To be published in the Proceedings of the Conference
FUNDAMENTAL PROBLEMS IN QUANTUM THEORY
June 19 - 22, Baltimore (MD), USA

Local Propensities and Quantum Theory

SW 9503

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INTRODUCTION

Three different approaches have been considered for the theoretical description of quantum phenomena:

The first one, based on standard quantum theory, is accepted by the large majority of theoretical physicists, who are impressed by the great predictive power of the theory. It is a fact, however, that several people believed that the presently available theory cannot be considered a reasonable representation of the atomic reality, in spite of its undeniable successes. The rather long list of dissenters includes Planck, Einstein, Ehrenfest, Schrödinger, de Broglie, and the late Dirac¹. There are also some contemporary physicists who feel in the same way, and it is rather clear that this line of dissent is not going to disappear soon.

The second approach that has been considered is a modified quantum theory in which nonfactorizable state vectors for two correlated systems spontaneously collapse to mixtures of factorizable state vectors when the distance between the two systems increases beyond a certain value. The idea was first studied by Furry² and by Schrödinger³ and could have come to an end when Bohm and Aharonov⁴ showed that it was in disagreement with the experiment carried out by Wu and Shaknov⁵. Actually it was later rediscovered by several other people, including Jauch⁶, de Broglie⁷, and Piccioni⁸, who probably did not know about the

experimental situation. Today the idea of spontaneous collapse has been falsified in many other experiments, including those performed with atomic photon pairs⁹. This is because "reduced quantum theory", as this approach is sometimes called, predicts not only the validity of the "weak" Bell-type inequalities¹⁰, but of the "strong" ones as well (for the distinction between weak and strong inequalities see the paper by Lepore and the present author¹¹). Therefore reduced quantum theory is completely ruled out.

The third approach is based on space-time causal models of atomic reality. If a new starting point is to be found after the discovery of Bell's theorem it cannot be a simple reinterpretation of the old theory, but must contain radically new ideas. Looking for a reformulation of quantum theory is therefore a bet about future developments: if Bell's inequalities were to be found violated in a new generation of experiments using nearly perfect detectors¹², there would be no point in seeking for a better theory, since the highly counterintuitive feature of action at a distance¹³ would then have been found to be present in nature. An "unreasonable" nature can of course be well represented by an "unreasonable" theory. Local realistic models have been developed for explaining the violations of the strong inequalities¹⁴, for the prediction of new effects in experiments with three polarizers¹⁵, for the interpretation of the Bohm-Aharonov effect¹⁶, for the understanding of two-photon interference experiments¹⁷. These models have the common feature of explaining the violations of strong inequalities, but of predicting the validity of the weak ones.

In the present paper it will be assumed that there is a concretely real propensity behind every probability which can be predicted correctly before it is measured: The previous 'reality criterion' will be shown to be compatible with Popper's idea of propensities. A suitable formalism will be introduced by starting from standard probability calculus and its frequency interpretation. A set of similar systems described as an eigenstate in quantum theory will be shown to be necessarily nonhomogeneous, if propensities are actually at work in nature. Therefore variable probability is the key to the quantum world. Local propensities lead to a new proof of Bell's theorem, and at the same time explain the well known experimental results concerning it. Naturally, for nearly ideal detectors the variable probability approach remains incompatible with the quantum mechanical predictions.

NEED FOR PROPENSITIES

The alternative starting point we are looking for should look so strong and natural that all physicists should find it easy to accept it. We will show that probabilistic local realism can be formulated in so general terms that it is a natural candidate for such a role. In fact, firstly, if there is no reality of any type even behind a stable and predictable probability, then no material reality is left at all, and, secondly, also the quantum mechanical probabilities are local in the sense advocated in the present paper.

A new description of the physical reality in probabilistic terms, based on the conception of propensity, was presented by Popper¹⁸. The main task to be achieved in this way is an objectivistic and realistic reformulation of quantum theory and of its probability calculus without relying heavily on idealistic elements. The same program was discussed in a paper by J. B. Bastos Filho and the present author¹⁹, and

it was shown that any objectivistic and local formulation of probabilities leads to the validity of Bell's theorem and thus to the impossibility of a full agreement with existing quantum theory. In spite of this it is interesting to work on the idea of locality, since the experimental evidence concerning the quantum mechanical violations of Bell's inequality is far from conclusive¹¹. We assume therefore that the reality behind every stable and predictable probability is always a local propensity (the meaning of "locality" is discussed in the next section).

