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PERCOLATION AND CLUSTER ANALYSIS FOR $\frac{\Delta T}{T}$ MAPS

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Abstract In this paper we discuss the method of filtering, cluster analysis and percolation for $\frac{\Delta T}{T}$ of the Cosmic Microwave Background maps. We show that this method provides unique information about the ionization history of the Universe and this is a realistic way for the investigation of the Gaussian nature of the Cosmic Microwave Background fluctuations.

Introduction

During the last several years the angular anisotropy of Cosmic Microwave Background (CMB) has certainly been widely investigated. First of all, we mention the results of COBE experiment which pointed out an anisotropy of CMB at the $\theta \geq 10^\circ$ (Smoot *et al.*, 1992; Bennett *et al.*, 1994). The distribution of the fluctuations of $\frac{\Delta T}{T}$ at $\theta \geq 10^\circ$ characterizes the fluctuations of the primordial density, velocity and gravitational field at the scale $r > r_{ls}$, where r_{ls} is the acoustic horizon at the moment of the last scattering. One of the most important problems of cosmology is the distribution of the CMB fluctuations at the angular scales $\theta \simeq 30' - 2^\circ$. This angular size corresponds to the scale of recombination and the distribution of angular CMB anisotropy preserves the information about ionization history of the Universe.

Before the recombination of the primordial plasma the adiabatic perturbations evolved as standing acoustic waves. Sakharov (1965) pointed out that the transitional phenomena lead to a periodic dependence of the perturbation amplitude on wavelength at the moment of transition. We call this dependence Sakharov's oscillations. The angular distribution of $\frac{\Delta T}{T}$ on the celestial sphere preserves the signature of Sakharov's oscillations at the epoch of recombination. Various scenarios of the ionization history of the Universe and different amounts of baryons in the Universe

(which is characterized by the present baryonic density Ω_b) lead to different properties of the anomaly in $\frac{\Delta T}{T}$ angular distribution.

Several authors propose different methods to detect the peak-like anomaly in the angular distribution of $\frac{\Delta T}{T}$. Note, that it is very difficult to distinguish a contribution of Sakharov's oscillations using the standard so called three- or two-beam experiments. These difficulties are due to the following reason. The characteristic angular scale of the peculiarities $\theta_{ls} \approx \frac{r_{ls}}{\xi_n} / (1 + \frac{3\rho_b}{4\rho_m})^{1/2}$ (where ρ_b and ρ_m are the densities of baryons and CMB at the epoch of the last scattering, and ξ_n is the horizon at the present time) is significantly greater than the characteristic correlation scale $\theta_c \simeq r_{silk}/\xi_n$, where r_{silk} is the Silk's damping scale. Recently Naselsky and Novikov (1993), Jørgensen *et al.* (1994) proposed a 4-beam scheme for detecting Sakharov's oscillations. They proposed detecting the second derivative of the correlation function $C(\theta)$ which emphasizes the peak-like anomalies more clearly. This method is analogous to that used by Watson and Gutierrez de la Cruz (1993). However, the level of the peculiarities of anisotropy, which has to be detected by using this method is equal to $\frac{\Delta T}{T} \approx 3 \cdot 10^{-6}$.

Recently a few groups have observed the anisotropy of CMB on the scale of the order 1° , see for example Clapp *et al.* (1994). However it is worth noting, that the experimental possibilities became very close to detecting of the CMB anisotropy in one of the most important diapason of the angular $\frac{\Delta T}{T}$ distribution, where so called Doppler-peak or Sakharov's oscillations manifest themselves.

In this paper we propose a new method of the investigation of $\frac{\Delta T}{T}$ distribution on the celestial sphere which is based on the percolation and cluster analysis of $\frac{\Delta T}{T}$ maps. The investigation of the topology of the structural systems including percolation and cluster analysis has a great bibliography. These models are widely used in the theory of formation of the Large Scale Structure of the Universe (see for review Dominik and Shandarin, 1992). The efficiency of the percolation method for these problems is based on the extremely sensitivity of the level of percolation to the redshift.

