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A COVARIANT APPROACH TO GAUGED 2D SELF-DUAL FIELDS*

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Abstract

It is shown that the chiral boson model of Floreanini and Jackiw (FJ) and of McClain, Wu and Yu (MWY) are two different representations of the same chiral theory, each displaying different aspects of the bosonic representation for chiral fields: while in the FJ side the spectrum is quite evident from the equations of motion, it lacks manifest covariance making the coupling to external fields particularly difficult. On the MWY side, a symmetry hidden in the FJ version is apparent due to the presence of infinite scalar Wess-Zumino fields that carry the chiral representation in this side of the problem. On the other hand the evidence of physical spectrum is lost. In this work we show how to go from one representation to the other and find that the FJ chiral field is a collective field built by a linear combination of all MWY scalar fields. Using then the manifest covariance of the MWY representation we study the problem of coupling bosonic chiral fields to a background gauge field.

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I INTRODUCTION

The quantization of self-dual scalar fields in two-dimensional theories is a very interesting theoretical problem. These objects, also known as chiral bosons, are receiving much attention in recent years mainly because of their presence in many string models.[1] Furthermore, in the description of the fractional quantum Hall effect, chiral bosons play an important role since there they appear as edge-states of a Hall fluid.[2] Quantum Hall fluids are incompressibles and possess no gapless bulk excitations. Therefore, it is believed that the only gapless excitations, which are observed on experiments with finite quantum Hall systems, are those localized at the edge of the sample. The dynamics on the edge is non-trivial: in the integer effect it is described by an one-dimensional fermi-liquid theory, and by a $U(1)$ Kac-Moody algebra in the fractional case.[3]

The usual Lagrangian route is to start with scalar fields and project out one of the chiralities by means of a chiral constraint $\partial_+ \phi \approx 0$. According to Dirac's theory[4] of constrained systems, this is a second-class constraint, but to avoid to Lagrange multiplier to become dynamical one needs it to be first-class. In fact it is the second-class nature of the chiral constraint that makes the problem particularly challenging. The replacement of the second-class chiral constraint by a first-class one has been done in two different ways. Siegel[5]

proposed to set to zero one component of the energy-momentum tensor, resulting an action where the chiral constraint appears squared and possess a reparametrization invariance called as Siegel symmetry. To square the chiral constraint to make it first-class results in a theory presenting (infinitely) reducible constraints.[6] Besides the reducibility problem, Siegel symmetry becomes anomalous at quantum level on account of the central extension of the conformal algebra of the energy-momentum tensor. This anomaly can be canceled with the introduction of a local counter-term, but the presence of this anomaly canceling piece makes the coupling with gravity rather difficult. In fact, only the N=2 supersymmetric extensions of Siegel action seems to have been consistently coupled to gravity, leading to a bosonic formulation for the heterotic string. Hull,[7] on the other hand, proposed to cancel the anomaly introducing auxiliary fields, called no-movers, but the coupling to gravity still remains an open problem.

An alternative route to covariantization is through introduction of auxiliary Wess-Zumino fields a la Faddeev-Shatashvili.[8] Contributions along this line have been given in Ref.[9] and [10], and, from these results, it seems that the price one has to pay to covariantization is to work with an infinite set of scalar fields: McClain, Wu and Yu[9] proposed a model containing infinite scalar fields coupled by a combination

of right and left chiral constraints, carefully adjusted to be first-class. The physical spectrum was shown to contain a single chiral boson by use of a very elegant group theoretical method based on the covariant extension of the BRST algebra. Alternatively, in [10] we started with the noncovariant chiral boson model of Floreanini and Jackiw[11][12] and iteratively changed the nature of the left over second-class constraint, introducing an infinite number of Wess-Zumino fields. As shown by Boyanovski et al,[13] the inclusion of Wess-Zumino type fields in a theory has no influence over the particle spectrum since these fields will affect only the unphysical sector of the Hilbert space of the theory.

