Di-Nuclear Molecular States in Oblate-Oblate System

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Narrow resonances in heavy-ion scattering of $^{28}\text{Si} + ^{28}\text{Si}$ are studied as an example of oblate-oblate di-nuclear systems. By the use of the molecular model, a stable configuration of the system is found to be an equator-equator touching one, due to the oblate shape of the ^{28}Si nuclei. Normal modes at the equilibrium are solved and various molecular levels are obtained, which appear to be in good correspondence with experiments.

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1. Introduction

Narrow high-spin resonances observed in heavy-ion scattering of $^{24}\text{Mg} + ^{24}\text{Mg}$, $^{28}\text{Si} + ^{28}\text{Si}$, etc. are striking phenomena, because they are in high excitation of 60 ~ 70 MeV in the compound nuclei [1 ~ 6]. Their origin is still an open question. In the excitation functions of $^{28}\text{Si} + ^{28}\text{Si}$ scattering many isolated resonances with very narrow widths of about 150 keV are observed, level density of which is more than one per MeV [2, 3]. Therefore it is expected that many other degrees of freedom other than the relative motion participate in formation of those resonance states. The resonances are observed correlatingly among the elastic and inelastic decay channels of single and mutual excitations. Hence they are considered to be eigenstates of the whole compound system.

Lund group calculated deformation dependence of energy for ^{56}Ni by Nilsson-Strutinsky prescription and found secondary minima at high spins and at very large deformation, the structure of which appears to correspond to a di-nuclear configuration [7]. For $^{24}\text{Mg} + ^{24}\text{Mg}$, Maass and Scheid investigated stability of similar configurations with two-center-shell model, and obtained stable di-nuclear configurations for several high spins [8]. However dynamical modes which are activated in such an equilibrium and responsible for many narrow resonances are not yet clarified. The present authors proposed a new molecular model [9] and successfully applied it to $^{24}\text{Mg} + ^{24}\text{Mg}$ (prolate-prolate system) [10 ~ 12]. It was found that a pole-pole configuration is a stable one, and around the equilibrium various dynamical modes appear, which would be responsible for observed narrow resonances. In an oblate-oblate system with high spins such as $40\hbar$ [1], a stable configuration is inferred to be an elongated one, namely, equator-equator touching (E-E) configuration, due to the dominant centrifugal force. Hence it is interesting to investigate molecular modes around such a new type of equilibrium. Very recently Schmidt and Scheid have applied a similar model with such a stable configuration to the $^{12}\text{C} + ^{12}\text{C}$ system [13]. They used the same solving method as that adopted to the prolate-prolate system [12]. At the E-E configuration, however, it is easy to introduce collective butterfly coordinates, with which we are able to directly solve twisting motion without coupling to β -excitation (de-excitation). Then the nature of the

normal modes would be transparently explored. The purpose of the present paper is to develop a formulation for the dynamical modes in oblate-oblate systems and to apply it to the $^{28}\text{Si} + ^{28}\text{Si}$ system.

2. Formulation

First, we define a coordinate system of the molecular model and give the kinetic energy operator in the coordinate system. For simplicity, we take up the system of two identical deformed nuclei with a constant deformation and axial symmetry. In fig.1, such an oblate-oblate configuration is displayed. We introduce a rotating molecular frame, z' -axis of which is parallel to the relative vector of two interacting nuclei. In the laboratory frame, the kinetic energy of the system consists of the relative motion between two nuclei and rotational energies of constituent nuclei. It can be easily transformed into the rotational energy of the total system and kinetic energies of internal motions referred to the molecular frame. We start with seven degrees of freedom as illustrated in fig.1. Three of them are from the relative vector $\mathbf{R} = (R, \theta_2, \theta_1)$. Collective degrees of freedom of deformed nuclei are the orientations of the symmetry axes, which are described by the Euler angles (α_1, β_1) and (α_2, β_2) . In the present model, they are referring to the molecular frame, and therefore they are a kind of internal collective variables. α_1 and α_2 are combined into $\theta_3 = (\alpha_1 + \alpha_2)/2$ and $\alpha = (\alpha_1 - \alpha_2)/2$. Then we have $(q_i) = (\theta_1, \theta_2, \theta_3, R, \alpha, \beta_1, \beta_2)$, where θ_i 's are the Euler angles of the molecular frame with four other internal variables. At first, the kinetic energy is written in terms of angular velocities. After expressing those angular velocities with time derivatives of the corresponding Euler angles, we obtain a classical kinetic energy expression $\frac{1}{2} \sum g_{ij} \dot{q}_i \dot{q}_j$, and then quantize the system by the usual prescription. (Details are given in ref. [12].)

