) = \frac{1}{8} \left\{ -(1 + 4E(t)) \pm \left[(1 + 4E(t))^2 + 64(1 + D(t)) \right]^{1/2} \right\} \quad (2.28)$$

It is easy to verify that $x_-(t \rightarrow \infty)$ and $x_+(t \rightarrow 0)$ will respectively be the ultraviolet (UV) and the infrared (IR) stable fixed point [13]. This means that we will have the relations

$$\dot{\lambda}(t \rightarrow \infty) = \dot{Z}_H(t \rightarrow \infty) x_-(t \rightarrow \infty) \quad (2.29)$$

and

$$\dot{\lambda}(t = 0) = \dot{Z}_H(t = 0) x_+(t = 0) \quad (2.30)$$

They determine the UV and IR boundary value of $\dot{\lambda}$ respectively. In view of the fact that in the model with heavier fermions than the top-quark, the momentum cut-off Λ may be

lower [7], it might well not be a good approximation to use Eq.(2.29) to determine the UV boundary value of $\tilde{\lambda}/\tilde{Z}_H$ through replacing $t \rightarrow \infty$ in Eq.(2.29) by $t = t_\Lambda = \ln(\Lambda/M_Z)$. Alternatively, a more reliable way may be to use Eq.(2.30) to obtain the IR boundary value of $\tilde{\lambda}/\tilde{Z}_H$. This means that the value of $\tilde{\lambda}(t=0)$ can be calculated from Eq.(2.30) if $\tilde{Z}_H$ and all the $\tilde{Z}_Q$ have been solved in advance from Eqs.(2.21) and (2.22). The problem merely lies in that whether the solution of $\tilde{\lambda}$ so obtained also satisfied the UV compositeness boundary condition $\tilde{\lambda}(\Lambda) = 0$ in Eq.(2.15). In fact, it is not difficult to find from Eq.(2.26) that as long as $x_-(t) < x \leq x_+(t)$ then we will have $dx/dt < 0$ i.e. x will decrease as t increases. This process can begin at $t = 0$ i.e. from $x \leq x_+(0)$ and x will eventually be driven toward the constant $x_-(t_\Lambda)$ when $t \rightarrow t_\Lambda$. As a result, we will have $\tilde{\lambda}(t_\Lambda) = \tilde{Z}_H(t_\Lambda)x_-(t_\Lambda) = 0$ owing to $\tilde{Z}_H(t_\Lambda) = 0$ by Eq.(2.15). Therefore, taking the IR boundary condition (2.30) will be able to guarantee the UV compositeness boundary condition of $\tilde{\lambda}(t)$ to be satisfied automatically. The above result also indicates that the UV boundary conditions $\tilde{Z}_H(\mu \rightarrow \Lambda) \rightarrow 0$ and $\tilde{\lambda}(\mu \rightarrow \Lambda) \rightarrow 0$ in Eq.(2.15) are actually not independent each other. As for the low-energy boundary values of the running gauge coupling constants, we will take that

$$\bar{g}_1^2(M_Z) = 0.127, \quad \bar{g}_2^2(M_Z) = 0.446, \quad \bar{g}_3^2(M_Z) = 1.44 \quad (2.31)$$

and $M_Z = 91.17 \text{ GeV}$, following Ref.[2].

Based on the results of the standard model, the fermion mass m_Q and the Higgs-boson mass m_{h^0} will be determined by the roots of the equations

$$m_Q = \frac{v}{\sqrt{2}} \bar{g}_Q(m_Q) \quad (2.32)$$

and

$$m_{h^0}^2 = v^2 \tilde{\lambda}(m_{h^0}^2) \quad (2.33)$$

respectively, where $v = (\sqrt{2}G_F)^{-1/2} = 246 \text{ GeV}$ is the standard-model Higgs vacuum expectation value and G_F is the Fermi constant [14]. By means of the definitions (2.12)-(2.14), Eqs.(2.32) and (2.33) can also be changed into the following forms:

$$\tilde{Z}_H(t_t) = (v^2/2M_Z^2)e^{-2t_t}, m_t = M_Z e^{t_t}, \text{ for top quark} \quad (2.34)$$

$$\tilde{Z}_Q(t_Q) = (v^2/2M_Z^2)e^{-2t_Q}, m_Q = M_Z e^{t_Q}, \text{ for fermion } Q \neq t \quad (2.35)$$

and

$$\tilde{\lambda}(t_{h^0}) = (M_Z^2/v^2)e^{2t_{h^0}} \tilde{Z}_H^2(t_{h^0}), m_{h^0} = M_Z e^{t_{h^0}}, \text{ for Higgs boson } h^0 \quad (2.36)$$

3 The model with the fourth generation of quarks and leptons

The fourth generation of quark- and lepton-doublet will be denoted as (U, D) and (N, E) respectively and the Yukawa coupling constants between the heavy fermion fields and the Higgs field now include $\bar{g}_t$ and $\bar{g}_Q (Q = U, D, N, E)$. Since all the fermions in this model have the $SU_c(3) \times SU_L(2) \times U_Y(1)$ quantum number assignments of the standard model, we can easily put down the equations satisfied by the unconventional normalization constants $\tilde{Z}_H(\mu)$, $\tilde{Z}_Q(\mu) (Q = U, D, N, E)$ and $\tilde{\lambda}(\mu)$ from Eqs.(2.21)-(2.23) as follows.

