Twoloop corrections to the fermionic decay rates of the Higgs boson

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and $z$ of the unbroken theory. The two-loop corrections are opposite in sign to the one-loop electroweak corrections. They exceed the one-loop corrections in magnitude for $M_H > 1114 \text{ GeV}$ and increase in relative magnitude as $M_H^2$ for larger values of $M_H$. We conclude that the perturbation expansion in powers of $G_F M_H^2$ breaks down for $M_H \approx 1100 \text{ GeV}$. We discuss briefly the QCD and the complete one-loop electroweak corrections to $H \to b\bar{b}, t\bar{t}$ and comment on the validity of the equivalence theorem. Finally we note how a very heavy Higgs boson could be described in a phenomenological manner.

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I. INTRODUCTION

One of the great puzzles of contemporary elementary particle research is whether nature makes use of the Higgs mechanism to generate the observed particle masses. In the minimal standard model (SM) of electroweak interactions, the symmetry breaking is implemented using this mechanism with one weak-isospin doublet of complex scalar fields with weak hypercharge $Y = 1$. With the spontaneous breaking of the $SU(2)_L \otimes U(1)_Y$ gauge symmetry, three of the four scalar degrees of freedom are absorbed to create the longitudinal polarization states of the intermediate bosons $W^\pm$ and $Z$. At the same time, the quarks and charged leptons acquire masses through their Yukawa interaction with the scalar doublet. There remains at the end one neutral scalar boson with positive parity and charge conjugation, the physical Higgs boson $H$.

Most of the properties of the scalar or Higgs sector of the SM are fixed experimentally, e.g., the vacuum expectation value $v = 2^{-1/4}G_F^{1/2} \approx 246 \text{ GeV}$ the coupling of the Higgs boson to the gauge bosons $g_{VVH} = 2^{5/4}G_F^{1/2} M_V^2$ where $V = W^\pm, Z$; and the coupling of the Higgs to the fermions $g_{f\bar{f}H} = 2^{1/4}G_F^{1/2} m_f$. However, the mass $M_H$ of the Higgs boson and its quartic self-coupling $\lambda = G_F M_H^2 / \sqrt{2}$ are unspecified. It is therefore of considerable interest to analyze processes which can give theoretical limits on $M_H$ for test the effects of the quartic coupling phenomenologically.

The range of possible Higgs masses is constrained from below both experimentally and theoretically. The non-detection of the $Z$-boson decay $Z \rightarrow f\bar{f}H$ at LEP 1 and SLC has ruled out a Higgs mass of less than 63.9 GeV at the 95% confidence level [1]. Depending on the mass of the top quark, the requirement that the vacuum be the true ground state could provide an even more stringent theoretical lower bound [2]. Other theoretical arguments bound the Higgs mass from above. Non-perturbative lattice computations [3–11] give an upper limit for $M_H$ of about 710 GeV [4]. Unitarity arguments in intermediate-boson scattering at high energies [5–6] and considerations concerning the range of validity of perturbation theory [7–8] establish an upper bound $M_H < (8\pi\sqrt{2}/3G_F)^{1/2} \approx 1 \text{ TeV}$ in a weakly interacting.
theory. The unitarity bound on $M_H$ is lowered significantly when the approach of [516] is extended to higher orders; see [9110] and references therein. However the improved bound depends on the energy scale up to which the SM is assumed to remain valid.

A violation of the unitarity bound on $M_H$ is presumably a signal for the onset of strong interactions in the Higgs sector of the SM, a possibility which is of considerable interest in its own right [11]. It would therefore be desirable to sharpen the bound by removing the uncertainty associated with the mass scale at which it is applied, or to find a separate scale-independent bound. In fact the work presented here on two-loop electroweak corrections to the fermionic decay modes of the Higgs boson $H \rightarrow f\bar{f}$ gives a scale-independent limit on $M_H$ in a weakly interacting theory. We find that the two-loop corrections to the fermionic decay rates exceed the one-loop corrections in magnitude for $M_H \approx 1114$ GeV and increase in relative magnitude proportionally to $M_H^2$ for larger Higgs-boson masses. We conclude as reported previously [12-14] that the perturbative expansion fails to converge satisfactorily and that the theory becomes effectively strongly interacting in the Higgs sector for $M_H \gtrsim 1100$ GeV.

The result noted above is a consequence of our calculation of the dominant two-loop electroweak corrections to the fermionic decay rates of the Higgs boson; the subject of this paper. The fermionic decay modes are of considerable phenomenological interest. For example the Higgs boson decays predominantly to $b\bar{b}$ pairs if $M_H \lesssim 135$ GeV. The search for a low- or intermediate-mass Higgs boson at future high-energy $e^+e^-$ linear colliders [15] will rely largely on this mode by tagging the $B$ mesons. Moreover in this mass range the branching fractions of all other decay channels depend sensitively on the $H \rightarrow b\bar{b}$ decay width. It has been argued that the low-mass Higgs boson might also be detectable at future hadron supercolliders through the $H \rightarrow \tau^+\tau^-$ signal [18] while the decay $H \rightarrow t\bar{t}$ will have an appreciable branching fraction for $M_H > 2m_t$. Future high-energy $e^+e^-$ colliders will also be able to measure the $Ht\bar{t}$ Yukawa coupling [19].

