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Time-domain Electronic-noise Generation for a Monte-Carlo Study of Signal Processing

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ABSTRACT

A general technique to generate electronic noise was developed in a time-domain approach with the aid of a deconvolution technique in a discrete-time sampling system. We found that the technique is applicable even for a system in which the electronic-noise charge is not well defined. The generated noise train was reconstructed in terms of a sample-correlated function and the frequency spectrum.

★ V1.2: submitted to Nucl. Instr. & Meth. section A

1. Introduction

We are interested in the electronic-noise properties at the output of the CMOS preamplifier for the silicon strip detector for BELLE collaboration.^[1] We tried to reproduce the time-domain behavior of the electronic noise in the output node. The generated voltage trains were employed to check their behavior in the time-domain in terms of a sample-correlated function and the frequency power spectrum to apply them to estimating the hit frequency of the comparator located at the output node of the preamplifier, which generated hit information for sparsification and/or triggering.^[2]

The problem raised in this paper is as follows. For a given transfer function, $h(t)$, or its Fourier transform, $H(\omega)$, as well as input-referred noise-generators, i_p^2 and v_g^2 , we tried to generate a train of noise impulses for each time interval τ . We describe a general procedure for obtaining access to the noise behavior in a discrete-time presentation. We separate the time coordinate by a segment of τ ,

$$\{k\tau | k = 0, 1, 2, \dots, n\}. \quad (1.1)$$

The timing interval τ could be a beam-crossing interval of a collider-type accelerator. The frequency response in the discrete sampling system can be described as the sum of the contributions from each discrete-time position, which tends to the continuous-time Fourier transformation,

$$H(\omega) = \sum_k^n \tau H_k e^{-jk\omega\tau} \rightarrow \int_0^\infty h(t) e^{-j\omega t} dt, \quad (1.2)$$

when $\tau \rightarrow 0$; H_k is taken to be $H_k = h(k\tau)$. The noise charge impulses can be described as

$$I(\omega) = \sum_k^n \tau i_k e^{-jk\omega\tau} \rightarrow \int_0^\infty i(t) e^{-j\omega t} dt \quad (1.3)$$

in the same context as Eq. (1.2). Taking the convolution between the noise current

and the impulse response we obtain

$$A(t) = \int_{-1/2\tau}^{1/2\tau} H(\omega)I(\omega)df \simeq \sum_k^n \tau i_k H_{n-k} \rightarrow \int_0^\infty i(t')h(t-t')dt'. \quad (1.4)$$

We find that the above equations become consistent in the limit of $\tau \rightarrow 0$. The noise-generation technique based on discrete-time sampling says more than the limiting procedure appearing in Eqs. (1.2), (1.3) and (1.4).

This paper is organized as follows. In chapter 2 we present the noise-generation technique for a signal processing chain with a CR-RC shaping amplifier in order to review the basic principle; we also adopt the technique to a charge-sensitive amplifier, where we resolve a problem of divergence of the noise power and show that the basic principle is still applicable. In chapter 3 we reconfirm the consistency of the noise-generation procedure with the theory of continuous-time signal processing, and try to estimate the hit frequency of the comparator. The signal-processing chain employed in this paper is shown in Figure 1, which is referred to when necessary.

2. Theoretical background regarding noise-generation

2.1. NOISE PROPAGATION IN A DISCRETE-TIME SAMPLING SYSTEM

The noise amplitude (A_n) for each time position is described as a superposition of the series noise component and the parallel noise component,

$$A_n = A_{pn} + A_{sn}. \quad (2.1)$$

Since A_{pn} and A_{sn} are statistically independent variables, i.e. $\langle A_{pn}A_{sn} \rangle = 0$, we can handle them separately. $\langle * \rangle$ represents an ensemble average over the statistical variable $*$, which could be a time ordered average when the statistical

variable has an ergodic nature. A_{pn} is described as a linear combination of i_{pk} with coefficients $H_{(n-k+1)}$,