$$(4\pi)^2 \frac{d}{dt} \ln \begin{pmatrix} \tilde{Z}_H \\ \tilde{Z}_U \\ \tilde{Z}_D \\ \tilde{Z}_N \\ \tilde{Z}_E \end{pmatrix} = - \begin{pmatrix} 9 & 6 & 6 & 2 & 2 \\ 6 & 9 & 3 & 2 & 2 \\ 6 & 3 & 9 & 2 & 2 \\ 6 & 6 & 6 & 5 & -1 \\ 6 & 6 & 6 & -1 & 5 \end{pmatrix} \begin{pmatrix} 1/\tilde{Z}_H \\ 1/\tilde{Z}_U \\ 1/\tilde{Z}_D \\ 1/\tilde{Z}_N \\ 1/\tilde{Z}_E \end{pmatrix} + \begin{pmatrix} 17/6 \\ 17/6 \\ 5/6 \\ 3/2 \\ 15/2 \end{pmatrix} \bar{g}_1^2 + \begin{pmatrix} 9/2 \\ 9/2 \\ 9/2 \\ 9/2 \\ 9/2 \end{pmatrix} \bar{g}_2^2 + \begin{pmatrix} 16 \\ 16 \\ 16 \\ 0 \\ 0 \end{pmatrix} \bar{g}_3^2 \quad (3.1)$$

$$(4\pi)^2 \frac{d}{dt} \tilde{\lambda} = 12 \left[\frac{\tilde{\lambda}^2}{\tilde{Z}_H^2} + \left(-\frac{1}{2\tilde{Z}_H} + \frac{2}{9}\bar{g}_1^2 + \frac{8}{3}\bar{g}_3^2 \right) \tilde{\lambda} + \frac{1}{16}(\bar{g}_1^4 + 2\bar{g}_1^2\bar{g}_2^2 + 3\bar{g}_2^4)\tilde{Z}_H^2 - \left(\frac{1}{\tilde{Z}_U^2} + \frac{1}{\tilde{Z}_D^2} + \frac{1}{3\tilde{Z}_N^2} + \frac{1}{3\tilde{Z}_E^2} \right) \tilde{Z}_H^2 - 1 \right] \quad (3.2)$$

By means of the variant t , the gauge coupling constants (2.9) become

$$\bar{g}_i^2(t) = \bar{g}_i^2(0) / \left[1 + \bar{g}_i^2(0) \frac{\beta_0^{(i)}}{8\pi^2} t \right], \quad i = 1, 2, 3 \quad (3.3)$$

where $\bar{g}_i^2(0) (i = 1, 2, 3)$ take the values given by Eq.(2.31) and

$$\beta_0^{(1)} = -\frac{163}{18}, \beta_0^{(2)} = \frac{11}{6}, \beta_0^{(3)} = \frac{17}{3} \quad (3.4)$$

Define

$$t_\Lambda = \ln \frac{\Lambda}{\mu} \quad (3.5)$$

then the compositeness boundary conditions (2.15) and (2.20) will take the following forms :

$$\tilde{Z}_H(t) \rightarrow 0, \tilde{Z}_Q(t) \rightarrow 0 (Q = U, D, N, E) \text{ and } \tilde{\lambda}(t) \rightarrow 0 \text{ when } t \rightarrow t_\Lambda \quad (3.6)$$

and

$$\frac{\tilde{Z}_H(t_\Lambda)}{\tilde{Z}_U(t_\Lambda)} = \eta_{Q_U} \quad (3.7a)$$

$$\frac{\tilde{Z}_H(t_\Lambda)}{\tilde{Z}_D(t_\Lambda)} = \eta_{Q_D} \left[\frac{1 - \frac{2}{9}b_1 + \frac{16}{3}b_3}{1 + \frac{4}{9}b_1 + \frac{16}{3}b_3} \right]^2 \frac{1 + \frac{4}{9}b_1 + \frac{4}{3}b_3}{1 + \frac{1}{9}b_1 + \frac{4}{3}b_3} \quad (3.7b)$$

$$\frac{\tilde{Z}_H(t_\Lambda)}{\tilde{Z}_N(t_\Lambda)} = \eta_{Q_N} \frac{1 + \frac{4}{9}b_1 + \frac{4}{3}b_3}{(1 + \frac{4}{9}b_1 + \frac{16}{3}b_3)^2} \frac{1 + \frac{1}{36}b_1 + \frac{3}{4}b_2 + \frac{4}{3}b_3}{1 + \frac{1}{4}b_1 + \frac{3}{4}b_3} \quad (3.7c)$$

$$\frac{\tilde{Z}_H(t_\Lambda)}{\tilde{Z}_E(t_\Lambda)} = \eta_{Q_E} \left[\frac{1 + 2b_1}{1 + \frac{4}{9}b_1 + \frac{16}{3}b_3} \right]^2 \frac{1 + \frac{1}{36}b_1 + \frac{3}{4}b_2 + \frac{4}{3}b_3}{1 + \frac{1}{4}b_1 + \frac{3}{4}b_2} \frac{1 + \frac{4}{9}b_1 + \frac{4}{3}b_3}{1 + b_1} \quad (3.7d)$$

where by Eq.(2.19),

$$b_i = \left[\beta_0^{(i)} + 8\pi^2/\bar{g}_i^2(0)t_\Lambda \right]^{-1}, i = 1, 2, 3 \quad (3.8)$$

and we have made the assumption that

$$\eta_{Q_i} = (g_Q^0/g_i^0)^2 = \text{constant, for } Q = U, D, N, E \quad (3.9)$$

i.e.