The measurement of the Higgs-boson mass and couplings in future experiments will
require an understanding of the radiative corrections to the fermionic decay rates of the Higgs boson. Much work has been done in this area and a recent review is given in [20]. There are important differences between the radiative corrections involved in Higgs physics and those familiar in the gauge sector of the SM. e.g., in Z-boson decays. It is well known that the quantum effects induced by virtual Higgs bosons are screened in Z-boson physics: they depend only logarithmically on $M_H$ at one loop and are quadratic in $M_H$ but with minute coefficients at two loops [22]. In contrast, the one-loop electroweak corrections to the partial decay widths [723–25] and production cross sections [26] of the Higgs boson are already dominated for $M_H \gg M_W$ by terms quadratic in $M_H$. These terms give rise to moderate enhancements of the rates for Higgs masses of up to 1 $TeV$. However, it is premature to conclude that the two-loop electroweak corrections will also be perturbatively small in the high-$M_H$ range since these corrections have terms quartic in $M_H$. It is of both theoretical and phenomenological interest to check the importance of these potentially large corrections by explicit calculation.

In this paper, we calculate the complete $O(G_F^2 M_H^2)$ corrections to the fermionic decay rates of a Higgs boson with $M_H \gg M_W$. These corrections, which are the leading two-loop electroweak corrections for $M_H \gg M_W$, are independent of the fermion flavor and as noted above, are larger than the one-loop corrections of $O(G_F M_H^2)$ for $M_H > 1114$ GeV. We compare our results with other known one-loop corrections. To obtain the full electroweak two-loop corrections for specific fermion channels in the limit $M_H \gg M_W$, one would have to calculate further flavor-dependent corrections of mixed orders in the Higgs and Yukawa couplings namely $O(G_F^2 M_H^2 m_f^2)$ and $O(G_F^2 m_f^4)$. These corrections are not universal. For example, different fermionic channels such as $H \rightarrow \tau^+\tau^-$, $b\bar{b}$, $t\bar{t}$ all have different dependence on $m_f$. 


II. CALCULATION OF THE $O(G_F^2 M_H^4)$ CORRECTIONS

In this section we sketch the calculation of the dominant flavor-independent electroweak corrections to the decay rate $H \to f\bar{f}$. The starting point of our analysis is the bare Lagrangian for the Higgs-fermion interaction

$$\mathcal{L}_{Yuk}^{f,0} = -\frac{m_{f,0}}{v_0} \bar{\psi}_{f,0} H_0 \psi_{f,0}, \quad (2.1)$$

where the subscript “0” denotes bare quantities. Our aim is to obtain the leading flavor-independent corrections to the $H f \bar{f}$ vertex in powers of $G_F M_H^2$. The mass and wave-function renormalization constants for the fermions as well as the loop corrections to the vertex depend on the Yukawa couplings for the fermions and are omitted consistently because of their subleading nature. (We will return to these corrections at the one-loop level in Sec. IIIB.) We may therefore replace the fermionic quantities $m_{f,0}$ and $\psi_{f,0}$ in Eq. (2.1) by $m_f$ and $\psi_f$. The contributions to the renormalization constants for the bare Higgs field $H_0$ and the bare vacuum expectation value $v_0$ in powers of $G_F M_H^2$ are determined entirely by the symmetry-breaking sector of the SM. The contributions of fermion loops to these two quantities depend on the Yukawa couplings and are again omitted as they do not contribute to the $O(G_F^2 M_H^4)$ corrections (see Sec. IIIB). We may therefore calculate the desired corrections to the decay $H \to f\bar{f}$ vertex with all Yukawa couplings set to zero.

The calculation can be simplified greatly in the limit of interest $M_H \gg M_W \Gamma$ through the use of the Goldstone-boson equivalence theorem [27]. This theorem states that the leading electroweak contribution to a graph in powers of $G_F M_H^2$ can be calculated by replacing the gauge bosons $W^\pm Z$ by the would-be Goldstone bosons $w^\pm z$ of the symmetry-breaking sector of the theory. Because $M_W/M_H \propto g v/M_H \Gamma$ we can simplify our calculation consistently in the limit of a heavy Higgs boson by neglecting the gauge couplings $g, g'$ and taking the Goldstone bosons to be massless. Adopting the conventions of [28] we can write the relevant Lagrangian for the symmetry-breaking sector of the SM in terms of bare quantities as follows:
\[ \mathcal{L}_0^{SBS} = \frac{1}{2} \partial_{\mu} w_0 \cdot \partial^{\mu} w_0 + \frac{1}{2} \partial_{\mu} H_0 \partial^{\mu} H_0 - \frac{1}{2} M_{w,0}^2 w_0^2 - \frac{1}{2} M_{H,0}^2 H_0^2 \]
\[ -\frac{\lambda_0}{4} \left( w_0^2 + H_0^2 \right)^2 - \lambda_0 v_0 \left( w_0^2 + H_0^2 \right) H_0, \]

where the real scalar triplet \( \Gamma w = (w_1, w_2, w_3) \) is related to the Goldstone bosons \( \Gamma w^\pm \) and \( \Gamma z \) by \( w^\pm = (w_1 \mp iw_2) / \sqrt{2} \) and \( z = w_3 \) respectively. The tadpole counterterm \( \Gamma \) which cancels all tadpole contributions of \( \mathcal{L}_0^{SBS} \) order by order has been omitted in writing Eq. (2.2); therefore all graphs which include tadpole contributions need to be dropped in calculations [28].