That is the status for the free theory. A consistent scheme to gauge Siegel symmetry has been presented by Bellucci, Golterman and Petcher (BGP).[14] Since the presence of gauge fields does not affect the Lagrange multiplier field λ , one can learn from this result how to couple gauge fields to the noncovariant Floreanini-Jackiw action by just extending the Bernstein-Sonnenschein proof[12] to the gauged case. On the other hand, Bastianelli and Van Nieuwenhuizen,[15] proposed to reduce the scalar field into a chiral object by projecting out in phase space, the undesirable component. Following this procedure they obtained the chiral boson coupled to super-gravity. Following Bastianelli-Van Nieuwenhuizen procedure Harada,[16] worked out

the bosonic version of the chiral Schwinger model to obtain the gauged version for Floreanini-Jackiw chiral boson. He obtained the same gauging scheme for the FJ model as that of the BGP procedure, the equivalence of these results being established by Bazeia.[17] Some alternative schemes to gauge both models have been proposed after that[18][19]

In this paper we study the problem of gauging chiral bosons in a covariant way. Firstly, we show that Floreanini-Jackiw and McClain-Wu-Yu theories are two different representations of the same chiral theory. The MWY representation besides being manifestly covariant, displays a symmetry that is hidden in the FJ side. Based on this equivalence, we then propose to gauge the chiral boson theory starting with the fully covariant MWY representation. Our results, when seen in the Floreanini-Jackiw non-covariant side, show complete consistency with that of BGP[14] and Harada[15]. The advantage of this formulation is twofold. First the MWY chiral boson is free of the criticism put forth upon Siegel's model. Furthermore, since it is manifestly covariant, permits the coupling to gauge fields to be done unambiguously. MWY model thus appears as a natural candidate to study the covariant gauging question. Since these are theories presenting constraints, we shall work within the symplectic formalism of Faddeev and Jackiw[20] for first-order constrained systems, that seems to present some practical benefit over Dirac's

theory. We will see that this has the advantage of providing a simpler derivation of the MWY's physical spectrum, both with and without gauge fields, not relying on the group structure of the model. It will be seen, in the free case first, that the application the reductive constraint process of Ref.[20], will result in a description of the MWY chiral boson as a collective field, built of by a combination of all the MWY scalar fields, and whose dynamics is controlled by a Floreanini-Jackiw action. Following the same process, it will be shown that when imersed in a region where a gauge field is present, the resulting collective field couples to just one chirality of the gauge field. Moreover, the action controlling the dynamics of this collective field will be that of BGP-Harada scheme.

II MWY CHIRAL BOSON

Initially, let us review the MWY proposal for a chiral boson theory, and study its symmetry properties under the symplectic quantization point of view for systems with true constraints, as described in Ref.[21]. The action proposed in [9] contains infinite scalar fields ϕ_k

$$(1) \quad S^{MWY} = \int d^2x \sum_{k=0}^{\infty} (-1)^k \partial_+ \phi_k \partial_- \phi_k$$

$(\partial_{\pm} = (\partial_0 \pm \partial_1)/\sqrt{2})$, coupled by a set of (infinite) irreducible constraints

$$(2) \quad T_k = \Omega_k^{(+)} - \Omega_{k+1}^{(-)}$$

where $\Omega_k^{(\pm)} = \pi_k \pm \phi_k'$ are right and left chiral constraints that satisfy two uncoupled U(1) Kac-Moody algebra

$$(3) \quad \begin{aligned} \{\Omega_k^{(+)}(x), \Omega_m^{(+)}(y)\} &= (\pm) 2\delta_{km} \delta'(x-y) \\ \{\Omega_k^{(+)}(x), \Omega_m^{(-)}(y)\} &= 0 \end{aligned}$$

where $\{A(x), B(y)\}$ represents the Poisson bracket of fields $A(x)$ and $B(x)$. Throughout this paper we shall be using the conventional notation adopted in (1+1) dimensional field theories where dot(prime) means time(space) partial derivative. The canonical Hamiltonian is easily obtained and reads

$$(4) \quad H = \frac{1}{2} \int d^2x \sum_{k=0}^{\infty} (-1)^k (\pi_k^2 + \phi_k'^2)$$

