On the measurement of the Hubble constant
in a local low-density universe

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ON THE MEASUREMENT OF THE HUBBLE CONSTANT
IN A LOCAL LOW-DENSITY UNIVERSE

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ABSTRACT

Astrophysical observations indicate that the “Local Universe” has a relatively lower matter density ($\Omega_0$) than the predictions of the standard inflation cosmology and the large-scale motions of galaxies which provide a mean mass density to be very close to unity. In such a local underdense region the Hubble expansion may not be representative of the global behaviour. Utilizing an underdense sphere embedded in a flat universe as the model of our “Local Universe”, we show that the local Hubble constant would be 1.2 – 1.4 times larger than the global value on scale of $\sim 80$ Mpc, depending on the variation of $\Omega_0$. This may account for the recent measurements of the unpleasantly large Hubble constant of $\sim 80$ km/s/Mpc using the Cepheid variables in the Virgo cluster and the relative distance between Virgo and Coma cluster and removes the resulted apparent paradox of the age of our universe.

Subject headings: cosmology: distance scale – large-scale structure of universe

1. INTRODUCTION

It is firmly established from the dynamical analysis that the mean matter density of the Local Supercluster on scale of $\sim 10$ Mpc is within a factor of two of $\Omega_0 = 0.1$, which is about an order of magnitude smaller than the global value of $\Omega_0 = 1$ favoured by the inflation cosmological model. Indeed, the large-scale motions of galaxies with the assumption of “light-trace-mass” suggest an increasing tendency of $\Omega_0$ with scale up to $\sim 100$ Mpc [see Dekel (1994) for a recent review]. It is likely that we are situated in a local
low-density “universe”. Therefore, the local properties can significantly differ from the
global ones and any local astrophysical measurements may not actually be representative
of the real universe. Although this general argument is not entirely a new one [Readers
are recommended to refer to Turner, Cen & Ostriker (1992, references therein)], not much
attention has been really concentrated on the issue, due to the unknown details of theo-
retical consideration on how large the differences between the local measurements and
the global ones would be.

The critical question arises from the two recent independent measurements of the Hub-
ble constant through the observations of Cepheid variables in the Virgo cluster (Pierce
et al. 1994; Freedman et al. 1994), both of which give rise to a high value of \( H_0 \approx 80 \)
km/s/Mpc. In the frame of standard cosmological model that assumes a completely homo-
geneous matter distribution everywhere in a flat universe, this large \( H_0 \) leads to a conflict
that the expansion age of the Universe (\( \sim 8 \) Gyr) is smaller than the age of globular
clusters of the Galaxy (\( \sim 16 \) Gyr), implying that either the standard cosmological model
needs to be revised or the present measurements of stellar ages need to be re-examined.
This situation is indeed unfortunate as the standard Big Bang model and the theories
of stellar evolution are two fundamentals of modern astrophysics and any modifications
may have significant impacts on our conventional views. To avoid the difficulties resulted
from the new measurements of \( H_0 \), we explore the possibility of attributing the high \( H_0 \)
to a local low-density region which expands faster than the rest of the universe. This may
survive the standard cosmological and stellar evolutionary theories.

2. MODEL AND RESULTS

The Tolman-Bondi metric is often used for the description of the space-time of an
overdense or an underdense spherical region embedded in an expanding Universe (e.g.
Zel’dovich & Grishchuk 1984; Arnau et al. 1993; Fang & Wu 1993; Wu & Fang 1994):

\[
ds^2 = \frac{r^2}{1 - \epsilon f^2(x)} dx^2 + r^2(x, t)(d\theta^2 + \sin^2 \theta d\phi^2) - dt^2. \tag{1}
\]
The evolution of \( r(x,t) \) is given by

\[
   r^2 = -\epsilon f^2(x) + \frac{F(x)}{r}
\]

and \( F(x) \) relates with the invariant mass \( M(x) \) within \( x \) through

\[
   F(x) = 2M(x) = \int_0^r 8\pi x^2 \rho(x,t) \, dx \equiv \frac{8\pi}{3} r_i^2 \bar{\rho}_i
\]

where the prime denotes a derivative with respect to \( x \), the dot with respect to \( t \), \( \rho \) and \( \bar{\rho} \) are the matter density and the mean matter density, respectively. We use the subscript “\( i \)” to stand for the “initial epoch” and “\( e \)” for the “present time”.