The kinetic energy operator obtained can be divided into three parts, that is, a rotational operator $\frac{\hbar^2}{2} \sum \mu_{ij} \hat{J}_i \hat{J}_j$ for the Euler angles θ_i , an internal kinetic energy operator for $(R, \alpha, \beta_1, \beta_2)$ and Coriolis coupling terms. Since the kinetic energy operators at this stage are not simple yet, nondiagonal parts $\hat{J}_i \hat{J}_j$ of T_{rot} with *small* coefficients are moved to T_C . Then, $T = T' + T'_C$ with $T' = T'_{\text{rot}} + T_{\text{int}}$. In the following, we restrict our discussion to rotation and vibration terms expressed by T' . Effects of the remaining terms T'_C will be discussed elsewhere. Consistently with the coordinate system, we

introduce a rotation-vibration type wave function as basis one,

$$\Psi_\lambda \sim D_{MK}^J(\theta_i) \chi_K(R, \alpha, \beta_1, \beta_2). \quad (1)$$

Then kinetic energy operators are expressed as follows:

$$T'_{\text{rot}}(J, K) = \frac{\hbar^2}{2} \left[\frac{1}{\mu R^2} \left(J(J+1) - \frac{3}{2} K^2 + \frac{1}{2} \cos 2\alpha \cot \beta_1 \cot \beta_2 (K^2 - 1) \right) + \frac{1}{4} \left(\frac{1}{I} + \frac{1}{\mu R^2} \right) \left(\frac{K^2 - 1}{\sin^2 \beta_1} + \frac{K^2 - 1}{\sin^2 \beta_2} - 2 \right) \right], \quad (2)$$

$$T_{\text{int}} = -\frac{\hbar^2}{2} \left[\frac{1}{\mu} \frac{\partial^2}{\partial R^2} + \left(\frac{1}{I} + \frac{1}{\mu R^2} \right) \left(\frac{\partial^2}{\partial \beta_1^2} + \frac{\partial^2}{\partial \beta_2^2} \right) + \frac{2 \cos 2\alpha}{\mu R^2} \frac{\partial^2}{\partial \beta_1 \partial \beta_2} + \frac{1}{4} \left(\left(\frac{1}{I} + \frac{1}{\mu R^2} \right) \left(\frac{1}{\sin^2 \beta_1} + \frac{1}{\sin^2 \beta_2} \right) - \frac{2}{\mu R^2} \right) \frac{\partial^2}{\partial \alpha^2} - \frac{\partial}{\partial \alpha} \frac{\cos 2\alpha}{2\mu R^2} \cot \beta_1 \cot \beta_2 \frac{\partial}{\partial \alpha} - \frac{1}{2\mu R^2} \left(\cot \beta_2 \frac{\partial}{\partial \beta_1} + \cot \beta_1 \frac{\partial}{\partial \beta_2} \right) \left(\sin 2\alpha \frac{\partial}{\partial \alpha} + \frac{\partial}{\partial \alpha} \sin 2\alpha \right) \right], \quad (3)$$

where μ denotes the reduced mass of the two nuclei, and I denotes the moment of inertia of a single constituent nucleus, the value of which is determined with the experimental excitation energy of the 2_1^+ state of the nucleus.