One can verify that $T_k(x)$ satisfy a first-class algebra under the Poisson bracket operation

$$(5) \quad \{T_k(x), T_m(y)\} = 0$$

To obtain the symmetry transformation properties of the physical variables under these first-class constraints and the physical spectrum of the MWY theory, we make use of the Faddeev-Jackiw method based on first-order Lagrangians. To this end we rewrite the MWY action in its first-order form

$$(6) \quad S^{MWY} = \frac{1}{2} \int d^2x \sum_{k=0}^{\infty} \left[\pi_k \phi_k + \frac{1}{2} (-1)^{k+1} (\pi_k^2 + \phi_k'^2) - \lambda_k T_k \right]$$

Certainly, due to the first-class character of the constraints $T_k(x)$, the symplectic matrix, as defined in [20], is singular, and therefore cannot be inverted to provide the Dirac brackets. However, as shown in Ref.[22], the zero-modes of the symplectic matrix display all the information about the transformation properties of the physical variables related to the symmetries generated by $T_k(x)$, which are, after all, the objects responsible for the singular nature of this matrix. To see how this works here, we follow Ref.[22] and relabel $-\lambda_n \rightarrow \dot{\eta}_n$ to construct the symplectic matrix $F_{km}(x, y)$, defined as

$$(7) \quad F_{km}(x, y) = \frac{\delta a_m(y)}{\delta \xi_k(x)} - \frac{\delta a_k(x)}{\delta \xi_m(y)}$$

where the symplectic variables are $\xi_k = \{\phi_k, \pi_k, \eta_k\}$ and the

$a_k(x) = \{\pi_k, 0, T_k\}$ are the components of the canonical one form.

After some algebra, we find the symplectic matrix to be,

$$(8) \quad F_{km}(x - y) = \begin{pmatrix} 0 & -1 & M\partial_x \\ 1 & 0 & N \\ \tilde{M}\partial_x & -\tilde{N} & 0 \end{pmatrix} \delta(x - y)$$

where 0 , 1 , M , and N are infinite dimensional matrices describing the null matrix, the identity matrix and

(9)

$$M = \begin{pmatrix} -1 & 0 & 0 & 0 & \dots \\ -1 & -1 & 0 & 0 & \dots \\ 0 & -1 & -1 & 0 & \dots \\ 0 & 0 & -1 & -1 & \dots \\ \cdot & \cdot & \cdot & \cdot & \dots \end{pmatrix}$$

$$N = \begin{pmatrix} 1 & 0 & 0 & 0 & \dots \\ -1 & 1 & 0 & 0 & \dots \\ 0 & -1 & 1 & 0 & \dots \\ 0 & 0 & -1 & 1 & \dots \\ \cdot & \cdot & \cdot & \cdot & \dots \end{pmatrix}$$

where $\tilde{M}$ and $\tilde{N}$ are the corresponding transposed matrices. This symplectic matrix has infinite zero-modes $v_k(x)$, whose general structure is

(10)

$$v_0 = \begin{pmatrix} \begin{pmatrix} -\epsilon_0 \\ \epsilon_0 \\ \cdot \\ \cdot \\ \cdot \end{pmatrix} \\ \begin{pmatrix} -\epsilon'_0 \\ -\epsilon'_0 \\ \cdot \\ \cdot \\ \cdot \end{pmatrix} \\ \begin{pmatrix} \epsilon_0 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \end{pmatrix} \end{pmatrix} \quad v_1 = \begin{pmatrix} \begin{pmatrix} 0 \\ -\epsilon_1 \\ \epsilon_1 \\ \cdot \\ \cdot \end{pmatrix} \\ \begin{pmatrix} 0 \\ -\epsilon'_1 \\ -\epsilon'_1 \\ \cdot \\ \cdot \end{pmatrix} \\ \begin{pmatrix} 0 \\ \epsilon_1 \\ \cdot \\ \cdot \\ \cdot \end{pmatrix} \end{pmatrix} \quad v_2 = \begin{pmatrix} \begin{pmatrix} 0 \\ 0 \\ -\epsilon_2 \\ \epsilon_2 \\ \cdot \end{pmatrix} \\ \begin{pmatrix} 0 \\ 0 \\ -\epsilon'_2 \\ -\epsilon'_2 \\ \cdot \end{pmatrix} \\ \begin{pmatrix} 0 \\ 0 \\ \epsilon_2 \\ \cdot \\ \cdot \end{pmatrix} \end{pmatrix} \dots$$