$$g_U^0 = g_D^0 = g_N^0 = g_E^0 \quad (3.10)$$

which also implies that $m_U^0 = m_D^0 = m_N^0 = m_E^0$, i.e. the massed of the fourth generation of fermions are degenerate in the bubble approximation. This assumption does not affect the essential results but it will simplify our calculations. In fact, under the assumption (3.9) only the unique free parameter η_{Q_i} is included in the total boundary conditions of $\tilde{Z}_H(t)$ and $\tilde{Z}_Q(t) (Q = U, D, N, E)$. In addition, we may note that in the IR boundary condition of $\tilde{\lambda}(t)$ obtained from Eq.(2.30) with the following form

$$\tilde{\lambda}(0) = \frac{\tilde{Z}_H(0)}{8} \left\{ \left([1 + 4E(0)]^2 + 64[1 + D(0)] \right)^{1/2} - [1 + 4E(0)] \right\},$$

$$E(0) = \tilde{Z}_H(0) \left\{ \frac{1}{2} \left[\frac{1}{\tilde{Z}_U(0)} + \frac{1}{\tilde{Z}_D(0)} \right] + \frac{1}{6} \left[\frac{1}{\tilde{Z}_N(0)} + \frac{1}{\tilde{Z}_E(0)} \right] - \frac{1}{72}\bar{g}_1^2(0) - \frac{3}{8}\bar{g}_2^2(0) + \frac{4}{3}\bar{g}_3^2(0) \right\},$$

$$D(0) = \tilde{Z}_H^2(0) \left\{ \frac{1}{\tilde{Z}_U^2(0)} + \frac{1}{\tilde{Z}_D^2(0)} + \frac{1}{3\tilde{Z}_N^2(0)} + \frac{1}{3\tilde{Z}_E^2(0)} - \frac{1}{16} [\bar{g}_1^4(0) + 2\bar{g}_1^2(0)\bar{g}_2^2(0) + 3\bar{g}_2^4(0)] \right\} \quad (3.11)$$

there is no free parameter if the $\tilde{Z}_H$ and all the $\tilde{Z}_Q$ have been found out through the equation system (3.1). Therefore, the parameter η_{Q_i} will also be the only free parameter contained in the total equations (3.1) and (3.2). This fact greatly reduces arbitrariness of the solutions.

A significant question deserves to be raised. What kind of result could be obtained if we make analysis of the fixed point behavior for the functions $\tilde{Z}_H$ and $\tilde{Z}_Q (Q = U, D, N, E)$ in Eq.(3.1), as was made for $\tilde{\lambda}$ and $\tilde{Z}_H$ in Eq.(2.23) through the function $x = \tilde{\lambda}/\tilde{Z}_H$? To answer this question, let us define the functions

$$y(t) = \frac{\tilde{Z}_H(t)}{\tilde{Z}_U(t)}, \quad z(t) = \frac{\tilde{Z}_U(t)}{\tilde{Z}_D(t)}, \quad w(t) = \frac{\tilde{Z}_H(t)}{\tilde{Z}_N(t)}, \quad u(t) = \frac{\tilde{Z}_N(t)}{\tilde{Z}_E(t)} \quad (3.12)$$

It is easy by means of Eq.(3.1) to verify that these functions obey the following equations:

$$\begin{aligned} (4\pi)^2 \frac{dy}{dt} &= \frac{3}{\tilde{Z}_U} (y - yz - 1) \\ (4\pi)^2 \frac{dz}{dt} &= \frac{6}{\tilde{Z}_D} (z - 1 + \frac{\bar{g}_1^2}{3y} \tilde{Z}_H) \\ (4\pi)^2 \frac{dw}{dt} &= \frac{3}{\tilde{Z}_N} (w - wu - 1 + [\frac{4}{9}\bar{g}_1^2 + \frac{16}{3}\bar{g}_3^2] \tilde{Z}_H) \\ (4\pi)^2 \frac{du}{dt} &= \frac{6}{\tilde{Z}_E} (u - 1 - \frac{\bar{g}_1^2}{w} \tilde{Z}_H) \end{aligned} \quad (3.13)$$

If the gauge interactions are all neglected (i.e. set $\bar{g}_i = 0 (i = 1, 2, 3)$), then it may turn out from Eq.(3.13) that $z = 1$ and $u = 1$ will be the IR stable fixed points of $z(t)$ and $u(t)$. However, substituting $z = 1$ and $u = 1$ into the other two equations in Eq.(3.13) we will obtain

$$(4\pi)^2 \frac{dy}{dt} = -\frac{3}{\tilde{Z}_U} \text{ and } (4\pi)^2 \frac{dw}{dt} = -\frac{3}{\tilde{Z}_N}$$

which do not lead to any fixed point of $y(t)$ and $w(t)$. Once the gauge interactions are opened, it is easy to see that the right-handed sides of all the equations in Eq.(3.13) contain no zero-point, hence no fixed point can be touched on here. In this case, extra

boundary conditions of $\tilde{Z}_H$ and $\tilde{Z}_Q(Q = U, D, N, E)$ must be imposed, as have been made above. However, the IR stable fixed point $z = 1$ and $u = 1$ which would exist if the gauge interactions are closed will be bound to lead to the results that $\tilde{Z}_U/\tilde{Z}_D \approx 1$ and $\tilde{Z}_N/\tilde{Z}_E \approx 1$ at low energy region, because the gauge interactions are certainly weak and will only bring about quite small corrections even if they are opened. Hence it may be believed that the fermion masses to be obtained must have such feature that $m_U \approx m_D$ and $m_N \approx m_E$.