The interaction energy V_{int} between two deformed nuclei is calculated by the double-folding model[14]. We expand a density of each deformed nucleus in the body-fixed frame as $\rho_i^{\text{B}}(\mathbf{r}'') = \sum_{l=\text{even}} \rho_l(r'') Y_{l0}(\hat{\mathbf{r}}'')$, and then in the molecular frame, $\rho_i(\mathbf{r}') = \hat{R}(\alpha_i \beta_i \gamma_i) \rho_i^{\text{B}}(\mathbf{r}') = \sum_{lm} \rho_l(r') D_{m0}^{l*}(\alpha_i \beta_i \gamma_i) Y_{lm}(\hat{\mathbf{r}}')$. In the frame, V_{int} is expressed as

$$V_{\text{int}}(R, \alpha, \beta_1, \beta_2) = \sum_{m \geq 0} f_m(R, \beta_1, \beta_2) \cos(2m\alpha), \quad (4)$$

where

$$f_m(R, \beta_1, \beta_2) = \sum_{l'l''l} (2\pi)^{-3} i^{l'-l''-l} \hat{l} \hat{l}'' (l' l'' 00 | l0) F_{l'l'l}(R) \times (-1)^m (2 - \delta_{m0}) (l' l'' m - m | l0) \times d_{m0}^{l''}(\beta_1) d_{m0}^{l''}(\beta_2), \quad (5)$$

$$F_{l'l'l}(R) = \int dk k^2 j_l(kR) \tilde{v}(k) \tilde{\rho}_{l'}(k) \tilde{\rho}_{l''}(k). \quad (6)$$

$\tilde{\rho}_l(k)$ and $\tilde{v}(k)$ in eq. (6) denote Fourier transforms of nuclear density and of a nucleon-nucleon interaction. As usual a density profile of $\rho_i^{\text{B}}(\mathbf{r}'')$ is assumed to be a Fermi

distribution. For ^{28}Si nucleus the existence of hexadecapole deformation is suggested from coupled-channel analyses for elastic and inelastic neutron scattering[15]. Therefore we take the radius of each nucleus as $R_N(r'') = r_0 A_i^{1/3} [1 + \beta_Q Y_{20}(\hat{r}'') + \beta_H Y_{40}(\hat{r}'')]$ including two parameters β_Q and β_H for deformation. The value of r_0 is taken to be 1.03fm[12]. β_Q and β_H are determined to be -0.46 and 0.22, respectively, for the ratio β_Q/β_H from the value suggested in ref. [15] and for their magnitudes with $B(E2)$ value of the ground-rotational band of ^{28}Si . The surface diffuseness is determined to be 0.48fm with RMS radius of the ground state. Density dependent M3Y potential[16] is used as a folded nucleon-nucleon interaction without renormalization factor[17].

It should be noted that simple folding-model potential is not accurate in fully overlapping region. We must take account of a repulsive effect by the Pauli principle, or of density overlap going beyond the normal density. In addition to the interaction from nucleon-nucleon potential, we introduce a repulsive potential due to higher overlapping density, which is estimated by a binding energy loss of nuclear matter due to overlapping of density, i.e., by the equation of state. By using the prescription given in ref. [12], the strength V_P is determined to be 330MeVfm³ as a coefficient of the delta-type nucleon-nucleon pseudo-potential, for the $^{28}\text{Si} + ^{28}\text{Si}$ system with DDM3Y interaction, with a sharp nuclear surface of diffuseness 0.25fm.

3. Internal collective modes

In order to know dynamical aspects of multi-dimensional internal motion, we calculate an effective potential with specified spin J and K , defined as follows:

$$V_{JK}(R, \alpha, \beta_1, \beta_2) = V_{\text{int}}(R, \alpha, \beta_1, \beta_2) + T_{\text{rot}}(J, K). \quad (7)$$

In fig.2, an $R - \beta(\beta_1 = \beta_2)$ energy surface, i.e., $V_{JK}(R, \pi/2, \beta, \beta)$ is displayed for $J = 38$ and $K = 0$. We find a local minimum point at $\beta_1 = \beta_2 = \pi/2$ and $R = 7.6\text{fm}$, namely, at the equator-equator configuration, with a rather deep potential well around the equilibrium. In fig. 3(a), α -dependence of V_{JK} in the E-E configuration at the equilibrium distance is shown. (Note that our definition of the domain of the variables are $0 \leq \alpha < \pi$ and $0 \leq \beta_1, \beta_2 \leq \pi$.) We find the α -dependence is extremely weak. Another point is that we have two local minima at $\alpha = 0$ and $\pi/2$. Those

