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**SOLAR AND ATMOSPHERIC NEUTRINOS IN THREE
GENERATIONS WITH A MAGNETIC MOMENT**

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ABSTRACT

A solution to the solar and atmospheric neutrino problems in three generations in the joint context of matter oscillations and the magnetic moment is investigated. An appropriate rotation of the evolution Hamiltonian reduces the three generation case to a two generation one. A convenient background for such a scenario with small neutrino masses and large magnetic moments is given by the Zee type models, in which the mass generation mechanism leads to a pair of separate orders of magnitude for the mass square differences between neutrino species. We obtain a ratio $\epsilon \simeq 10^{-2} - 10^{-3}$ between these orders of magnitude, so that one of them $[(0.3 - 3) \times 10^{-2} \text{ eV}^2]$ is suitable for the atmospheric neutrino solution and the other $[0(10^{-5} \text{ eV}^2)]$ for the solar neutrino solution. The magnetic moment leads to a decrease of the survival probability with solar neutrino energy. Such a decrease is consistent with the experimental situation.

The usually acknowledged solar neutrino puzzle consists in the discrepancy between the observed neutrino flux from the Sun¹⁾ and the solar standard model prediction²⁾. The attempts to explore the solutions to the solar neutrino puzzle, whether based on vacuum³⁾, matter (MSW) oscillations⁴⁾ or the magnetic moment⁵⁾, concentrate normally on the possibility of two neutrino generations. On the other hand, the experimental observation of atmospheric neutrinos⁶⁾ seems to indicate a ratio μ/e ⁷⁾ which is below the theoretical prediction. As in the case of solar neutrinos, this deficit can also be explained via MSW oscillations but appears to be independent of a magnetic moment assignment.

If one considers two neutrino generations, only two parameters are relevant for the physics of oscillations or the magnetic moment, namely the mass square difference between flavours and the vacuum mixing angle. Extending to three generations, five parameters are implied: two mass square differences between two pairs of the three neutrinos $\nu_{1,2,3}$ and three vacuum mixing angles. The orders of magnitude of the mass square difference parameter required to provide a solution are very different in the case of the solar and atmospheric neutrino problems. Thus, a joint explanation of both requires three neutrino families.

In this paper we propose a solution to the solar and atmospheric neutrino problems with three generations using MSW oscillations and the magnetic moment. The Zee type models⁸⁾, in particular the one developed in ref.9), where neutrino masses and a sizeable magnetic moment are generated radiatively and a large suppression for the masses is ensured, provide a natural scenario for the implementation of such a solution. In fact in the model of ref.9) with three Higgs doublets and a discrete symmetry, m_ν is doubly suppressed relative to the magnetic moment and can be as small as 10^{-3} eV, whereas $\mu_\nu \sim 10^{-11} \mu_B$. It will be seen that as a consequence of the special form of the neutrino mass generation mechanism⁹⁾, the three family case can be conveniently approximated by the much simpler case of two families, via an appropriate rotation of the Hamiltonian. Furthermore, as a consequence of the magnetic moment, the transition probability from active

to sterile neutrinos decreases smoothly with increasing energy, a fact consistent with experimental evidence.

We start with the neutrino Majorana mass matrix generated at two loop level on flavour basis^{9),10)}

$$M = \begin{pmatrix} 0 & m_{e\mu} & m_{e\tau} \\ m_{e\mu} & 0 & m_{\mu\tau} \\ m_{e\tau} & m_{\mu\tau} & 0 \end{pmatrix} \quad (1)$$

whose eigenvalues can be written as

$$\begin{aligned} m_1 &= \varepsilon m_0 \sin 2\theta \\ m_2 &= -m_0 \left(1 + \frac{\varepsilon}{2} \sin 2\theta\right) \\ m_3 &= m_0 \left(1 - \frac{\varepsilon}{2} \sin 2\theta\right) \end{aligned} \quad (2)$$

where $m_0 = \sqrt{m_{e\tau}^2 + m_{\mu\tau}^2}$, $\sin \theta = \frac{m_{e\tau}}{m_0}$, $\varepsilon = \frac{m_{e\mu}}{m_0}$ and the mixing matrix relating flavour and mass eigenstates is¹⁰⁾

$$S = \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2}c & s & s \\ -\sqrt{2}s & c & c \\ 0 & 1 & -1 \end{pmatrix} \quad (2a).$$

