GROUND MOTION IN LEP AND LHC

L. VOS

CERN, CH-1211 Geneva 23, Switzerland

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The subject of ground motion has been introduced in the field of accelerator physics by G. Fischer\(^1\),\(^2\) of SLAC about 10 years ago. At that moment several colliders, linear and circular, are at various stages in the design phase. Their common characteristics are large machine dimensions and extremely small beam sizes. The extreme dimensions make these colliders more prone to ground motion effects while the minute beam size increases their sensitivity. The object of this paper is to report on the results obtained with a dedicated beam position monitor, concerning ground motion effects on the LEP beams. It starts with some properties of ground vibrations which are relevant to the observation of the phenomenon in LEP. The results of the observations will be presented both at the betatron frequencies and at very low frequencies. They are compared with published data to check the plausibility of the hypothesis that ground motion is indeed the primary cause of the observed signals. Finally, a discussion on the influence of the effect on the LHC beams is presented.

1 INTRODUCTION

It is a fair statement to say that the subject of ground motion was introduced in the field of accelerator physics by G. Fischer\(^1\),\(^2\) of SLAC about 10 years ago. At that time several colliders, linear and circular, were at various stages in the design phase but sufficiently advanced to have been baptized: SLC, CLIC, VLEPP, UNK, SSC, LHC, Eloisatron. Their common characteristics are large machine dimensions and extremely small beam size. The extreme dimensions make these colliders more prone to ground motion effects while the minute beam size increases their sensitivity. The effect of seismic perturbations on machine performance has been the subject of several studies.\(^2\)\(^4\)

This paper reports on the results obtained with a beam-position monitor dedicated to ground motion effects on the LEP beams. Since the ultimate objective of this study is the LHC, it is appropriate to say that LEP is used as a test bench for LHC. The report starts with some properties of ground vibrations which are relevant to the observation in LEP. Then the transformation of ground motion power into beam motion power is calculated. The expected performance of the observation system is discussed in some detail. Indeed the ambition is not only to show the presence of the phenomenon but also to measure its magnitude so that its consequences can be evaluated with sufficient precision. The results of the observations will be presented both at the betatron frequencies and at very low frequencies centred around the common mode. They are compared with published data to check the plausibility of the hypothesis that ground motion is indeed the primary cause of the observed signals. Finally, a discussion on the influence of the effect on the LHC beams is presented.
2 GROUND MOTION

A compendium on ground motion data from Lebedev (1992)\textsuperscript{5} is shown in Figure 1. The spectral density $S_{gm}$ falls off very quickly with frequency while the dispersion in the results spans several decades. However, local measurements have been performed\textsuperscript{6–9} albeit at frequencies below 100 Hz. A typical result is shown in Figure 2. The combination of the two makes extrapolation possible in the kHz region that is of prime importance.

From correlation measurements between two probes for varying distances as a function of frequency, it is possible to derive the velocity of the ground waves. Indeed, the correlation...
between two probes at distance $l$ drops to zero for a frequency such that this distance is a quarter wavelength.

$$l = \frac{\lambda}{4} = \frac{v}{4f}.$$  

It is interesting to note that in the TT2A tunnel an average speed of 1500 m/s is found, while in two points in the LEP tunnel this speed has increased to 4000 m/s. This is due to a different quality of the rock in which the tunnels have been excavated. This also provides at least a partial explanation for the dispersion of the world-wide results.

The effect of ground motion in a large accelerator is transported by the uncorrelated motion of quadrupoles of a focusing family F or D. It follows that the effect will decrease quickly for frequencies above the coherence limit:

$$f_m = \frac{v}{4l},$$

where $l$ is the cell length. For LEP $l = 79$ m so that the lower limit of the coherence is reached for a frequency of 12.6 Hz (upper limit) or 4.7 Hz (lower limit). In LHC $l = 90$ m and the coherent limits are 11 Hz and 4 Hz. The absolute cut-off frequency is reached when the whole machine fits within a quarter wave length, that is for $l = \text{machine diameter}$. This limit for LEP/LHC is 1/8 Hz. The famous ‘seven-second hum’ will be next to invisible.

It may be interesting to make an extrapolation of the measured ground motion power from 10 Hz to 2000 Hz, a frequency close to the tunes of LEP.