Note that a full equivalence-theorem calculation would also require the complete Yukawa Lagrangian for the interactions of fermions with the massless Goldstone bosons and the Higgs boson [29]. Because we are not interested in corrections due to the Yukawa couplings we do not give the complete Yukawa Lagrangian here except for the piece given in Eq. (2.1).

The on-mass-shell renormalization is carried out in such a way that the physical mass of the Higgs boson \( \Gamma \) defined in terms of the position of the pole in the Higgs propagator \( \Gamma \) is \( M_H \).

The three Goldstone bosons remain massless to all orders in the perturbation expansion in \( \lambda_0 \Gamma \) and satisfy a residual SO(3) symmetry [30]. Requiring that the residues of the physical on-shell propagators be unity fixes the wave-function renormalization constants defined by the relations \( w^\pm_0 = Z_{w}^{1/2} w^\pm \Gamma z_0 = Z_{z}^{1/2} z \Gamma \) and \( H_0 = Z_{H}^{1/2} H \Gamma \) where \( w^\pm \Gamma z \Gamma \) and \( H \) are the physical fields. The result is [28]

\[ \frac{1}{Z_w} = 1 - \frac{d}{dp^2} \Pi_w^0(p^2) \bigg|_{p^2=0}, \]
\[ \frac{1}{Z_z} = 1 - \frac{d}{dp^2} \Pi_z^0(p^2) \bigg|_{p^2=0}, \]
\[ \frac{1}{Z_H} = 1 - \frac{d}{dp^2} \text{Re} \Pi_H^0(p^2) \bigg|_{p^2=M_H^2}, \]

where \( \Pi_w^0(p^2) \Gamma \Pi_z^0(p^2) \Gamma \) and \( \Pi_H^0(p^2) \) are the self-energy functions for the bare fields calculated from the Lagrangian in Eq. (2.2). Because of the SO(3) symmetry \( \Pi_w^0 = \Pi_z^0 \) and \( Z_w = Z_z \).

Explicit expressions for the \( \Pi^0 \)'s correct to two loops may be found in Eqs. (11) and (12) of [28]. Furthermore we have [28]

\[ M_{H,0}^2 = M_H^2 - \text{Re} \Pi_H^0 \left( M_H^2 \right), \]
\[ M_{w,0}^2 = -Re \Pi_w^0(0) = -\Pi_w^0(0), \]
\[ v_0 = Z_{w}^{1/2} v, \]
\[ \lambda_0 = \frac{\lambda}{Z_w} \left( 1 - \frac{Re \Pi_{H}^0(M_H^2) - \Pi_w^0(0)}{M_H^2} \right). \]

Using these results, we can write the Lagrangians above entirely in terms of renormalized physical quantities. The physical vacuum expectation value is fixed in terms of the Fermi constant \( G_F \) by the familiar relation \( v = 2^{-1/4} G_F^{-1/2} \Gamma \) while the physical quartic coupling is given by \( \lambda = G_F M_H^2 / \sqrt{2} \). The renormalized symmetry-breaking Lagrangian is given in Eq. (7) of [28] while the renormalized form of the Higgs-fermion Lagrangian is given for our purposes by

\[ \mathcal{L}_{Yuk} = \frac{G_F}{Z_w} \left( \frac{H}{Z_{w}^{1/2}} \right) \bar{\psi}_f H \psi_f. \]  

The radiativelty corrected fermionic decay rate of the Higgs boson is consequently given by

\[ \Gamma \left( H \to f \bar{f} \right) = \frac{Z_H}{Z_{w}^{1/2}} \Gamma_B \left( H \to f \bar{f} \right), \]

where [31]

\[ \Gamma_B \left( H \to f \bar{f} \right) = \frac{N_c m_f^2 M_H}{8\pi v^2} \left( 1 - \frac{4m_f^2}{M_{H}^2} \right)^{3/2} \]

is the Born result. Here \( N_c = 1 \) (3) for lepton (quark) flavors.