We will solve Eqs.(3.1) and (3.2) with the boundary conditions (3.6), (3.7) and (3.11) numerically. The zeroes of $\tilde{Z}_H$, $\tilde{Z}_Q$ and $\tilde{\lambda}$ at the UV boundary $t = t_\Lambda$ will be approximated by 10^{-6} . Then the mass equations (2.34)-(2.36) will be used to find out the masses of the fermions t, U, D, N, E and the Higgs boson h^0 . The only free parameter η_{Qt} in the boundary conditions will be chosen so as to fit it to the values of the top-quark mass m_t . We list in Table I the results of the masses of the fourth generation of quarks and leptons and the Higgs boson for different momentum cut-off Λ when the top-quark mass m_t is taken to be the present experimental value 174 GeV [15] and part of these results have appeared in Ref.[10]. In order to examine the variation of the results induced by the change of m_t , we list in Table II the particle masses for the momentum cut-off $10^7 - 5 \times 10^3 \text{ GeV}$ when $m_t = 160 \text{ GeV}, 170 \text{ GeV}$ and 180 GeV respectively.

Table I. The masses of the fourth generation of fermions U, D, N, E and the Higgs boson h^0 for different values of the momentum cut-off Λ . The values of parameter η_{Qt} have been chosen so as to fit them to the top- quark mass $m_t = 174 \text{ GeV}$. The dashes in the Table mean that no solution bigger than the Z -boson mass $M_Z = 91.17 \text{ GeV}$ exists.

$\Lambda(\text{GeV})$	10^{15}	10^{10}	10^8	10^7	10^6	10^5	5×10^4	2.5×10^4	10^4	5×10^3
η_{Qt}	0.0111	0.0277	0.053	0.083	0.149	0.33	0.44	0.60	0.942	1.376
$m_U(\text{GeV})$	152	176	195	209	229	261	275	293	326	366
$m_D(\text{GeV})$	148	174	193	207	228	260	274	292	325	365
$m_N(\text{GeV})$	---	---	---	90	110	142	156	174	207	244
$m_E(\text{GeV})$	---	---	---	93	113	144	159	176	209	246
$m_{h^0}(\text{GeV})$	193	219	242	260	287	333	353	379	427	481

Table II. The masses of the top-quark, the fourth generation of quark-leptons U, D, N, E and the Higgs boson h^0 for different values of the momentum cut-off Λ . The values of the parameter η_{Qt} have been chosen so as to fit them to the top-quark mass $m_t = 160 \text{ GeV}, 170 \text{ GeV}$ and 180 GeV in each value of Λ . As a comparison, in the last line we also give the mass $m_{h^0}^0$ of the Higgs boson in the bubble approximation when $m_t = 160 \text{ GeV}$.

$\Lambda(\text{GeV})$	10^7			10^6			10^5		
η_{Qt}	0.155	0.1	0.064	0.275	0.177	0.116	0.56	0.385	0.262
$m_t(\text{GeV})$	160	170	180	160	170	180	160	170	180
$m_U(\text{GeV})$	215	211	206	235	231	226	266	262	258
$m_D(\text{GeV})$	214	210	204	233	229	225	265	261	257
$m_N(\text{GeV})$	93	91	88	112	110	108	145	143	140
$m_E(\text{GeV})$	96	94	92	115	113	111	147	145	143
$m_{h^0}(\text{GeV})$	264	261	259	291	288	286	336	334	331
$m_{h^0}^0(\text{GeV})$	—	—	—	324	—	—	395	—	—

$\Lambda(\text{GeV})$	5×10^4			10^4			5×10^3		
η_{Qt}	0.72	0.505	0.353	1.38	1.05	0.8	1.91	1.51	1.19
$m_t(\text{GeV})$	160	170	180	160	170	180	160	170	180
$m_U(\text{GeV})$	279	276	273	330	327	324	369	367	364
$m_D(\text{GeV})$	278	275	272	329	326	323	369	366	363
$m_N(\text{GeV})$	159	157	155	209	208	206	247	245	243
$m_E(\text{GeV})$	161	159	157	212	210	208	249	247	245
$m_{h^0}(\text{GeV})$	356	354	352	430	428	426	482	482	481
$m_{h^0}^0$	—	—	—	567	—	—	686	—	—

It can be seen from Table I and Table II that for the full four-generation quark-lepton condensate scheme of the minimal electroweak symmetry breaking, it is entirely possible to accomodate itself to the present experimental top-quark mass $m_t = 174 \text{ GeV}$ and its possible scope of change $160 - 180 \text{ GeV}$ for the acceptable momentum cut-off $\Lambda = 10^6 - 5 \times 10^3 \text{ GeV}$ [7] by appropriately choosing values of the parameter η_{Qt} . Under the assumption that the fourth generation of fermions are mass-degenerate in the bubble approximation, once m_t is specified then the masses of all the other heavy particles in the scheme will be uniquely determined by the RG equations.