$$A_{pn} = \sum_{k=1}^n \alpha \sqrt{\tau} i_{pk} H_{(n-k+1)}, \quad (2.2)$$

while A_{sn} could be described as a linear combination of v_{sk} with coefficients $H'_{(n-k+1)}$,

$$A_{sn} = \sum_{k=1}^n C_D \beta \sqrt{\tau} v_{sk} H'_{(n-k+1)}. \quad (2.3)$$

Here, C_D is the detector capacitance, i_{pk} and v_{sk} are statistical variables whose values are obtained by a Monte-Carlo technique from the Gaussian probability distribution for each time position with standard deviations for the parallel noise,

$$i_p^2 = 2qI_L + \frac{4kT}{R_B}, \quad (2.4)$$

and for the series noise,

$$v_s^2 = \frac{2}{3} \frac{4kT}{g_m}. \quad (2.5)$$

Here, q is the electronic charge, I_L the detector's leakage current, k the Boltzmann constant, T the absolute temperature in Kelvin, R_B the bias resistor, and g_m the transconductance of the input FET of the preamplifier. The effect of the $1/f$ noise^[3] is not taken into account. Due to introducing the factor $\sqrt{\tau}$, α and β can be made dimensionless variables when amplitudes A_{pn} and A_{sn} are measured in Coulomb for convention in this paper. We also note that the amplifier system is set up so that the output has a peak amplitude of 1 Coulomb when an input impulse charge of 1 Coulomb is fed into the amplifier. Here we mention the statistical properties of the variables $\sigma_p^2 = \langle A_{pn}^2 \rangle$, $\sigma_s^2 = \langle A_{sn}^2 \rangle$, $\langle i_{pk}^2 \rangle = i_p^2$, $\langle v_{sk}^2 \rangle = v_s^2$, $\langle i_{pk}v_{sk'} \rangle = 0$, $\langle i_{pk}i_{pk'} \rangle = 0$, and $\langle v_{sk}v_{sk'} \rangle = 0$ for $k \neq k'$. σ_p and σ_s represent the continuous-time equivalent noise charge from the parallel noise and the series

noise, respectively. H_k and H'_k are related to the impulse response ($h(x)$) of the system as

$$H_k = h((k-1)\tau/t_0) \quad (2.6)$$

and

$$H'_k = \frac{1}{t_0} h'((k-1)\tau/t_0), \quad (2.7)$$

where t_0 is the specific time constant for the impulse response. H_k and H'_k are listed in Table 1 (a) for $h(x) = xe^{1-x}$ and in Table 1 (b) for $h(x) = 1 - e^{-x}$, where x stands for t/t_0 . We write $\alpha\sqrt{\tau}i_{pk}$ and $C_D\beta\sqrt{\tau}v_{sk}$ as s_{pk} and s_{sk} , respectively. In order to evaluate α and β , we write the inverse relation of Eqs. (2.2) and (2.3),

$$s_{pk} = \sum_{l=1}^k A_{pl}w_{k-l+1} \quad (2.8)$$

and

$$\sum_{l=1}^k s_{sl}u_{k-l+1} = \sum_{l=1}^k A_{sl}w_{k-l+1}, \quad (2.9)$$

where w_{k-l} is a weighting function. In general, this procedure is called "discrete-time deconvolution."^[4] For the systems which we are going to discuss in detail we can find that only three weighting functions, (w_1 , w_2 , and w_3), have non-zero values, which are listed in Table 2(a). The s_{pk} for parallel noise reduces to

$$s_{pk} = A_{p(k-2)}w_3 + A_{p(k-1)}w_2 + A_{pk}w_1. \quad (2.10)$$

We evaluate the power for s_{pk} ,

$$\langle s_{pk}^2 \rangle = (w_1^2 + w_2^2 + w_3^2)\sigma_p^2 + 2(w_1w_2 + w_2w_3)C_{p1} + 2w_1w_3C_{p2}, \quad (2.11)$$

where we note that $\langle A_k^2 \rangle = \langle A_{k-1}^2 \rangle = \langle A_{k-2}^2 \rangle = \sigma_p^2$, $\langle A_{k-2}A_{k-1} \rangle = \langle A_{k-1}A_k \rangle = C_{p1}$, and $\langle A_{k-2}A_k \rangle = C_{p2}$. C_{p1} and C_{p2} stand for auto-correlated