The wave-function renormalization constants \( Z_H \) and \( Z_w \) were calculated to two loops \( O(G_F^2 M_H^2) \) in [28] using dimensional regularization. The two-loop diagrams that contribute to \( Z_H \) and \( Z_w \) at that order through the derivatives of the self-energy functions in Eq. (2.3) are shown in Fig. 1; no single diagram gives an exceptionally large contribution. The results of the calculation can be written in the form

\[ \frac{1}{Z_\sigma} = 1 + \hat{\lambda} \xi^\sigma \left( a_\sigma + O(\epsilon) \right) + \hat{\lambda}^2 \xi^{2\epsilon} \left( \frac{3}{\epsilon} + b_\sigma + O(\epsilon) \right) + O(\hat{\lambda}^3) \quad (\sigma = w, H), \]

where \( \hat{\lambda} = (\lambda/16\pi^2) \Gamma \xi = 4\pi \mu^2 / M_H^2 \Gamma \epsilon = (4 - D)/2\Gamma D \) is the dimensionality of space-time \( \Gamma \) and \( \mu \) is the arbitrary scale parameter introduced in the interaction to keep \( \lambda \) dimensionless for \( \epsilon \neq 0 \). The coefficients in the expansions above are:
\[ a_w = 1, \]
\[ b_w = \frac{3}{2} + 2\zeta(2) - 6\gamma - 3\pi\sqrt{3} + 12\text{Cl}_2\left(\frac{\pi}{3}\right)\sqrt{3} \]
\[ \approx 6.098, \]
\[ a_H = -12 + 2\pi\sqrt{3} \]
\[ \approx -1.12, \]
\[ b_H = \frac{291}{2} - 96\zeta(2) + 90\zeta(3) - 6\gamma - 48\pi\text{Cl}_2\left(\frac{\pi}{3}\right) + 116\pi\sqrt{3} - 216\text{Cl}_2\left(\frac{\pi}{3}\right)\sqrt{3} - 162K_5 \]
\[ \approx 41.12. \]

The constant \( K_5 = 0.92363 \ldots \) was evaluated numerically from Eq. (A86) of [28]. The Riemann \( \zeta \) function takes the values \( \zeta(2) = \pi^2/6 \) and \( \zeta(3) = 1.20205 \ldots \) \( \Gamma \text{Cl}_2 \) is Clausen’s function \( \Gamma \text{Cl}_2(\sqrt{3}/3) = 1.01494 \ldots \) \( \Gamma \) and \( \gamma = 0.57721 \ldots \) is Euler’s constant. The one-loop coefficients \( a_H \) and \( a_w \) are similar in magnitude but the two-loop coefficients \( b_H \) and \( b_w \) differ in magnitude by roughly a factor of \( 7 \Gamma \) despite the fact that almost the same number of diagrams with similar structures and magnitudes contribute. It is also interesting that the coefficients in \( Z_h^{-1} \) alternate in sign; those in \( Z_w^{-1} \) do not. We note that the results above revised relative to our previous analysis [12] are now in complete agreement with the those of Ghinculov [13] which have also been revised [14].

Because the decay width \( \Gamma \left( H \rightarrow f \bar{f} \right) \) is a physical quantity and all radiative corrections that depend only on \( G_F M_H^2 \) are contained in the factor \( Z_H/Z_w \) in Eq. (2.6) this factor must be finite for \( \epsilon \rightarrow 0 \). The \( Z \)’s are finite at one loop but not at two loops. Hence the parts of the two-loop contributions to \( Z_H \) and \( Z_w \) that are proportional to \( 1/\epsilon \) must cancel in the ratio. The cancellation is clear if the \( Z \)’s are written in factored form

\[
\frac{1}{Z_\sigma} = \left(1 + a_\sigma \lambda \xi^\epsilon + b_\sigma \lambda^2 \xi^{2\epsilon}\right) \left(1 + \frac{3}{\epsilon} \lambda^2 \xi^{2\epsilon}\right) + O\left(\lambda^3\right) \quad (\sigma = w, H),
\]

and is exact to all orders in \( \lambda \). The complete cancellation of the divergent terms allows us to take the limit \( \epsilon \rightarrow 0 \). In this limit \( \Gamma \xi^\epsilon \rightarrow 1 \) with no pieces left over and the final ratio is independent of the scale \( \mu \) introduced in the process of dimensional regularization.
The $O(G_F^2 M_H^4)$ electroweak corrections to the fermionic decay rates emerge naturally in this formalism as the finite ratio

$$
\frac{Z_H}{Z_w} = \frac{1 + a_w \lambda + b_w \lambda^2}{1 + a_H \lambda + b_H \lambda^2}.
$$

(2.11)

This expression for $Z_H/Z_w$ automatically resums one-particle-reducible Higgs-boson self-energy diagrams in a way that conforms with the standard procedure in $Z$-boson physics; see, e.g., $\Gamma[32]$. However, it is clear that the resummation contains only limited information on higher-order terms. Since we actually have no control of terms beyond $O(\hat{\lambda}^2)$ and are not aware of a physical principle which would select this as an optimum resummation scheme, we expand Eq. (2.11) and discard terms beyond $O(\hat{\lambda}^2) = O(G_F^2 M_H^4)$. This gives the alternative representation

$$
\frac{Z_H}{Z_w} = 1 + (a_w - a_H)\hat{\lambda} + \left(b_w - b_H - a_w a_H + a_H^2\right)\hat{\lambda}^2
$$

(2.12)

$$
\approx 1 + 2.12\hat{\lambda} - 32.66\hat{\lambda}^2
$$

$$
\approx 1 + 0.013\lambda - 0.0013\lambda^2
$$

$$
\approx 1 + 11.1\% \left(\frac{M_H}{1 TeV}\right)^2 - 8.9\% \left(\frac{M_H}{1 TeV}\right)^4.
$$