It is noted that for most values of Λ except the lower values $\Lambda = 10^4 - 5 \times 10^3 \text{ GeV}$ we always have the parameter $\eta_{Qt} < 1$. This implies that the masses of the fourth generation of fermions do not need always to be bigger than the top-quark mass in the bubble approximation so as to accomodate the model to the physical top-quark mass $m_t = 174 \text{ GeV}$ and to obtain the heavier fourth generation of fermions. This remarkable fact only reflects the strong correlation effect of the running Higgs-Yukawa coupling constants between the fourth generation of fermions and the top-quark. In addition, as is shown in Table II, it is natural that for a fixed value of Λ , we must reduce the value of η_{Qt} so as to obtain a bigger m_t .

When $m_t = 174 \text{ GeV}$, for the fourth generation of quarks U and D , we always have $m_U, m_D > m_t$ for even $\Lambda = 10^{10} - 5 \times 10^3 \text{ GeV}$. For the fourth generation of leptons N and E , however, we may have either $m_N, m_E < m_t$ when $\Lambda > 2.5 \times 10^4 \text{ GeV}$ or $m_N, m_E > m_t$ when $\Lambda = 2.5 \times 10^4 - 5 \times 10^3 \text{ GeV}$. In any case, the masses of the fourth generation of fermions will decrease as the momentum cut-off Λ increases. In particular, when $\Lambda > 10^7 \text{ GeV}$ we will be led to $m_N, m_E < m_Z$, as can be seen from Table I. This is because Eq.(2.35) for $Q = N, E$ will have no solution bigger than m_Z in this case. Therefore, based on the requirement that $m_N, m_E > m_Z$, the acceptable cut-off $\Lambda = 10^6 - 5 \times 10^3 \text{ GeV}$ are still the optimal selection in the scheme with four generations of quarks and leptons when the full dynamics has been included in. On the other hand, it can be seen from Table II that when m_t increase from 160 GeV to 180 GeV , the masses

of the fourth generation of fermions will decrease for a fixed value of Λ . At the same time, the scope of the Λ value where we may have $m_N, m_E > m_t$ will become narrower and shrink toward to the direction of $5 \times 10^3 \text{ GeV}$, for instance, from $\Lambda < 5 \times 10^4 \text{ GeV}$ for $m_t = 160 \text{ GeV}$ to $\Lambda < \sim 2 \times 10^4 \text{ GeV}$ for $m_t = 180 \text{ GeV}$. Hence, the requirement that the fourth generation of leptons are heavier than the top-quark will confine the compositeness scale Λ of the Higgs boson to quite a low value. This conclusion also coincides with the results of $\Lambda < 10 \text{ TeV}$ obtained from the analyses of the fourth generation fermion condensate scheme in Ref.[5] when the fourth generation left-handed neutrino is required to have large Dirac mass.

The results in Table I and Table II also show that we always have $m_U \approx m_D$ and $m_N \approx m_E$, i.e. the quark-doublet and the lepton-doublet in the fourth generation respectively have only very small mass-splittings even after the full interactions have been included. This verifies the conclusion drawn from the above discussions about the IR fixed points and is also consistent with the result of the scheme in Ref.[5].

The Higgs boson mass m_{h^0} will rapidly increase as the momentum cut-off Λ decreases. For example, when $m_t = 174 \text{ GeV}$, m_{h^0} will vary in the region $287 - 481 \text{ GeV}$ for $\Lambda = 10^6 - 5 \times 10^3 \text{ GeV}$. In the same region of Λ , m_U will vary in the region $228 - 365 \text{ GeV}$. The results coincide with the general pattern that both m_{h^0} and m_U increase as Λ decreases in the bubble approximation [9]. In addition, the mass inequalities $2(m_Q)_{\min} < m_{h^0} < 2(m_Q)_{\max}$ ($Q = t, U, D, N, E$) valid in the bubble approximation are also satisfied in present case. This mass constraint seems to be a general feature of the composite Higgs boson composed of many generations of fermions. On the other hand, we find that the Higgs boson mass m_{h^0} is not sensitive to a possible change of m_t . For instance, when m_t is changed from 160 GeV to 180 GeV , m_{h^0} will reduce only by 5 GeV , and as Λ decreases such reduction of m_{h^0} will become smaller and smaller and eventually be down to $< 1 \text{ GeV}$ for $\Lambda = 5 \times 10^3 \text{ GeV}$. This insensitivity of m_{h^0} to the variation of m_t may be explained by the following fact that the top-quark mass m_t is now not the biggest one among the heavy fermion masses m_Q ($Q = t, U, D, N, E$). When Λ decreases, the biggest fermion mass

$m_U \approx m_D$ will increase and m_{h^0} will depend upon m_U and m_D more and more but upon m_t less and less. From the last line of Table II it can also be seen that when $m_t = 160 \text{ GeV}$, if Λ varies from 10^6 GeV to $5 \times 10^3 \text{ GeV}$ then the difference between the tree and the physical Higgs-boson mass $m_{h^0}^0 - m_{h^0}$ will be in the region $33 - 204 \text{ GeV}$. Therefore, the low-energy gauge interactions and Higgs dynamics have quite strong effect on the Higgs-boson mass and, the lower is Λ , the stronger is this effect.

