**DETERMINATION OF THE CHARGED PION-NUCLEON COUPLING
CONSTANT FROM $\bar{p}p \rightarrow \bar{n}n$ DIFFERENTIAL CROSS-SECTION**

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A recent experiment at LEAR, the CERN Low Energy Antiproton Ring, has provided very precise differential cross-section data for the $\bar{p}p \rightarrow \bar{n}n$ charge-exchange reaction. Extrapolating the data to the pion pole we have determined the charged-pion coupling constant to be $f_c^2 = 0.071 \pm 0.002$ from the data at 601 MeV/c. Details of the fitting procedure are given, as well as results obtained when applying the same method to recent $np \rightarrow pn$ data.

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It was suggested almost thirty years [1] ago that accurate differential cross-section data for the $\bar{p}p \rightarrow \bar{n}n$ charge-exchange reaction could be used to determine the charged-pion nucleon coupling-constant f_c^2 through the extrapolation procedure to the pion pole originally proposed by G. F. Chew [2] and successfully applied to the $np \rightarrow pn$ reaction by several authors [3, 4, 5]. Quite recently the Nijmegen group [6] has shown that a phase-shift analysis could be performed on the existing database on the $\bar{p}p \rightarrow \bar{n}n$ reaction below 925 MeV/ c incident $\bar{p}$ momentum (which included about 2000 data points) and a value for the coupling constant could be extracted ($f_c^2 = 0.0751 \pm 0.017$) with an accuracy comparable to that provided by other methods. Since in the Nijmegen analysis data coming from many different experimental results are used, which are affected by quite different systematic errors, it is not easy to evaluate the accuracy of the result.

On top of this, recent analysis of the NN [7-9] and πN [10] data have provided for f_c^2 values considerably lower than the value ($f_c^2 = 0.079 \pm 0.001$) which had been generally accepted since many years [11], and presently a controversy is going on on what the true value of this quantity is [12, 13].

Given the relevance of the issue, an experiment was proposed to CERN [14] for a precision measurement of the differential cross-section and possibly a model-independent determination of f_c^2 from a single set of data along the lines suggested in ref. [1]. The experiment was performed in 1993 at LEAR, the Low Energy Antiproton Ring at CERN by the PS206 collaboration, and the results have just been published [15]. The data have percent accuracy point-to-point, and overall normalization errors of 3% and 4% for the two measured momenta, 601 and 1202 MeV/ c respectively. As pointed out by the authors, the data exhibit an interesting t -dependance, and the extrapolation to the pion pole looks smooth and very well feasible.

In this paper we detail the work we have done on the extrapolation procedure and give estimates for f_c^2 . Some general comments on the physical significance of the measurement and on the comparison with the $np \rightarrow pn$ line-reversed reaction are given in a separate paper [16].

Our analysis follows closely the work done in the past on the $np \rightarrow pn$ reaction, and in particular the simplest one [3]. We have made no assumption whatsoever on the structure of the scattering amplitude other than the presence of the pion pole, which contributes with a term:

$$\left(\frac{d\sigma}{d\Omega}\right)_\pi = \frac{1}{s} \left[\frac{g_c^2 t}{m_\pi^2 - t} \right]^2$$

to the cross-section. Therefore we have not introduced either cuts along the real axis, nor backward poles, like in ref. [4, 5]. The fact that the pion term interferes with a smooth background amplitude comes out naturally when, following the suggestion of ref. [2], we multiply the measured $d\sigma/d\Omega$ data by $x^2 = (m_\pi^2 - t)^2$, and we fit the quantities

$$y = \frac{s}{m_\pi^4 g_c^4} x^2 \left(\frac{d\sigma}{d\Omega} \right)$$

with a polynomial in x :

$$P(x) = (a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \dots)$$

In the above formulas $g_c^2 = \frac{4M^2}{m_\pi^2} f_c^2$, t and s are the Mandelstam variables, m_π and M the pion and proton masses. We have factorized g_c^4 , and put $g_c^2 = 13.6$ as in ref. [9], so that a_0 should come out from the fitting procedure close to one.

The choice of the order of the polynomial $P(x)$ is critical for the fitted a_0 value. Also the range in t over which the quantities y are fitted is critical in determining the order of the polynomial. Therefore the result depends somewhat on the choice of the range in t for the fit.

If a sufficiently small t -interval is chosen, from $t = 0$ to $-t = 2 \div 3m_\pi^2$ good fits are already obtained with a second order polynomial, indicating that the pion term interfering with a constant background already reproduces the data. In this case the extrapolation error is quite large. We have decided to fit the whole t -range measured by the experiment, and to choose the order of the polynomial, as the minimum value which produces an acceptable fit. The quantities y are plotted in fig.1, for the two measured momenta, 601 MeV/ c (a) and 1202 MeV/ c (b). Also shown are the 3-rd, 4-th and 5-th order polynomial fits. The corresponding χ^2 and a_0 values are given in table 1, but it suffices to look at the curves in fig.1 to realize that 5-th order polynomials are necessary to give good fits over the entire measured range. Since the χ^2 is already very good with the 5-th order polynomial fits, we believe that there is no point to go to higher orders.