two configurations are, however, exactly the same, namely, *parallel* E-E configuration (z'' -axes of the constituent nuclei are parallel). Therefore it is necessary to impose symmetrization on the wave functions. In fig. 3(b), β -dependences of V_{JK} with $\beta_1 = \beta_2$ are compared between at $\alpha = 0$ and at $\alpha = \pi/2$, where solid line is for $\alpha = \pi/2$ (the cross section of fig.2 at $R = R_e = 7.6\text{fm}$) and dashed line for $\alpha = 0$. (Note that configurations with $\beta_1 = \beta_2 \neq \pi/2$ at $\alpha = 0$ are not the same as those with the same β_i -values at $\alpha = \pi/2$, but are the same as those with $\beta_1 = \pi - \beta_2$ at $\alpha = \pi/2$.) The β -well at $\alpha = \pi/2$ is seen to be rather shallow, compared with that at $\alpha = 0$. Hence, in addition to the weak α -dependence of V_{JK} in the E-E configuration, we have significantly α -dependent restoring force for β -motions around the E-E configuration.

In order to solve normal modes for four variables $(R, \alpha, \beta_1, \beta_2)$, we expand V_{JK} into a quadratic form for R , β_1 and β_2 , at the equilibrium E-E configuration, i.e., at $R = R_e$ and $\beta_1 = \beta_2 = \pi/2$, while for α we keep its dependence exactly in a form of $\cos(2m\alpha)$ series, such as those given in the interaction potential of eq. (4). Then the effective potential is expressed as

$$\begin{aligned} V_{JK}(R, \alpha, \beta_1, \beta_2) = & V_{JK}(R_e, \alpha, \frac{\pi}{2}, \frac{\pi}{2}) + \frac{k_R}{2}(R - R_e)^2 \\ & + \frac{1}{2}k_\beta^{11}(\alpha)\Delta\beta_1^2 + \frac{1}{2}k_\beta^{22}(\alpha)\Delta\beta_2^2 \\ & + k_\beta^{12}(\alpha)\Delta\beta_1\Delta\beta_2 + (\text{higher order}), \end{aligned} \quad (8)$$

where $\Delta\beta_i = \beta_i - \pi/2$. $k_\beta^{ij}(\alpha)$ denotes the second derivative $\partial^2 V_{JK}/\partial\beta_i\partial\beta_j$, $k_\beta^{11}(\alpha)$ being equal to $k_\beta^{22}(\alpha)$. Although $k_\beta^{ij}(\alpha)$ is a coefficient of $\Delta\beta_i\Delta\beta_j$ in the expansion, it is a function of α , i.e., we take into account α -dependence of the coefficient, in addition to the α -dependence of $V_{JK}(R_e, \alpha, \frac{\pi}{2}, \frac{\pi}{2})$. As $k_\beta^{11}(\alpha)$ consists of $\cos(2m\alpha)$ series with $m = \text{even}$ including zero, the major part is a constant k_0 from $m = 0$. We write $k_\beta^{11}(\alpha) = k_\beta^{22}(\alpha) = k_0 + k_2(\alpha)$, $k_2(\alpha)$ being a sum of contributions from terms with $m = \text{even} > 0$.

We introduce new coordinates in order to eliminate cross products of β_1 and β_2 both in T_{int} and in the quadratic expansion of V_{JK} . The new variables are *butterfly* $\beta_+ = (\Delta\beta_1 + \Delta\beta_2)/\sqrt{2} = (\beta_1 + \beta_2 - \pi)/\sqrt{2}$ and *anti-butterfly* $\beta_- = (\Delta\beta_1 - \Delta\beta_2)/\sqrt{2} = (\beta_1 - \beta_2)/\sqrt{2}$. Furthermore the inertia masses of three variables α, β_+ and β_- are

approximated by the values given at E-E configuration. Combining the kinetic energy operator and expanded effective potential, the total hamiltonian is given as follows:

$$H = H_0 + T'_C + (\text{higher order}), \quad (9)$$

$$H_0 = H_R + H_{\text{angl}}(\beta_+, \beta_-, \alpha), \quad (10)$$

$$H_R = -\frac{\hbar^2}{2\mu} \frac{\partial^2}{\partial R^2} + \frac{k_R}{2}(R - R_e)^2, \quad (11)$$