In our calculations we have used the perturbative RG equations (3.1) and (3.2) but non-perturbative UV boundary conditions (3.6), since the latter means that $\tilde{g}_Q^2(t_\Lambda) \rightarrow \infty$ ($Q = t, U, D, N, E$). Hence the problem of applicability of the perturbation theory should be examined, in particular, in present case when the scheme has the lower values of the momentum cut-off Λ . In fact, we may point out that the total results of particle masses will remain to be in the region where the perturbation theory is applicable. To verify this point, we notice that the scales where the perturbation theory is applicable may be assumed to satisfy the conditions

$$\frac{\tilde{g}_Q^2(t)}{4\pi} = \frac{1}{4\pi\tilde{Z}_Q(t)} < 1, \quad Q = t, U, D, N, E \quad (3.14)$$

hence the root of the equation

$$\frac{1}{4\pi\tilde{Z}_Q(t)} = 1, \quad (3.15)$$

denoted by t_{Qp} , can be considered as a critical scale which will distinguish the perturbative momentum region with $t < t_{Qp}$ from the non-perturbative one with $t > t_{Qp}$, since all the $\tilde{Z}_Q(t)$ are actually monotonically descent functions of the argument t . The scales of the obtained particle masses may be expressed by

$$t_Q = \ln \frac{m_Q}{m_Z}, \quad \text{for } Q = t, U, D, N, E \quad (3.16)$$

In Table III we give a comparison between t_{Qp} and t_Q for three typical momentum cut-off $\Lambda = 10^6 \text{ GeV}, 5 \times 10^3 \text{ GeV}$ and $\Lambda_c = 1092 \text{ GeV}$ [7].

Table III. A comparison between the critical perturbative scales t_{Qp} and the particle mass scales $t_Q(Q = t, U, D, N, E)$ for the momentum cut-off $\Lambda = 10^6 \text{ GeV}, 5 \times 10^3 \text{ GeV}$ and $\Lambda_c = 1092 \text{ GeV}$ when $m_t = 174 \text{ GeV}$. The dashes represent that no solution can be found in that case.

$\Lambda(\text{GeV})$	10^6	5×10^3	1092
η_{Qt}	0.149	1.376	3.72
$m_t(\text{GeV})$	174	174	174
t_{tp}	8.769	3.764	2.345
t_{Up}	8.655	3.210	1.651
t_{Dp}	8.656	3.211	1.652
t_{Np}	8.946	3.484	1.893
t_{Ep}	8.939	3.482	1.891
t_t	0.647	0.647	0.647
t_U	0.920	1.389	---
t_D	0.915	1.387	---
t_N	0.184	0.985	2.283
t_E	0.210	0.993	2.293

It is seen from Table III that in the case with $\Lambda = 10^6 \text{ GeV}$ and $\Lambda = 5 \times 10^3 \text{ GeV}$ we always have $t_Q < t_{Qp}(Q = t, U, D, N, E)$. This implies that the physical results are indeed in the region where Eq.(3.14) is valid thus the perturbation theory is applicable. It is also not difficult to find that the higher is Λ , the better is applicability of the perturbation theory. Conversely, when Λ go down to $\Lambda_c = 1092 \text{ GeV}$ (the critical momentum cut-off), if m_t is fixed at 174 GeV , then there will be no solution for m_U and m_D owing to absence of the root of the mass equation (2.35) and for m_N and m_E although there could be a solution respectively, their values appear in the non-perturbative energy region. This indicates a breakdown of the perturbation theory at $\Lambda = \Lambda_c$.

4 The model without the fourth generation of leptons

In this section we will consider a model which contains the fourth generation of extra quarks but no corresponding leptons. In order to avoid the $SU_L(2) \times U_Y(1)$ gauge anomaly[16], the extra left-handed quark-doublet $Q_L^T = (U, D)_L$ will have $U_Y(1)$ charge $Y_{Q_L} = 0$ and the extra right-handed quarks U_R and D_R will have $U_Y(1)$ charge $Y_{U_R} = 1$ and $Y_{D_R} = -1$ respectively. Thus the extra quarks U and D will have electric charges $e_U = 1/2$ and $e_D = -1/2$. These quarks are all also assumed in the triplet representations of the $SU_c(3)$ group. Owing to absence of the extra leptons we will obtain some different results from ones of the model in Sect.3.

We still ignore the Yukawa coupling constant $\bar{g}_b$ between the bottom-quark and the Higgs boson. Thus we will have the Higgs-Yukawa coupling constants $\bar{g}_t, \bar{g}_U$ and $\bar{g}_D$ as well as a Higgs quartic self-coupling constant $\bar{\lambda}$ to be left. These coupling constants obey Eqs.(2.4) and (2.5). However, it is not difficult to find that since Eqs.(2.4) contains only the squared quantum numbers e_Q^2 and $Y_{Q_L}^2$ of the electroweak gauge group, the equations obeyed by $\bar{g}_U$ and $\bar{g}_D$ will be in entirely symmetric position, i.e. the two equations may exchange each other under the transformation $\bar{g}_U \leftrightarrow \bar{g}_D$. Hence, the subtraction between the two equations will give that

$$(4\pi)^2 \mu \frac{d}{d\mu} (\bar{g}_U - \bar{g}_D) = \left[\frac{9}{2} (\bar{g}_U^2 + \bar{g}_D^2) + 3\bar{g}_U \bar{g}_D + 3\bar{g}_t^2 - \frac{3}{4} \bar{g}_1^2 - \frac{9}{4} \bar{g}_2^2 - 8\bar{g}_3^2 \right] (\bar{g}_U - \bar{g}_D) \quad (4.1)$$