functions with distances τ and 2τ , respectively. We mention here general properties of the auto-correlation function. In the limit $x \rightarrow \infty$, which is the no-correlation limit, the correlation functions tend to zero. In the limit $x \rightarrow 0$, which is the perfect correlation limit, the correlation function tends to σ_p^2 or σ_s^2 for the parallel-noise and series-noise parts, respectively. Regarding Eq. (2.9), we can find that only two elements of u_k have non-zero values, which are u_1 and u_2 , as listed in Table 2 (b). Eventually, for the series noise we can write $\langle s_{sk}^2 \rangle$ as

$$u_2s_{s(k-1)} + u_1s_{sk} = A_{k-2}w_3 + A_{k-1}w_2 + A_kw_1. \quad (2.12)$$

Taking into account additional notes, i.e. $\langle s_{s(k-1)}s_{sk} \rangle = 0$ and $\langle s_{s(k-1)}^2 \rangle = \langle s_{sk}^2 \rangle$, we have

$$\langle s_{sk}^2 \rangle = \frac{(w_1^2 + w_2^2 + w_3^2)\sigma_s^2 + 2(w_1w_2 + w_2w_3)C_{s1} + 2w_1w_3C_{s2}}{u_1^2 + u_2^2}, \quad (2.13)$$

where C_{s1} and C_{s2} are auto-correlation functions for the series noise, as similarly defined for the parallel noise. Table 3 summarizes σ_p^2 , σ_s^2 , C_{pk} , and C_{sk} , whose derivations are discussed in detail in the next chapter. The matrix presentation of the formula is described in Appendix A to help understanding the nature of the inverse linear combination appearing in Eqs. (2.10) and (2.12). There appears to be something confusing about the relation between the noise-generation procedure and the derivation of the equivalent noise charge. In order to evaluate the equivalent noise charge we evaluate the output noise power, and then move it back to the input node based on the assumption that we measure the noise charge in terms of the shot noise equivalent. Eventually the equivalent noise charge for a discrete-time sampling system is described as

$$\langle s_{pk}^2 \rangle + (u_1^2 + u_2^2) \langle s_{sk}^2 \rangle. \quad (2.14)$$

2.2. NOISE GENERATION USING A CR-RC SHAPER

The impulse response of a CR-RC shaper system with a charge-sensitive preamplifier is

$$H(\omega) = \frac{e}{t_0 (j\omega + 1/t_0)^2}, \quad (2.15)$$

where t_0 is the time constant of the shaping amplifier, which has already appeared in Eqs. (2.6) and (2.7). The output node of this system is "Node-2" in Fig. 1. The charge-sensitive preamplifier is assumed to have a very fast response time. The impulse response for this system is

$$h(x) = xe^{1-x}. \quad (2.16)$$

We know that the noise amplitudes of the CR-RC shaper system can be written as

$$\sigma_p^2 = \frac{i_p^2 t_0}{2} \int_0^\infty h^2(x) dx \rightarrow \frac{e^2}{8} t_0 i_p^2 \quad (2.17)$$

and

$$\sigma_s^2 = \frac{v_s^2 C_D^2}{2t_0} \int_0^\infty h'^2(x) dx \rightarrow \frac{e^2 v_s^2 C_D^2}{8t_0} \quad (2.18)$$

where $h'(x)$ is

$$h'(x) = (1-x)e^{1-x}. \quad (2.19)$$

The auto-correlation functions are evaluated as

$$C_{pk} = \frac{i_p^2}{2} \int_{-\infty}^\infty e^{j\omega k\tau} |H(\omega)|^2 df \rightarrow \sigma_p^2 (1+kx)e^{-kx} \quad (2.20)$$

and

$$C_{sk} = \frac{v_s^2 C_D^2}{2} \int_{-\infty}^\infty e^{j\omega k\tau} \omega^2 |H(\omega)|^2 df \rightarrow \sigma_s^2 (1-kx)e^{-kx}. \quad (2.21)$$