By removing data points at the extreme of the fitted t -range we have checked the stability of the result. The result at 601 MeV/ c turns out to be perfectly stable, but this is not true for the data at 1202 MeV/ c . Removing data points in the forward direction (obviously very important to reduce the error in the extrapolation), we have obtained for the 5-th order polynomial fit the χ^2 and the a_0 values plotted in fig.2. The trend of the χ^2 suggests that the first points are not correct: taking into account the fact that those points are affected by fairly large acceptance corrections we have decided to exclude the first 6 points from the fit, and anyway not to use these data for the determination of the coupling constant.

Taking into account the normalization error of the data, our result for the πNN coupling constant as obtained from the data at 601 MeV/ c is

$$f_c^2 = 0.0708 \pm 0.0016 \pm 0.0011 = 0.071 \pm 0.002$$

where the first error comes from the extrapolation procedure, and the second from the overall normalization of the data. This result is quite compatible with the estimate of ref. [6] and with the outcome ($f_c^2 = 0.0732 \pm 0.0011$) of a more recent phase-shift analysis [17], which uses all the available $\bar{N}N$ data, and not only the charge-exchange channel. As well as the quoted references, it suggest a “low” value for the coupling constant.

In order to investigate possible systematic errors in our fitting procedure, we have applied the same analysis to the $n p \rightarrow p n$ reaction. Recently new accurate cross-section data have been published, and several precision experiments are presently going on. The comments we made about the polynomial fits are still valid in this case. As an example in fig.3 we show the y values for the recent data points at 1040 MeV/ c [5], and the 3-rd, 4th, and 5-th order polynomial fits. These data are particularly interesting for us because the authors have performed the same extrapolation procedure to the pion pole we have done to our $\bar{p} p \rightarrow \bar{n} n$ data, and we can compare the results of the two fits. Both the χ^2 and the a_0 values we obtain with our fit turn out to be identical with the values they quote in ref. [5], and their choice of the order of the polynomial, which uses the F-test for statistical significance [18], coincides with our criterion. Their data give a value $f_c^2 = 0.069 \pm 0.002$, but it is interesting to note that the authors, given the difficulty of measuring the neutron flux, regard the extrapolation procedure as a method to fix the normalization of the data (as originally suggested in ref. [19]) rather than as a new determination of the coupling constant.

Although our result can not contribute much to the present controversy which is going on over the magnitude of the π NN coupling constant, we believe that our analysis is useful because

- this is the first time the $\bar{p}p \rightarrow \bar{n}n$ cross-section data are used to this purpose, and the method can surely be improved. In particular more data at 601 MeV/c (by a factor of ~ 8) are still being analyzed by the PS206 collaboration, and the accuracy of the final results should approach ± 0.001 on f_c^2 .
- potentially the $\bar{p}p \rightarrow \bar{n}n$ channel is more sensitive to the pion-exchange term than the $np \rightarrow pn$ reaction. Experimentally one can have a better control of the overall normalization, and theoretically the background contribution from short-range terms is smaller, as already pointed out in ref. [1]).
- in $\bar{N}N$ scattering what is measured is the product of the πN and $\pi \bar{N}$ coupling constants, therefore these measurement also provide a nice verification of the charge-conjugation invariance.

This work has been possible thanks to the contribution of a large number of people, and we would like to express our gratitude to all of them: it was largely stimulated by the original determination of the π NN coupling constant from the $\bar{p}p \rightarrow \bar{n}n$ data by the Nijmegen group, and it could be carried through only thanks to the joint effort of the whole PS206 collaboration.

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Table 1: χ^2 and a_0 values for the polynomial fits to the $\bar{p}p \rightarrow \bar{n}n$ data, for polynomials of third, fourth, and fifth degree.

Data sample	Fit results	3-rd order	4-th order	5-th order
601	χ^2/dof	21	1.28	0.90
MeV/ c	a_0	0.263 ± 0.010	0.755 ± 0.020	$0.891 \pm .038$
1202	χ^2/dof	47	2.62	0.95
MeV/ c	a_0	0.089 ± 0.008	0.606 ± 0.014	$0.732 \pm .020$

Figure 1: The quantity y , which is proportional to the product of the $\bar{p}p \rightarrow \bar{n}n$ differential data and $x^2 = (m_\pi^2 - t)^2$, as a function of x/m_π^2 at 601 MeV/c (a) and 1202 MeV/c (b). The constants are so chosen that at the pion pole the data should extrapolate to one if $g_c^2 = 13.6$. Also shown are low-order polynomial fits to the data with the indication of the order.

Figure 2: χ^2 (left axis, open points) and a_0 (right axis, closed points) for the 5-th degree polynomial fit to the $\bar{p}p \rightarrow \bar{n}n$ data at 1202 MeV/ c as a function of the number of forward points removed from the fit.

Figure 3: The quantity y , which is proportional to the product of the $np \rightarrow pn$ differential data at $1040 \text{ MeV}/c$ and $x^2 = (m_\pi^2 - t)^2$, as a function of x/m_π^2 . The constants are so chosen that at the pion pole the data should extrapolate to one if $g_c^2 = 13.6$. Also shown are low-order polynomial fits to the data with the indication of the order.