Since the mass matrix elements (1) are proportional to the heaviest charged lepton mass squared exchanged in the corresponding diagram (e.g. $m_{e\mu} \sim m_\mu^2/M_W$), the quantity ε represents indeed a small number. Typically $\varepsilon \approx 10^{-2} - 10^{-3}$. Hence the eigenvalue spectrum consists of a light state of mass m_1 and two heavier ones whose spacing is small

$$\begin{aligned} \Delta^2 m_{31} &\approx \Delta^2 m_{21} \approx m_0^2 \\ \Delta^2 m_{23} &= 2\varepsilon m_0^2 \sin 2\theta \end{aligned} \quad (3).$$

Furthermore, on the same basis and in matter, the Hamiltonian can be written as¹⁰⁾

$$H = \frac{m_0^2}{2E} \begin{pmatrix} s^2 + \rho & sc & \epsilon c \\ sc & c^2 & \epsilon s \\ \epsilon c & \epsilon s & 1 \end{pmatrix} \quad (4).$$

Here $s = \sin\theta$, $c = \cos\theta$, $\rho = \frac{a_{\nu_e} - a_{\nu_\mu}}{m_0^2/2E}$ where a_{ν_e} denotes the interaction potential for electron neutrinos¹¹⁾ and we recall that $a_{\nu_\mu} = a_{\nu_\tau}$. In (4) we have subtracted the identity and neglected terms of order ϵ^2 . Rotating this matrix to a basis where the 12 and 21 elements vanish one obtains

$$H' = \frac{m_0^2}{2E} \begin{pmatrix} (s^2 + \rho) c_m^2 - 2s_m c_m s c + & 0 & \epsilon(c_m c - s_m s) \\ + s_m^2 c^2 & & \\ 0 & s_m^2 (s^2 + \rho) + 2s_m c_m s c + & \epsilon(s_m c + c_m s) \\ & + c_m^2 c^2 & \\ \epsilon(c_m c - s_m s) & \epsilon(s_m c + c_m s) & 1 \end{pmatrix} \quad (5)$$

where $s_m = \sin\theta_m$, $c_m = \cos\theta_m$ and θ_m is the rotation angle given by

$$\tan 2\theta_m = \frac{\sin 2\theta}{\cos 2\theta - \rho} \quad (6).$$

From the definition of ρ given above and the orders of magnitude of m_0^9 , a_{ν_e} ¹¹⁾ and the neutrino energy $E(0.1 - 10\text{MeV})$, it is clear that $\rho \ll \cos 2\theta$ so that $\theta_m \approx \theta$. Consequently the 11, 22 and 13 elements of this matrix can be approximated by

$$H'_{11} \approx \rho c^2, H'_{12} \approx 1 + \rho s^2, H'_{13} \approx \epsilon(c^2 - s^2) \quad (7)$$

so that

$$|H'_{11} - H'_{33}| \gg |H'_{13}| \quad (8).$$

The submatrix defined by the elements $H'_{11}, H'_{13}, H'_{33}$ is therefore very close to diagonal. No such conclusion can be drawn as regards the other submatrix defined by $H'_{22}, H'_{23}, H'_{33}$. Thus it is seen that the Hamiltonian (5) has in fact the structure

submatrix defined by $H'_{22}, H'_{23}, H'_{33}$. Thus it is seen that the Hamiltonian (5) has in fact the structure

$$H' = \begin{pmatrix} H'_{11} & 0 & 0 \\ 0 & H'_{22} & H'_{23} \\ 0 & H'_{23} & 1 \end{pmatrix} \quad (9).$$