Obviously, $\bar{g}_U = \bar{g}_D$ is a solution of Eq.(4.1) at all momentum scale μ . Hence we can take $\bar{g}_U = \bar{g}_D$ and reduce a Higgs-Yukawa coupling constant. The remaining coupling constants will be $\bar{g}_t, \bar{g}_U$ and $\bar{\lambda}$. Correspondingly, we will have only three unconventional wave function normalization constants $\tilde{Z}_H, \tilde{Z}_U$ and $\tilde{\lambda}$ respectively defined by Eqs.(2.12), (2.14) and (2.13). The equations (2.21)-(2.23) obeyed by them now take the following forms:

$$(4\pi)^2 \frac{d}{dt} \tilde{Z}_H = -9 - 12\tilde{Z}_H/\tilde{Z}_U + \left(\frac{17}{6} \bar{g}_1^2 + \frac{9}{2} \bar{g}_2^2 + 16\bar{g}_3^2 \right) \tilde{Z}_H \quad (4.2)$$

$$(4\pi)^2 \frac{d}{dt} \tilde{Z}_U = -12 - 6\tilde{Z}_U/\tilde{Z}_H + \left(\frac{3}{2} \bar{g}_1^2 + \frac{9}{2} \bar{g}_2^2 + 16\bar{g}_3^2 \right) \tilde{Z}_U \quad (4.3)$$

$$(4\pi)^2 \frac{d}{dt} \tilde{\lambda} = 12 \left[\tilde{\lambda}^2/\tilde{Z}_H^2 + \left(\frac{2}{9} \bar{g}_1^2 + \frac{8}{3} \bar{g}_3^2 - \frac{1}{2\tilde{Z}_H} \right) \tilde{\lambda} + \frac{1}{16} (\bar{g}_1^4 + 2\bar{g}_1^2 \bar{g}_2^2 + 3\bar{g}_2^4) \tilde{Z}_H^2 - 2\tilde{Z}_H^2/\tilde{Z}_U^2 - 1 \right] \quad (4.4)$$

where the running gauge coupling constants are expressed by

$$\bar{g}_i^2(t) = \bar{g}_i^2(0) / \left[1 + \bar{g}_i^2(0) \frac{\bar{\beta}_0^{(i)}}{8\pi^2} t \right], i = 1, 2, 3 \quad (4.5)$$

with the constants

$$\bar{\beta}_0^{(1)} = -\frac{47}{6}, \bar{\beta}_0^{(2)} = \frac{13}{6} \text{ and } \bar{\beta}_0^{(3)} = \frac{17}{3} \quad (4.6)$$

The UV compositeness boundary conditions (2.15) and (2.16) become that

$$\tilde{Z}_H(t_\Lambda) = \tilde{Z}_U(t_\Lambda) = \tilde{\lambda}(t_\Lambda) = 0 \quad (4.7)$$

$$\frac{\tilde{Z}_H(t_\Lambda)}{\tilde{Z}_U(t_\Lambda)} = \eta_{Ut} \left(\frac{1 + \frac{16}{3} \bar{b}_3}{1 + \frac{4}{9} \bar{b}_1 + \frac{16}{3} \bar{b}_3} \right)^2 \frac{(1 + \frac{1}{36} \bar{b}_1 + \frac{3}{4} \bar{b}_2 + \frac{4}{3} \bar{b}_3)(1 + \frac{4}{9} \bar{b}_1 + \frac{4}{3} \bar{b}_3)}{(1 + \frac{3}{4} \bar{b}_2 + \frac{4}{3} \bar{b}_3)(1 + \frac{1}{4} \bar{b}_1 + \frac{4}{3} \bar{b}_3)} \quad (4.8)$$

with

$$\bar{b}_i = \left[\bar{\beta}_0^{(i)} + \frac{8\pi^2}{\bar{g}_i^2(0)t_\Lambda} \right]^{-1}, i = 1, 2, 3 \quad (4.9)$$

and t_Λ is defined by Eq.(3.5). In order to overcome the difficulty that $\tilde{\lambda}(t_\Lambda)/\tilde{Z}_H(t_\Lambda)$ in Eq.(4.4) has the indefinite form of 0/0 which is encountered when the equation is solved numerically, we will replace the UV boundary condition $\tilde{\lambda}(t_\Lambda) = 0$ in Eq.(4.7) by the IR boundary condition

$$\tilde{\lambda}(0) = \frac{\tilde{Z}_H(0)}{8} \left\{ \left([1 + 4\bar{E}(0)]^2 + 64 [1 + \bar{D}(0)] \right)^{1/2} - [1 + 4\bar{E}(0)] \right\} \quad (4.10)$$

where

$$\bar{E}(0) = \frac{\tilde{Z}_H(0)}{\tilde{Z}_U(0)} - \left[\frac{1}{72} \bar{g}_1^2(0) + \frac{3}{8} \bar{g}_2^2(0) - \frac{4}{3} \bar{g}_3^2(0) \right] \quad (4.11)$$

$$\bar{D}(0) = 2 \frac{\tilde{Z}_H^2(0)}{\tilde{Z}_U^2(0)} - \frac{1}{16} [\bar{g}_1^4(0) + 2\bar{g}_1^2(0)\bar{g}_2^2(0) + 3\bar{g}_2^4(0)] \tilde{Z}_H^2(0) \quad (4.12)$$

In Table IV we list the results of the masses of the heavy fermions and the Higgs boson obtained by numerically solving the RG equations (4.2)-(4.4) and the mass equations (2.34)-(2.36).

