HIGH-FREQUENCY AND PULSE RESPONSE OF COAXIAL TRANSMISSION CABLES
WITH CONDUCTOR, DIELECTRIC AND SEMICONDUCTOR LOSSES

H. Riege

GENEVA
1970
Propriété littéraire et scientifique réservée pour tous les pays du monde. Ce document ne peut être reproduit ou traduit en tout ou en partie sans l'autorisation écrite du Directeur général du CERN, titulaire du droit d'auteur. Dans les cas appropriés, et s'il s'agit d'utiliser le document à des fins non commerciales, cette autorisation sera volontiers accordée.

Le CERN ne revendique pas la propriété des inventions brevetables et dessins ou modèles susceptibles de dépôt qui pourraient être décrits dans le présent document; ceux-ci peuvent être librement utilisés par les instituts de recherche, les industriels et autres intéressés. Cependant, le CERN se réserve le droit de s'opposer à toute revendication qu'un usager pourrait faire de la propriété scientifique ou industrielle de toute invention et tout dessin ou modèle décrits dans le présent document.

© Copyright CERN, Genève, 1970

Literary and scientific copyrights reserved in all countries of the world. This report, or any part of it, may not be reprinted or translated without written permission of the copyright holder, the Director-General of CERN. However, permission will be freely granted for appropriate non-commercial use.

If any patentable invention or registrable design is described in the report, CERN makes no claim to property rights in it but offers it for the free use of research institutions, manufacturers and others. CERN, however, may oppose any attempt by a user to claim any proprietary or patent rights in such inventions or designs as may be described in the present document.
HIGH-FREQUENCY AND PULSE RESPONSE OF COAXIAL TRANSMISSION CABLES

WITH CONDUCTOR, DIELECTRIC AND SEMICONDUCTOR LOSSES

H. Riege

GENEVA
1970
ABSTRACT

The distortion of arbitrary pulses is computed for coaxial transmission cables with different kinds of losses. Starting from cables with normal skin effect losses, formulae for attenuation and rise-time of rectangular and nearly rectangular pulses are developed. The theory of combined conductor and dielectric losses is reviewed. Furthermore, the contribution of additional conducting or semiconducting layers to the losses in high-voltage pulse cables is investigated. By use of Maxwell's equations, the propagation constants in the frequency domain are calculated. For coaxial cables with semiconducting layers also an approximate solution in the time domain is presented, by which the distortion of arbitrary pulse shapes can be numerically computed.
CONTENTS

1. INTRODUCTION ................................................................. 1
2. TRANSMISSION LINE EQUATIONS ........................................ 1
3. HIGH-FREQUENCY APPROXIMATION FOR COAXIAL CABLES WITH LOSSES
   3.1 Coaxial cables with conductor losses only ..................... 2
   3.2 Coaxial cables with conductor and dielectric losses ............ 11
4. LOSSES BY CONDUCTING AND SEMICONDUCTING LAYERS
   IN COAXIAL HIGH-VOLTAGE PULSE CABLES ......................... 13
   4.1 Skin effect in conducting layers ................................ 14
   4.2 Losses in semiconducting layers ................................ 15
APPENDIX I: FREQUENCY ANALYSIS OF SOME TYPICAL PULSE SHAPES .... 23
APPENDIX II: SKIN EFFECT IN TWO CONDUCTING LAYERS
              WITH DIFFERENT CONDUCTIVITIES ......................... 27
APPENDIX III: CALCULATION OF THE COMPLEX PROPAGATION CONSTANT \gamma_s
              IN THE CASE OF AN IDEAL TRANSMISSION LINE WITH A
              SEMICONDUCTING LAYER ..................................... 29
REFERENCES ........................................................................... 33
1. INTRODUCTION

In the fast ejection systems of high-energy accelerators the transmission of pulses through coaxial cables plays an important role. One problem is the power transmission from the storage line pulse generators to the fast kicker magnets, which deflect the particle beam. Rise-time and attenuation of the transmitted pulses have to fulfill certain minimum conditions in order to guarantee a clean ejection. A second point of interest is the distortion of pick-up signals (beam diagnostics, monitoring), which need to be known if one wants to eject efficiently. It is, therefore, very useful to have available theoretical methods, which allow the computation of the pulse response for different types of transmission cables.

The exact calculation of the response of coaxial cables to an input signal with an arbitrary frequency spectrum is analytically a complex problem. Fortunately, however, the frequency content of signals practically applied is usually concentrated within a limited frequency range (see Appendix I). Then it is often possible to find rather simple solutions, which describe the cable behaviour with a good approximation.

2. TRANSMISSION LINE EQUATIONS

The purpose of this Section is to review briefly the well-known general transmission line theory [see, for example, Johnson], Matlick], and Guillemin,] and to give some definitions which are used in the following Sections. The time response of a transmission line to an input signal can be calculated from the transmission line equations for voltage \( V \) and current \( I \) as functions of time \( t \) and space coordinate \( z \):

\[
\frac{\partial V}{\partial z} = -\left(R + L \frac{\partial}{\partial t}\right) I
\]

\[
\frac{\partial I}{\partial z} = -\left(G + C \frac{\partial}{\partial t}\right) V .
\]

Here \( R \) (resistance/length), \( L \) (inductance/length), \( G \) (conductance/length) and \( C \) (capacity/length) are constant parameters.

For the spectral amplitudes of voltage and current, \( V_\omega \), \( I_\omega \), it follows from Eq. (1) that

\[
\frac{dV_\omega(z)}{dz} = -(R + j\omega L) \cdot I_\omega(z) = -Z_\omega(z) \cdot I_\omega(z)
\]

\[
\frac{dI_\omega(z)}{dz} = -(G + j\omega C) \cdot V_\omega(z) = -Y_\omega(z) \cdot V_\omega(z) .
\]

These equations are also valid if the series impedance/length, \( Z_\omega \), and the parallel admittance/length, \( Y_\omega \), are arbitrary functions of \( \omega \). The general solutions of Eq. (2) are given by

\[
V_\omega(z) = V_{\omega0} \ exp (-\gamma z) + V_{\omega+} \ exp (+\gamma z)
\]

\[
I_\omega(z) = \frac{1}{Z_\omega} \left[V_{\omega0} \ exp (-\gamma z) - V_{\omega+} \ exp (+\gamma z)\right] ,
\]
where $V^x$, $V^z$ are the voltage amplitudes of the waves moving in the $+z$ or $-z$ direction, respectively. The characteristic impedance $Z_0$ is

$$Z_0 = \sqrt{Z_s(\omega) / Y_p(\omega)}$$  \hspace{1cm} (4)

and $\gamma$ represents the complex propagation constant, which is given by

$$\gamma = \sqrt{Z_s(\omega) \cdot Y_p(\omega)}.$$  

If $Z_s$ and $Y_p$ are simple functions of frequency $\omega$, sometimes a solution can be found in the time domain with the aid of Laplace or Fourier transformation.