Consulting Tables 2 and 3, and applying them to Eqs. (2.11) and (2.13) we obtain

$$\langle s_{pk}^2 \rangle = \sigma_p^2 \frac{e^{-2}(e^{2x} - 4x - e^{-2x})}{x^2} \quad (2.22)$$

and

$$\langle s_{sk}^2 \rangle = \sigma_s^2 \frac{e^{-2}(e^{2x} + 4x - e^{-2x})}{e^{2x} + (x+1)^2}. \quad (2.23)$$

Once again we mention here that $\langle s_{pk}^2 \rangle = \alpha^2 \tau i_p^2$ and $\langle s_{sk}^2 \rangle = \beta^2 \tau v_s^2 C_D^2$, and we can now assign the following values for α and β :

$$\alpha = \sqrt{\frac{1}{8x^3}(e^{2x} - 4x - e^{-2x})} \quad (2.24)$$

and

$$\beta = \sqrt{\frac{1}{8x} \frac{(e^{2x} + 4x - e^{-2x})}{(e^{2x} + (x+1)^2)}}. \quad (2.25)$$

We can now go back to Eqs. (2.2) and (2.3) to generate a noise impulse train in practice.

2.3. NOISE GENERATION FOR A CHARGE-SENSITIVE PREAMPLIFIER

When we employ a charge-sensitive amplifier's response function

$$H(\omega) = \frac{1}{j\omega t_0(j\omega + 1/t_0)}, \quad (2.26)$$

whose impulse response in the time-domain presentation is

$$h(x) = 1 - e^{-x}. \quad (2.27)$$

Here, t_0 is the preamplifier's rise time instead of the shaping time, as in the previous section. The output node of this system is designated as "Node-1" in Fig. 1. Since

we can find that $w_1 + w_2 + w_3 = 0$, we can modify Eqs. (2.11) and (2.13) as

$$\langle s_{pk}^2 \rangle = 2(C_{p1} - \sigma_p^2)(w_1 w_2 + w_2 w_3) + 2(C_{p2} - \sigma_p^2)w_1 w_3 \quad (2.28)$$

and

$$\langle s_{sk}^2 \rangle = \frac{2(C_{s1} - \sigma_s^2)(w_1 w_2 + w_2 w_3) + 2(C_{s2} - \sigma_s^2)w_1 w_3}{u_1^2 + u_2^2}. \quad (2.29)$$

Unfortunately, the integrals used to evaluate σ_p^2 , C_{p1} , and C_{p2} do not converge. We know that the noise power for parallel noise could meet convergence when we use a double-correlated sampling or a difference filter. The output node for the difference filter appears as "Node-3" in Fig. 1. The power for the double-correlated sampling, i.e. $\langle (A_{k+1} - A_k)^2 \rangle$, gives a finite result, which is evaluated for the parallel-noise part, $2(\sigma_p^2 - C_{p1})$, and for the series-noise part, $2(\sigma_s^2 - C_{s1})$, separately. The first one is

$$\frac{i_p^2}{2} \int_{-\infty}^{\infty} |(1 - e^{-j\omega\tau})H(\omega)|^2 df \rightarrow \frac{i_p^2 t_0}{2} (x - 1 + e^{-x}), \quad (2.30)$$

and the latter one is

$$\frac{v_s^2 C_D^2}{2} \int_{-\infty}^{\infty} \omega^2 |(1 - e^{-j\omega\tau})H(\omega)|^2 df \rightarrow \frac{v_s^2 C_D^2}{2t_0} (1 - e^{-x}). \quad (2.31)$$

Since σ_s^2 , C_{s1} , and C_{s2} give finite results, as shown in Table 3, the difference filter is not necessarily a mandatory part for them. Then, Eq. (2.28) leads to

$$\langle s_{pk}^2 \rangle = \frac{t_0}{2} i_p^2 \{x(1 + e^{-2x}) - (1 - e^{-2x})\} / (1 - e^{-x})^2, \quad (2.32)$$

and Eq. (2.29) leads to

$$\langle s_{sk}^2 \rangle = \frac{t_0}{4} v_s^2 C_D^2 (1 - e^{-2x}). \quad (2.33)$$