The idea of propensities is best introduced by observing that the classical theory of probability is based upon the following definition: "The probability of an event is the number of favourable possibilities divided by the number of all the (equal) possibilities". Thus the probability of the event "tails turning up" would be 1 divided by 2 because there are altogether two equal possibilities and only one is "favourable". Similarly, the probability of throwing an odd number smaller than five with a perfect die is 2 divided by 6 since there are six equal possibilities and only two of them (the sides marked "1" and "3") are favourable to the event "an odd number smaller than five".

But what happens if the die is loaded? Then according to the classical theory of Laplace we can no longer say that its six sides represent equal possibilities and we cannot speak anymore of probabilities in the classical numerical sense. If we insert in a wooden die a small piece of lead near, say, the face bearing the number "3", then this number will turn up less frequently than it does in throws with a fair die, and the number of the opposite face (which is "4") will turn up more frequently.

A good theory of probability must of course include weighted possibilities, where the statistical weight can be interpreted as a measure of the propensity or tendency of the figure "3" to turn up upon repetition of the throws, always in the example of the loaded die. The weight, or frequency, of the event "three turning up" can be measured by repeating many throws with the loaded die and counting the number of times the face "3" shows up. If we find this frequency to be 0.05, instead of $1:6=0.1667$, we can conclude that the die is not perfect and take this frequency as a quantitative measure of the propensity the die has to produce the event "three turning up".

There is an important difference between the classical situation and the one met in atomic physics. When dealing with photons, electrons, or atoms one cannot perform more than one measurement on a single individual object because its quantum state is modified. A statistical experiment in the atomic domain is performed with a large set of objects of the same type prepared in the same quantum state. Obviously this is the same as performing many experiments with one object only if the atomic systems of the same type (photons, electrons, ...) are identical.

Quantum theory is usually assumed to describe identical systems, for example in connection with the Bose-Einstein and the Fermi-Dirac statistics, but realistically speaking the perfect identity is difficult to conceive and the explanation of the quantum statistics could for example be along the lines proposed by Tersoff and Bayer²⁰, that is by introducing variable statistical weights of the allowed states. One can also observe that unstable "identical" systems, e.g. neutrons, live different individual lives before disintegrating and that such a concrete behaviour is not quite compatible with their presumed identity.

It turns out that in atomic physics one deals with situations similar to a set of dice loaded differently from one another, with each of which only one throw can be made. From a single act of measurement one cannot deduce a probability and the

individual propensities are thus not measurable. One can however introduce a propensity of a set of atomic systems to give different results with different frequencies. The latter propensity is easily measurable and can be considered as some kind of physical average of (undetectable) individual propensities.

LOCAL PROPENSITIES AND PROBABILITIES

The words "probability" and "frequency" are essentially identifiable, because the probability of an event can be considered as nothing but its measured (or predicted) frequency in a given set of similar objects. A probabilistic reality criterion will now be given stating essentially that there is a real propensity behind every probability which can be predicted correctly before it is measured²¹. An example can be the loaded die of the previous section: if one can predict that the side marked "3" will turn up in many throws with frequency 0.05, one can infer the existence of a certain objectively real propensity of the die to give the result "3" with this particular frequency.

Let S and T be two sets of N physical objects of the same type (photons, neutrons, kaons, ...):

$$\begin{aligned} S &= \{\alpha_1, \alpha_2, \dots, \alpha_N\} \\ T &= \{\beta_1, \beta_2, \dots, \beta_N\} \end{aligned} \quad (1)$$

Let the objects be produced in pairs, α_1 with β_1 , α_2 with $\beta_2, \dots$, α_N with β_N , and different pairs be totally independent from one another.