3. HIGH-FREQUENCY APPROXIMATION FOR COAXIAL CABLES WITH LOSSES

For coaxial cables with small losses, $Z_0$ can nearly be regarded as a real constant and $\gamma$ is a rather simple function of frequency in the frequency range between several hundreds of kHz and the cut-off frequency. From Eqs. (2) and (4) we see, if $R << \omega L$ and $G << \omega C$, that

$$Z_0 = \sqrt{\frac{1}{C}} \sqrt{\frac{1 + R/\omega L}{1 + G/\omega C}} \approx \sqrt{\frac{1}{C}}$$  \hspace{1cm} (6)

and

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)} \approx j\omega \sqrt{\frac{1}{C} \left[ 1 + \frac{1}{2} \left( \frac{R}{\omega L} + \frac{G}{\omega C} \right) \right]}$$  \hspace{1cm} (7)

or, if $\gamma = \alpha + j\beta$

$$\alpha \approx \frac{1}{2} \frac{R}{Z_0} + \frac{1}{2} \frac{G}{C} \cdot$$  \hspace{1cm} (8)

$$\beta \approx \omega \sqrt{1/C}.$$  \hspace{1cm} (9)

The first term of the attenuation constant $\alpha$ represents the losses in the cable conductors, whilst the second term is the loss contribution of the dielectric insulator. Below a few hundreds of MHz the dielectric losses are negligible. However, at very high frequencies (more than 1 GHz) they may predominate.

3.1 Coaxial cables with conductor losses only

Let us consider first a coaxial cable with an ideal dielectric insulator, but with conductors, which have very small, however finite, conductivities $\sigma_1$, $\sigma_a$ (see Fig. 1). The currents produced by the longitudinal field component in the conductors therefore lead to losses, which increase with frequency on account of skin effect. The series impedance $Z_s$ [Eq. (2)] in this case can be easily calculated with the aid of the Maxwell equations for cylindrical fields. Here the exact solution [which can be found in Johnson$^1$ and Fidecaro$^2$] is not presented, because already at fairly low frequencies there holds a very simple, but rather accurate, approximation for $Z_s$. One can write

$$Z_s = R_1 + j\omega (L_1 + L_a) = Z_{s1} + j\omega L_a,$$  \hspace{1cm} (10)
where $R_i$ is the resistance of the conductors per length, and $L_i$ and $L_a$ are the internal and external inductances per length.

![Diagram of coaxial cable configuration](image)

**Fig. 1** Configuration of a coaxial cable

The approximate expression for $Z_{si}$ obtained with the Maxwell equations is given by

$$Z_{si} = \frac{\sqrt{\mu_0}}{2\pi} \left( \frac{\mu_i}{\sigma_i} \frac{1}{R_i} + \frac{\mu_a}{\sigma_a} \frac{1}{R_a} \right), \quad (11)$$

with $R_i$, $R_a$, $\sigma_i$, $\sigma_a$ and $\mu_i$, $\mu_a$ being the radii, conductivities and permeabilities of the inner and outer conductor, respectively (see Fig. 1). From Eqs. (4), (5), (10) and (11) and with the well-known formulas for $L_a$ and $C$

$$L_a \approx \frac{\mu_0}{2\pi} \ln \frac{r_a}{r_i}, \quad C = \frac{2\pi \varepsilon}{\ln(r_a/r_i)},$$

where $\varepsilon = \text{dielectric constant of the insulator}$

$\mu_0 = \text{permeability of the vacuum} = \text{permeability of the dielectric}$

and furthermore with $\mu_i = \mu_a = \mu_0$, $\sigma_i = \sigma_a = \sigma_c$ we get the following expressions for $\gamma$, $\alpha$ and $\beta$

$$\gamma = A\sqrt{\beta} + B\beta \quad (12)$$

$$\alpha = \sqrt{\frac{\varepsilon}{2}} A \quad (13)$$

$$\beta = B\beta + \sqrt{\frac{\varepsilon}{2}} A \quad (14)$$

where

$$A = \frac{1}{2} \sqrt{\frac{\varepsilon}{\sigma_c}} \left( \frac{1}{R_a} + \frac{1}{R_i} \right) \frac{1}{\ln(r_a/r_i)} \quad (15)$$

$$B = \sqrt{\varepsilon \mu_0} \quad (16)$$

These formulae are valid as far as the skin depth $\delta = \sqrt{2/\mu_0 \sigma_c}$ is small with respect to the radial thickness of the inner and outer conductors.
For a matched or infinitely long coaxial cable it is now possible to calculate the distortion of arbitrary input signals \( V_1(t) \), the frequency content of which does not extend considerably beyond the limits of validity of Eqs. (13), (15) and (16). From Eqs. (3) and (12) one gets the following relation between the Fourier components \( V_\omega(k) \) and \( V_\omega(0) \) of the output voltage \( V_{\text{out}}(t,k) \) and the input voltage \( V_1(t) \)

\[
V_\omega(k) = V_\omega(0) e^{-A\sqrt{2}\omega k} e^{-B\omega k}.
\]  

(17)

If \( V_1(t) = 0 \) for \( t < 0 \), we find the output voltage \( V_{\text{out}}(t,k) \) appearing after the length \( \ell \) of cable by use of the convolution theorem as

\[
V_{\text{out}}(t,\ell) = \frac{A\ell}{2\sqrt{\pi}} \int_0^{t-B\ell} V_1(t-B\ell-x) \exp \left( -\frac{(A\ell)^2}{4x} \right) \frac{1}{x^{3/2}} \, dx \, H(t-B\ell),
\]  

(18)

where \( H(t) \) is the Heaviside function. If one neglects the pure cable delay, \( B_1 \), and restricts oneself to the distortion of the signal and if the "cable rise-time" \( \tau_0 = (A\ell)^2 / 2 \) is introduced, one can write Eq. (8) with \( y = t/\tau_0, z = x/\tau_0 \) also as

\[
V_{\text{out}}(y) = \frac{1}{2\sqrt{\pi}} \int_0^y V_1(y-z) \exp \left( -\frac{1}{4z} \right) \frac{dz}{z^{3/2}}.
\]  

(19)

The output voltage \( V_{\text{out}} \), according to Eqs. (18) and (19), can be numerically evaluated for any input signal by a FORTRAN program on the computer. However, in several cases one can easily find an exact solution.

A very simple response comes out for a delta pulse of weight 1, \( V_1(y) = \delta(y)/\tau_0 \), in the input of the cable. By definition of the delta function \( \delta(y) \) the delta response \( d_c(y) \) of a cable with conductor losses is expressed as

\[
V_{\text{out}}(y) = d_c(y) = \frac{1}{\tau_0} \frac{1}{2\sqrt{\pi}} \exp \left( -\frac{1}{4y} \right) \frac{1}{y^{3/2}} H(y).
\]  

(20)

The delta response \( d_c(y) \) is a good approximation for rectangular pulses of weight 1, if the pulse length is much smaller than \( \tau_0 \). In Fig. 2, \( d_c(y) \) is plotted for \( \tau_0 = 1 \). The response \( u_c(y) \) to a unit step input pulse, \( V_1(y) = H(y) \), follows directly from Eq. (19) as

\[
V_{\text{out}}(y) = u_c(y) = \operatorname{erfc} \left( \frac{1}{2\sqrt{y}} \right) H(y) = \left[ 1 - \operatorname{erf} \left( \frac{1}{2\sqrt{y}} \right) \right] H(y),
\]  

(21)

where

\[
\operatorname{erf}(x) = 1 - \operatorname{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-u^2} \, du
\]

is the well-known error function.

