Cosmic Fluctuations and Dark Matter from Scalar Field Oscillations

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It is argued that scale-invariant cosmic fluctuations and cosmic dark matter might originate from two different modes of a single scalar field. The dynamical relaxation of a multicomponent classical scalar field in an expanding universe excites two generic types of global modes: massless Goldstone oscillations, which lead to scale-invariant isentropic fluctuations, and massive Higgs modes, which lead to cold dark matter with isocurvature fluctuations in density. Estimates are given of these effects in a simple generic model where the large scale fluctuations and dark matter density match those observed; it is shown that the former requires a heavy scale \( \phi_0 \approx 10^{30}\) GeV for the potential minimum, and the latter requires an extremely small self coupling \( \lambda \approx 10^{-63} \). An additional prediction of the model is then the formation of isocurvature fluctuations, leading to the early collapse of dense \( \approx 100 M_{\odot} \) dark matter "miniclusters", formation of dense halos on the scale of compact dwarf galaxies, and possibly observable gravitational microlensing of quasars.

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I. INTRODUCTION

It is generally acknowledged that new physics is required to explain the origin of cosmic structure, both in the cosmic background radiation and in the galaxy distribution. The favored approach at present is to invoke quantum fluctuations in the fields driving cosmic inflation; the amplitude of the fluctuations comes out about right if the inflaton self-coupling is very small, \( \lambda \approx 10^{-15} \) [1-4]. Another possibility however is that the inflaton field is so weakly coupled that inflation leaves behind a smooth universe, and that structures are introduced later by large-scale classical motions of scalar fields. These motions are most commonly discussed in the context of distinctive topological defects such as cosmic strings or textures [5,6]. At the same time, new physics is also required to produce the apparent prevalence of nonbaryonic, nonrelativistic dark matter in the universe. Among the many possibilities are scalar and pseudoscalar bosons such as the axion, which condense into nonrelativistic dark matter during vacuum phase transitions [7-9]. In general, dark matter candidates have little to do with the sources of cosmic fluctuations.

In this paper I explore the possibility that the two unresolved phenomena—large scale fluctuations and dark matter—might arise from two different modes of oscillation of a single scalar field. I use a simple Mexican-hat model potential to describe the generic (topology-independent) classical dynamical behavior of multicomponent scalars in an expanding universe, in which classical modes are excited by the Kibble mechanism. I argue that in general the Goldstone modes of the fields produce scale-invariant fluctuations and the Higgs modes produce cold dark matter with small-scale isocurvature fluctuations in density. The behavior is determined quantitatively by two standard parameters describing the shape of the potential \( V(\phi) \). The width of the hat, \( \phi_0 \), controls the strength of the scale-invariant gravitational perturbations; the self-coupling, \( \lambda \), controls the amount of dark matter, in this simple model where a smooth phase transition is paced just by the natural timescale of the potential. We fix \( \phi_0 \) to match the COBE/DMR fluctuation amplitude on large scales [4,10]; to achieve the present density of dark matter then requires a self-coupling much weaker even than inflation, \( \lambda \approx 10^{-83} \). The condensation of the dark matter occurs late enough to produce isocurvature fluctuations on astrophysically interesting scales.