Suppose that one measures a dichotomic physical quantity $A(a) = \pm 1$ on the α 's of S , and that one can predict the frequencies $f(a+)$ of $A(a) = +1$ and $f(a-)$ of $A(a) = -1$ in a subset S' of S identified by performing some operation on the β 's of T , when S and T are physically separated from one another. It is natural to conclude that these predicted frequencies $f(a\pm)$ reflect some reality of S' , in the sense that they are necessary consequences of some concrete physical property ("propensity") of S' and, possibly, of the physical apparatus used. We can therefore assume the following probabilistic reality criterion (PRC):

Given a set S of N α particles, if it is possible: [1] To predict the existence of a subset S' of S such that future measurements of $A(a)$ on the α 's of S' will give the results $+1$ and -1 with the probabilities(frequencies) $f(a+)$ and $f(a-)$, respectively; [2] To predict the population N' of S' ($0 < N' \leq N$); [3] To make the previous predictions without disturbing in any way the α objects of S and S' ; Then a propensity λ_a belonging to S' is assumed to fix the probabilities:

$$f(a+) = f(a+, \lambda_a) \quad ; \quad f(a-) = f(a-, \lambda_a) \quad (2)$$

Actually Popper also considered competing propensities for different outcomes of a process (e.g., of a measurement)²². In fact, in the present connection he would probably have introduced two different propensities $\lambda_{a\pm}$ for the two outcomes $A(a) = \pm 1$, but our symbol λ_a actually includes this since it can be written as a set of two propensities:

$$\lambda_a = \{ \lambda_{a+}, \lambda_{a-} \} \quad (3)$$

We can thus say that the frequency $f(a+)$ of the outcome $A(a) = +1$ is the synthetic result of all the propensities actively present in S' , and write it as in (2), and similarly for $f(a-)$. The above PRC is no attempt to "define" a physical reality, but only a criterion useful for recognizing the existence of an objective physical property, called propensity, once a realistic attitude has been adopted. The previous PRC provides a natural generalisation of the famous deterministic reality criterion of Einstein, Podolsky and Rosen²³. In this connection the PRC can be applied to the subsets $S(b\pm)$ defined below that will thus play the role of S'

In a typical EPR experiment there are two observers, O_α who measures the dichotomic observable $A(a)$ on the α particles of S , and O_β who measures a similar observable $B(b)$ on the β particles of T . Assuming that O_β performs his measurements before O_α (in the laboratory frame), his results split T into two subsets, $T(b+)$ (all cases $B(b) = +1$) and $T(b-)$ (all cases $B(b) = -1$). Corresponding to this splitting of T one can also consider:

i) A splitting of S into $S(b+)$ and $S(b-)$ composed of the α particles correlated individually with the β particles contained in $T(b+)$ and $T(b-)$, respectively;

ii) A splitting of the set E of (α, β) pairs

$$E = \{(\alpha_1, \beta_1), (\alpha_2, \beta_2), \dots, (\alpha_N, \beta_N)\} \quad (4)$$

into two parts $E(b+)$ and $E(b-)$, such that $S(b+)$ and $T(b+)$ compose physically $E(b+)$, $S(b-)$ and $T(b-)$ compose $E(b-)$.

Let $N(b+)$ be the population of $E(b+)$, $S(b+)$, and $T(b+)$, and $N(b-)$ the population of $E(b-)$, $S(b-)$, and $T(b-)$, with

$$N(b+) + N(b-) = N \quad (5)$$

These populations are related to the probabilities $g(b\pm)$ of measuring $B(b)$ in T and finding ± 1 , respectively:

$$g(b\pm) = \frac{1}{N} N(b\pm) \quad (6)$$

Given enough previous experimental activity with repetitions of the sets E, S, T it is clear that O_β himself will be able to *predict* that (later) O_α will find: $A(a) = \pm 1$, with the frequencies $f(a\pm|b+)$ in $S(b+)$; and $A(a) = \pm 1$, with the frequencies $f(a\pm|b-)$ in $S(b-)$. But $f(a\pm|b+)$ in $S(b+)$ and $f(a\pm|b-)$ in $S(b-)$ are in general different from the probabilities $f(a\pm)$ of finding $A(a) = \pm 1$ in the whole set S . Therefore, if O_α does find experimentally the frequencies earlier predicted by O_β , he can conclude that in the physical reality of $S(b\pm)$ there is something (namely a propensity!) that generates these frequencies. By applying Eq. (2) to $S(b\pm)$ one has:

$$\begin{aligned} f(a\pm|b+) &= f[a\pm, \lambda_a(b+)] \\ f(a\pm|b-) &= f[a\pm, \lambda_a(b-)] \end{aligned} \quad (7)$$