Figure 3 shows a plot of \( u_c(y) \). The response \( s_c(y) \) to a square pulse of length \( T \) can be expressed with \( a = T/\tau_0 \) as

\[
V_{\text{out}}(y) = s_c(y) = \operatorname{erfc} \left( \frac{1}{2\sqrt{y}} \right) H(y) - \operatorname{erfc} \left( \frac{1}{2\sqrt{y-a}} \right) H(y-a).
\]  

(22)
Fig. 2 Delta response of a cable with conductor losses only. $y = t/\tau_0 = \epsilon = \text{normalized time}; \ t = \text{time}; \ \tau_0 = (\Delta \xi)^2; \ \xi = \text{cable length}.$

A plot of square pulses with the same initial length $T$ distorted by different lengths of cable is given in Fig. 4.

The next example is a trapezoidal pulse (see Appendix I) with a flat top length $T$ and linear rise and fall $T'$. With $a = T/\tau_0$, $a' = T'/\tau_0$ the response $\text{tr}_c(y)$ to such a pulse can be written

$$V_{out}(y) = \text{tr}_c(y) = \frac{1}{a'} \left[ \int_0^y \text{erfc} \left( \frac{1}{2\sqrt{x}} \right) dx \ H(y) - \int_0^{y-a'} \text{erfc} \left( \frac{1}{2\sqrt{x}} \right) dx \ H(y-a') - \int_0^{y-a-a'} \text{erfc} \left( \frac{1}{2\sqrt{x}} \right) dx \ H(y-a-a') + \int_0^{y-a-2a'} \text{erfc} \left( \frac{1}{2\sqrt{x}} \right) dx \ H(y-a-2a') \right].$$

(23)

For $a' < a$ ($T' < T$) and $y > a' = T'/\tau_0$, an expansion of $\text{tr}_c(y)$ in a Taylor series leads to the approximation

$$\text{tr}_c(y) \approx s_c(y) - \frac{T'}{2} \left[ d_c(y) H(y) - d_c(y-a) H(y-a) \right].$$

(24)
Fig. 3 Unit step response of a cable with conductor losses only. 
y = t/τ₀ = normalized time; t = time; τ₀ = (Aℓ)²; ℓ = cable length.

Fig. 4 Distortion of a square pulse after different lengths of a cable 
with conductor losses only. T = pulse length; t = time; τ₀ = (Aℓ)²; 
ℓ = cable length.

The distortion of a trapezoidal pulse with a'/a = 0.2 can be seen in Fig. 5. Figure 6 
shows the response to a parabolic input pulse (see Appendix I), which is computed numeri-
cally from Eq. (19). For small rise and fall times, Eq. (24) is also a good approximation 
for the parabolic pulse response. Finally we regard the distortion of a pulse without 
flat top, namely V₁(y) = sin²(πy/a), the response of which is presented in Fig. 7.
For practical purposes it is often not necessary to know the total shape of the distorted signal. Generally it is sufficient to know the attenuation and the rise-time of the pulse. Since the pulse, after it has passed through a certain length of cable, does not anymore present a flat top, the most convenient definition of attenuation of square pulses with and without initial rise-time is \( \text{att} = 1 - \frac{V_{\text{max}}}{V_0} \), where \( V_0 \) is the flat top voltage of the input pulse and \( V_{\text{max}} \) the maximum voltage of the distorted pulse.

Fig. 5 Response of a cable with conductor losses only to a trapezoidal input pulse and a square pulse of flat top length \( T \). \( T' = \text{rise-time} \); \( t = \text{time} \); \( \tau_0 = (\Lambda \xi)^2 \); \( \xi = \text{cable length} \).

Fig. 6 Response of a cable with conductor losses only to a parabolic input pulse and a square pulse of flat top length \( T \). \( T' = \text{rise-time} \); \( t = \text{time} \); \( \tau_0 = (\Lambda \xi)^2 \); \( \xi = \text{cable length} \).
A rough idea of the rise-time of a distorted pulse is given by the "cable rise-time" 
\( \tau_0 = (At)^2 \). For a unit step \( \tau_0 \) indicates the time after which the output signal is reaching 48% of the input voltage \( V_0 \). With this value, however, one does not take into account the strong bending of the response function between \( V_0/2 \) and \( V_{\text{max}} \). Therefore it seems more convenient to define the rise-time \( t_{\text{RX}} \) as the time in which the output signal is increasing from 0 to a certain ratio \( x \) of the maximum voltage \( V_{\text{max}} \). For a square pulse of length \( T \) the maximum nearly coincides with the starting point of the second term of \( s_c(y) \) [see Eq. (22) and Fig. 4] provided \( T > \tau_0 \). Therefore it holds approximately with \( q = T/\tau_0 \) that

\[
V_{\text{max}} \approx V_0 \left[ 1 - \text{erf} \left( \frac{1}{2\sqrt{q}} \right) \right], \quad q > 1
\]

and

\[
\text{att} \approx \text{erf} \left( \frac{1}{2\sqrt{q}} \right), \quad q > 1.
\]

We can also calculate the relative rise-time for a rectangular pulse \( y_{\text{RX}} = t_{\text{RX}}/T \), from Eqs. (22) and (25).

\[
y_{\text{RX}} \approx \left[ (1 - x) \sqrt{xq} + x \right]^{-2},
\]

valid only for \( q > 1, qy_{\text{RX}} > 1, x > 0.5 \).

In Fig. 8 the attenuation of a square pulse is plotted as a function of \( q \). Figure 9 shows a graph of normalized rise-time \( y_{\text{RX}} = t_{\text{RX}}/T \) versus \( q \) for different ratios 
\( x = V(y_{\text{RX}})/V_{\text{max}} \). When using these graphs practically one only has to determine the
Fig. 8 Attenuation (att) of rectangular pulses in dependence of pulse length $T$ and of the "cable rise-time" $\tau_0$ for cables with conductor losses only. $\tau_0 = (A\lambda)^2$; $\lambda$ = cable length.
Fig. 9 Normalized rise-time $y_x$ (from 0 to $x \cdot v_{\text{max}}$) of distorted rectangular pulses for cables with conductor losses only. $y_x = t_{rx}/T$; $t_{rx}$ = rise-time from 0 to $x \cdot v_{\text{max}}$; $T$ = pulse length; $\tau_0 = (A\lambda)^2$; $\lambda$ = cable length.
attenuation factor $\Lambda$ of the cable, either theoretically [Eq. (15)] or from a measured attenuation curve $\alpha(\omega)$ [Eq. (13)].