The power measured at 10 Hz is $S_{gm} = 5 \times 10^{-9} \mu^2/\text{Hz}$, while the logarithmic slope with frequency is about $-2.5$. Therefore, the expected power at 2 kHz is

$$S_{gm}(2000) = 0.9 \times 10^{-14} \mu^2/\text{Hz}.$$  

3 FROM GROUND MOTION TO BEAM MOTION

Ground vibrations at frequencies higher than 12.6 Hz will cause uncorrelated motions of the quadrupoles in LEP. A quadrupole displacement provokes a displacement of the beam. When the frequency of the vibration lies in the $\beta$ frequency of the beam, the beam will oscillate with an amplitude that depends on the power of the exciter (ground motion) and on the frequency spread in the beam. The expected response is computed in several steps as follows.

3.1 Beam displacement due to a kick caused by a quadrupole movement

The kick $\theta$ by a magnetic field $B$ active over a length $ds$ imposed on a particle with rigidity $B_\rho$ is given by:

$$\theta = \frac{Bds}{B_\rho}.$$  

In the case of a quadrupole that is displaced by an amount $\varepsilon$ the kick follows from:

$$\theta = \varepsilon \frac{Gds}{B_\rho} = \varepsilon (K\ell).$$
where $G$ is the gradient and $K\ell$ is the integrated normalized quadrupole strength. This kick will cause a $\beta$ oscillation. Call $\beta_Q$ and $\beta_O$, respectively, the $\beta$ function at the quadrupole and at the observation point. The amplitude at the observation station will be:

$$x = \sqrt{\beta_O\beta_Q K\ell\epsilon}.$$  

The contributions of many quadrupoles add quadratically:

$$x^2 = \beta_O \Sigma \beta_Q (K\ell)^2 \epsilon^2.$$

3.2 Response of a beam to external excitation

3.2.1 Response at $\beta$ frequencies for beam with tune spread

Consider a beam with tune spread $\delta Q$. The external excitation is $x$ and the frequency covers the $\beta$ oscillation of the beam. From the study of the transverse stability diagram and for a reasonable distribution of the frequencies, the following expression holds for the oscillation amplitude of the ensemble:

$$\tilde{x} = \frac{x}{2\delta Q}.$$  

This can be applied to the previous result:

$$\tilde{x}^2 = \frac{\beta_O \Sigma \beta_Q (K\ell)^2}{4\delta Q^2} \epsilon^2.$$

3.2.2 Changes of closed orbit

The optic amplification factor $1/\delta Q$ for the $\beta$ amplitudes is replaced by the optic orbit amplification factor $1/|\sin(\pi q)|$, yielding:

$$x_{co}^2 = \frac{\beta_O \Sigma \beta_Q (K\ell)^2}{4 \sin^2(\pi q)} \epsilon^2.$$  

3.2.3 Relating beam response and input power

The data on ground motion are given in the form of spectral density. The previous formulae can be rewritten taking this aspect into account. Indeed,

$$S_{gm}(f) = \frac{\epsilon^2}{df}$$

and

$$\frac{\tilde{x}^2}{df} = \frac{\beta_O \Sigma \beta_Q (K\ell)^2}{4\delta Q^2} S_{gm}(f). \quad (1)$$

$$\frac{x_{co}^2}{df} = \frac{\beta_O \Sigma \beta_Q (K\ell)^2}{4 \sin^2(\pi q)} S_{gm}(f). \quad (2)$$
These expressions relate the response of a beam at a position monitor to the spectral density of ground motion. It is worth pointing out that \( \bar{x} \) and \( x_{co} \) are quantities that are measurable by a beam-position monitor.

The next step is to estimate the various constants in order to verify the necessary resolution of the observation system. An estimate of \( S_{gm}(f) \) has already been made.

The parameter \( \beta_o \Sigma Q (K \ell)^2 \) depends on the machine optics. It has been computed for the 1993 pretzel optics used in LEP during physics runs. The contributions of the four LOBSs (physics insertions), the four HIBLs (insertions not used for physics) and the lattice quadrupoles are included. The result for the vertical plane is 18 700 and for the horizontal one 16 600.

As far as the tune spread \( \delta Q \) is concerned, a pragmatic attitude is adopted in the sense that it is taken to be 0.015 which is some average value based on the actual observations. It is realised that this is in contradiction with the expectations for beams influenced by a beam–beam tune spread generated by four interaction points.