The b dependence of $\lambda_a(b\pm)$ is due to the selection made by O_β . The notation

$\lambda_a(b\pm)$ only means that the sets $S(b\pm)$ and their propensities are defined by the measurement of $B(b)$, and is not meant to imply the existence of an action at a distance from T to S capable of modifying λ_a . In fact to the PRC one can naturally add the following locality assumption:

Measurements performed on the β objects of T do not in any way generate or modify the propensities of the α objects of S, and viceversa.

The propensities $\lambda_a(b\pm)$ must then be real properties of existing subsets $S(b\pm)$ of S even if no measurement of $B(b)$ is performed. Not to admit this would imply that the propensities and the probabilities for the α particles are created via some action-at-a-distance mechanism by the measurements on the β -particles, which is precisely what the locality assumption excludes. Actually there should be many equivalent splittings of S into $S(b\pm)$, since in a truly probabilistic situation it cannot be fixed a priori which particular β particles will give the results +1 or -1. This complication is irrelevant for our purposes: knowing that at least one splitting of the previous type exists is all we need.

LIMITATION ON CONDITIONAL PROBABILITIES

Other probabilities can similarly be introduced for a different splitting of E into $E(b'+)$ and $E(b'-)$ arising from the actual (or the possible) measurements of $B(b')$ on the β particles. Naturally also S splits into $S(b'+)$ and $S(b'-)$. Considering again the observable $A(a)$ of the α -particles, from the application of the PRC to the present case one has

$$f(a\pm|b'+) = f[a\pm, \lambda_a(b'+)] \quad (8)$$

$$f(a\pm|b'-) = f[a\pm, \lambda_a(b'-)]$$

where the first frequency holds in $S(b'+)$ and the second one in $S(b'-)$. In general different subsets imply different probabilities, and $\lambda_a(b'+)$ will thus be different from $\lambda_a(b+)$ since different effects (probabilities) imply different causes (propensities). Clearly

$$S = S(b+) \cup S(b-) \quad (9)$$

and therefore $S(b'+)$, which is a subset of S, is necessarily composed partly of objects of $S(b+)$ and partly of objects of $S(b-)$. In fact by (9) one has:

$$S(b'+) = S(b'+) \cap S = S(b'+) \cap [S(b+) \cup S(b-)] = [S(b'+) \cap S(b+)] \cup [S(b'+) \cap S(b-)] \quad (10)$$

the last step being a consequence of the fact that $S(b+)$ and $S(b-)$ do not overlap.

Consider now four of the eight probabilities (7)-(8), those referring to $A(a) = +1$. If $S(b+)$ were homogeneous, in the sense that every subset of it had a propensity $\lambda_a(b+)$ for the measurements of $A(a)$, and similarly for $S(b-)$, it would follow from (10) that $f[a+, \lambda_a(b+)]$ would apply to $S(b'+) \cap S(b+)$, containing a fraction γ of the population of $S(b'+)$, and that $f[a+, \lambda_a(b-)]$ would apply to $S(b'+) \cap S(b-)$, containing

a fraction $(1-\gamma)$ of the population of $S(b'+)$ (with $0 \leq \gamma \leq 1$), so that one would necessarily have:

$$f[a+, \lambda_a(b'+)] = \gamma f[a+, \lambda_a(b+)] + (1-\gamma) f[a+, \lambda_a(b-)] \quad (11)$$

This is a consequence of locality: If probabilities were instead created via action at a distance they would exist literally only after a measurement on T is performed. But $B(b)$ and $B(b')$ are incompatible and the previous equation could not be deduced. Eq. (11) makes full sense in the local realistic approach, and gives the left-hand side as a weighted average of other two probabilities. Therefore the left-hand side must be *internal* to the interval of positive numbers in the right-hand side, and one has

$$f[a+, \lambda_a(b+)] \geq f[a+, \lambda_a(b'+)] \geq f[a+, \lambda_a(b-)] \quad (12)$$