Approximative formulae for attenuation and rise-time of distorted pulses with finite, but, compared with the pulse length $T$, small initial rise-times $T'$ can be derived from Eq. (24). With $q = T/\tau_0$ and $a'' = T'/T$ the attenuation is given by

$$\text{att} \approx \text{erf}\left(\frac{1}{2\sqrt{q}}\right)\left[1 - \frac{a''}{4} \exp\left(-\frac{1}{4q}\right)\right] \approx \frac{1}{\sqrt{\pi q}} \left(1 - \frac{a''}{4}\right),$$  \hspace{1cm} (28)

valid for $a'' \ll 1$, $q > 1$.

The expression for the relative rise-time $y'_{rx} = t'_{rx}/T$ from 0 to $x = V(y'_{rx})/V_{\text{max}}$ of such an output pulse can be written

$$y'_{rx} = \frac{t'_{rx}}{T} \approx y_{rx} + \frac{a''}{2} \left(1 + x \cdot y_{rx}^2/2\right) = y_{rx} + \frac{a''}{2},$$  \hspace{1cm} (29)

valid only for $a'' \ll 1$, $q > 1$, $q y'_{rx} > 1$.

Thus, in a first approximation and under the specified limitations, the output rise-time $t'_{rx}$ of a pulse with initial rise-time $T'$ is nearly the sum of the rise-time for an ideal square pulse plus half of the initial rise-time.

3.2 Coaxial cables with conductor and dielectric losses

At frequencies higher than 100 MHz the dielectric insulator of a coaxial cable is contributing more and more to the total losses. Hysteresis and relaxation of the polarization in the dielectric material are acting in the same way, as if a real conductivity were present. Losses of this kind are linked with the radial electric field component and are quantitatively described by the loss angle $\theta$ of the insulating material. Often $\theta$ is found to be nearly constant in the frequency range between several hundreds of kHz and 1 GHz. If we replace $G/\omega C$ in Eq. (7) by $\text{tg} \theta$ and $R$ by $Z_{si}$ [see Eqs. (10) and (11)], the complex propagation constant $\gamma$ can be written

$$\gamma \approx j \omega B + A \sqrt{\text{jo}} + \frac{1}{2} B \omega \text{tg} \theta,$$  \hspace{1cm} (30)

Physically this expression cannot be completely correct, since any transfer function should contain $j$ and $\omega$ only in the combination $j \omega$. Nevertheless, Eq. (30) is often used\textsuperscript{6,7} and seems to be a rather good approximation.

The distortion of arbitrary input signals on a cable with conductor and dielectric losses can again be computed according to the convolution theorem. But before doing this let us first deal with the pulse distortion by dielectric losses only. Disregarding the cable delay $B \kappa$, it holds for the ratio of output and input spectral amplitudes that

$$\frac{V_{o}(\omega)}{V_{i}(0)} = \exp\left(-B \omega \kappa\right),$$  \hspace{1cm} (31)

where $B = \frac{1}{2} B \text{tg} \theta$. For an arbitrary input pulse $V_{i}(t)$, Eq. (31) can be solved by Fourier transformation. The response to a delta pulse of weight 1, $V_{i}(t) = \delta(t)$ follows then
simply as
\[ V_{ou}\{t,k\} = \frac{2}{\pi} \frac{1}{b k} \frac{1}{1 + (t/bk)^2} , \]  
(32a)
or with \( \tau_d = b \lambda \), \( t/\tau_d = u \)
\[ V_{ou}(u) = d_d(u) = \frac{2}{\pi} \frac{1}{\tau_d} \frac{1}{1 + u^2} . \]  
(32b)

One can find the calculation and a plot of \( d_d(u) \) in Ref. 6. With Eq. (32b) the square pulse response \( s_d(u) \) can be calculated as
\[ s_d(u) = \frac{2}{\pi} \left[ \arctg(u) H[u] - \arctg(u - a) H[u - a] \right] , \]  
(33)
where \( a = T/\tau_d \) and \( T = \) pulse length (Fig. 10).

The response to an arbitrary input pulse in a cable with conductor and dielectric losses can be computed by the convolution
\[ V_{ou}(y) = \int_0^y V_i(x) \delta(y - x) \, dx , \]  
(34)
where \( \delta(y) \) is the response to a delta pulse of weight 1 of a cable with conductor and dielectric losses. \( \delta(y) \) can be computed by the convolution
\[ \delta(y) = \tau_0^2 \int_0^y d_d(y' \cdot x) \, dx , \]  
(35)
using Eqs. (20) and (32b). The parameter \( \gamma' = \tau_0/\tau_d \) has to be introduced, if the time variable \( t \) is normalized to \( \tau_0 \) as in Eqs. (34) and (35). The convolution theorem allows, however, different modes of calculation of the general pulse response. For example, the square pulse response \( s_d(y) \) for conductor and dielectric losses can be computed from a

Fig. 10 Distortion of a square pulse after different lengths of a cable with dielectric losses only. \( T = \) pulse length; \( \tau_d = b \lambda = \) "cable rise-time"; \( k = \) cable length.
convolution between the square pulse response \( s_d(u) \) for dielectric losses only and the delta response \( d_c(y) \) for conductor losses only:

\[
V_{\text{out}}(y) = s_{cd}(y) = \tau_0 \int_0^y s_d(y' \cdot x) \, d_c(y - x) \, dx .
\] (36)

In Fig. 11 a few distorted square pulses of constant initial length \( T \) are presented for different values of \( y' \).

**Fig. 11** Response of a cable with conductor and dielectric losses to a square input pulse for constant \( \tau_0 \) but different \( \tau_d \). \( T = \) pulse length; \( t = \) time; \( \tau_0 = (Aw)^2 = \) const.; \( \tau_d = \) const.; \( L = \) cable length; \( y' = \tau_0/\tau_d \).