The result agrees at $O(\hat{\lambda})$ with the known one-loop result $[7][23]$$

$$
\frac{Z_H}{Z_w} = 1 + \frac{G_F M_H^2}{8\pi^2\sqrt{2}} \left(\frac{13}{2} - \pi\sqrt{3}\right).
$$

(2.13)

III. RESULTS

A. Limits on perturbation theory

We are now in a position to explore the phenomenological implications of our results. In Fig. 2 we show the leading electroweak corrections to $\Gamma(H \rightarrow f \bar{f})$ in the one- and two-loop approximations with and without resummation of one-particle-reducible higher-order terms plotted as functions of $M_H$. We will concentrate first on the expanded results given in Eq. (2.12). While the $O(G_F^2 M_H^4)$ term (upper solid line in Fig. 2) gives a modest increase of
the rates e.g. $\Gamma$ by 11% at $M_H = 1$ TeV the situation changes when the two-loop term is included. The importance of this term which grows as $M_H^4$ increases with $M_H$ in such a way that it cancels the one-loop term completely for $M_H = 1114$ GeV and is twice the size of the one-loop term with the opposite sign for $M_H = 1575$ GeV. The total correction shown by the lower solid line in Fig. 2 is then negative and has the same magnitude as the one-loop correction alone. The perturbation series for the corrections to $\Gamma (H \rightarrow f \bar{f})$ clearly ceases to converge usefully if at all for $M_H \approx 1100$ GeV or equivalently for $\lambda \approx 10$. A Higgs boson with a mass larger than about 1100 GeV effectively becomes a strongly interacting particle. Conversely $M_H$ must not exceed approximately 1100 GeV if the standard electroweak perturbation theory is to be predictive for the decays $H \rightarrow f \bar{f}$. Note that one cannot use the usual unitarization schemes invoked in studies of $W_L^\pm, Z_L, H$ scattering [6B3] to restore the predictiveness for the heavy-Higgs width as no unitarity violation is involved.

One might expect to improve the perturbative result in the upper range of $M_H$ somewhat by resumming the one-particle-reducible contributions to the Higgs-boson wave-function renormalization by using Eq. (2.11) rather than Eq. (2.12). This leads to an increase of the one-loop correction (upper dotted line in Fig. 2) while the negative effect of the two-loop correction is lessened (lower dotted line) for large values of $M_H$. However in the mass range below $M_H = 1400$ GeV this effect is too small to change our conclusions concerning the breakdown of perturbation theory. Moreover the resummed expression for the one-loop terms in the perturbation expansion when reexpanded to $O(G_F^2 M_H^4)$ does not yield a proper estimate for the size of the two-loop terms. There is consequently no reason to favor this approach to the present problem.

It might be argued that the apparent breakdown in the perturbation expansion as judged by a comparison of the one- and two-loop terms is an artifact of a small one-loop contribution rather than a consequence of large two-loop terms. However we see no evidence in the calculation that there are unusual cancellations in the one-loop corrections. In fact the one-loop contributions $a_H$ and $a_w$ add in magnitude in the ratio $Z_H/Z_w \Gamma$ whereas the two-loop contribution $b_w$ is in magnitude subtracted from $b_H$ [see Eqs. (2.9) and (2.12)].
In a previous publication, we looked at other processes to which \( Z_w \) and \( Z_H \) contribute, e.g., the scattering of Higgs bosons and longitudinally polarized \( W^\pm \) and \( Z \) bosons [10]. In the latter case, even larger coefficients appear in the perturbation expansion when it is expressed as above in a series in the (running) parameter \( \hat{\lambda}_s = (\lambda_s/16\pi^2) \) [34]. The contributions due to the finite parts of \( Z_H \) and \( Z_w \) are rather insignificant in comparison with the finite parts of the unrenormalized two-loop scattering graphs.

While the factors of \((1/16\pi^2)\) occur naturally at each order in perturbation theory, their incorporation into the natural parameter \( \hat{\lambda} \) is misleading: the coefficients of the zero-, one-, and two-loop terms in the diagonal partial-wave scattering amplitudes and the one- and two-loop terms in the Higgs-boson decay rate calculated above all have similar magnitudes when the series are rewritten as expansions in the physical parameters \( \lambda \) and \( \lambda = G_F M_H^2/\sqrt{2} \) as seen in Eq. (2.12) and the comment [34]. It appears therefore that \( \lambda \) and not \( \hat{\lambda} = \lambda/(16\pi^2) \) is the natural expansion parameter.

The high-energy scattering processes give strong evidence for a breakdown of the perturbation series for a running coupling \( \lambda_s(\sqrt{s}) \approx 2.3 \) at either the one-loop [9] or two-loop [10] level. This translates to \( M_H \approx 380 \) GeV if the SM is assumed to remain valid for energies \( \sqrt{s} \) up to \( \sim 5 \) TeV [10]. The \( M_H \) upper bound obtained in the present analysis is considerably less stringent than the one found in [10]. However, we emphasize that the result obtained here does not depend on the extra assumption about an energy scale; the breakdown of perturbation theory is fixed solely by the physical value of \( M_H \).