Here, α and β can be assigned as

$$\alpha = \frac{1}{1 - e^{-x}} \sqrt{\frac{1}{2x} \{x(1 + e^{-2x}) - (1 - e^{-2x})\}} \quad (2.34)$$

and

$$\beta = \frac{1}{2} \sqrt{\frac{1 - e^{-2x}}{x}}. \quad (2.35)$$

We are now ready to generate electronic noise for the preamplifier's output. For our reference we mention the equivalent noise charge. It is quite interesting to note that the electronic noise for a charge-sensitive amplifier produces a finite amount of noise power in the discrete-time presentation, while in the continuous-time presentation the noise power shows divergence.

3. Experiment with simulation

3.1. APPEARANCE OF THE GENERATED NOISE

The typical parameters for a computer simulation are listed in Table 4, where the value for the leakage current is exaggerated for demonstration purposes. An example of the generated noise is shown in Figure 2. The time scale covers 0 to 500 μs with 500 nsec increments. We can see that the CR-RC shaper delivers a well-whitened output for both the series noise and the parallel noise. In the case for the preamplifier's output, the parallel noise shows a Brownian-like behavior, while the series noise component still appears "white". The parallel noise can be whitened by applying a difference filter, as shown in Figure 2 (e).

3.2. SAMPLE-CORRELATION FUNCTION VERSUS AUTO-CORRELATION FUNCTION

In order to estimate the power spectrum, the first thing to do is to calculate a sample-correlation function,

$$r_v(l) = \frac{1}{N} \sum_{k=1}^{N-1-l} v_k v_{k+l}, \quad (3.1)$$

where N is the number of sampling points, l runs from 0 to a predetermined value (m). m was taken as 100 with $N=10000$. The sampling interval was taken as 50 nsec so as to be compatible with the higher end of the frequency response. The sample-correlation functions were compared with Eqs (2.20) and (2.21) for the CR-RC shaper. For the noise at the output node of the preamplifier, the series noise part of the auto-correlation function gave a finite value, as shown in Table 3. In order to have access to parallel noise we employed a difference filter for the noise amplitude train, for which we calculated a sample-correlation function. The mathematical operation of the difference filter was

$$v_k \leftarrow v_k - v_{k-1}, \quad (3.2)$$

where v_1 was kept as it was. The analytic form for the auto-correlation function after the difference filter is

$$\frac{t_p^2 t_0}{2} \left\{ \frac{e^{-|k-1|x} + e^{-(k+1)x}}{2} - e^{-kx} + \frac{(|k-1| - k + 1)x}{2} \right\}. \quad (3.3)$$

Figure 3 shows sample-correlated functions for each option together with an analytic estimation based on continuous-time theory. The agreement between the sample- and auto- correlation function is excellent for the series noise, while for the parallel noise the sample-correlation function shows a smaller value than the auto-correlated function by 20%, which is possibly due to an insufficient time coverage and/or statistics.

3.3. POWER SPECTRUM

Employing discrete-time sampling theory,^[9] we reconstructed the noise power spectrum as

$$S_v(\omega_k) = \tau \{ r_v(0) + 2 \sum_{l=1}^{m-1} w_b(l) r_v(l) \cos(\omega_k \tau l) \}, \quad (3.4)$$

where ω_k is

$$\omega_k = \frac{\pi k}{m\tau}. \quad (3.5)$$

We employed a Bartlett window for $w_b(l)$,

$$w_b(l) = 1 - \frac{l}{m}, \quad (3.6)$$

where $w_b(l)$ was zero for $l > m$. The noise power spectrum $S_v(\omega_k)$ should be compared to

$$\frac{1}{2} (i_p^2 + v_s^2 C_D^2 \omega_k^2) |H(\omega_k)|^2. \quad (3.7)$$

From Eqs. (2.15) and (2.26), $|H(\omega)|^2$ for systems with and without the CR-RC shaping amplifier are