4. **LOSES BY CONDUCTING AND SEMICONDUCTING LAYERS IN COAXIAL HIGH-VOLTAGE PULSE CABLES**

The simple theory of skin effect and dielectric losses enables us to predict the behaviour of normal commercial coaxial cables rather accurately. Unfortunately, it fails for many of the available high-voltage pulse cables, in which well-conducting (graphite or carbon loaded paper) or semiconducting (conducting polythene) layers are present between the dielectric insulator and the conductors. These layers are used to increase the lifetime of the cables, especially if they are constructed with braided conductors. Conducting layers (generally between outer conductor and dielectric) guarantee a better electrical contact between the single wires and a better mechanical contact with the dielectric insulator, which is also protected from the sharp edges of the metallic conductors. Release of high electric point stresses at the wires of the braided internal conductor and elimination of voids caused by thermal expansion of the insulator are the main reasons for using a semiconducting layer between internal conductor and insulator. On the other hand these layers may considerably increase the losses and deteriorate the pulse response. In the following sections some approximative estimates are made concerning the contribution of conducting or semiconducting layers to the total loss of coaxial high-voltage pulse cables.
4.1 Skin effect in conducting layers

The losses arising in an additional conducting layer are mainly due to skin effect as in the metal conductors. The total loss of a combination of a well-conducting (metal) and a worse-conducting medium (for instance graphite) will now be studied for a plane case as shown in Fig. 12. We assume having a longitudinal electrical field in z direction,

\[ E_z(y) \exp(j \omega t), \] the amplitude of which only depends on the y coordinate. Since \( \sigma_1 \ll \sigma_2 \), at low frequencies the main current is flowing in medium 2 with a skin depth of \( \delta_2 = \sqrt{2/\nu_{\sigma_2} \omega} \). Then the resistance per unit length (in z direction) of a strip of width \( \Delta x \) (in x direction) is given by

\[ R_2 = \frac{1}{\sigma_2 \delta_2 \Delta x} = \frac{\sqrt{\nu_{\sigma_2} \omega}}{2 \sigma_2} \frac{1}{\Delta x}. \] \( 37 \)

At frequencies high enough, where \( \delta_2 \) is going to zero, the current is forced to flow mainly in medium 1, where again skin effect is taking place resulting in the higher resistance

\[ R_1 = \frac{1}{\sigma_1 \delta_1 \Delta x} = \frac{\sqrt{\nu_{\sigma_1} \omega}}{2 \sigma_1} \frac{1}{\Delta x}. \] \( 38 \)

if \( \delta_1 \ll d \) = thickness of the conducting layer 1. The exact solution for the total series impedance \( Z_{S1} \) of medium 1 and 2 is given in Appendix II. The frequency interval where the real part of \( Z_{S1} \), the resistance \( R_2(\omega) \), is rising from \( R_2 \) to \( R_1 \), is determined by the parameters \( d, \sigma_1, \sigma_2 \). Figure 13 shows the exact frequency dependence of \( R_2(\omega) \) for some special values of these parameters and for \( \Delta x = 1m \).
Fig. 13 Real part, $R_\delta(\omega)$, of the series impedance $Z_{\delta1}$ calculated for skin effect in two conducting media (e.g. copper and graphite) according to Eq. (AII.8). $d$ = thickness of the bad conducting layer (medium 1); $\sigma_1$ = conductivity of the medium 1.

If we come back to a coaxial configuration (Fig. 1) and assume having a conducting layer of thickness $d \ll r_a - r_1$ between the dielectric insulator and the outer conductor, we find the complex propagation constant from Eq. (7) as

$$\gamma \approx j\omega \sqrt{L_a C + \frac{1}{2} \frac{Z_{\delta1}}{Z_0} + \frac{1}{2} GZ_0}. \quad (39)$$

With Eq. (8) and Eq. (AII.8) the attenuation constant $\alpha$ can be expressed for $\Delta x = 2\pi r_a$ as

$$\alpha \approx \frac{1}{4\pi Z_0} \left[ \sqrt{\frac{Z_{\delta1}}{2Z_0} \frac{1}{r_1} + 2\pi R_\delta(\omega)} + \frac{1}{2} GZ_0 \right]. \quad (40)$$

In the time domain the computation of the distortion of arbitrary pulse shapes by a cable with a propagation constant $\gamma$, as given by Eq. (40), can be done only in a rough approximation.

4.2 Losses in semiconducting layers

Whereas in conducting materials very small electric fields can produce considerable losses, in semiconductors only high fields will give rise to counting losses on account of their small conductivity. Therefore, in a semiconducting layer of a coaxial cable mainly the radial electric field is contributing to the total loss. For the coaxial structure as shown in Fig. 14, we can determine the complex propagation constant $\gamma_s$, if we suppose the
Fig. 14 Coaxial cable with a semiconducting layer between insulator and internal conductor.

conductors and the dielectric insulator to be ideal \((\sigma_c = \infty, \varepsilon_{\text{die}} = 0)\) and the semiconductor to have the same permittivity \(\varepsilon\) and permeability \(\mu\) as the insulator \((\mu = \mu_c = \mu_{\text{die}} = \mu_s = \mu_o)\).

In Appendix III, \(\gamma_s\) is calculated with

\[
\Delta = \frac{\ln(1 + d/r)}{\ln(r_a/r)} \approx \frac{d/r}{\ln(r_a/r)}
\]

as follows

\[
\gamma_s = j\omega \sqrt{\varepsilon\mu} \sqrt{\frac{1}{1 - (\Delta/\omega \varepsilon \sigma_s)^2}}.
\]  \hspace{1cm} (41)

For \(\Delta \ll 1\) we can simplify

\[
\gamma_s \approx j\omega \sqrt{\varepsilon\mu} \left(1 + \frac{\Delta}{2} \left(\frac{1}{1 + \frac{\omega \varepsilon \sigma_s}{2\omega \varepsilon \sigma_s}}\right)\right).
\]  \hspace{1cm} (42)

Separating this expression into real and imaginary parts it holds, with \(B = \sqrt{\varepsilon\mu}\), that

\[
\alpha = \frac{\Delta}{2} B \left(\frac{\sigma_s}{\varepsilon} \frac{1}{1 + \left(\frac{\omega \varepsilon \sigma_s}{2\omega \varepsilon \sigma_s}\right)^2}\right),
\]  \hspace{1cm} (43)

and

\[
\beta = \omega B \left(1 + \frac{\Delta}{2} \left(\frac{1}{1 + \frac{\omega \varepsilon \sigma_s}{2\omega \varepsilon \sigma_s}}\right)\right).
\]  \hspace{1cm} (44)

These formulae are equivalent to the circuit shown in Fig. 15.

Equations (41) to (44) were developed under the condition of infinite conductivity of the conductors. If we assume now having also losses in the metal there will not be very much change in the field distribution. The longitudinal electrical field components will not be exactly zero at the interfaces between insulator and metal and between semiconductor and metal. Nevertheless they are so small that one can linearly superpose them on the semiconductor losses. So we can write the total propagation constant

\[
\gamma \approx j\omega B \left(1 + \frac{\Delta}{2} \left(\frac{1}{1 + \frac{\omega \varepsilon \sigma_s}{2\omega \varepsilon \sigma_s}}\right)\right) + A \sqrt{j\omega}
\]  \hspace{1cm} (45)
and the attenuation constant

$$\alpha \approx \frac{\Delta}{2} B \frac{c_s}{\varepsilon} \frac{1}{1 + (c_s/\varepsilon)^2} + \frac{A}{\sqrt{2}} \sqrt{\omega} ,$$

(46)

where \(A\) is given again by Eq. (15). Equations (45) and (46) can easily be modified, if there is also (or only) a semiconducting layer in contact with the outer conductor. If also dielectric losses or losses by an additional conducting layer are present, the corresponding terms have to be added or to be modified.