Assuming a flat universe as the background, we can write the solution to an underdense region as \( (\epsilon = -1) \)

\[
   \frac{S}{S_i} = \frac{1 - \delta_i \cosh \eta - 1}{\delta_i} \\
   \sinh \eta - \eta = \frac{4}{3} \frac{\delta_i^{1/2} (\frac{t}{t_i} - 1)}{1 - \delta_i} + \frac{2\delta_i^{1/2}}{1 - \delta_i} - \cosh^{-1} \frac{1 + \delta_i}{1 - \delta_i}.
\]

Here \( S \) is the expansion factor of the universe defined by \( r = S(x,t) x \), \( \delta_i \), the mean initial matter perturbation given by \( \bar{\rho}_i = \rho_s(1 - \delta_i) \), and \( \rho_c \), the critical mass density of the universe. The present Hubble constant is introduced through \( H_0 = \dot{S}_0/S_0 \), which is generally a function of the radial coordinate \( x \), i.e., the expansion rate may vary not only with time but also with position. We obtain the relation between the present local Hubble constant \( (H_L) \) in the underdense region and the present global Hubble constant \( (H_G) \) in the background universe to be

\[
   \frac{H_L}{H_G} = \frac{(1 + z_i)^{3/2}}{(S_0/S_i)} \left[ \delta_i + \frac{1 - \delta_i}{(S_0/S_i)} \right]^{1/2}
\]

in which \( z_i \) is the redshift at the epoch \( t = t_i \) when the initial density perturbation occurred. For simplicity, we choose \( t_i \) to be the decoupling time that corresponds to \( z_i \approx 1000 \). The present density contrast \( \frac{\Delta \rho}{\rho} \) stemming from the initial density perturbation profile \( \delta(x) \) is found to be

\[
   \frac{\Delta \rho}{\rho} = 1 - \left[ \frac{1 + z_i}{(S_0/S_i)} \right]^3 \frac{1 - \delta(x)}{1 + d \ln S_0/d \ln x}
\]
and the present density parameter is $\Omega_0 = 1 - \frac{\Delta \rho}{\rho}$.

We first adopt a constant density perturbation profile $\delta(x) = \delta_i = \delta_0$ in the local underdense region. In this case, the present Hubble constant inside the underdense sphere depends uniquely on the present matter density $\Omega_0$, as is shown in Figure 1. In fact, this corresponds to an ensemble of solutions to the expanding “universe” with different $\Omega_0(<1)$. Utilizing this model to our local universe, a present matter density of $\Omega_0 = 0.2$ results in a local Hubble constant of 1.33 times larger than the global one and the $\Omega_0 = 0.5$ gives $H_L = 1.19 H_G$. In the extreme case of $\Omega_0 = 0$, the local expansion rate is 1.5 times larger than that of the background universe of $\Omega_0 = 1$. Therefore, if we are unfortunately situated in a local low-density universe with a mass density of a few tenth of the critical value of the background universe, the local measurements like the two recent observations (Pierce et al. 1994; Freedman et al. 1994) utilizing the Cepheid variables of the Virgo cluster and the recession velocity of the Coma cluster would provide a relatively higher Hubble constant than the true value in the background flat universe.

Nevertheless, the local universe on scale of as large as the distance to Coma cluster cannot be well described by a constant matter perturbation. It appears that $\Omega_0$ varies from $\sim 0$ to $\sim 1$ with the increase of scale. We have then tested two initial density perturbation profiles that give rise to the similar shape of $\Omega_0$ to the observed one: $\delta(x) = \delta_0/[1 + (x/a)^2]$ (the isothermal sphere with a core) and $\delta(x) = \delta_0/[1 + (x/a)^2]^{3/2}$ (the King model), where $\delta_0$ determines the maximum initial density contrast and $a (a_0)$ is the initial (present) scale length of the perturbed region. Our computations show that these two profiles don’t provide significantly different results of the present density contrast within $\sim 100$ Mpc. Figure 2 demonstrates the variations of $\Delta \rho/\rho$ with distance for three sets of parameters in the King model. Although these curves might not exactly fit to the true distribution of $\Omega_0$ which has been unknown to date, they essentially represent the variation tendency of $\Omega_0$ with scale, which provide $\Omega_0 \sim 0.1$ on scale of 10 Mpc and $\Omega_0 \sim (0.4 - 0.9)$ on scale of 100 Mpc. The Hubble constant variations with distance are shown in Figure 3 for the same parameters in Figure 2. At the distance of $\sim 80$ Mpc where the Coma cluster
locates as were indicated by the recent observations, the local Hubble constant may be estimated to be 1.2–1.4 times larger than the global one, depending on the local matter content.