$$\frac{e^2}{t_0^2 (\omega^2 + 1/t_0^2)^2} \quad (3.8)$$

and

$$\frac{1}{t_0^2 \omega^2 (\omega^2 + 1/t_0^2)}, \quad (3.9)$$

respectively. Figure 4 shows the reconstructed noise power spectrums. The solid lines are from Eq. (3.7) with an appropriate filtering function, as shown in Eqs. (3.8) and (3.9). We require special care to reconstruct the parallel noise for the preamplifier, which tends to infinity at the limit of very low frequency. We apply a difference filter for each data point. The difference filter moderates the behavior

at the low-frequency region. At the end of the procedure we correct the power spectrum by the factor

$$\frac{1}{2(1 - \cos\omega\tau)} \quad (3.10)$$

to reproduce the original power spectrum. The agreement between the reconstructed power spectrum and the analytic formula from continuous-time theory is excellent for the series noise, but for the parallel noise it degrades by 20% in the same context discussed regarding the sample-correlation function.

3.4. HIT FREQUENCY OF A COMPARATOR AT THE PREAMPLIFIER

The comparator attached at the output node of the preamplifier samples the input voltage for each 500 nsec. Once the comparator fires, it quickly samples the input level to update the baseline and to be ready for the next firing. When the comparator does not deliver any hit until 10 msec after the baseline update, the comparator delivers an artificial hit so as to move back to the initial condition. The hit frequency obtained from above-mentioned procedure is shown in Figure 5. While the hit for the series noise shows a steep fall-off for a higher threshold above 1000 electrons, the hit frequency due to parallel noise continues to stay at around 1 KHz or less. We can say that the hit frequency at the output node of the preamplifier is dominated by the parallel noise as long as the threshold level is set at a few thousand electrons, even though the simulated events are only 100 and the statistics are very poor.

4. Summary

We developed a scheme to generate electronic noise with a given timing granularity. The generated noise was examined for its sample-correlated function and power spectrum, which revealed good agreement with the continuous-time formula used in analytic presentations. Although the output noise of the charge-sensitive preamplifier was known to be inadequate to define the equivalent noise charge, we could provide a procedure to deliver noise amplitude trains, which were employed to estimate the average hit frequency for the comparator located at the output node of the preamplifier. The procedures developed here are applicable for the class of signal-processing systems in which the input referred electronic charge is at the heart of interest. Once we find an impulse response ($h(t)$), we can derive deconvolution coefficients $\{w_k\}$ and $\{u_k\}$ as well as the auto-correlation functions, which are inputs used to specify the properties of the noise-generation engine.

APPENDIX A

Matrix presentation The matrix presentation of the impulse response is

$$H = \begin{pmatrix} H_1 & 0 & 0 & 0 & \dots \\ H_2 & H_1 & 0 & 0 & \dots \\ H_3 & H_2 & H_1 & 0 & \dots \\ H_4 & H_3 & H_2 & H_1 & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}. \quad (A.1)$$

Here, H characterizes the response for the parallel noise, while H' ,

$$H' = \begin{pmatrix} H'_1 & 0 & 0 & 0 & \dots \\ H'_2 & H'_1 & 0 & 0 & \dots \\ H'_3 & H'_2 & H'_1 & 0 & \dots \\ H'_4 & H'_3 & H'_2 & H'_1 & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}, \quad (A.2)$$

gives characterization for the series noise.

$$W = \begin{pmatrix} w_1 & 0 & 0 & 0 & \dots \\ w_2 & w_1 & 0 & 0 & \dots \\ w_3 & w_2 & w_1 & 0 & \dots \\ 0 & w_3 & w_2 & w_1 & \dots \\ \cdot & \cdot & \cdot & \cdot & \dots \end{pmatrix} \quad (\text{A.3})$$

is essentially the inverse matrix of H , but the exact relation between H and W is described as

$$I' = WH = \begin{pmatrix} 0 & 0 & 0 & 0 & \dots \\ 1 & 0 & 0 & 0 & \dots \\ 0 & 1 & 0 & 0 & \dots \\ 0 & 0 & 1 & 0 & \dots \\ \cdot & \cdot & \cdot & \cdot & \dots \end{pmatrix}. \quad (\text{A.4})$$