The transfer function \(f_T\) of a matched or infinitely long coaxial cable with conductor and semiconducting losses is found from Eqs. (3) and (45) with \(j \omega\) replaced by \(p\), \(\omega_s = c_s/\varepsilon\), \(\tau_n = B\varepsilon\), \(\tau_p = \Delta B\varepsilon/2\) and \(\tau_0 = (\Delta k)^2\)

$$f_T(p) = e^{-\tau_p} e^{-\sqrt{4p}} e^{-p\omega_p[1 - \omega_p/(p + \omega_s)]}$$

(47)

The antitransformation of Eq. (47) into the time domain is possible by using the inverse Laplace transform \(J_0(\alpha \sqrt{t})\) of the image function \(\exp(-\alpha^2/4p)/p \) [see Ref. 5]. \(J_0\) is the normal Bessel function of order zero. Disregarding the normal cable delay \(\tau_n\) we find the transfer function \(f_T(t, \varepsilon)\) in the time domain as

$$f_T(t, \varepsilon) = \frac{1}{2\sqrt{\pi}} \int_0^\infty \frac{\exp[-\tau_p/4(t-x)]}{(t-x)^{1/2}} \exp[-\omega_p(x + \tau_p)] I_1(\omega_p \sqrt{4\tau_p x}) \frac{dx}{\sqrt{x}}$$

(48)

where \(I_1\) is the modified Bessel function of order 1. Then the response to an arbitrary input pulse \(V_1(t)\) can be computed from the convolution

$$V_{out}(t, \varepsilon) = f_T(t, \varepsilon) \cdot V_1(t) .$$

(49)
Fig. 16 Real and theoretically calculated attenuation constant $\alpha$ as function of frequency for BICC 40P3/20 $\Omega$ cable with a semi-conducting layer.
As an example, Eqs. (45) and (49) are applied to a BICC 40P3/20 Ω high-voltage pulse cable, which has a semiconducting layer of d = 0.7 mm nominal thickness between the insulator and the inner conductor. The characteristic parameters of this cable are given by $r_1 = 6.3$ mm, $r_2 = 10.6$ mm, $\sigma_C = 2 \times 10^{-2} (\text{l/mm})$, $\epsilon/\epsilon_0 = 2.3$. Since the conductivity $\sigma_S$ of the semiconducting layer was not exactly known, the frequency response (attenuation) of the cable was measured (see Fig. 16). A theoretical curve according to Eq. (45) was fitted to the experimental one by choosing the parameters $\omega_s = 5 \times 10^9$ (l/sec) or $\sigma_S = 10^{-1}$ (l/mm), respectively, $\Delta = 0.25$ and $\Lambda = 2.70 \times 10^{-7}$ (sec$^2$/m). The effective values of $\Delta$ and $\Lambda$ are higher than those calculated from the nominal values of the characteristic cable parameters. This may be mainly due to the braided conductors.

With the parameter values given above the unit step response

$$V_{\text{out}}(\tau, \lambda) = \int_{\tau_0}^{\tau} F(x, \lambda) \, dx$$

(50)

was computed for a cable length of 80 m ($\tau_0 = 0.46$ nsec, $\tau_D = 46$ rsec). Figure 17 shows a comparison between the computed response and the response experimentally measured after the same length of cable.

Figure 18 shows a few distorted pulses computed with different linear input rise-times. As for cables with conductor losses only [Eq. (29)], it holds approximately that the rise-time of the output pulse (e.g., from 10 to 90%) is the sum of the cable rise-time for a unit step pulse plus half of the initial rise-time of the input pulse.

From Figs. 17 and 18 it appears that the pulse rise-time is much more deteriorated by a cable with a semiconducting layer than by a cable with conductor losses only. Besides the normal delay $\tau_n$, a considerable additional delay is produced, which can be quantitatively

![Diagram](image)

Fig. 17 Real and theoretically calculated unit step response of an 80 m long BICC 40P3/20 Ω cable. Characteristic parameters according to Eq. (48): $\tau_0 = 0.46$ nsec, $\tau_D = 46$ nsec, $\omega_s = 5 \times 10^3$ l/sec.
described by $\tau_p = \Delta B\ell/2$ [for definition see Figs. 19 and 20; compare also with the approximation of Eq. (47) for small p]. The attenuation of a long pulse is mainly determined by the conductor losses (Fig. 20). The semiconductivity $\sigma_s$ influences only the rise-time of the pulse (Fig. 21). Figures 19, 20 and 21 present the unit-step response for constant cable length, but different sets of the parameters $\tau_p$, $\tau_0$ and $\omega_s$.

Fig. 10 Distortion of pulses with linear rise-time in a coaxial cable with conductor and semiconductor losses. Parameters: $\tau_0 = 0.46$ nsec, $\tau_p = 46$ nsec; $\omega_s = 5 \times 10^3$ 1/sec, representing a BICC 40P3/20 $\Omega$ cable.

Fig. 19 Unit step response of coaxial cables with semiconducting layers of different thickness. (Constant parameters: $\tau_0 = 0.23$ nsec, $\omega_s = 2.8 \times 10^3$ 1/sec.) The parameter $\tau_p = \Delta(d)B\ell/2$ quantitatively describes an additional delay, which, for small $\tau_0$, is counted from 0 to 50% of the flat top voltage.
Increasing $\Delta$, which means the thickness of the semiconducting layer, results in an increasing additional delay only. Higher skin effect attenuation ($\Lambda$) leads to more attenuation and higher rise-time, whereas decreasing the semiconductivity $\sigma_s$ increases the rise-time only.

**Fig. 20** Unit step response of cables with a semiconducting layer and different amounts of skin effect losses represented by the skin effect "cable rise-time" $\tau_0$. Constant parameters: $\tau_D = 47.5 \text{ nsec}$; $\omega_s = 2.8 \times 10^9 \text{ 1/sec}$.

**Fig. 21** Unit step response of cables with a semiconducting layer with different conductivities ($\omega_s = \sigma_s / \epsilon$). Constant parameters: $\tau_0 = 0.23 \text{ nsec}$, $\tau_D = 47.5 \text{ nsec}$.
Acknowledgements

I would like to thank Mr. B. Kuiper for encouraging this work and for his helpful criticism.

It is also a pleasure to make acknowledgement to Mr. A. Messina for several stimulating discussions and his help with experimental work. I am grateful to Mr. U. Berger for the preparation of the figures.
FREQUENCY ANALYSIS OF SOME TYPICAL PULSE SHAPES

The harmonic content of the transmitted pulse gives an indication for the applicability of a certain transmission line theory. Therefore, the frequency spectra $A(\omega)$ of some typical pulse shapes $V(t)$ computed by

$$A(\omega) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} V(t) \exp(-j\omega t) \, dt \quad (A1.1)$$

are presented.

a) Rectangular pulse

![Rectangular pulse](image1)

$$|A(\omega)| = \frac{V_0 T}{2\pi}$$

![Magnitude of A(\omega)](image2)

Fig. A.1 The rectangular pulse in (a) the time and (b) the frequency domain.