The fractional tune \( q \) in LEP varies between 0.1 and 0.3. Hence the orbit amplification factor \( \sin(\pi q) \) varies between 0.3 and 0.8, say an average of 0.5.

This leads finally to a value for the expected spectral density of the beam motion at the \( \beta \) frequencies of:

- **Vertically**
  \[ \frac{\bar{x}}{\sqrt{df}} = 0.43 \text{ nm}/\sqrt{\text{Hz}} \]

- **Horizontally**
  \[ \frac{\bar{x}}{\sqrt{df}} = 0.41 \text{ nm}/\sqrt{\text{Hz}} \]

while the orbit motion around 100 Hz will have a spectral density of:

- **Vertically**
  \[ \frac{x_{co}}{\sqrt{df}} = 0.55 \text{ nm}/\sqrt{\text{Hz}} \]

- **Horizontally**
  \[ \frac{x_{co}}{\sqrt{df}} = 0.52 \text{ nm}/\sqrt{\text{Hz}} \]

## 4 PROPERTIES OF THE HIGH-RESOLUTION BEAM-POSITION MONITOR SYSTEM

A beam-position monitor of the directional coupler type had been installed in LEP for general-purpose use. The properties of the monitor and its associated acquisition system can be analysed in great detail together with the characteristics of a single lepton bunch coasting in LEP. This leads to the following set of expressions relating the monitor response with the spectral density of ground motion. For the observation of the betatron frequencies this yields:

- **Vertically**
  \[ S_{gm} = \left( \frac{\Delta x \delta Q}{i_b} \right)^2 \times 21.7 \times 10^{-14} \mu^2/\text{Hz} \]

- **Horizontally**
  \[ S_{gm} = \left( \frac{\Delta x \delta Q}{i_b} \right)^2 \times 243 \times 10^{-14} \mu^2/\text{Hz} \]

where \( \Delta \) is the difference signal in volts measured on the FFT plot, \( i_b \) the bunch current in amperes and \( \delta Q \) the observed frequency spread in terms of tune.
A similar expression can be derived for observations around the common mode signal (closed orbit):

vertical \[ S_{gm} = \left( \frac{\Delta_v \sin(\pi q)}{i_b} \right)^2 21.7 \times 10^{-14} \mu^2/\text{Hz} \]

horizontal \[ S_{gm} = \left( \frac{\Delta_h \sin(\pi q)}{i_b} \right)^2 243 \times 10^{-14} \mu^2/\text{Hz} . \]

5 OBSERVATIONS

5.1 Observations at $\beta$ frequencies

A typical FFT plot of the $\beta$ oscillations is shown in Figure 3. The top trace is horizontal and the bottom trace is vertical.

It may be worth-while to work out this example. The bunch intensity is 300 $\mu$A. The vertical noise level is 12.7 bits where the expected level was 10 bits. In the horizontal plane a noise level of 11.3 bits is found while the expected level was 8 bits.

The central frequency of the vertical signal is 1800 Hz. The peak signal is 81 bits, correction for noise by quadratic subtraction yields 80 bits. The tune spread $\delta Q = 0.014$. The spectral density for ground motion is:

\[ S_{gm}(1800) = 710^{-14} \mu^2/\text{Hz} . \]

![FIGURE 3: Typical FFT plot of beam signals excited by ground motion.](image-url)
TABLE 1: Results from vertical $\beta$-oscillation amplitudes.

<table>
<thead>
<tr>
<th>$i_b$ ($\mu A$)</th>
<th>Frequency (Hz)</th>
<th>$\delta Q$</th>
<th>Peak signal (bit)</th>
<th>$S_{gm}$ ($10^{-14}\mu^2$/Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>260</td>
<td>1720</td>
<td>0.016</td>
<td>540</td>
<td>550</td>
</tr>
<tr>
<td>240</td>
<td>1760</td>
<td>0.012</td>
<td>84</td>
<td>8.8</td>
</tr>
<tr>
<td>300</td>
<td>1800</td>
<td>0.014</td>
<td>80</td>
<td>7</td>
</tr>
<tr>
<td>117</td>
<td>1870</td>
<td>0.006</td>
<td>40</td>
<td>2.1</td>
</tr>
<tr>
<td>289</td>
<td>1900</td>
<td>0.014</td>
<td>67</td>
<td>5.3</td>
</tr>
<tr>
<td>300</td>
<td>1940</td>
<td>0.02</td>
<td>45</td>
<td>4.5</td>
</tr>
<tr>
<td>230</td>
<td>1940</td>
<td>0.015</td>
<td>93</td>
<td>18.4</td>
</tr>
<tr>
<td>324</td>
<td>2210</td>
<td>0.018</td>
<td>65</td>
<td>6.5</td>
</tr>
</tbody>
</table>

TABLE 2: Results from horizontal $\beta$-oscillation amplitudes.