There is little new here apart from the particular way a lot of familiar ideas are spliced together from many other scenarios which create cosmic structure with active scalars. The Goldstone-mode part of the model discussed here corresponds to massive global cosmic strings with mass per length \( \approx \phi_0^4 \)—even though the strings are much thicker, and much less dense, than the usual situation considered with potentials of central height \( V_0 \approx \phi_0^4 \). However, the relaxation dynamics create scale-invariant fluctuations without regard to topological defects; nothing in the analysis depends on topological stability, and similar horizon-scale fluctuations occur with a different (noncircular) topology, indeed for any continuous manifold of degenerate true vacua. The Higgs-mode condensate resembles the direct-condensation process for forming cosmic axions [2,7-9], and the interaction with Goldstone modes corresponds to radiating axions from cosmic strings [11,12]. The formation of the isocurvature fluctuations is by the same process envisioned for "axon miniclusters" [13,14], except that the scale is now large enough to be more astrophysically interesting. The same condensation process was also considered in a late-phase-transition model [15] with still weaker coupling; in that scenario only the isocurvature fluctuations and not the Goldstone modes play a significant astrophysical role.
so the COBE/DMR fluctuations are not explained. In both cases, as in inflation models, the extremely weak self-coupling required is not motivated by particle physics, but by astrophysical phenomenology, although models have been discussed which can accommodate such a potential [16], and are contemplated to allow a natural nonzero cosmological constant [17]. Here, as in the inflationary picture, two parameters are used to determine two quantities; the present scheme has at least the economy of requiring only one new field to produce both structure and dark matter. A unique new feature of the present scenario is that isocurvature and isentropic fluctuations from a single process are both predicted to be astrophysically significant—the former on small scales, the latter on large scales.

II. HIGGS AND GOLDSTONE MODES

For definiteness we consider the behavior of a complex classical scalar field described by Lagrangian density [2]

$$L = \partial_\mu \partial^\mu \phi/2 - V(\phi),$$

with a potential of the familiar form

$$V(\phi) = (\lambda/4)(\phi^2 - \phi_0^2)^2,$$

assuming as usual $c = h = 1$. The potential has the form of a Mexican hat of height at center $V_0 = \lambda \phi_0^4/4$ and a set of degenerate minima forming a circle at $|\phi| = \phi_0$, characterised by an internal phase angle $\theta$. The field obeys the evolution equation

$$\dot{\phi} + 3H \dot{\phi} - \nabla^2 \phi + \partial V/\partial \phi = 0,$$

where $H = \dot{a}/a$ and $a$ denotes the cosmic scale factor.

The system supports two kinds of oscillation. The first are “classical Goldstone modes,” corresponding to motion within the circle of minima. Quantum mechanically these modes are massless Goldstone bosons; the classical modes are oscillations in which the gradient term $\nabla^2 \phi$ plays the role of a restoring force which tries to correct misalignments in $\phi$. In these modes the field is not perfectly aligned but has spatial variations in $\theta$; the misalignments propagate in space at unit velocity, with a characteristic frequency determined by the wavelength.

The second type of oscillations are “classical Higgs modes,” corresponding to harmonic motion in the hat’s radial direction. The characteristic frequency is

$$\omega^2 = V''(\phi_0) = 2m^2 = 2\lambda \phi_0^2$$

where $m$ is the mass of the Higgs particle. The zero momentum modes are spatially uniform and do not propagate.

Both types of modes carry energy with distinctive equations of state. The density and pressure are [2]

$$\rho_\phi = \dot{\phi}^2/2 + V(\phi) + (\nabla \phi)^2/2$$

$$p_\phi = \dot{\phi}^2/2 - V(\phi) - (\nabla \phi)^2/6$$

where the gradient term contributes an an isotropic pressure in the direction of $\nabla \phi$. A static uniform $\phi$ field yields the familiar inflationary (de Sitter, steady state, cosmological constant) equation of state, $p = -\rho = -V$; a changing $\phi$ contributes an ultra-stiff component, $p = \rho = \dot{\phi}^2$; and a stationary spatial gradient contributes $p = -\frac{1}{2} \rho = -\frac{1}{6}(\nabla \phi)^2/6$. (Which incidentally in spite of carrying energy, has zero Newtonian gravity; a universe made of such material mimics an open universe even with zero space curvature).