An ideal rectangular pulse of length $T$ and amplitude $V_0$ has the spectrum

$$A(\omega) = \frac{V_0}{\pi \omega} \sin \omega \frac{T}{2} \quad (A1.2)$$
The main frequencies are concentrated in the first few peaks of the frequency distribution (Fig. A.1). The frequency spectrum of a delta pulse, which is a special case of a square pulse with \( V_0 \rightarrow \infty \), \( T \rightarrow 0 \), extends from 0 to \( \pm \infty \) with the same amplitude, \( A(\omega) = V_0 T / 2\pi = \text{const.} \).

b) Trapezoidal pulse

![Trapezoidal pulse diagram](image)

The frequency spectrum of a trapezoidal pulse with a linear rise and fall of length \( T' \) can be expressed as

\[
A(\omega) = \frac{2V_0}{\pi\omega^2 T'} \sin \left( \frac{1 + T'}{2} \right) \sin \frac{\omega T'}{2}.
\]  

(Al.3)

Compared with the ideal square pulse the finite rise-time \( T' \) leads to a concentration of
the harmonic content in the first few peaks of the frequency distribution (Fig. A.2b). This is even more true for a parabolic pulse.

c) **Parabolic pulse**

A parabolic pulse (Fig. A.3) is an even better, and rather simple, mathematical representation of a real "rectangular" pulse.

![Diagram](image)

**Fig. A.3** Parabolic pulse in (a) the time and (b) the frequency domain.

Rise, fall and top of the parabolic pulse are given in the time domain by

\[
V = \frac{V_o}{2} \frac{(t + T/2 + T')^2}{(T'/2)^2} \quad \text{for} \quad \frac{T}{2} + T' \leq t \leq \frac{3T}{2} \\
V = \frac{V_o}{2} \left( 1 + 2 \frac{(t + T/2 + T'/2)^2}{T'/2} - \frac{(t + T/2 + T'/2)^2}{(T'/2)^2} \right) \quad \text{for} \quad \frac{T}{2} + T' \leq t \leq \frac{T}{2} \\
V = V_o \quad \text{for} \quad -\frac{T}{2} < t < +\frac{T}{2}
\]

AI.4)
Then the frequency spectrum can be written

$$A(\omega) = \frac{16 V_0}{\pi \omega^3 T^2} \sin^2 \left(\frac{\omega T}{4}\right) \sin \left[\frac{\omega(T + T')}{2}\right].$$

(A1.5)
SKIN EFFECT IN TWO CONDUCTING LAYERS WITH DIFFERENT CONDUCTIVITIES

The series impedance $Z_{si}$ of the arrangement shown in Fig. 12 can be calculated with Maxwell's equations

$$\text{rot } \vec{H} = -\mu \frac{\partial}{\partial t} \vec{E}$$  \hspace{1cm} (AII.1)
$$\text{rot } \vec{E} = \sigma \frac{\partial}{\partial t} \vec{H} + \frac{\partial}{\partial t} \vec{E}$$  \hspace{1cm} (AII.2)

Elimination of the magnetic field vector $\vec{H}$ delivers

$$\Delta \vec{E} = \mu \sigma \frac{\partial}{\partial t} \vec{E} + \mu \epsilon \frac{\partial^2}{\partial t^2} \vec{E}$$  \hspace{1cm} (AII.3)

We assume having only a longitudinal electrical field component $E_x(y) \exp(j\omega t)$ in the conducting layers. For $\sigma_1, \sigma_2 >> \omega$ it holds that

$$\frac{\partial^2 E_x(y)}{dy^2} = j\omega \mu \sigma E_x(y)$$  \hspace{1cm} (AII.4)

Observing the following boundary conditions with $\mu_1 = \mu_2 = \mu$

$$E_x^{(1)}(0) = E_0$$
$$E_x^{(1)}(d) = E_x^{(2)}(d)$$
$$H_x^{(1)}(d) = H_x^{(2)}(d)$$

and introducing the skin-depths

$$\delta_1 = \frac{2}{\mu \omega \sigma_1}, \quad \delta_2 = \frac{2}{\mu \omega \sigma_2}$$

one obtains the solution for the two media 1 and 2

$$E_x^{(1)}(y) = A_1 \exp\left[(1+j)y/\delta_1\right] + A_2 \exp\left[-(1+j)y/\delta_1\right]$$  \hspace{1cm} (AII.5)
$$E_x^{(2)}(y) = A_3 \exp\left[-(1+j)(y-d)/\delta_2\right]$$  \hspace{1cm} (AII.6)

The integration constants are given by

$$A_1 = \frac{\Delta \exp\left[-(1+j)y\right]}{\tau' \exp\left[(1+j)y\right] + \Delta' \exp\left[-(1+j)y\right]} E_0$$
$$A_2 = \frac{\tau' \exp\left[(1+j)y\right]}{\Delta \exp\left[-(1+j)y\right]} E_0$$
$$A_3 = \frac{2\delta_2}{\Delta} E_0$$
where $\Delta' = \delta_2 - \delta_1$, $\Sigma' = \delta_2 + \delta_1$, $\nu = d/\delta_1$ and $IN$ is the denominator of $A_1$. The total current $J$ flowing in the two conductors (1) and (2) is calculated by

$$J = \int_0^d \sigma_1 E_1'(y) \, dy + \int_0^d \sigma_2 E_2'(y) \, dy.$$  \hspace{2cm} \text{(AII.7)}

The total series impedance $Z_{si} = E_0/(J\Delta x)$ of a strip of width $\Delta x$ per unit length can be expressed as

$$Z_{si} = \frac{(1 + j)[\Sigma' \exp(j\nu) \exp(j\Delta') \exp(-\nu) \exp(-j\Delta')]}{\Delta x \{\delta_1 \sigma_1 \Sigma' \exp(j\nu) \exp(j\Delta') \exp(-\nu) \exp(-j\Delta') - 2\Delta_1 + 2\Delta_2\}}$$ \hspace{2cm} \text{(AII.8)}

$$= R_s(\omega) + j\omega L_1(\omega),$$

where $R_s(\omega)$ is the real part of $Z_{si}$ and $L_1(\omega)$ the internal series inductance per length.
APPENDIX III

CALCULATION OF THE COMPLEX PROPAGATION CONSTANT $\gamma_S$ IN THE CASE OF AN IDEAL TRANSMISSION LINE WITH A SEMICONDUCTING LAYER

We apply the Maxwell equations to the coaxial structure shown in Fig. 15. Conductors and insulator are assumed to be ideal. Only the longitudinal electric field component $E_z$, the radial component $E_r$ and the azimuthal magnetic field component $H_\phi$ are different from zero. All derivatives $\partial/\partial \phi$ in the Maxwell equations (AII.1) and (AII.2) are equal to zero.