with $\epsilon_k(x)$ being arbitrary parameters. It is easy to see[mw] that in order to keep the action invariant, the fields must transform as $\delta\xi_k = v_k^a \epsilon_a$, which gives us, immediately, that

$$\begin{aligned}
 (11) \quad \delta\phi_k &= \epsilon_{k-1} - \epsilon_k \\
 \delta\pi_k &= -\epsilon'_{k-1} - \epsilon'_k \\
 \delta\eta_k &= \epsilon_k
 \end{aligned}$$

To obtain the transformation for the Lagrange multipliers λ_k we undo the transformation $-\lambda_k \rightarrow \dot{\eta}_k$ and remember to include the structure function coming from

$$(12) \quad [T_k(x), H] = (-1)^{k+1} T'_k(x)$$

to obtain

$$(13) \quad \delta\lambda_k = -\dot{\epsilon}_k - (-1)^k \epsilon'_k$$

As explained in [9] these transformations correspond to infinite abelian gauge transformations existing in this model, meaning that all the fields ϕ_k contribute to guarantee the existence of this symmetry. There exists a delicate pairwise canceling mechanism, which makes the coupling among the fields, although weak, just strong enough to produce the desired gauge symmetry. The result of this canceling mechanism is the existence of a net chirality for the whole set, that will be carried by the collective chiral field Φ , to be defined below. The gauge invariance of the MWY model is due in part to this canceling between pairs

among neighbors fields shown in Eq.(11), the net difference being compensated by the transformation of the Lagrange multipliers λ_k , Eq.(13).

III PHYSICAL SPECTRUM AND COVARIANT GAUGING

Let us investigate now this theory's particle contents. This will be done by reducing the MWY constraints using the technique described by Faddeev and Jackiw. The reduction of the constraints can be done iteratively, starting with $T_0(x) \approx 0$. Implementing the first constraint as $\pi_0 = \pi_1 - \phi'_1 - \phi'_0$ gives the following Lagrange density

$$(14) \quad \mathcal{L} = -[\dot{\phi}_0 \phi'_0 + \phi'^2_0 - \phi'_1(\dot{\phi}_0 + \phi'_0)] + \pi_1(\dot{\phi}_0 + \dot{\phi}_1 + \phi'_0 + \phi'_1) - \sum_{n=2}^{\infty} \frac{(-1)^k}{2} (\pi_n^2 + \phi'^2_n) - \sum_{n=1}^{\infty} \lambda_n T_n$$

This reductive process destroys the original invariance of the model, but that is the price for the simplicity in the spectrum. Repeating this procedure for the remaining (infinite) constraints, we find

$$(15) \quad S^{MWY} = \int d^2x \sum_{n=0}^{\infty} \left[-\dot{\phi}_n \phi'_n - \phi'^2_n - 2(\phi'_n + \dot{\phi}_n) \sum_{m=n+1}^{\infty} \phi'_m \right]$$

A simple algebraic manipulation shows that MWY action can also be written as

$$(16) \quad \begin{aligned} S^{MWY} &= \int d^2x \sum_{n=0}^{\infty} \left[-(\dot{\phi}_n \phi'_n + \phi'^2_n) - 2\phi'_n \sum_{m=1}^{n-1} (\dot{\phi}_m + \phi'_m) \right] \\ &= \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} (-\dot{\phi}_m \phi'_n - \phi'_m \phi'_n) \end{aligned}$$