On timescales comparable to the oscillation period, the pressure and density of the modes fluctuates between these various extreme equations of state. On timescales long compared to an oscillation period, the equations of state of the two types of modes are harmonic time averages of these expressions over the oscillation. The Goldstone modes have $\langle V \rangle = 0$ so their equation of state averages $\dot{\phi}^2/2$ and $\langle (\nabla \phi)^2 \rangle$, with the latter multiplied times $+1/2$ and $-1/6$ for the density and pressure respectively, to produce simple relativistic matter with $p = \rho/3$. In the the zero-momentum Higgs modes the gradient vanishes and the average of $V$ and $\dot{\phi}^2/2$ produces a cancellation, yielding pressureless matter $p = 0$; this phenomenon is familiar from the formation of a cosmological axion condensate. In both cases the density and pressure are proportional to the squared amplitude of the oscillations.
III. FLUCTUATIONS AND DARK MATTER

Now consider the evolution of the classical field in an expanding universe. The effective potential at high temperatures is as usual driven to have a minimum at \( \phi = 0 \), which we take as the initial condition of the system apart from small fluctuations. For \( T < 2\theta \), this minimum disappears, so that the classical system rolls down from the center of the hat to someplace on the circle of minima. This process in general excites both types of modes, but in different ways and at different times.

The Goldstone modes are excited by the same Kibble mechanism responsible for the formation of cosmic strings. (In fact, for the specific potential used here strings actually form as well; but the Goldstone oscillations are in addition to any such topological defects). Widely separated portions of the universe choose independently the value of \( \theta \) they relax to; thus any random initial conditions of the field naturally produce large-amplitude (in \( \theta \)) spatial gradients in \( \phi \) on all scales. These gradients are approximately preserved, frozen in comoving coordinates, on each comoving scale as long as the wavelength exceeds \( H^{-1} \), they excite propagating Goldstone modes of unit amplitude on each scale when the wavelength comes within \( H^{-1} \).

The expansion rate is related to the cosmic density \( \rho \) by the Friedmann equation \( H^2 = \rho /m_P^2 \) where \( m_P \equiv (3\pi^8/8\pi G)^{1/2} = 4.5 \times 10^{18}\text{GeV} \) is the Planck mass. The Goldstone modes on each scale contribute density fluctuations when they cross the horizon of the order of their fluctuating density,

\[
\delta \rho_{\phi} = \delta \rho \approx (\nabla \phi)^2 \approx H^2 \phi_0^2 \delta \theta^2 = \rho (\phi_0 /m_P)^2 \delta \theta^2,
\]

independent of the composition of \( \rho \) controlling the expansion. Since the modes on each scale are excited with \( \delta \theta \approx 1 \), this processes scale-free fluctuations with amplitude \( \delta \rho /\rho \approx (\phi_0 /m_P)^2 \). For many purposes — and almost certainly in data we have at present — these have probably in distinguishable from the inflationary/quadratic fluctuations usually considered. To match the COBE/DMR amplitude, we require \((\phi_0 /m_P)^2 \approx 10^{-5} \), or \( \phi_0 \approx 10^{16}\text{GeV} \). In general, gravitational effects of strings or textures, if any, are comparable to (and additional to) the Goldstone effect.

One extra effect is to generate extra coherent relativistic waves. After they start propagating, the Goldstone amplitudes decrease (by “Hubble damping”), with energy density \( \rho \propto a^{-2} \), where \( a \) denotes the cosmic scale factor. This leaves a stochastic background of relativistic waves, which however is practically undetectable.

The radial Higgs mode is also excited in this system, just by starting at the top of the hat (at the origin), rolling off and overshooting the minimum. The natural timescale for initial relaxation to produce the zero-momentum mode is just the oscillation time \( \omega^{-1} \). Even if the rolling starts very early (\( T \approx \phi_0 \)), the oscillations as such do not start until \( \omega \approx H \), which is also when the Higgs and Goldstone modes decouple from each other. We can estimate the temperature \( T_{\text{w}} \) when this happens by setting \( \omega H = 2\lambda \delta \theta^2 \) equal to \( H^2 = a_4 T_{\text{w}}^4 /m_P^2 \), yielding

\[
T_{\text{w}} /m_P = (2/a_4)^{1/4} \lambda^{1/4} (\phi_0 /m_P)^{1/2}.
\]