Hence in this basis, which we denote by $(\nu'_e, \nu'_\mu, \nu'_\tau)$, one of the neutrinos (namely ν'_e) decouples from the other two and propagates unchanged. Moreover $\nu'_\tau \equiv \nu'_\mu$. We emphasize that this decoupling effect is a consequence of the relative orders of magnitude of the mass eigenvalues (2), with m_2 and m_3 being almost degenerate and m_1 much smaller. One can therefore study separately the propagation of the two neutrino state (ν'_μ, ν'_τ) thereby reducing the three generation case to the two generation one.

In the class of models⁹⁾ underlying this analysis, neutrinos are Majorana particles, as we have seen, with a large transition magnetic moment which plays of course an important role in their propagation. So one must enlarge this two dimensional basis to include the corresponding antiparticles. Using the notation

$$H'_2 = \begin{pmatrix} H'_{22} & H'_{23} \\ H'_{23} & 1 \end{pmatrix} \quad (10)$$

the (4x4) Hamiltonian is then

$$H'_4 = \begin{pmatrix} H'_2 & \mu B \\ \mu B & H'_{-2} \end{pmatrix} \quad (11).$$

Here H'_2 (H'_{-2}) denotes the (2x2) Hamiltonian submatrix between particles (antiparticles). The notation μB in (11) represents the submatrix

$$\mu B = \begin{pmatrix} s_2 \theta \mu_{e\mu} B & (s\mu_{e\tau} + c\mu_{\mu\tau})B \\ (s\mu_{e\tau} + c\mu_{\mu\tau})B & 0 \end{pmatrix} \quad (11a)$$

where μ_{ij} denotes the transition moment between v_i and $\bar{v}_j$ and as in (7) we work in the approximation $\theta_m = \theta$. Using eq.(3) and adding the identity matrix multiplied by a_{v_μ} , H_2 can be converted to

$$H_2' = \begin{pmatrix} \frac{\Delta_{31}^2}{2E} + s^2 a_{v_e} + c^2 a_{v_\mu} & \frac{\Delta_{23}^2}{4E} \\ \frac{\Delta_{23}^2}{4E} & \frac{\Delta_{31}^2}{2E} + a_{v_\mu} \end{pmatrix} \quad (12)$$

with obvious notation. For H_{-2}' one just performs the replacements $a_{v_i} \rightarrow -a_{v_i}$.

Finally we rotate the matrix H_4' defined by (11), (11a) and (12) to a new basis where for simplicity H_2' and H_{-2}' are diagonal. The angle α that diagonalizes H_2' is given by the standard expression

$$\tan 2\alpha = \frac{2(H_2')_{12}}{(H_2')_{22} - (H_2')_{11}} = \frac{\Delta_{23}^2/2E}{s^2(a_{v_\mu} - a_{v_e})} \quad (13).$$

Correspondingly, the diagonalizing angle for H_{-2}' is defined to be $\bar{\alpha} = \frac{\pi}{2} - \alpha$, so that the two coincide in the vacuum ($a_{v_e} = a_{v_\mu} = 0$). Hence, using the simplifying notation $a_v = a_{v_e} - a_{v_\mu}$, H_4' becomes