We can write the field components:

$$E_z = E_z(r) \exp(-\gamma_S z) \exp(j\omega t)$$  \hspace{0.5cm} (AIII.1)

$$E_r = E_r(r) \exp(-\gamma_S z) \exp(j\omega t)$$  \hspace{0.5cm} (AIII.2)

$$H_\phi = H_\phi(r) \exp(-\gamma_S z) \exp(j\omega t),$$  \hspace{0.5cm} (AIII.3)

where $E_z$, $E_r$ and $H_\phi$ are the amplitudes of the waves spreading in $z$ direction with the propagation constant $\gamma_S$. For the component $E_z$ it follows from Eqs. (AII.3) and (AIII.1) that

$$\frac{d^2E_z(r)}{dr^2} + \frac{1}{r} \frac{dE_z(r)}{dr} + \lambda^2 E_z(r) = 0,$$  \hspace{0.5cm} (AIII.4)

where

$$\lambda^2 = k^2 + \gamma_S^2$$  \hspace{0.5cm} (AIII.5)

$$k^2 = \mu \varepsilon \omega^2 - j\omega \sigma.$$  \hspace{0.5cm} (AIII.6)

The solutions $E_z^{(1)}$, $E_z^{(2)}$ in the two media 1 (semiconductor) and 2 (insulator) can be written with the Bessel function $J_\lambda$ and the Neumann function $N_\lambda$ as

$$E_z^{(1)}(r) = A_1 J_\lambda(j\lambda r) + B_1 N_\lambda(j\lambda r)$$  \hspace{0.5cm} (AIII.7)

$$E_z^{(2)}(r) = A_2 J_\lambda(j\lambda r) + B_2 N_\lambda(j\lambda r),$$  \hspace{0.5cm} (AIII.8)

where $\lambda_1$ and $\lambda_2$ are valid for medium 1 and medium 2 respectively.

Using the boundary conditions

$$E_z^{(1)}(r_1) = E_z^{(2)}(r_a) = 0$$

$$E_z^{(1)}(r_1 + d) = E_z^{(2)}(r_1 + d),$$

the integration constants $B_1, A_2$ and $B_2$ can be expressed by $A_1$ as

$$B_1 = -A_1 \frac{J_\lambda(j\lambda r_1)}{N_\lambda(j\lambda r_1)}$$  \hspace{0.5cm} (AIII.9)
\[
A_2 = \frac{N_0(\beta_2 r_a) \{N_0(\beta_1 r_1) J_0(\beta_1 (r_1 + d)) - J_0(\beta_1 r_1) N_0(\beta_2 (r_1 + d))\}}{N_0(\beta_1 r_1) \{N_0(\beta_2 r_a) J_0(\beta_2 (r_1 + d)) - J_0(\beta_2 r_a) N_0(\beta_1 (r_1 + d))\}} A_1 \tag{AIII.10}
\]
\[
B_2 = \frac{J_0(\beta_2 r_a) \{N_0(\beta_1 r_1) J_0(\beta_1 (r_1 + d)) - J_0(\beta_1 r_1) N_0(\beta_2 (r_1 + d))\}}{N_0(\beta_1 r_1) \{N_0(\beta_2 r_a) J_0(\beta_2 (r_1 + d)) - J_0(\beta_2 r_a) N_0(\beta_1 (r_1 + d))\}} A_1 \tag{AIII.11}
\]

The boundary condition for the magnetic field component \(H_2\) at the interface of insulator and semiconductor,
\[
H_2^{(1)}(r_1 + d) = H_2^{(2)}(r_1 + d) , \tag{AIII.12}
\]
gives the equation for the determination of \(\gamma_s\). With the aid of the Maxwell equations (AII.1) and (AII.2), one can verify
\[
H_2(r) = -\frac{k^2}{\omega\mu^2} \frac{d E_2(r)}{dr} \tag{AIII.13}
\]
and together with Eqs. (AIII.7) to (AIII.12) the following equation for \(\gamma_s\) is obtained:
\[
\frac{k^2}{k^2} \sqrt{k^2 + \gamma_s^2} = \frac{J_0(\beta_2 r_a) \{N_0(\beta_2 (r_1 + d)) - N_0(\beta_2 r_a) J_0(\beta_2 (r_1 + d))\}}{J_0(\beta_1 r_1) \{N_0(\beta_1 (r_1 + d)) - N_0(\beta_1 r_1) J_0(\beta_1 (r_1 + d))\}} \times
\]
\[
\frac{N_0(\beta_1 r_1) J_0(\beta_1 (r_1 + d)) - J_0(\beta_1 r_1) N_0(\beta_2 (r_1 + d))}{N_0(\beta_2 r_a) J_0(\beta_2 (r_1 + d)) - J_0(\beta_2 r_a) N_0(\beta_1 (r_1 + d))} . \tag{AIII.14}
\]

\(J_1\) is the Bessel function and \(N_1\) the Neumann function of order 1. Generally it is difficult to solve this complex transcendental equation, which contains also the higher modes of wave propagation. For our case we try to find an approximate solution. For an ideal dielectric insulator (2) it holds that
\[
k^2 = \mu \varepsilon_2 \tag{AIII.15}
\]
and for the semiconducting layer (1)
\[
k^2 = \mu \varepsilon_1 - \omega \mu \sigma_s . \tag{AIII.16}
\]

Both media are assumed to have the same \(\varepsilon\) and \(\mu\). We now consider only the principal propagation mode at frequencies not higher than a few hundred MHz. For semiconductivities \(\sigma_s \) of less than \(10^{-5}\) (1/\(\mu m\)) and usual cable dimensions, we can use the following approximations of the Bessel functions for small arguments \(|z| \ll 1\) (see Ref. 8)
\[
J_0(z) \approx \frac{z^0}{\sqrt{12}^0} , \quad N_0(z) \approx -\frac{(v-1)!}{\pi} \left(\frac{v}{2}\right)^v , \quad N_0(z) \approx -\frac{2}{\pi} \ln \frac{2}{\gamma z} ,
\]
where \(\gamma = 1.78107\). Then Eq. (AIII.14) is reduced to
\[
\frac{k^2}{k^2} \sqrt{k^2 + \gamma_s^2} \approx \frac{\lambda_1}{\lambda_2} \frac{\Delta}{\Delta - 1} , \tag{AIII.17}
\]
with $\Delta = \ln \left(1 + \frac{d}{r_i}\right)/\ln \left(r_a/r_i\right)$. Solving for $\gamma_s$ and observing Eqs. (AIII.15) and (AIII.16) results finally in

$$\gamma_s = j\omega \sqrt{\frac{\mu\epsilon}{1 - \left[\Delta/(1 + j\omega\epsilon/\sigma_s)\right]}}$$  (AIII.18)

or for $\Delta \ll 1$

$$\gamma_s \approx j\omega \sqrt{\mu\epsilon \left(1 + \frac{\Delta}{2} \frac{1}{1 + j\omega\epsilon/\sigma_s}\right)}$$  (AIII.19)
REFERENCES

5) G. Doetsch, Anleitung zum praktischen Gebrauch der Laplace-transformation (Oldenburg, München, 1961).