(Here we have assumed radiation domination \( \rho_{\text{rad}} = a_4 T^4 \), where \( a_4 = \pi^2 N_{\text{eff}}/15 \) and \( N_{\text{eff}} \) is the number of effective photon degrees of freedom). At this time, the density in the Higgs modes, like the Goldstone modes at all times as they enter the horizon, is

\[
(\rho_{\phi} /\rho_{\text{rad}})_{\text{w}} \approx V_0 /a_4 T_{\text{w}}^4 = (\phi_0 /m_P)^2 \approx 10^{-5}.
\]

The subsequent behavior is very different however. Although subsequent excitation of Higgs modes is very inefficient, these initial excitations, like those of the Goldstone modes, have their amplitudes reduced only by the expansion. However, as the Higgs correspond to pressureless matter, they do not lose energy as quickly as the relativistic matter: \( \rho_{\text{H}} \propto \rho_{\text{rad}} /T \). They contribute a component of pressureless dark matter today whose ratio to the radiation density at the present temperature \( T_0 \approx 5 \times 10^{-3} m_P \) is then

\[
(\rho_{\phi} /\rho_{\text{rad}})_{\text{w}} \approx (\phi_0 /m_P)^2 (T_{\text{w}} /T_0) \approx (\phi_0 /m_P)^2 \cdot 5 \times 10^{-5}(m_P /T_0)^3/4).
\]

Let \( \Omega_{\phi} \) denote the fraction of critical density today in the form of cosmic cold dark matter in the Higgs oscillations, and \( \Omega_{\phi} \approx 4 \times 10^{-5} h^{-2} \) the fraction in radiation. We find that a reasonable density requires a tiny coupling: \( \Omega_{\phi} \rho_0^2 \approx 4 \times 10^{-20} \lambda^{1/4} /3 \approx 2 \times 10^{-2} (\Omega_{\phi} h^2) \lambda /10^{-3}(\Omega_{\phi} h^2)^2 \), yielding a tiny Higgs mass, \( m = \sqrt{2} \lambda^{1/2} \phi_0 \approx \phi_0 \lambda^{1/2} 6 \times 10^{-17}(\Omega_{\phi} h^2)^2 \text{eV} \). The characteristic energy scale of the unbroken vacuum is \( V_0 /\phi_0 \approx 6 \times 10^{-11}(\Omega_{\phi} h^2)^2 \text{keV} \). The formation of the Higgs condensate occurs at redshift \( z_{\text{f}} H \approx (\rho_0 /\rho_{\text{rad}}) (\phi_0 /m_P)^2 \approx 2.5 \times 10^{-5} (\Omega_{\phi} h^2)^2 \), at a temperature \( T_{\text{w}} H \approx 1 \times 10^{6} (\Omega_{\phi} h^2)^{-2/5} K \), or about \( 600 (\Omega_{\phi} h^2) \text{keV} \). The Compton wavelength of the Higgs particles is \( 2\pi /m \approx 2\pi /H \approx 2 \times 10^{12} (\Omega_{\phi} h^2)^{-2/5} \text{cm} \), corresponding to a characteristic oscillation period of about \( 60 (\Omega_{\phi} h^2)^{-2/5} \text{sec} \).
The Higgs is not excited in a spatially uniform zero momentum mode. The initial mixing with the Goldstone modes ensures fractional variations in the order of unity in the rolloff time, since the spatial gradients in the field accelerate rolloff in some places and retard it in others, leading to different phases of oscillation and different initial epochs for the oscillations. The amplitude of the Higgs oscillation therefore fluctuates spatially, with about unit fractional amplitude on the scale of the Hubble length at $T_{eq} H$. This variation leads to dark matter forming in lumps. The lumps are created with a distribution of dark matter masses roughly in the range $[1 \text{ to } (2\pi)^3] \times V_{eq}/m^2$ $\approx [2 \text{ to } 550](\Omega_0 h^2)^{-2} M_\odot$. We do not compute the distribution here, but simply take $100(\Omega_0 h^2)^{-2} M_\odot$ as a typical value. These “isocurvature fluctuations” in dark matter density form in addition to the Goldstone modes considered already. Since the lumps are laid down with no large-scale correlations (according to original hypothesis of the Kibble domain formation), the fluctuation spectrum corresponds to white noise on larger scales.