**B. Comparison with complete one-loop corrections**

The leading corrections discussed so far are independent of the flavor of the final-state fermion. However, from an experimental point of view, they are relevant only for the \( t\bar{t} \) and perhaps the \( b\bar{b} \) and \( \tau^+\tau^- \) decays of the Higgs boson. It is therefore interesting to compare the corrections calculated here with the full one-loop electroweak corrections [25] and the QCD corrections that are available for these decay channels [35,36]. In particular, the subleading
two-loop electroweak corrections those of $O(G_F^2 M_H^2 m_t^2)$ and $O(G_F^2 m_t^4)$ are still unknown but one may estimate their likely importance by comparing the top-quark Yukawa-coupling correction to the Higgs-coupling correction at one loop. The QCD corrections to the $H \rightarrow q\bar{q}$ modes where $q$ denotes a quark flavor are known to $O(\alpha_s)$ for arbitrary values of $m_q$ [35] and to $O(\alpha_s^2)$ in the limit $m_q \ll M_H$ [36]. Their main effect is to replace the pole mass of the quark $q$ by its $\overline{MS}$ mass evaluated at the scale $M_H$. For completeness we mention that the $O(\alpha_s G_F m_t^2)$ corrections to the fermionic decay rates are also now available [37].

The results in the preceding section were derived using the Goldstone-boson equivalence theorem and neglecting further electroweak corrections of orders $g^2\Gamma g^2\Gamma$ etc. as well as contributions that involve the Yukawa and QCD couplings of the fermions. Since $\lambda^2\Gamma \alpha_s\Gamma$ and the Yukawa couplings are independent parameters of the theory our conclusion that the perturbation series in $\lambda = G_F M_H^2/\sqrt{\sigma}$ fails to converge satisfactorily is independent of further corrections involving powers of $g$ and the other independent couplings though those further corrections may be numerically important in applications of the results.

The use of the equivalence theorem provides a correct framework for calculating the leading electroweak corrections—those enhanced by the maximum powers of $M_H/M_W$—at each order. By neglecting subleading corrections that involve $g$ or the Yukawa couplings $g_f = \sqrt{2m_f}/v\Gamma$ we expect to obtain a good approximation to the full result provided that $M_W/M_H \propto g v/M_H \ll 1$ and $m_f/M_H \ll 1$. Because of the high mass of the $t$ quark [38] it is interesting to test the accuracy of the approximation. In Fig. 3 we compare the $O(G_F M_H^2)$ correction to $\Gamma (H \rightarrow t\bar{t})$ already shown in Fig. 2 to the full one-loop electroweak correction including the effects of fermions [25]. The full correction was evaluated in the on-shell renormalization scheme using $m_t = 174$ GeV [38]. We see that the $O(G_F M_H^2)$ term underestimates the full one-loop electroweak correction term by 32% (24%) at $M_H = 500$ GeV (1 TeV). To check that the difference arises primarily from the inclusion of the top quark in the full calculation and not from the supposedly small contributions from the gauge sector of the SM that is from a failure of the equivalence theorem we have carried out a complete one-loop calculation using the equivalence theorem with the gauge couplings
set to zero [29] but the top-quark Yukawa coupling retained. The extra contributions are $O(G_F m_t^2) \Gamma$ and are independent of those considered above. As shown in Fig. 3 the result of the calculation reproduces the full one-loop electroweak result very well. The result obtained using the equivalence theorem with $g_t \neq 0$ is only 3.9% (1.8%) larger than the full electroweak one-loop term at $M_H = 500$ GeV (1 TeV) for $m_t = 174$ GeV. The use of the equivalence theorem therefore gives a quite accurate approximation to the full theory even for the rather low values of $M_H$ with which we are concerned. The small residual differences away from the decay threshold at $M_H = 2m_t$ can be accounted for by the transverse gauge couplings, the nonzero masses of the $W$ and $Z$ bosons, and the finite masses and Yukawa couplings for the remaining fermions. The extra structure close to the threshold is the result of virtual-photon exchange in QED. This generates a Coulomb singularity and a correction that behaves near threshold as $1 + \alpha_{em} Q_t^2[(\pi/2\beta) + O(1)] \Gamma$ where $Q_t$ and $\beta$ are the top-quark electric charge and velocity; see left end of the dashed line in Fig. 3.

In Fig. 4 we compare the electroweak and QCD corrections. The latter were calculated with the asymptotic scale parameter of QCD adjusted to give $\alpha_s(M_Z) = 0.118$ [39]. The one-loop QCD correction to $\Gamma (H \rightarrow t\bar{t})$ in Fig. 4(a) shows the expected color-Coulomb threshold singularity with $\alpha_{em} Q_t^2$ replaced in the expression above by $(4/3) \alpha_s(M_H)$. This singularity is associated with the nonrelativistic motion of the quarks. The set of correction terms in powers of $(2\pi \alpha_s/3\beta)$ corresponds to the expansion of a Coulomb wave function at zero quark separation. For $M_H \gg 2m_t \Gamma$ the one-loop QCD correction is negative with a magnitude which increases logarithmically. The two-loop QCD corrections to $\Gamma (H \rightarrow t\bar{t})$ are unknown. They are expected to be large close to the $t\bar{t}$ production threshold at $M_H \gtrsim 2m_t$. At $M_H \gg 2m_t \Gamma$ the potentially large logarithmic contributions in all higher orders can be resummed by using the top-quark $\overline{MS}$ mass evaluated at the scale $M_H \Gamma$ and the residual corrections should be small (see the discussion given below for the $b\bar{b}$ decay).