Table IV. The masses of the fourth generation of extra quarks $U(D)$ and the Higgs boson h^0 for different values of the momentum cut-off Λ in the model without the fourth generation of leptons. The values of the parameter η_{U_t} have been chosen so as to fit them to the top-quark mass $m_t = 174 \text{ GeV}$ in each value of Λ .

$\Lambda(\text{GeV})$	10^{19}	10^{15}	10^{12}	10^{11}	10^{10}	10^9
η_{U_t}	0.00738	0.0112	0.018	0.0221	0.0282	0.0377
$m_U(m_D)(\text{GeV})$	143	154	168	175	182	192
$m_{h^0}(\text{GeV})$	187	198	212	219	228	240

$\Lambda(\text{GeV})$	10^8	10^7	10^6	10^5	5×10^4	10^4	5×10^3
η_{U_t}	0.0538	0.085	0.155	0.352	0.476	1.066	1.59
$m_U(m_D)(\text{GeV})$	204	220	243	280	297	359	407
$m_{h^0}(\text{GeV})$	255	277	309	363	388	478	546

The results in Table IV indicate an important difference between the models without and with the fourth generation of leptons. The latter has quite severe limitation to the momentum cut-off Λ but the former has much looser one when we require that the fourth generation of fermions are heavier than the top-quark. For instance, in the model without the fourth generation of leptons, if we demand $m_U > m_t = 174 \text{ GeV}$ then Λ can be quite high and even up to 10^{11} GeV . This result also break the constraint of the acceptable momentum cut-off $\Lambda \leq 10^6 \text{ GeV}$ in the bubble approximation[7]. Except this difference, both models are very similar in all the other hands. For the model without the fourth generation of lepton, we may also find the feature that when m_t is fixed m_U and m_{h^0} will increase as Λ decreases. For example, if $m_t = 174 \text{ GeV}$ then when Λ varies from 10^{11} GeV to $5 \times 10^3 \text{ GeV}$, m_U will varies from 175 GeV to 407 GeV . At the same time, the Higgs boson mass m_{h^0} will increase from 219 GeV to 546 GeV .

5 Conclusions

We have made the detailed renormalization group analyses of the masses of the heavy fermions and the Higgs boson in the model of minimal electroweak symmetry breaking of NJL-type with full four generations of quarks and leptons. Under the assumption that the fourth generation of fermions are mass-degenerate in the bubble approximation, by appropriate selections of the only free parameter η_{Q_t} , the squared mass-ratio between the fourth generation of fermions and the top-quark in the bubble approximation, the model could accomodate itself to the top-quark mass $m_t = 160 - 180 \text{ GeV}$.

The results indicate that the masses of the fourth generation of fermions will generally increase as the momentum cut-off Λ decreases and the quark-doublet and the lepton-doublet in the fourth generation are respectively approximately mass-degenerate. The latter result is consistent with the analysis of the IR stable fixed points for the coupling constants. The requirement that the fourth generation of leptons are heavier than the Z-boson and the top-quark will limit Λ to the region $10^6 - 5 \times 10^3 \text{ GeV}$ and $2.5 \times 10^4 - 5 \times 10^3 \text{ GeV}$ respectively when $m_t = 174 \text{ GeV}$. In the latter case, the masses of the fourth

generation of quarks and leptons will vary in the region $292 - 366 \text{ GeV}$ and $174 - 246 \text{ GeV}$ separately.

The Higgs boson mass m_{h^0} in the model is predicted in the region $287 - 481 \text{ GeV}$ for $\Lambda = 10^6 - 5 \times 10^3 \text{ GeV}$ when $m_t = 174 \text{ GeV}$ and it is not sensitive to the possible change of m_t . For $\Lambda = 10^6 - 5 \times 10^3 \text{ GeV}$, m_{h^0} will decrease only by $5 - 1 \text{ GeV}$ when m_t from 160 GeV to 180 GeV and the lower is Λ , the smaller is the variation of m_{h^0} .

We also find that in a model without the fourth generation of leptons the limitation to the momentum cut-off Λ imposed by the requirement that the masses of the fourth generation of extra quarks are bigger than $m_t = 174 \text{ GeV}$ could be greatly loosen and up to 10^{11} GeV . In this case the masses of the extra quarks and the Higgs boson will vary respectively in the region $175 - 407 \text{ GeV}$ and $219 - 546 \text{ GeV}$ for $\Lambda = 10^{11} - 5 \times 10^3 \text{ GeV}$.

The experimental test of the full four-generation fermion condensate scheme discussed in this paper depends on finding the heavy fourth generation of fermions and, in particular, the Higgs boson with the masses predicted above. Since the predicted particle masses are only in the region $200 \sim 500 \text{ GeV}$, it is expected that the future experiments operated on Tevaron and LHC could give some answer.

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