$$H_{4R}' = \begin{pmatrix} c_\alpha^2 s^2 a_v + a_{v_\mu} - & 0 & [\frac{1}{2}s_{2\alpha} s_{2\theta} - & c_\alpha^2 s_{2\theta} \mu B \\ -s_{2\alpha} \Delta_{23}^2/4E & & -(s+c)] \mu B \\ 0 & s_\alpha^2 s^2 a_v + a_{v_\mu} + & s_\alpha^2 s_{2\theta} \mu B & [\frac{1}{2}s_{2\alpha} s_{2\theta} + \\ +s_{2\alpha} \Delta_{23}^2/4E & & & +(s+c)] \mu B \\ [\frac{1}{2}s_{2\alpha} s_{2\theta} - & s_\alpha^2 s_{2\theta} \mu B & -s_\alpha^2 s^2 a_v - a_{v_\mu} - & 0 \\ -(s+c)] \mu B & & -s_{2\alpha} \Delta_{23}^2/4E \\ c_\alpha^2 s_{2\theta} \mu B & [\frac{1}{2}s_{2\alpha} s_{2\theta} + & 0 & -c_\alpha^2 s^2 a_v - a_{v_\mu} + \\ & +(s+c)] \mu B & & +s_{2\alpha} \Delta_{23}^2/4E \end{pmatrix} \quad (14)$$

on the basis $(v_2 v_3 \bar{v}_2 \bar{v}_3)$ and where we subtracted $\Delta_{31}^2/2E$ times the identity. As before we work in the approximation $\theta_m = \theta$. Furthermore, since

all Yukawa couplings to the $SU(2)_L$ Higgs singlet are in Zee type models within the same order of magnitude^{8),9)}, we take in H_{4R}' a common value μ for all transition moments.

We turn next to the survival probability in matter for electron neutrinos. To this end we first recall the relationships among the three bases we used, namely

$$\begin{pmatrix} \nu_e' \\ \nu_\mu' \\ \nu_\tau' \end{pmatrix} = \begin{pmatrix} c_m & s_m & 0 \\ -s_m & c_m & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} \quad (15)$$

and

$$\begin{pmatrix} \nu_2 \\ \nu_3 \\ \bar{\nu}_2 \\ \bar{\nu}_3 \end{pmatrix} = \begin{pmatrix} c_\alpha & s_\alpha & 0 & 0 \\ -s_\alpha & c_\alpha & 0 & 0 \\ 0 & 0 & s_\alpha & c_\alpha \\ 0 & 0 & -c_\alpha & s_\alpha \end{pmatrix} \begin{pmatrix} \nu_\mu' \\ \nu_\tau' \\ \bar{\nu}_\mu' \\ \bar{\nu}_\tau' \end{pmatrix} \quad (15a).$$

The electron neutrinos produced in the Sun are obviously a superposition of these states. At the production point (time $t = 0$) we have

$$\nu_e(0) = c_m \nu_e' - s_m \nu_\mu'(0) = c_m \nu_e' - s_m(c_\alpha \nu_2(0) - s_\alpha \nu_3(0)) \quad (16).$$

We have omitted in (16) the argument of ν_e' since this propagates unchanged (see eq.(9)). It is therefore easily seen that the possible transitions are contained in the Hamiltonian (14). They are $\nu_2 \rightarrow \bar{\nu}_3$ and $\nu_3 \rightarrow \bar{\nu}_2$. Let us denote by b_{23} and b_{32} respectively the amplitudes for these transitions and b_{22} , b_{33} the corresponding survival amplitudes. The submatrices of (14) involved in this evolution are therefore

$$\begin{pmatrix} (H_{4R}')_{11} & (H_{4R}')_{14} \\ (H_{4R}')_{41} & (H_{4R}')_{44} \end{pmatrix} \text{ and } \begin{pmatrix} (H_{4R}')_{22} & (H_{4R}')_{23} \\ (H_{4R}')_{32} & (H_{4R}')_{33} \end{pmatrix} \quad (16).$$