if, for instance, $f[a+, \lambda_a(b+)] \geq f[a+, \lambda_a(b-)]$. One can obviously repeat the previous argument for $f[a+, \lambda_a(b'-)]$, by considering that also for $S(b'-)$ an expression similar to (10) can be deduced, and therefore that $S(b'-)$ is composed of a fraction γ' of elements of $S(b+)$ and of a fraction $(1-\gamma')$ of elements of $S(b-)$, with $0 \leq \gamma' \leq 1$. Under the hypothesis that $S(b\pm)$ are homogeneous one gets now:

$$f[a+, \lambda_a(b+)] \geq f[a+, \lambda_a(b'-)] \geq f[a+, \lambda_a(b-)] \quad (13)$$

We thus see from (12) and (13) that probabilities $f[a+, \lambda_a(b'\pm)]$ must be internal to the interval defined by the other two probabilities $f[a+, \lambda_a(b\pm)]$. But b' is arbitrary and the previous conclusion must hold for all possible b' . But also b is arbitrary and it cannot be true that it is always that value for which one gets the largest interval of probabilities, unless of course the probabilities are actually constant when b is varied. The latter condition is however far too restrictive for theoretical and practical reasons, in particular because correlation experiments of the EPR-type have shown that probabilities of the previous type do actually vary with a and b . Therefore the sets $S(b\pm)$ in general cannot be homogeneous. The only exception to this conclusion is the trivial case of conditional probabilities $f(a\pm|b\pm)$ which do not really depend on the argument b . Otherwise the conclusion holds in total generality, and can be applied for example to a set of particles described in quantum mechanics by the "singlet" state, as shown in the next section.

"SINGLET" STATE AS AN EXAMPLE

The considerations carried out up to now are totally general and apply to sets S , T and E of any nature. It is however interesting to see how far the idea of local propensities applies to quantum systems. Of course it is out of the question to reproduce totally the predictions of the quantum singlet state, because of Bell's theorem. Nevertheless it is useful to learn directly from quantum theory some needed features of the propensity approach. Consider the quantum observables

$$A(a) = \sigma \cdot a \quad ; \quad B(b) = \tau \cdot b \quad (14)$$

where σ_i and τ_j ($i, j = 1, 2, 3$) are the Pauli matrices for two spin-1/2 particles, α and β respectively. They are dichotomic observables with ± 1 as eigenvalues and - apart from a trivial factor - they represent the spin components of particles α and β along the directions a and b . The well known singlet state is given by:

$$\eta_s = \frac{1}{\sqrt{2}} [u(b+) v(b-) - u(b-) v(b+)] \quad (15)$$

where

$$\sigma \cdot b u(b\pm) = \pm u(b\pm) \quad ; \quad \tau \cdot b v(b\pm) = \pm v(b\pm) \quad (16)$$

so that η_s is an eigenstate of $\sigma \cdot b + \tau \cdot b$ with eigenvalue zero. Since the singlet state is rotationally invariant the previous conclusion holds for an arbitrary direction b . Suppose next that $\tau \cdot b$ is measured on the set T of β particles: According to quantum theory the results ± 1 will be found with the same frequency 1/2. With the notation of Eq. (5) one has:

$$N(b+) = N(b-) = N/2$$

These populations $N(b\pm)$ of course refer not only to the subsets $T(b\pm)$, but also to $E(b\pm)$ and $S(b\pm)$. According to quantum theory, during the measurement the "reduction of the state vector" takes place and one must consider the new states:

$$u(b-) v(b+) \quad \text{in } E(b+) \quad ; \quad u(b+) v(b-) \quad \text{in } E(b-) \quad (17)$$

It is now a simple matter to calculate the quantum-mechanically predicted probabilities $f(a\pm | b\pm)$ of the previous section. One has for example

$$f_{qm}(a+ | b+) = \langle u(b-) v(b+) | (I + \sigma \cdot a) / 2 | u(b-) v(b+) \rangle = \cos^2(a-b).$$