In Fig. 4(b) we repeat the comparison of Fig. 4(a) for the case $H \rightarrow b\bar{b}$ assuming $m_b = 4.72$ GeV [40]. The difference between the full one-loop electroweak correction and the $O(G_F M_H^2)$ result is again accounted for at large values of $M_H$ by the omission of top-quark...
effects in the latter. The spikes in the full correction at $M_H = 2M_W$ and $2M_Z$ originate in threshold singularities of the Higgs-boson wave-function renormalization. The dent at $M_H = 2m_t$ is not accompanied by such a divergence. These features may be understood as artifacts of the underlying approximation of treating the unstable Higgs boson as an asymptotic state. The QCD corrections are calculated to $O(\alpha_s^2)$ in the $\overline{MS}$ scheme [35B36]; see Eq. (37) of [41]. When the quark pole mass is used as a basic parameter the largest part of the one-loop QCD correction comes from a large logarithmic term $-(4\alpha_s/\pi)\ln(M_H/m_b)$ [35]. In general the large logarithms are of the form $(\alpha_s/\pi)^n\ln^m(M_H/m_b)\Gamma$ with $n \geq m$. By exploiting renormalization-group techniques these logarithms may be absorbed completely into the $\overline{MS}$ quark mass $m_t(\mu)\Gamma$ evaluated at the scale $\mu = M_H$ [20]. The logarithms are resummed to all orders and the remaining perturbative expansion converges more rapidly. The offset seen in the Fig. 4(b) with the QCD-corrected decay considerably below the Born decay rate results mainly from the use of $m_t(M_H)$ instead of $m_t$ in the prefactor in Eq. (2.7). While the effect is large it is controlled by the resummation. The remaining part of the QCD correction at two loops is rather small.

C. Handling a nonperturbative Higgs

One would like to be able to describe the Higgs-boson decay to fermions phenomenologically even in the case of a strongly interacting Higgs sector. In the case of the QCD corrections discussed above the renormalization group provided a physical principle that could be used to motivate and organize a resummation of higher-order effects to obtain a controlled final expression even though the corrections could be large when viewed order-by-order in $\alpha_s$. We do not know of a similar physical organizing principle to use in the resummation of higher-order corrections in $\lambda$ especially as the leading corrections only depend on one energy scale namely the mass of the Higgs boson. Any summation of the perturbation series will therefore be speculative and will necessarily be based on the mathematical structure of the series rather than a physical argument. We note in this connection that
the correction terms in Eq. (2.12) alternate in sign. This suggests that Padé summation of the series might be reasonable. In particular, we can use the two-loop information given in Eq. (2.12) to rewrite the perturbative series using a \[1 \Pi\] Padé approximant \[1\Pi\] that is as a ratio of two first-degree polynomials in \(\lambda\) with the coefficients adjusted to fit the expansion in Eq. (2.12) through order \(\lambda^2\). We obtain

\[
\frac{Z_H}{Z_w} = 1 + \frac{(a_w - a_H)\lambda}{1 - [(b_w - b_H - a_w a_H + a_H^2)/(a_w - a_H)]\lambda}.
\]  

(3.1)

In Fig. 5\Gamma we compare the Padé-summed correction factor \(\Gamma\) Eq. (3.1) \(\Gamma\) with the earlier results from Fig. 2 or Eqs. (2.11) and (2.12). The result suggests that the leading electroweak corrections to \(\Gamma (H \rightarrow f \bar{f})\) will be quite small even for values of \(M_H \gtrsim 1.5\) TeV. How far the result can be trusted is a matter of speculation. A similar Padé summation of the partial-wave scattering amplitudes for \(W_L^\pm, Z_L, H\) scattering turns out to give a fairly good prediction for the two-loop contribution in terms of the zero- and one-loop terms [43] so the method may be more reliable in this rather similar case than our limited input information would suggest. If so, the leading electroweak corrections to \(H \rightarrow f \bar{f}\) in powers of \(\lambda\) or \(G_F M_H^2\) will be negligible when resummed relative to the corrections introduced by the Yukawa couplings and QCD. Only experimental results or reliable nonperturbative calculations can resolve this speculation.

We note in this connection that recent lattice simulations of certain Yukawa models for the interaction of the Higgs boson with mirror or reduced staggered fermions suggest that the Yukawa couplings cannot be strong unless the regularization scale is unacceptably low [44].