For adiabatic propagation we have¹¹⁾

$$b_{22} = \tilde{c}'_1 \tilde{c}'_0 \exp \left[i \int_{r_0}^{r_1} (H'_{4R})_{11} dr \right] + \tilde{s}'_1 \tilde{s}'_0 \exp \left[i \int_{r_0}^{r_1} (H'_{4R})_{44} dr \right] \\ b_{33} = \tilde{c}'_1 \tilde{c}'_0 \exp \left[i \int_{r_0}^{r_1} (H'_{4R})_{22} dr \right] + \tilde{s}'_1 \tilde{s}'_0 \exp \left[i \int_{r_0}^{r_1} (H'_{4R})_{33} dr \right] \quad (17)$$

where $\tilde{c}(\tilde{c}')$ and $\tilde{s}(\tilde{s}')$ denote the cosine and sine of the corresponding rotation angle $\tilde{\theta}(\tilde{\theta}')$ that diagonalizes (16) and is given by the standard expression (see e.g. eq.(13)). The subscripts 0,1 refer to the initial and final location of the neutrino (production point and vacuum) and r is the distance measured from the Sun's centre. Thus, after a time t

$$\nu_e(t) = c_m \nu_e' - s_m [c_{\alpha_1} [b_{22} \nu_2(0) + b_{23} \bar{\nu}_3(0)] - s_{\alpha_1} [b_{33} \nu_3(0) + b_{32} \bar{\nu}_2(0)]] \quad (18)$$

where we used (15), (15a). Combining (16) and (18), one obtains for the survival amplitude

$$(\nu_e(t), \nu_e(0)) = c^2 + s^2 \left[c_{\alpha_1} c_{\alpha_0} b_{22} + s_{\alpha_1} s_{\alpha_0} b_{33} \right] \quad (19).$$

The probability in the adiabatic approximation is the square modulus of this quantity. Non-adiabatic effects in the vicinity of each resonance corresponding to processes $\nu_2 \rightarrow \bar{\nu}_3$ and $\nu_3 \rightarrow \bar{\nu}_2$ must be taken into account. The analytic expression for the probability then takes the propagation through the Sun as adiabatic with the exception of a short segment in the vicinity of the resonance¹¹⁾. Denoting by P_c, P'_c the probability for the non-adiabatic "jumping" at each resonance, we have

$$P(\nu_e \rightarrow \nu_e) = c^4 + s^4 \left[c_{\alpha_0}^2 c_{\alpha_1}^2 \left[\frac{1}{2} + \left(\frac{1}{2} - P_c \right) c_{2\tilde{\theta}_0} c_{2\tilde{\theta}_1} \right] + \right. \\ \left. + s_{\alpha_0}^2 s_{\alpha_1}^2 \left[\frac{1}{2} + \left(\frac{1}{2} - P'_c \right) c_{2\tilde{\theta}_0}' c_{2\tilde{\theta}_1}' \right] \right] \quad (20).$$

There are several possible approximate formulas for P_c . In the case of the Sun, which is our concern here, the very simple Landau-Zener one for linearly decreasing density is accurate for the purpose¹²⁾. For two generations^{11),12)}

$$P_c = \exp \left(- \frac{\mu^2 B^2}{\frac{\Delta^2}{2E}} R_0 \right) \quad (21).$$

The parameter $\Delta^2/2E$ fixes the location of the resonance, B is the magnetic field at the resonance and $R_0 = 0.09 \times$ (solar radius). In order to investigate the high and low energy limits of eq.(20), we first note that in the vacuum, since $a_{\nu_\mu} = a_{\nu_e} = 0$, we have $s_{\alpha_1}^2 = c_{\alpha_1}^2 = \frac{1}{2}$ (see eq.(13)). Further, the angles $\tilde{\theta}_1, \tilde{\theta}_1'$ which diagonalize the submatrices (16) in the vacuum satisfy $\tan 2\tilde{\theta} \approx 0$, as B in the vacuum is either zero or very small and $\Delta^2/4E$ is fixed. Also $\tilde{\theta}_0, \tilde{\theta}_0'$ satisfy $\tan 2\tilde{\theta} \approx 0$, because at the production point, in dense matter, a_{ν_e}, a_{ν_μ} are at least one order of magnitude above μB ¹¹⁾. Since a resonance is assumed to exist in each case, $\tilde{\theta}$ and $\tilde{\theta}'$ must rotate from $\pi/2$ to zero from the production point to the solar surface, or conversely. Hence in (20) one can perform the replacements $c_2 \tilde{\theta}_0^{(1)} c_2 \tilde{\theta}_1^{(1)} = -1$.