I' is equivalent to a unit matrix, but not in fact, since the H matrix is singular, i.e. $H_1 = 0$. The product between W and H' gives a matrix (U),

$$U = WH' = \begin{pmatrix} u_1 & 0 & 0 & 0 & \dots \\ u_2 & u_1 & 0 & 0 & \dots \\ 0 & u_2 & u_1 & 0 & \dots \\ 0 & 0 & u_2 & u_1 & \dots \\ \cdot & \cdot & \cdot & \cdot & \dots \end{pmatrix}. \quad (\text{A.5})$$

The vectors $\mathbf{A}_s$, $\mathbf{A}_p$, $\mathbf{i}_p$, and $\mathbf{v}_s$ are defined in their transposed from as

$${}^t\mathbf{A}_s = (A_{s1} \ A_{s2} \ A_{s3} \ A_{s4} \ \dots) \quad (\text{A.6})$$

$${}^t\mathbf{A}_p = (A_{p1} \ A_{p2} \ A_{p3} \ A_{p4} \ \dots) \quad (\text{A.7})$$

$${}^t\mathbf{i}_p = (i_{p1} \ i_{p2} \ i_{p3} \ i_{p4} \ \dots) \quad (\text{A.8})$$

and

$${}^t\mathbf{v}_s = (v_{s1} \ v_{s2} \ v_{s3} \ v_{s4} \ \dots) \quad (\text{A.9})$$

Eqs. (2.2) and (2.3) lead to

$$\mathbf{A}_p = \alpha H \mathbf{i}_p \quad (\text{A.10})$$

and

$$\mathbf{A}_s = \beta H' \mathbf{v}_s, \quad (\text{A.11})$$

respectively. Eqs. (2.8) and (2.9) lead to

$$\alpha I' \mathbf{i}_p = W \mathbf{A}_p \quad (\text{A.12})$$

and

$$\beta U \mathbf{v}_s = W \mathbf{A}_s, \quad (\text{A.13})$$

respectively.

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TABLE CAPTIONS

- 1: Element for the transfer matrix
(a) is for a CR-RC shaper, and (b) is for a charge sensitive preamplifier
- 2: Weighting functions
- 3: Auto-correlation functions
- 4: Typical parameters for benchmark

Table 1. Transfer matrix (a)

H_1	0	H'_1	e/t_0
H_2	xe^{1-x}	H'_2	$(1-x)e^{1-x}/t_0$
H_3	$2xe^{1-2x}$	H'_3	$(1-2x)e^{1-2x}/t_0$
H_4	$3xe^{1-3x}$	H'_4	$(1-3x)e^{1-3x}/t_0$
H_5	$4xe^{1-4x}$	H'_5	$(1-4x)e^{1-4x}/t_0$
H_n	$(n-1)xe^{1-(n-1)x}$	H'_n	$(1-(n-1)x)e^{1-(n-1)x}/t_0$

Table 1. Transfer matrix (b)

H_1	0	H'_1	$1/t_0$
H_2	$1 - e^{-x}$	H'_2	e^{-x}/t_0
H_3	$1 - e^{-2x}$	H'_3	e^{-2x}/t_0
H_4	$1 - e^{-3x}$	H'_4	e^{-3x}/t_0
H_5	$1 - e^{-4x}$	H'_5	e^{-4x}/t_0
H_n	$1 - e^{-(n-1)x}$	H'_n	$e^{-(n-1)x}/t_0$

Table 2. Weighting functions (a)