More generally, one has:

$$\begin{aligned} f_{qm}(a+ | b+) &= f_{qm}(a- | b-) = \cos^2(a-b) \\ f_{qm}(a+ | b-) &= f_{qm}(a- | b+) = \sin^2(a-b) \end{aligned} \quad (18)$$

where $a-b$ is the angle between the directions a and b . The main point deduced in the previous section is that conditional probabilities emerge as necessary consequences of objectively existing propensities. Of course propensities are always there, but probabilities become concrete frequencies only when they are measured. One has from (7):

$$\begin{aligned} f_{lr}(a+ | b+) &= f[a+, \lambda_a(b+)] = F(a-b) \\ f_{lr}(a+ | b-) &= f[a+, \lambda_a(b-)] = 1 - F(a-b) \end{aligned} \quad (19)$$

where F is the (presently unknown) prediction of probabilistic local realism, and the dependence on $a-b$ can be justified on the basis of rotational invariance. Of course Bell's theorem does not allow F to equal the quantum mechanical prediction. From (8) it follows:

$$\begin{aligned}
f_{1r}(a+|b'+) &= f[a+, \lambda_a(b'+)] = F(a-b') \\
f_{1r}(a+|b'-) &= f[a+, \lambda_a(b'-)] = 1 - F(a-b')
\end{aligned}
\tag{20}$$

Eq. (10) shows that $S(b'+)$ splits into two subsets. By assuming the homogeneity of $S(b\pm)$ it would now follow, as a consequence of Eqs.(13) or (14):

$$F(a-b) \geq F(a-b') \geq 1 - F(a-b) \tag{21}$$

if, for instance, $F(a-b) \geq 1 - F(a-b)$. But (21) cannot be valid for arbitrary values of a, b, b' , if the predictions (19) and (20), while respecting Bell's inequality, are even vaguely similar to the quantum theoretical expressions. Therefore the subsets $S(b+)$ and $S(b-)$ cannot be homogeneous. Since to these subsets we have attributed the quantum mechanical states $u(b-)$ and $u(b+)$, respectively [see Eq.(17)] we are forced to the conclusion that in an EPR situation the eigenvectors of the spin observables cannot represent homogeneous ensembles. This conclusion follows rigorously by starting from the singlet state, but one can guess that it could be valid for all states of spin-1/2 particles independently of EPR correlations.

We have so discovered a new and more general sense in which quantum theory is incomplete. J. von Neumann showed²⁴ that the assumption of quantum mechanical completeness implies that a set described as an eigenstate is homogeneous (that is, everyone of its subsets must be described by the same eigenstate). We have instead come to the conclusion that at least in some cases quantum mechanical eigenstates actually describe inhomogeneous ensembles, if our assumptions concerning realism(propensities) and locality are true in nature. In conclusion :

If local probabilities are consequences of propensities belonging to the sets $S(b\pm)$ for which they are predicted, they must result from averages of other probabilities, in general different for different subsets of $S(b\pm)$, but constant within every such subset.

These probabilities of homogeneous subsets of S and T are local, and therefore totally independent of what is eventually measured on the other set of particles (T and S respectively).

HOMOGENEOUS SETS OF SIMILAR SYSTEMS

The conclusion of a previous section concerning the existence of a splitting of S into $S(b+)$ and $S(b-)$ even if no measurement of $B(b)$ is made is now our starting point; the same splitting can be attributed to E which consists of S and T .

Previously we have also been led to the conclusion that $S(b+)$ and $S(b-)$ - and therefore S that represents their union - cannot be homogeneous as far as the probabilities of finding $A(a) = \pm 1$ are concerned. A symmetrical conclusion is obtained for the probabilities of finding $B(b) = \pm 1$ in T by repeating the proof with the roles of particles α and β interchanged. Recalling the results of the previous section, we consider now a splitting of E (arising from a similar splitting of S) into the subsets

$$\sigma_1(a), \sigma_2(a), \dots, \sigma_r(a) \tag{22}$$

$\tau_1(b_n), \tau_2(b_n), \dots, \tau_{s_n}(b_n)$ homogeneous for $B(b_n)$.

The homogeneous nature of the subsets (26) and (27) has the usual meaning: All the subsets of $\sigma_1(a_1)$ have the same probabilities $f_1(a_1 \pm, \lambda_{a_1})$ for the results $A(a_1) = \pm 1$, where λ_{a_1} is a local propensity. All the subsets of $\sigma_2(a_1)$ have the same probabilities $f_2(a_1 \pm, \lambda_{a_1})$ for the results $A(a_1) = \pm 1$. And so on.