$$H_{\text{angl}}(\beta_+, \beta_-, \alpha) = H_+(\beta_+, \alpha) + H_-(\beta_-, \alpha) - \frac{\hbar^2}{4I} \frac{\partial^2}{\partial \alpha^2} + V_{JK}(R_e, \alpha, \frac{\pi}{2}, \frac{\pi}{2}), \quad (12)$$

$$H_{\pm}(\beta_{\pm}, \alpha) = -\frac{\hbar^2}{2} \left(\frac{1}{I} + \frac{1 \pm \cos 2\alpha}{\mu R_e^2} \right) \frac{\partial^2}{\partial \beta_{\pm}^2} + \frac{k_{\pm}(\alpha)}{2} \beta_{\pm}^2, \quad (13)$$

where $+$ or $-$ sign of $\pm$ in eq. (13) corresponds to β_+ and β_- degrees of freedom, respectively, with $k_+(\alpha) = k_0 + k_2(\alpha) + k_{\beta}^{12}(\alpha)$ and $k_-(\alpha) = k_0 + k_2(\alpha) - k_{\beta}^{12}(\alpha)$.

Now we solve the Schrödinger equation with the hamiltonian H_0 for the internal four degrees of freedom, which is separated into two parts. One is a hamiltonian H_R for radial motion and nothing but that of a simple one dimensional harmonic oscillator. Another is H_{angl} for the angle variables α, β_+ and β_- , which is also *almost separable* into H_+ of β_+ , H_- of β_- and remaining hamiltonian for α . H_+ and H_- again represent harmonic oscillators, although the masses and the restoring forces depend on α . Hence we analytically obtain wave functions for $H_{\pm}$ and their energy quanta $\hbar\omega_{\pm}$ with the frequencies

$$\omega_{\pm} = \sqrt{k_{\pm}(\alpha) \left(\frac{1}{I} + \frac{1 \pm \cos 2\alpha}{\mu R_e^2} \right)}. \quad (14)$$

Taking into account those vibrational energies from β -degrees of freedom, we introduce a reduced potential for α -motion, and obtain a Schrödinger equation for α -motion as follows:

$$\left[-\frac{\hbar^2}{4I} \frac{\partial^2}{\partial \alpha^2} + V_{JK}(R_e, \alpha, \frac{\pi}{2}, \frac{\pi}{2}) + E_{n_+, n_-}^{\beta}(\alpha) \right] \phi(\alpha) = E_{\text{angl}} \phi(\alpha), \quad (15)$$

where $E_{n_+, n_-}^{\beta}(\alpha)$ denotes vibrational energy $(n_+ + 1/2)\hbar\omega_+ + (n_- + 1/2)\hbar\omega_-$ from $H_+ + H_-$, added as a part of the reduced potential. Note that, in order to obtain

analytic form of $\hbar\omega_{\pm}$ in $\cos(2m\alpha)$ series, we expand square root in eq. (14) supposing $\omega_0 = \sqrt{k_0(1/I + 1/\mu R_e^2)}$ to be the leading term. We do not know simple analytic solutions of eq. (15). As the reduced potential $V_{JK}(R_e, \alpha, \frac{\pi}{2}, \frac{\pi}{2}) + E_{n_+, n_-}^{\beta}(\alpha)$ is described by a sum of $\cos(2m\alpha)$, we consider a solution $\phi(\alpha)$ described by cosine and sine functions of α , i.e., Fourier series. Then eq. (15) is reduced to a secular equation, which is easily solved. Thus the eigenenergy of the system is given as follows, specified by the quantum numbers $(n, n_+, n_-, K, (\nu, \pi_{\alpha}))$,

$$E^J(n, n_+, n_-, K, (\nu, \pi_{\alpha})) = E_0(R_e) + \frac{\hbar^2}{2} \left[\frac{J(J+1) - K^2 - 1}{\mu R_e^2} + \frac{K^2 - 2}{2I} \right] + \left(n + \frac{1}{2} \right) \hbar\omega_R + (n_+ + n_- + 1) \hbar\omega_0 + E_{\nu}^{\alpha}(\pi_{\alpha}), \quad (16)$$

where ν denotes a dominant frequency of α -motion with π_{α} for a parity concerning a reflection in the equilibrium of $\alpha = \pi/2$. The first and second terms in r.h.s. of eq. (16) are constant energies from the interaction potential and the centrifugal energy included in V_{JK} at the equilibrium, respectively. $(n_+ + n_- + 1)\hbar\omega_0$ and $E_{\nu}^{\alpha}(\pi_{\alpha})$ are vibrational energies for β -motions without α -dependence and the energy for α -motion, respectively.