Furthermore, the atmospheric neutrino solution requires the mass difference between neutrino flavours to be of order $(0.3-3) \times 10^{-2} \text{ eV}^2$ ⁷⁾, while for the solar neutrino solution it should be 10^{-5} eV^2 or smaller¹³⁾. Therefore on the grounds of eq.(3), one must choose

$$\Delta_{31}^2 \approx \Delta_{21}^2 \approx 10^{-2} \text{ eV}^2 \text{ and } \Delta_{23}^2 \approx 10^{-5} \text{ eV}^2 \quad (22).$$

Recalling the range of solar neutrino energies $E = 0.1-10 \text{ MeV}$, one has, for the low energy sector,

$$\tan 2\alpha_0 \approx \frac{10^{-5}/5 \times 10^5}{0(10^{-11} - 10^{-12})} \quad (23a)$$

and for the high energy sector

$$\tan 2\alpha_0 \approx \frac{10^{-5}/2 \times 10^7}{0(10^{-11} - 10^{-12})} \quad (23b)$$

implying for low energy solar neutrinos $\alpha_0 \approx \frac{\pi}{4}$ and for high energy ones $\alpha_0 \approx 0$. Thus eq.(20) can be finally simplified to

$$P(\nu_e \rightarrow \nu_e) = c^4 + \frac{1}{4} s^4 (P_c + P_c') \quad (24a)$$

in the low energy sector of solar neutrinos ($E = 0.1 - 0.4$ MeV) and

$$P(\nu_e \rightarrow \nu_e) = c^4 + \frac{1}{2} s^4 P_c \quad (24b)$$

in the high energy sector ($E \approx 10$ MeV).

The decreasing behaviour of P_c with energy (see eq.(21)) still of course holds for three generations, so equations (24) indicate that the survival probability is also smaller for high energies than for low ones. The experimental situation is on the other hand not conclusive regarding the possibility of energy dependence of solar neutrino suppression¹²⁾. Nevertheless, if such a dependence exists, the data are consistent with high energy neutrinos being more suppressed than low energy ones and clearly inconsistent with the opposite¹²⁾. Our analysis also obviously favours this possibility.

The atmospheric neutrino solution requires a vacuum mixing angle in the range⁷⁾

$$\sin^2 2\theta \approx 0.4 - 0.8$$

hence

$$c^4 \approx 0.52 - 0.79$$

(25)

indicating that the neutrino suppression is at most approximately 48% both for the high and low energy solar neutrinos.

To summarize, we have investigated a scenario with a large neutrino transition magnetic moment and a small radiative mixing mass in three generations based on Zee type models. Along with the MSW effect, the magnetic moment resonant transition is expected to play an essential role in the conversion from active to sterile neutrinos. Since three generations are assumed to be involved, there are two independent mass square differences between flavours to be considered whose orders of magnitude lie far apart. This separation is determined by the mass generation mechanism and as a

consequence the mass differences can be used to explain both the atmospheric and the solar neutrino problems. The survival probability obtained for solar neutrinos decreases with increasing neutrino energy, a phenomenon which appears to be consistent with experimental data.

Finally, the particle physics implementation of this scenario is most naturally done in the context of the Zee type model with three Higgs doublets⁹⁾. In fact, although the more economical two Higgs doublet model is enough to generate a large magnetic moment while keeping the electron and muon neutrinos almost degenerate and light with respect to the tau neutrino, it is plagued by a two loop mass diagram¹⁴⁾. This gives a contribution to neutrino masses which is too large, the problem being avoided by including a third doublet.

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