<table>
<thead>
<tr>
<th>$i_b$ ($\mu A$)</th>
<th>Frequency (Hz)</th>
<th>$\delta Q$</th>
<th>Peak signal (bit)</th>
<th>$S_{gm}$ ($10^{-14}\mu^2$/Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>190</td>
<td>1090</td>
<td>0.004</td>
<td>400</td>
<td>400</td>
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<tr>
<td>240</td>
<td>3150</td>
<td>0.006</td>
<td>44</td>
<td>6.8</td>
</tr>
<tr>
<td>260</td>
<td>3160</td>
<td>0.015</td>
<td>140</td>
<td>370</td>
</tr>
<tr>
<td>300</td>
<td>3200</td>
<td>0.008</td>
<td>43</td>
<td>7.4</td>
</tr>
<tr>
<td>340</td>
<td>3300</td>
<td>0.01</td>
<td>32</td>
<td>5</td>
</tr>
<tr>
<td>300</td>
<td>3300</td>
<td>0.008</td>
<td>30</td>
<td>3.6</td>
</tr>
</tbody>
</table>

The horizontal frequency is 3200 Hz and the peak signal is 44.5 bits corrected to 43 bits. The tune spread is 0.008 leading to:

$$S_{gm}(3200) = 7.410^{-14}\mu^2$/Hz .$$

In Table 1 and 2 results obtained at various tune values are summarized.

5.2 Observation at the common mode (closed orbit)

A typical FFT plot of the closed orbit spectrum is shown in Figure 4.

A clear valley at $q = 0.001$, or $f = 11.2$ Hz can be seen. It is very likely that this corresponds to the coherence limit that was calculated before and found to be at 12.6 Hz. The sudden rise of the noise power for frequencies lower than this can probably be attributed to the noise of power supplies of dipole corrector magnets.
In Table 3 vertical measurements at frequencies of 45 Hz and 90 Hz are shown for a few cases. Owing to a reduced resolution limit in this frequency range, only the point at 45 Hz was significant in the horizontal plane. It should be remembered that \( \sin(\pi q) \) was 0.5 vertically and 0.8 horizontally.

This is the moment to make a small digression. In a previous paragraph a possible contender for the origin of the signals was ruled out. That contender was Schottky noise. A second contender can now be ruled out for the excitation of the beam at a few kHz. This is power supply noise. Power supplies in LEP are essentially low-frequency devices.

<table>
<thead>
<tr>
<th>( i_b )</th>
<th>Peak signal</th>
<th>( S_{gm} )</th>
<th>Peak signal</th>
<th>( S_{gm} )</th>
<th>Peak signal</th>
<th>( S_{gm} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mu A )</td>
<td>bit</td>
<td>( 10^{-10} \mu^2/Hz )</td>
<td>bit</td>
<td>( 10^{-10} \mu^2/Hz )</td>
<td>bit</td>
<td>( 10^{-10} \mu^2/Hz )</td>
</tr>
<tr>
<td>340</td>
<td>170</td>
<td>3.1</td>
<td>1400</td>
<td>210</td>
<td>230</td>
<td>170</td>
</tr>
<tr>
<td>200</td>
<td>86</td>
<td>2.3</td>
<td>970</td>
<td>290</td>
<td>210</td>
<td>400</td>
</tr>
<tr>
<td>300</td>
<td>120</td>
<td>2.0</td>
<td>1280</td>
<td>230</td>
<td>230</td>
<td>220</td>
</tr>
<tr>
<td>80</td>
<td>40</td>
<td>3.1</td>
<td>370</td>
<td>260</td>
<td></td>
<td></td>
</tr>
<tr>
<td>135</td>
<td>70</td>
<td>3.4</td>
<td>450</td>
<td>140</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
with a bandwidth of only a few Hz. While it is not unreasonable that their effect may be visible at these low frequencies, it is to be expected that their strength decreases at a rate of at least two orders of magnitude per decade. At 2 kHz the skin effect of the aluminium chamber will cause a further attenuation of the order of a factor of 10. All this adds up to a signal density which is a lot smaller than the resolution of the system.