For the investigation of the CMB anisotropy the topologic method was used by Zabotin and Naselsky (1985), Sazhin (1985), Bond and Efstathiou (1987), Gott *et al.* (1990), Smoot *et al.*, (1994). Based on the results of these papers, we show that the methods of percolation and cluster analysis can be very successful for detection Sakharov's modulations of the spectrum of the primordial adiabatic perturbations in the angular distribution of CMB.

In this paper we discuss the following problems:

First, we propose a new test to determine whether the $\frac{\Delta T}{T}$ field is Gaussian or not. It is well known that for a random Gaussian field of fluctuations the critical point of percolation takes place when the areas of "cold" ($\frac{\Delta T}{T} < 0$) and "hot" ($\frac{\Delta T}{T} > 0$) zones are equal, and each of them contains a half of the whole area of the map. If the areas of the zones are not equal it means that the $\frac{\Delta T}{T}$ field is not Gaussian, or there is an additional noise in the system. The non Gaussian character of the $\frac{\Delta T}{T}$ field can be caused, for example by cosmic strings.

Second, we propose a new method of filtering the Gaussian $\frac{\Delta T}{T}$ field with the subsequent cluster analysis of the filtered field to determine the existence of Sakharov's oscillations. This allows us to investigate the ionization history of the Universe.

2. Behavior of the correlation function of $\frac{\Delta T}{T}$ at angular scales $30' - 2^\circ$

It is well known that the general properties of the CMB anisotropy for the Gaussian initial perturbations are completely determined by the correlation function $C(\theta)$ and its derivatives (Bond and Efstathiou, 1987). Let us introduce the local temperature of CMB on the celestial sphere, which is measured by an observer in the direction $\vec{q}$

$$T(\vec{q}) = \langle T(\vec{q}) \rangle + \sum_{l=1}^{\infty} \sum_{m=-l}^l a_l^m C_l^{1/2} Y_l^m(\vec{q}) \quad (1)$$

where the coefficients $\{a_l^m\}$ of the spherical harmonics $Y_l^m(\vec{q})$ expansion are the stochastic variables of the observed position with zero mean and the dispersion $\langle a_l^m a_{l'm'}^* \rangle = \delta_{ll'} \delta_{mm'}$; δ_{lm} is the Kronecker symbol, C_l is the spectrum in l -space.

The correlation function $C(\theta)$ can be written in the following form

$$C(\theta) = \langle \frac{\Delta T}{T}(\vec{q}_1) \frac{\Delta T}{T}(\vec{q}_2) \rangle = \frac{1}{4\pi} \sum_{l=2}^{\infty} \sum_{m=-l}^l (a_l^m)^2 C_l P_l(\cos \theta), \quad (2)$$

where the monopole and dipole terms are removed. The coefficients C_l in (2) are connected with the primordial spectrum of the perturbations $P(k)$ by the following relationship (see, for example, Bond and Efstathiou, 1987, Gorski, 1993):

$$C_l = A \int_0^\infty dk k^2 P(k) T^2(k) j_l^2(k\xi_n). \quad (3)$$

Here $P(k)$ is the initial power spectrum of the metric perturbations, $j_l^2(k\xi_n)$ is the spherical Bessel function; ξ_n is the conform time ($\xi \simeq f \frac{dt}{a}$) scaled at the present time; a is the scale factor for the Universe; A is the constant of normalization, $T(k)$ is the transfer function.

In this paper we discuss only one kind of initial spectrum $P(k) \propto k^{-3}$ (the so-called Harrison-Zeldovich spectrum) which is produced in the simplest models of inflation. The transfer function $T(k)$ describes the following main mechanisms of the $\frac{\Delta T}{T}$ anisotropy generation: the Silk's (1968), Sachs-Wolfe (1967) and Doppler effects. For the standard CDM ($\Omega = 1$) models of the Universe the numerical calculations of $T^2(k)$ were published by many authors (see for review Gorski, 1993). Below we introduce the following analytical approximation for the transfer function which is convenient for the further calculations:

$$T^2(k) = \exp \left[21.6 \Omega_b h^2 \sin(kR) \times \exp(-0.06kR - 0.05(kR)^2) \right], \quad (4)$$

where $R = r_{ls}$ is the scale of the acoustic horizon at the moment of recombination. This approximation describes the main behaviors of $T^2(k)$ for models with $\Omega_b > 0.001$ for the standard ionization history of the Universe.