There is a selection rule $\nu = \text{even}$ for $K = \text{even}$ and $\nu = \text{odd}$ for $K = \text{odd}$ ($K \pm \nu = \text{even}$), for α -motion. If n_+ is equal to n_- , then $K \pm \nu = 4m$, m being an integer. Because of the parity and boson symmetries, n_+ can be taken to be larger than or equal to n_- . For β -vibrational modes, we have a rule $(-1)^{n_+ + n_-} = (-1)^K$ due to symmetry of each constituent nucleus under space inversion. n_+ is also related to the parity of α -motion as $\pi_{\alpha} = (-1)^{n_+}$. As a special case for $K = 0$, both n_+ and n_- are even, i.e., π_{α} is positive, due to parity symmetry.

In fig. 4, molecular normal modes of $^{28}\text{Si} + ^{28}\text{Si}$ with spin 38 is displayed, where a pair of quanta (n_+, n_-) is given below the levels. Also given at the upper right-hand-side of the levels is a dominant quantum number ν for α -motion, which means α -motion is approximately described by $\cos \nu \alpha$ (or $\sin \nu \alpha$). Apparently K -excitation and twisting rotational mode appear to be lower than β -vibrational modes. The excitation energy for $K = 2$ is very small, smaller than 1 MeV, and even those for $K = 4$ or $\nu = 4$ are smaller than 3 MeV. In fig. 5(a), a few examples of wave functions for α -motion are shown, where

$\nu = 0$, and β -modes are in zero-point oscillation or 2quanta excitation of butterfly. We see that, with zero quanta for β -modes, the amplitude is wriggling around the value of a unit, the equilibria $\alpha = 0$ and $\pi/2$ being slightly favored. (With exact $\nu = 0$ we have a constant behavior.) With 2quanta for the butterfly mode, however, we find surprisingly strong concentration around the equilibrium of $\alpha = \pi/2$. In fig. 5(b), we inspect the reduced α -potential for quanta (2,0). Compared with the potential for (0,0), we find that the minimum at $\alpha = 0$ disappears, and the potential well at $\alpha = \pi/2$ is extended to wider region, which sustains the localization of the amplitude. One may wonder why the difference between $\alpha = 0$ and $\pi/2$ exists. The reason is as follows: at $\alpha = 0$, due to the definition of $\beta_{\pm}$, β -motion with $(n_+, n_-) = (2, 0)$ does not imply butterfly excitation but anti-butterfly one with 2quanta. Such a characteristic of $\beta_{\pm}$ coordinates gives larger excitation energy for (2,0) at $\alpha = 0$ than at $\alpha = \pi/2$. In fig. 5(c), energy quanta $\hbar\omega_{\pm}$ is shown versus α , where we are able to confirm the point. Returning back to di-nuclear configuration, for a configuration with $\beta_1 = \beta_2 < \pi/2$, for example, we obtain a butterfly one at $\alpha = \pi/2$, such as displayed in fig.1, while at $\alpha = 0$ we obtain an anti-butterfly one with the same values of β_i . Hence the localization around $\alpha = \pi/2$, seen in fig. 5(a), indicates nothing but a realization of a physical butterfly excitation. Thus we are able to classify the levels in fig. 4 into two groups, i.e., twisting mode and butterfly mode (or anti-butterfly), for which (t) or (b) mark is assigned in the lower part of the figure, respectively.

The molecular states obtained above are intrinsic states of the system, with which rotational bands are associated. Hence many molecular bands are expected to exist. It, however, should be noted that all of them cannot be observed as resonances in $^{28}\text{Si} + ^{28}\text{Si}$ collisions, because states with $K \neq 0$ and/or $\nu \neq 0$ carry no partial width of the elastic channel due to the selection rule in angular momentum and therefore they cannot be excited through the $^{28}\text{Si} + ^{28}\text{Si}$ entrance channel. Of course some of them may have $K = \nu = 0$ components due to the Coriolis coupling included in T'_C and thus have some resonance strengths. In fig. 6, the energy spectrum with $K = 0$ is shown in the resonance energy region. Experimental resonance levels are shown for comparison at the rightmost column of fig. 6. We see the density of the calculated levels including

β -excited states is very high. Thus we consider that, taking into account a little more increase of the level density due to the Coriolis coupling, the number of the experimental levels would be well reproduced by the present model.