In Figure 5 the measurement results with beam are assembled in a comparison plot with direct seismic measurements.

The correspondence between the direct seismic measurement results and the beam measurements is astonishing. Even the logarithmic slope of 2.5 shows up clearly in the beam measurements. Of course, the vertical scale is very compressed. It is not even sure that the scatter of the results is due to the measurement. Indeed, there is no good reason why the spectral density of the seismic activity should be constant in time. On several occasions, highlighted in Tables 1 and 2, a much higher activity was noted.

6 CONSEQUENCES FOR THE LHC

The transverse excitation source for LEP and LHC is obviously the same \( S_{gm} \). The effect of uncorrected ground motion in the LHC is beam blow-up since damping is very small. The blow-up can be computed with the following formula:

\[
\tau^{-1} = \left( \frac{f_{\text{rev}} \delta Q}{\sigma} \right)^2 \left( \frac{\dot{x}^2}{df} \right) = \beta_0 \Sigma \beta_Q (K \ell)^2 \left( \frac{f_{\text{rev}}}{2\sigma} \right)^2 S_{gm}(f)
\]

\[
= \Sigma \beta_Q (K \ell)^2 \left( \frac{f_{\text{rev}}}{2} \right)^2 \frac{\gamma}{\varepsilon} S_{gm}(f),
\]

where \( \varepsilon \) is the normal beam emittance and \( \gamma \) the mass-to-rest-mass ratio.
The optical parameter $\Sigma \beta Q (K \ell)^2$ in LEP is 143 m$^{-1}$ in the horizontal plane and 434 m$^{-1}$ in the vertical plane. For the LHC it is assumed that the focusing strength is about equal to the horizontal case of LEP, hence $\Sigma \beta Q (K \ell)^2 = 142$ m$^{-1}$. In order to get some feeling for this it is assumed that the growth rate should not be smaller than 40 hours knowing that the damping time is around 25 hours. The following limit for $S_{gm}$ is found for an emittance of 3.75 mrad m and $\gamma = 8000$:

$$S_{gm} \leq 70 \times 10^{-14} \mu^2/\text{Hz}. $$

This result shows that most of the time at fractional tunes of 0.15 and above, this condition is met. For the late SSC the limit on $S_{gm}$ would have been a factor of around 15 lower! However, seismic activity is not constant and cases were observed where the limit had been considerably exceeded over many hours. The impressive scatter of the results shown in Figure 1 is perhaps partially due to power variations of seismic vibrations. Moreover, even if the source of the effect is the same in LEP and LHC, it is not sure that the motion of the quadrupoles is identical. It is well known that the magnet supports can enhance the motion considerably.\textsuperscript{1,2} Therefore, it may be wise to consider a narrow-band, low-power, and low-noise transverse feedback system that will keep the persistent oscillation amplitude $\bar{x}$ below a level that corresponds to an acceptable growth rate. The feasibility study of such a system goes beyond the scope of this report.

7 CONCLUSIONS

The excitation of the beam in LEP by seismic vibrations has been measured rather accurately. Its effect on the blow-up of the LHC coasting beams has been estimated. In view of the large temporal variations of the power of seismic vibrations, it is suggested that a special feedback should be envisaged to keep the effect within acceptable bounds at all times.

This work has led to useful side-effects. The first one is the permanent measurement of tunes in fine detail in the LEP control room. The second one is related to the high-resolution observation of the effect of $k$-modulation\textsuperscript{11} which is operating in the frequency coherence desert below 14 Hz.

ACKNOWLEDGEMENTS

It is a pleasure to express my gratitude to my colleagues. B. Halvarsson participated in the installation and played a vital role as guide in the LEP environment which was new for the rest of us. J-P. Papis is responsible for the beam monitor and its modifications and collaborated closely with C. Boccard and T. Bogey to make the analog electronics work in unorthodox circumstances. H. Jakob installed and connected the acquisition system, while I. Milstead adapted the resident software. The authors of the application software are W. Fischer and F. Schmidt. C. Bovet is thanked for his interest and encouragement.

REFERENCES