Using (2)-(4) we have calculated the correlation function of $\frac{\Delta T}{T}$ fluctuations for the CDM models with the following parameters $\Omega_b h^2 = 0.001$ (model I) and $\Omega_b h^2 = 0.1$ (model 2). In both cases the total mass density in the Universe is $\Omega = 1$. Model II has the peak-like anomaly at the angular scale $\theta_{ls} \approx R/\xi_n$. Model I has no such anomaly. This anomaly was discussed by Naselsky and Novikov (1993) and Jørgensen *et al.*, (1994). The models with the secondary ionization of plasma are examples of the models without the peak-like anomaly (they are analogous to the model I). These models are the basis for our analysis of the topology peculiarities in the CDM Universe.

3. The $10^\circ \times 10^\circ$ map of the $\frac{\Delta T}{T}$ distribution and percolation.

This section is devoted to the manifestation of topological peculiarities in two basic models of the primordial $\frac{\Delta T}{T}$ fluctuations with $\Omega_b h^2 = 0.001$ and $\Omega_b h^2 = 0.1$. Let us imagine an experiment when an observer measures the distribution of $\frac{\Delta T}{T}$ on the celestial sphere in the region $10^\circ \times 10^\circ$. We suppose that the width of the antenna beam θ_A is much less than the correlation angle θ_c of $C(\theta)$ and there is no additional noise.

We generated two maps for the models I and II (see Fig.1) using the algorithm of Bond and Efstathiou (1987)

$$\frac{\Delta T}{T}(x, y) = \sum_{i,j} a_{ij} C^{1/2}(m_{ij}) \cos \left(2\pi \frac{ix + jy}{L} + \varphi_{ij} \right), \quad (5)$$

where $C(m_{ij})$ is the power spectrum from Eq.(2); a_{ij} are the independent Gaussian values normalized to 1; $m_{ij} = \frac{2\pi}{L} \sqrt{i^2 + j^2}$; φ_{ij} are random values which are uniformly distributed in the interval $(0, 2\pi)$; L is the angular size of the investigated area ($\sim 10^\circ$).

We will use the following notation $\nu = \alpha\sigma$, where $\sigma = \sqrt{\langle (\frac{\Delta T}{T})^2 \rangle_{L^2}}$ and α is the crossing parameter. We define an area limited by the line $\frac{\Delta T}{T} = \nu$ inside which $\frac{\Delta T}{T}(x, y) > \nu$ as a "hot" zone, and the "cold" zone has the area with $\frac{\Delta T}{T}(x, y) < \nu$.

For each cross level we calculate the value of area of the "cold" - S_c and "hot" - S_h zones. For the highest ($\nu \sim \sigma$) cross levels there is a percolation over "cold" zones (see Fig.1a,b). Here we have $S_h/L^2 \ll 1$ and $S_c/L^2 \sim 1$. Under decreasing of the cross level the topology of light and dark zones begins to change, followed by increasing of S_h/L^2 and decreasing of S_c/L^2 . Finally, at the level $\frac{\Delta T}{T}(x, y) = 0$ there arises a point of the phase transition when any small fall of the cross level of the map leads to the percolation over "hot" zones. The critical point of the phase transition corresponds to the condition $\frac{S_h}{L^2} = \frac{S_c}{L^2} = \frac{1}{2}$, $\nu = 0$ (see Fig.2). This property is determined by the Gaussian character of the distribution of the initial fluctuations only.