**IV. CONCLUSIONS**

In summary, we have calculated the leading two-loop electroweak corrections to the fermionic decay rates of a high-mass Higgs boson in the SM which are of \(O(G_F^2 M_H^4)\). The corrections are negative and exceed the positive \(O(G_F M_H^2)\) one-loop corrections in magnitude for \(M_H > 1114\) GeV. For larger values of \(M_H\) the perturbation series is clearly...
unreliable, and the theory becomes effectively strongly interacting. We conclude given the lack of a physical principle that would allow a convincing resummation of the perturbation expansion that a value \( M_H \sim 1100 \text{ GeV} \) has to be considered as a theoretical upper bound on \( M_H \) beyond which the fermionic decay width of the Higgs cannot be calculated perturbatively. This result is independent of speculations regarding the energy scale up to which the SM is valid as the center-of-mass energy in the Higgs decay is fixed \( \sqrt{s} = M_H \). However there is indication that high-energy interactions in the Higgs sector of the SM can become effectively strong and are not usefully calculable in perturbation theory for even smaller Higgs mass. For scattering processes with \( \sqrt{s} \sim 5 \text{ TeV} \) the critical value for \( M_H \) is about 380 GeV and for scattering at GUT energies the critical value is less than 160 GeV [10]. Clearly the present-day precision tests of the gauge sector of the SM are not affected by such nonperturbative effects.

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The SO(3) symmetry of the \((w^\pm, z)\) Goldstone bosons is broken by the interactions of the \(w^\pm\) and \(z\) with fermions but this does not affect the dominant two-loop corrections. The complete renormalization scheme for use with the Goldstone-boson equivalence theorem is discussed in [29].
For example, the largest of the diagonalized $J=0$ amplitudes for $W_L^\pm$, $Z_L$, $H$ scattering is given up to a negligible anomalous-dimension correction by

$$a_1(s) = \lambda_s \left[ -6 + \lambda_s (185.6 + 113.1 i) - \lambda_s^2 (9523.5 + 6892.3 i) + O \left( \lambda_s^3 \right) \right]$$

$$= -\frac{6\lambda_s}{16\pi} \left[ 1 - \lambda_s (0.1959 + 0.1194 i) + \lambda_s^2 (0.0637 + 0.0461 i) + O \left( \lambda_s^3 \right) \right].$$


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FIGURES

FIG. 1. The two-loop diagrams that contribute to the wave-function renormalization constants $Z_H$ and $Z_w$ at $O(G_F^2 M_H^4)$ through derivatives of the self-energy functions $\Pi_H^0$ and $\Pi_w^0$ Eq. (2.3). Heavy (light) lines represent Higgs ($w^\pm$ and $z$) bosons. The statistical weights of the diagrams are not shown but may be read off from Eqs. (11) and (12) in [28]. See Appendix A in [28] for the results.

FIG. 2. Complete $O(G_F M_H^3)$ and $O(G_F^2 M_H^4)$ correction factors for $\Gamma(H \rightarrow f \bar{f})$ for $100 \text{GeV} \leq M_H \leq 1700 \text{GeV}$. These corrections are universal i.e., they are independent of the flavor of the final-state fermions. In each order the expanded result given in Eq. (2.12) is compared to the calculation where the one-particle-reducible Higgs-boson self-energy diagrams are resummed as shown in Eq. (2.11). The two-loop correction cancels the one-loop correction at $M_H = 1114 \text{GeV}$ and is twice as large as the latter with an opposite sign at $M_H = 1575 \text{GeV}$.

FIG. 3. Comparison of the one-loop results for the ratio $\Gamma(H \rightarrow t \bar{t})/\Gamma_B(H \rightarrow t \bar{t})$ obtained in various approximations with the full one-loop electroweak result ($g_1$, $g_2$, $g_t$, $g_b \neq 0$). The solid curve (EQT) gives the result obtained using the equivalence theorem with vanishing gauge couplings ($g_1$, $g_2 \neq 0$) and a nonzero top-quark Yukawa coupling corresponding $g_t$ to $m_t = 174 \text{GeV}$. The dot-dashed curve shows the $O(\lambda) = O(G_F M_H^2)$ correction from Fig. 2 and it is equivalent to an EQT curve with $m_t = 0$.

FIG. 4. Electroweak and QCD correction factors for (a) $\Gamma(H \rightarrow t \bar{t})$ and (b) $\Gamma(H \rightarrow b \bar{b})$ as a function of $M_H$: universal $O(G_F M_H^3)$ term without resummation (solid line); full one-loop electroweak corrections (dashed line); QCD corrections (dot-dashed line); and universal $O(G_F M_H^3)$ plus $O(G_F^2 M_H^4)$ terms without resummation (solid line). The QCD corrections are evaluated in (a) to $O(\alpha_s)$ in the on-mass-shell scheme and in (b) to $O(\alpha_s^2)$ in the \overline{MS} scheme. The pole-mass values $m_t = 174 \text{GeV}$ and $m_b = 4.72 \text{GeV}$ [40] are used and the asymptotic scale parameter of QCD is adjusted so that $\alpha_s(M_Z) = 0.118$ [39]. Note the different scales used in the two plots.
FIG. 5. Comparison of the Padé-resummed correction for $\Gamma (H \rightarrow ff)$ with the universal electroweak correction factors calculated to $O (G_F M_H^2)$ and to $O (G_F^2 M_H^4)$. 