Obviously, the union of the sets of every line of (26) and (27) gives always E , just as in (24).

By means of suitable intersections one can introduce smaller subsets in which all the considered observables have constant probability. One can write:

$$E_k(a_1, \dots, a_m, b_1, \dots, b_n) = \sigma_{i_1}(a_1) \cap \dots \cap \sigma_{i_m}(a_m) \cap \tau_{j_1}(b_1) \cap \dots \cap \tau_{j_n}(b_n) \quad (28)$$

for a typical subset homogeneous for all the probabilities of the ± 1 results of the $(m+n)$ considered observables. In (28) the single index k has been chosen, for simplicity, to correspond in a one-to-one way to the set of indices

$$\{i_1, i_2, \dots, i_m, j_1, j_2, \dots, j_n\} \quad (29)$$

The number $t(m, n)$ of such sets can easily be calculated if one notices that i_1 can be chosen in r_1 different ways, $\dots$ i_m in r_m different ways, j_1 in s_1 different ways, $\dots$ j_n in s_n different ways, all choices being independent. Therefore

$$t(m, n) = r_1 r_2 \dots r_m s_1 s_2 \dots s_n \quad (30)$$

Notice, however, that $t(m, n)$ is in general expected to change if some arguments $a_1, \dots, b_n$ are modified, as it is also clear from the notation (26) and (27) where s_1 [which is the number of subsets of E homogeneous for $B(b_1)$] has the same index as b_1 precisely because they depend on one another and a modification of b_1 can give rise to a change of s_1 . Therefore the set I of indices k [set of integers from 1 to $t(m, n)$] depends on the arguments of the observables:

$$I = I(a_1, \dots, a_m, b_1, \dots, b_n) \quad (31)$$

Finally let us define

$$N_k(a_1, \dots, a_m, b_1, \dots, b_n) \quad (32)$$

to be the population of E_k as given by (28).

PROBABILITIES OF HOMOGENEOUS SETS

The basic probabilities, which are all constant for pairs belonging to a subset E_k are :

$f_k(a_1 \pm, \lambda_{a_1})$, probabilities for $A(a_1) = \pm 1$,

$f_k(a_2 \pm, \lambda_{a_2})$, probabilities for $A(a_2) = \pm 1$,

$$\dots\dots\dots (33)$$

$$f_k(a_m \pm, \lambda_{a_m}), \text{ probabilities for } A(a_m) = \pm 1;$$

and

$$g_k(b_1 \pm, \lambda_{b_1}), \text{ probabilities for } B(b_1) = \pm 1,$$

$$g_k(b_2 \pm, \lambda_{b_2}), \text{ probabilities for } B(b_2) = \pm 1,$$

$$\dots\dots\dots (34)$$

$$g_k(b_n \pm, \lambda_{b_n}), \text{ probabilities for } B(b_n) = \pm 1.$$

Of course the probabilities f_k belong to the α particles, and the g_k to the β -particles. Each of them depends only on the propensity of the measured observable. These propensities are assumed to be strictly local. Thus λ_{a1} does not depend in any way on the particular b_i which is eventually being measured on the β -particles, not even on the fact that a b_i is being measured or not.

It is important to stress that the present formulation of locality is strictly analogous to the quantum-mechanical locality of probabilities (which is well known to hold in spite of the overall nonlocal nature of the theory). It is true that the present formulation of locality is only applied to propensities, but nonlocal propensities are fully expected to generate nonlocal probabilities and therefore the locality condition for propensities is physically equivalent to a similar condition for probabilities.

An important feature of quantum probabilities holds also for (33)-(34) : A measurement of $f(a_\mu \pm)$ in a set S destroys the possibility of measuring $f(a_\nu \pm)$ with $\nu \neq \mu$ in the same S , since the observables $A(a_\mu)$ and $A(a_\nu)$ are in general incompatible. Similarly for $g(b_\sigma \pm)$ and $g(b_\tau \pm)$ in a set T , if $\sigma \neq \tau$.