In a separate realization of the Gaussian process the condition $\frac{S_h}{L^2} = \frac{S_c}{L^2} = \frac{1}{2}$ arises when ν is slightly different from zero because of the statistical character of the process. The possible difference of ν from zero can be evaluated as $\frac{\theta_c}{L}$ (see Fig.2).

Let us consider how the situation changes in the case of the existence of arbitrary noise connected with, for example, unresolved discrete sources. This problem will be described in detail in a separate paper. Here we describe the simplest model of such noise. Thus let the $\frac{\Delta T}{T}(10^\circ \times 10^\circ)$ map contain a primordial signal $\frac{\Delta T}{T}(\text{primordial})$ and a noise component $\frac{\Delta T}{T}(\text{noise})$, then:

$$\frac{\Delta T}{T} = \frac{\Delta T}{T}(\text{primordial}) + \frac{\Delta T}{T}(\text{noise}). \quad (6)$$

We introduce the new fluctuation field $\frac{\tilde{\Delta T}}{T}$ in such a way that $\langle \frac{\tilde{\Delta T}}{T}(x, y) \rangle_{L^2} = 0$. Then:

$$\frac{\tilde{\Delta T}}{T}(x, y) = \frac{\Delta T}{T}(\text{primordial}) + \frac{\Delta T}{T}(\text{noise}) - \langle \frac{\Delta T}{T}(\text{noise}) \rangle_{L^2}. \quad (7)$$

It is clear that for the field $\frac{\tilde{\Delta T}}{T}(x, y)$ the cross section $\frac{\tilde{\Delta T}}{T}(x, y) = 0$ corresponds to the following condition

$$\frac{\Delta T}{T}(\text{primordial}) = \langle \frac{\Delta T}{T}(\text{noise}) \rangle - \frac{\Delta T}{T}(\text{noise}). \quad (8)$$

Let us assume that the noise “signal” has the form of a narrow peak with height $h \sim \sigma$ and the halfwidth $\theta_n \sim \theta_A$. In this case the averaging (8) over the map leads to the following expression

$$\langle \left[\frac{\Delta T}{T}(\text{primordial}) \right]^2 \rangle_{L^2} = \langle \left[\frac{\Delta T}{T}(\text{noise}) \right]^2 \rangle - \langle \frac{\Delta T}{T}(\text{noise}) \rangle^2 = M^2. \quad (9)$$

As one can see from (9) the map of $\frac{\tilde{\Delta T}}{T}(x, y) = 0$ is approximately equivalent to the map $\langle \frac{\Delta T}{T}(\text{primordial}) \rangle = M$, and if $M > \frac{\epsilon_c}{L}$ we can say that additional noise is present in this map.

Our discussion demonstrates the sensitivity of the percolation method for the determination of the Gaussian character of the signal under consideration.

4. Filtering and the cluster analysis

In this section we investigate some special peculiarities of $\frac{\Delta T}{T}$ maps which are connected with Sakharov's oscillations of the spectrum. As it was shown in section 3 the distribution of $\frac{\Delta T}{T}$ anisotropy of CMB can be written as in Eq.(5). Using standard methods of filtering we introduce the smoothed field $\frac{\overline{\Delta T}}{T}(x, y)$;

$$\frac{\overline{\Delta T}}{T}(x, y) = \frac{1}{8\pi^2\delta^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} d\mu d\xi \frac{\Delta T}{T}(\mu, \xi) \exp\left(\frac{-[(x-\xi)^2 + (y-\mu)^2]}{2\delta^2}\right) \quad (10)$$

where δ is the half-width of the filter.