IV. ASTROPHYSICAL CONSEQUENCES

Isocurvature fluctuations [18] grow at a rate of the order of $(\rho_\phi/m_p^2)^{-1/2}$ — slower than $H$ until the epoch of equal matter and radiation densities $t_{eq}$, at the usual rate $H$ thereafter. Eventually the perturbations grow to be nonlinear and collapse into bound dark matter “miniclusters” [13,14] in virial equilibrium. A spherical model [14] estimates the density of the virialized system after collapse, $\rho_{eq} = 140\Phi^2(\Phi + 1)\rho_{eq}$, where $\rho_{eq} = 3 \times 10^{-15}(\Omega_0 h^2)^{2/3} g \text{ cm}^{-3}$ is the matter density at $t_{eq}$, and $\Phi$ is the initial fractional overdensity. For the first minicluster, $\Phi$ is of the order of unity, so they are already virialized at $t_{eq}$.

Linear initial fluctuations on scales larger than the original miniclusters have the white noise spectrum of rms fluctuations in spheres of mean mass $M$,

$$\Phi_{rms}(M) = \delta \rho/\rho \approx [M/10^2(\Omega_0 h^2)^{-2} M_\odot]^{-1/2} \Omega_0/\Omega_b.$$  

These fluctuations cause hierarchical clustering earlier than the standard CDM spectrum, starting with the minicluster formation and proceeding to larger scales. Applying the spherical model for each mass scale in the hierarchy of clustering, we find that the dark matter forms into virialized systems which have a virial radius

$$10^{16} \text{ cm} [M/10^2(\Omega_0 h^2)^{-2} M_\odot]^{-5/6} [(\Omega_b h^2)/(\Omega_0 h^2)^{5}]^{-1/3},$$

and a virial velocity $\approx \sqrt{GM/R}$

$$\approx 10 \text{ km sec}^{-1} [M/10^2(\Omega_0 h^2)^{-2} M_\odot]^{1/12} (\Omega_b/\Omega_0)^{1/6},$$

which shows that the specific binding energy of collapsing clusters slowly increases with time and mass. The characteristic velocity is high enough that any of these clusters can accrete $10^6K$ gas during collapse, so the baryons can collapse and cool at high redshift, immediately after decoupling at $z \approx 1000$, and form stars in these dark potentials.

Although structures typically get destroyed in a gravitational hierarchy as they are incorporated into larger systems, some isolated clusters can survive from early times to the present. The computed velocity dispersion approximately matches that of compact dwarf galaxies [19,20]. These systems have the densest dark halos yet found, with mean dark matter densities over $10^8\rho_{crit}$; in the spherical model, these systems must have turned around at $z \approx 30$. Such systems are easier to make here than in the usual CDM picture where the typical systems do not turn around until after $z \approx 10$. By extrapolation we predict that even denser, lower mass systems of dark matter, with somewhat smaller velocity dispersion, should have turned around at higher redshifts up to $z \approx 1000$, though these might now be very rare.

If any of the original miniclusters survive they might be detected by gravitational microlensing of quasars. The Einstein radius at the Hubble distance is [21,22]

$$R_E = 2\sqrt{GM/H} = 8 \times 10^{17} h^{-1/2} (M/10^2 M_\odot)^{1/2} \text{ cm.}$$

The smallest miniclusters are themselves smaller than this, so individual miniclusters are compact enough to amplify distant objects significantly. (Usually this is not the case for exotic dark matter; microlensing is usually only a useful probe of baryonic matter, such as stars, MACHOs and black holes.) Since $R_E$ is larger than the typical continuum emitting region of quasars, the miniclusters will tend to amplify them; this could lead to observable statistical associations of quasars with foreground dark matter [23]. In addition, motion of lensing miniclusters can lead to variations in amplification, which can produce an observable microlensing signature in a quasar light curve [21,22]. Finally, since the Einstein radius for the miniclusters is likely smaller than the narrow line emitting region of the quasars, they can lead to a scatter in quasar spectral properties [24].
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