3. DISCUSSION

Recent measurements of the Hubble constant using the Cepheid variables in the Virgo cluster and the relative distance between the Virgo and the Coma cluster result in $H_0 = 87 \pm 7$ km/s/Mpc (Pieroe et al. 1994) and $H_0 = 80 \pm 17$ km/s/Mpc (Freedman et al. 1994), which disagree with the other two HST measurements (Sandage et al. 1994; Saha et al. 1994) of $H_0 = 52 \pm 9$ km/s/Mpc using the Cepheid variables and the brightness of the type Ia supernovae in two relatively closer galaxies (distance $= 4.1 – 4.7$ Mpc). The former determines actually the expansion rate of the Coma cluster which is so distant that the peculiar velocity contributes a negligible component and furthermore, the different methods produce the same mean relative Coma-Virgo distance modules. Whilst the latter may suffer from the local calibration of the brightness of type Ia supernovae, leading to an overestimate of distance (Hogan 1994). We believe that the Coma recession velocity and the Coma-Virgo relative distance measurements are likely to reflect the nature of the local universe.

The apparent paradox of the “young” age of the universe may have arisen from the misuse of the local Hubble constant $H_L$ as the global value $H_G$. The Hubble constant of the background universe can be estimated by reducing the measured Hubble constant by a factor of 1.2–1.4 at the distance of Coma cluster, leading to an increase of the currently estimated age of the universe by the same factor. Thus, the cosmological conflict between the expansion age of the universe, predicted in the standard cosmological model using the recent measurements of the large Hubble constant of $H_0 \sim 80$ km/s/Mpc, and the ages of the oldest globular clusters of the Galaxy may vanish, or at least is partially resolved.

It appears that the true Hubble constant of the universe can be directly measured only
when the observations are made beyond the local low-density region. It then remains to be promising that the time delay between the images of gravitationally lensed quasars at the redshift of $\sim 1$ may provide the reliable value of $H_0$. From the optical/radio monitoring of the double quasar 0957+561A,B ($z = 1.41$) over $\sim 10$-years coverage (Vanderriest et al. 1989; Roberts et al. 1991), which exhibits a time delay of $415/513$ days, and the theoretical modeling of the lensing galaxies, one finds a Hubble constant of $H_0 = 48^{+16}_{-7}/39^{+15}_{-6}$. Another evidence for supporting a low value of $H_0 \sim 50$ km/s/Mpc obtained in the distant universe comes from the measurement of the Sunyaev-Zel’dovich effect in Abell cluster A2218 ($z = 0.171$) (Jones et al. 1993). Indeed, these two measurements of the Hubble constant at the cosmological distance yield a value apparently smaller than the local one, indicative of an expansion age of the universe comparable with the age of the oldest globular clusters. Bartlett et al (1994) have even claimed for a Hubble constant of as small as 30 km/s/Mpc and found that the small $H_0$ can overcome most of the difficulties in the current standard cosmological model. Further measurements of dynamical properties of the local universe and the future HST observations of the Cepheid variables in more distant galaxies are needed to confirm our arguments.

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References

Figure Captions

Figure 1  The ratio of the local Hubble constant $H_L$ to the global value $H_G$ versus $\Omega_0$. The local underdense region is modelled by a sphere with constant matter density $\Omega_0$ embedded in a flat universe.

Figure 2  Variations of local matter density $\Omega_0$ with scale for the King model as the local negative density perturbations. Two parameters determine the model: the maximum initial density fluctuation $\delta_0$ and the present length scale $a_0$.

Figure 3  Variations of $H_L/H_G$ with scale for the three sets of parameters in Figure 2.