Formally the limits of integrating in Eq.(10) have to be $(-\frac{L}{2}, \frac{L}{2})$, but if $\delta \ll L$ we can use the infinite interval of integrating. Using (5) and (10) we have

$$\frac{\overline{\Delta T}}{T}(x, y) = \sum_{ij} a_{ij} C^{1/2}(m_{ij}) e^{-\frac{m_{ij}^2 \delta^2}{2}} \cos\left[\frac{2\pi(ix + jy)}{L} + \varphi_{ij}\right]. \quad (11)$$

Now we increase the halfwidth of the filter δ from $\delta \simeq \theta_A$ up to $\delta \gg \theta_A$ and plot the $\frac{\overline{\Delta T}}{T}$ maps for different parameters δ . On Fig.3 we plotted the dynamics of varying of the correlation function $C(\theta)$ for different values of δ . As one can see on Fig.3 the second maximum of $\psi(\theta, \delta) = \frac{C(\theta, \delta)}{C(0, \delta)}$ connected with Sakharov's oscillations, disappears when the filter δ becomes equal to $\delta = \theta_{ls}$, where $\theta_{ls} = \frac{1}{2} \frac{R}{\xi_n}$.

In order to investigate the influence of the specific shape of the spectrum of $\frac{\Delta T}{T}$ with Sakharov's oscillations on the topology of the distribution of $\frac{\Delta T}{T}$, we generated the filtered maps using Eq.(11) for models I and II. We recall that Sakharov's oscillations are absent in model I and they are present in model II. We use the same Gaussian process for the construction of maps for different kinds of spectra. When δ is much less than all characteristic scales of correlations, the peak of the correlation function corresponding to the oscillating spectrum is much greater than the correlation radius (see Fig.3). Practically, this peak does not influence the topology of $\frac{\Delta T}{T}$ which is determined by the behavior of the correlation function in the vicinity of the correlation radius (Bond and Efstathiou, 1987) (see Fig.1). When δ increases to the critical value $\delta = \theta_{ls}$ then the difference of the behaviors of the two correlation functions becomes perceptible on scales comparable with the correlation radius (see Fig.3). After filtering, the topological properties of models I and II become different.

In order to give the qualitative characteristics of the different topologies, we made two maps cross at identical levels ν (maps 1c,d on Fig.1). On these maps, solid curves are the line of level $\frac{\overline{\Delta T}}{T}(x, y) = \nu$, and the asterisks mark all the maxima which are higher than a threshold value of $\nu_{th} > 2\sigma$.

Let us define a cluster of length k as the totality of k maxima within the closed line of level. After performing a statistic analysis of these clusters for different values of ν we see that for the same values of ν and ν_{th} the number of big clusters is essentially greater for the model II than for the model I. We plotted the histograms interpreting these results on Fig.4.

Conclusions

We have shown that percolation and cluster analysis are useful for the determination of the Gaussian nature of $\frac{\Delta T}{T}$ fluctuations and for the detection large scale correlation. This method can be used for processing the real signal for the determination of amount of baryons in the Universe and its ionization history. Of course, one needs for that to analyse a signal together with noise as we mentioned and discussed in Section 3 and will be discussed in some details in a separate paper. The questions connected with the deformation of the topology of primordial $\frac{\Delta T}{T}$ distribution will be discussed in a separate paper also.

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Figure captions.

Fig.1 *a, b* -maps of CMB for oscillating and smoothed spectrum respectively $\theta_c \ll \theta_{ls}$; *c, d* - the same as *a, b* but after filtering $\sigma \sim \theta_{ls}$, * -maxima with height more than 2σ and maxima with height more than 1.5σ . Lines correspond to $\frac{\Delta T}{T}(x, y) = \nu = 1\delta$.

Fig.2 Dependence of black area on ν . Deviation from 0.5 at zero point due to the relation $\frac{\Delta T}{T} = 0.5 \pm \frac{\theta_c}{L}$ and in our case $\theta_c \sim 0.1L$

Fig.3 Correlation function for different filters δ : Dashed line for $\delta = 0$, dot line for $\delta = 0.2\theta_{ls}$, solid line for $\delta = 0.3\theta_{ls}$, dashed-dot line for $\delta = \theta_{ls}$

Fig.4 Number of cluster containing k maxima With height equal to 2δ for the different value of ν . Dot line for smoothed spectrum, solid line for oscillating one. N is the number of maxima, N_k the number of clusters with the length equal to k

Fig.1b

Fig.1c