Obviously if one writes:

$$\rho_k(a_1, \dots, a_m, b_1, \dots, b_n) = \frac{1}{N} N_k(a_1, \dots, a_m, b_1, \dots, b_n) \quad (35)$$

then

$$\sum_{k \in I} \rho_k(a_1, \dots, a_m, b_1, \dots, b_n) = 1 \quad (36)$$

and ρ_k is the "statistical weight" of the subset E_k in the full set E .

JOINT PROBABILITIES AND BELL'S THEOREM

The definition (35) is useful for calculating all types of probabilities in the set E , for example those needed for a new proof of Bell's theorem. These are typically joint probabilities for observations carried out on correlated systems. The joint probability for $A(a_\mu) = \pm 1$ and $B(b_\nu) = +1$, is given by:

$$\Omega(a_\mu \pm, b_\nu +) = \sum_{k \in I} \rho_k(a_1, \dots, a_m, b_1, \dots, b_n) \Omega_k(a_\mu \pm, b_\nu +) \quad (37)$$

where Ω_k is the joint probability calculated in the homogeneous set E_k . One can obviously apply Bayes' formula and write

$$\Omega_k(a_{\mu\pm}, b_{v+}) = \omega_k(a_{\mu\pm}, \lambda_{a\mu}/b_{v+}) g_k(b_{v+}, \lambda_{bv}) \quad (38)$$

where $g_k(b_{v+}, \lambda_{bv})$ has been introduced in (34) and $\omega_k(a_{\mu\pm}, \lambda_{a\mu}/b_{v+})$ is the conditional probability of $A(a_{\mu}) = \pm 1$, given that $B(b_v) = +1$. One can also say that $\omega_k(a_{\mu\pm}, \lambda_{a\mu}/b_{v+})$ is the probability of $A(a_{\mu}) = \pm 1$ in

$$E_k(a_1, \dots, a_m, b_1, \dots, b_n) \cap E(b_{v+}) \quad (39)$$

since it has been concluded that at least one set $E(b_{v+})$ exists with the propensity $\lambda(b_{v+})$ even if $B(b_v)$ is not measured. Remember now that E_k is homogeneous, meaning that a probability valid for the whole of E_k applies as well to every conceivable part of it, for example to the intersection (39). Remembering also (33) this gives immediately

$$\omega_k(a_{\mu\pm}, \lambda_{a\mu}/b_{v+}) = f_k(a_{\mu\pm}, \lambda_{a\mu}) \quad (40)$$

Inserting (40) in (38) one gets

$$\Omega_k(a_{\mu\pm}, b_{v+}) = f_k(a_{\mu\pm}, \lambda_{a\mu}) g_k(b_{v+}, \lambda_{bv}) \quad (41)$$

and consequently from (37) it follows

$$\Omega(a_{\mu\pm}, b_{v+}) = \sum_{k \in I} \rho_k(a_1, \dots, a_m, b_1, \dots, b_n) f_k(a_{\mu\pm}, \lambda_{a\mu}) g_k(b_{v+}, \lambda_{bv}) \quad (42)$$

With a trivial generalisation of the reasoning leading from (37) to (42) one gets:

$$\Omega(a_{\mu\pm}, b_{v-}) = \sum_{k \in I} \rho_k(a_1, \dots, a_m, b_1, \dots, b_n) f_k(a_{\mu\pm}, \lambda_{a\mu}) g_k(b_{v-}, \lambda_{bv}) \quad (43)$$

The joint probabilities (42) and (43) look like Clauser-Horne factorizability formulae²⁵, the role of the hidden variable λ being played by the physically more transparent index k . There are however important differences, because now the parameters of the considered observables enter in the probability density ρ_k . This does not imply the introduction of nonlocality, because in principle all the conceivable observables should enter in ρ_k . In practice however one can take into account only observables measured in the considered experiment, the other ones being averaged away. A similar situation has been described by Wigner²⁶ with his deterministically based proof of Bell's theorem.

In spite of this dependence, it is very easy to deduce from (42) and (43) the usual inequalities of the Bell type²⁷. It is amusing to notice that we never needed to introduce joint probabilities for two incompatible observables, described in quantum mechanics by noncommuting operators. Observables like $A(a_{\mu})$ and $B(b_v)$

are instead relative to two different objects and are therefore always compatible, so that the previous obstacle does not exist and the joint probabilities (42)-(43) are empirically meaningful.

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