	For CR-RC system	For charge amplifier
$w_1 = \frac{1}{h_2}$	$\frac{e^x - 1}{x}$	$\frac{1}{1 - e^{-x}}$
$w_2 = -\frac{w_1 h_3}{h_2}$	$-\frac{2e^{-1}}{x}$	$-\frac{1 + e^{-x}}{1 - e^{-x}}$
$w_3 = -\frac{w_2 h_3 + w_1 h_4}{h_2}$	$\frac{e^{-x} - 1}{x}$	$\frac{e^{-x}}{1 - e^{-x}}$
$w_1^2 + w_2^2 + w_3^2$	$\frac{e^{-2}}{x^2}(e^{2x} + 4 + e^{-2x})$	$\frac{1 + (1 + e^{-x})^2 + e^{-2x}}{(1 - e^{-x})^2}$
$w_1 w_2 + w_2 w_3$	$-\frac{2e^{-2}}{x^2}(e^{-x} + e^x)$	$-\frac{(1 + e^{-x})^2}{(1 - e^{-x})^2}$
$w_1 w_3$	$\frac{e^{-2}}{x^2}$	$\frac{e^{-x}}{(1 - e^{-x})^2}$

Table 2. Weighting functions (b)

	For CR-RC shaper	For charge amplifier
u_1	$\frac{e^x}{t_0 x}$	$\frac{1}{t_0(1 - e^{-x})}$
u_2	$-\frac{x+1}{t_0 x}$	$-\frac{1}{t_0(1 - e^{-x})}$
$u_1^2 + u_2^2$	$\frac{e^{2x} + (x+1)^2}{t_0^2 x^2}$	$\frac{2}{t_0^2(1 - e^{-x})^2}$

Table 3. Correlation functions

	For CR-RC system	For charge amplifier
C_{pk}	$\frac{e^2 t_0}{8} (1 + kx) e^{-kx} i_p^2$	$\sigma_p^2 - \frac{t_0}{4} (kx - 1 + e^{-kx}) i_p^2$
C_{sk}	$\frac{e^2}{8 t_0} (1 - kx) e^{-kx} v_s^2 C_D^2$	$\sigma_s^2 - \frac{1 - e^{-kx}}{4 t_0} v_s^2 C_D^2 = \frac{1}{4 t_0} v_s^2 C_D^2 e^{-kx}$
σ_p^2	$\frac{e^2 t_0 i_p^2}{8}$	Undefined
σ_s^2	$\frac{e^2}{8 t_0} v_s^2 C_D^2$	$\frac{1}{4 t_0} v_s^2 C_D^2$

Table. 4 Typical parameter for benchmark

Specific time constant(t_0)	200 ns
Sampling interval (τ)	500 ns
Detector capacitance (C_D)	20 pF
Transconductance of the preamplifier (g_m)	2.0 mS
Detector's leakage current (I_L)	10 nA
Bias resistor (R_B)	20 M Ω
Ambient temperature (T)	300 K

FIGURE CAPTIONS

- 1) Signal-processing chain employed in the text
- 2) Example of the generated noise:
 - (a) parallel noise with CR-RC filter
 - (b) Series noise with CR-RC filter
 - (c) parallel noise without CR-RC filter
 - (d) Series noise without CR-RC filter
 - (e) parallel noise without CR-RC filter, but with difference filter
- 3) Sample correlation functions:
 - (a) parallel noise with CR-RC filter
 - (b) Series noise with CR-RC filter
 - (c) parallel noise without CR-RC filter, but with difference filter
 - (d) Series noise without CR-RC filter
- 4) Noise power spectrum:
 - (a) parallel noise spectrum with CR-RC filter
 - (b) Series noise spectrum with CR-RC filter
 - (c) parallel noise spectrum without CR-RC filter
 - (d) Series noise spectrum without CR-RC filter
- 5) Hit frequency of the output node of the preamplifier

The open circles are for the parallel noise, and the asterisks for the series noise. The solid lines are to guide the eyes.

Fig. 1

-26-

FIG. 2(B)

-25-

FIG. 2(A)

-28-

FIG.2(D)

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FIG.2(C)

- 30 -

FIG.3(A)

- 29 -

FIG.2(E)

-32-

FIG.3(C)

-31-

FIG.3(B)

FIG. 4(A)

FIG. 3(D)

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FIG.4(C)

- 35 -

FIG.4(B)

FIG.5

FIG.4(D)