4. Concluding Remarks

We have applied the molecular model to oblate-oblate system, i.e., to the $^{28}\text{Si} + ^{28}\text{Si}$ system. The stable configurations with high spins have been found to be equator-equator ones. We have developed a formulation for the dynamical modes around the E-E configuration, in which butterfly and anti-butterfly vibrations are firstly solved at each configuration with a specified value of α , and subsequently twisting motion is solved under the reduced α -potential including those vibrational energies. K -rotation and twisting rotational motion without butterfly or anti-butterfly excitation have appeared in low intrinsic excitation, while butterfly excitation has been found in higher energy, with a localization around $\alpha = \pi/2$ in the α -amplitude. Calculated molecular states for several high spins have been found to be as many as the observed resonances. Experimental information such as spins of the narrow resonances is strongly desired.

The results obtained above are extremely encouraging for further developments of the model. Effects of the Coriolis coupling terms T'_C are now under investigation. Up to now, a dynamical change of deformation of the constituent nuclei themselves is not taken into account, neither the axial asymmetry of ^{28}Si . Extension of the model, which includes such degrees of freedom, seems to be very interesting. In this respect, decays to the members of the strongly prolate deformed band of ^{28}Si observed by Sanders et al. would be suggestive[18].

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Figure captions

- Fig. 1 Di-nuclear configuration and the coordinates in the rotating molecular frame.
- Fig. 2 Effective potential energy V_{JK} for the $^{28}\text{Si} + ^{28}\text{Si}$ system with $J = 38$ and $K = 0$ is displayed, for $R - \beta(\beta_1 = \beta_2)$ degrees at $\alpha = \pi/2$. A local energy minimum exists at $R = 7.6\text{fm}$. Contours are in MeV.
- Fig. 3 (a) α -dependence of the effective potential V_{JK} with $J = 38$ and $K = 0$, for the E-E configuration at $R = R_e = 7.6\text{fm}$. (b) V_{JK} versus $\beta = \beta_1 = \beta_2$ at $\alpha = 0$ and at $\alpha = \pi/2$, which are displayed by dashed and solid lines, respectively.
- Fig. 4 Molecular normal modes for the $^{28}\text{Si} + ^{28}\text{Si}$ system for $J = 38$. The quantum states are specified by $(n, n_+, n_-, K, (\nu, \pi_\alpha))$, where $n = 0$ except for one level ($n = 1, \nu = 0$) displayed with dashed line. The quanta (n_+, n_-) of β -motions are given below the levels, and K at the bottom. ν for α -motion is given at the upper right-hand-side of the levels. (t) and (b) marks are assigned in the lower part of the figure, which indicate twisting rotational and butterfly modes, respectively.
- Fig. 5 (a) Wave functions for α -motion with $\nu = 0$, with zero-point oscillation (0,0) and butterfly excitation (2,0) for β -degrees, respectively, are displayed for $J = 38$ and $K = 0$. (b) The reduced α -potential $V_{JK}(R_e, \alpha, \frac{\pi}{2}, \frac{\pi}{2}) + E_{n_+, n_-}^\beta(\alpha)$. (c) α -dependence of β -energy quanta $\hbar\omega_\pm$.
- Fig. 6 Di-nuclear spectrum with $K = 0$ for the $^{28}\text{Si} + ^{28}\text{Si}$ system in the resonance energy region. Levels with $J = 38$ are connected by thin line for eye-guide. From the left, lowest states with specified spins, radially excited states (not well bound), twisting rotational ones with $\nu = 4$, and butterfly ones with $(n_+, n_-) = (2, 0)$ and $\nu = 0, 2$ and 4 are displayed. We selected resonance levels from peaks in the elastic and inelastic excitation functions according to a statistical analysis on their correlation[3]. They are shown on the right-hand-side, with an indicative bar for the available energy region of the data.

Fig.1

Fig.2

Fig.3

Fig.4

Fig.5

Fig.6