It is quite amazing to see how the MWY scalar fields have completely decoupled from each other as a result of the reductive process, this being a consequence of the weak coupling these constraints impose over the fields. Therefore, if we introduce the collective field

$$(17) \quad \Phi = \sum_{k=0}^{\infty} \phi_k$$

the chiral boson theory of McClain, Wu and Yu becomes

$$(18) \quad S^{MWY} = \int d^2x (-\dot{\Phi}\Phi' - \Phi'^2)$$

which is just the Floreanini-Jackiw action for the collective field Φ introduced above. As shown in [11], the presence of a single chiral excitation comes directly from its equation of motion. In this sense, it is the collective behavior of all the scalar fields that guarantees the presence of just one chiral excitation, while the (infinitely many) individual fields are necessary for the presence of reparametrization symmetry displayed in the original formulation proposed in [9].

We can say now that the FJ and MWY theories are two different representations of the same physical situation, each of which displaying different aspects of the chiral boson theory. In the FJ side the spectrum becomes quite clear, while the MWY side displays the symmetry aspect hidden in the other representation. Consequently, concerning the gauging question, this last repre-

sentation seems to be the more appropriated.

Next, we want to consider the coupling of MWY scalar fields with background gauge fields. Due to the explicit covariance displayed this can be done simply by substitution of the ordinary derivative by the covariant one

$$(19) \quad \partial_{\pm} \phi_k \rightarrow \partial_{\pm} \phi_k \pm (-)^k A_{\pm} \phi_k$$

where $A_{\pm} = (A_0 \pm A_1)/\sqrt{2}$. We will see that as a result of the alternating right-left coupling among the fields only the right component of the gauge field remains coupled to the collective (chiral) field ϕ .

In the presence of an abelian gauge field, the MWY action becomes[9],

$$(20) \quad \begin{aligned} S^{MWY} &= \int d^2x \sum_{k=0}^{\infty} (-)^k [(\partial_{+} \phi_k + (-)^k e A_{+})(\partial_{-} \phi_k - (-)^k e A_{-})] \\ &= \int d^2x \sum_{k=0}^{\infty} \left[(-)^k \frac{1}{2} (\dot{\phi}_k^2 - \phi_k'^2) + e(A_1 \dot{\phi}_k - A_0 \phi_k') \right] \end{aligned}$$

In its first-order form it reads

$$(21) \quad S^{MWY} = \int d^2x \sum_{k=0}^{\infty} \left\{ \pi_k \dot{\phi}_k - (-)^k \frac{1}{2} [(\pi_k - e A_1)^2 + \phi_k'^2] - e A_0 \phi_k' \right\}$$

Repeating the Faddeev-Jackiw reductive procedure used in the free case we obtain, after some algebra

$$(22) \quad S^{MWY} = \int d^2x \sum_{k=0}^{\infty} \left\{ -\dot{\phi}_k \phi_k' - \phi_k'^2 - 2\phi_k' \left[e \frac{1}{2} (A_0 + A_1) + \sum_{m=0}^{k-1} (\dot{\phi}_m \phi_m') \right] \right\}$$

Introducing, as before, the collective chiral field Φ , the MWY action results to be

$$(23) \quad S^{MWY} = \int d^2x \left(-\dot{\Phi}\Phi' - \Phi'^2 - eA_+ \Phi' \right)$$

presenting the chiral collective field Φ coupled just to A_+ as claimed. Notice that this is consistent with the result of Ref.[14] and [15]. It is interesting to see how the presence of the alternating chiral constraints, connecting neighboring fields conspire to eliminate one chirality of the gauge field. In conclusion, it seems to us that when discussing the question of coupling chiral bosons to background fields the MWY representation should be used as the starting point, getting the FJ version, if needed by iterative elimination of constraints. It would be interesting to see how this scheme works if a the chiral boson field is coupled to a background gravitational field, and to supergravity. Work in this direction is in progress.[23]

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