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The Compressibility of Finite Nuclei*

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Abstract

A simple formula without adjustable parameters is derived for the ratio of the compressibility of a finite charged nucleus to the compressibility of standard nuclear matter.

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1. Introduction

The effective compressibility coefficient $K(A,Z)$ of a finite nucleus of mass number A and atomic number Z can be considerably lower than the compressibility K_0 of standard, uncharged nuclear matter. The reduction is typically by some 35%. Existing semi-empirical formulae for $K(A,Z)/K_0$, based on expansions in powers of $A^{-1/3}$ and containing corrections for the Coulomb energy, suffer from poor convergence and the presence of several poorly determined parameters. (See, in particular, Refs. 1–3 and references therein.) In what follows, we shall derive a formula for $K(A,Z)/K_0$ which is both simple and free of adjustable parameters.

2. Compressibility and Binding Energy

The physical input in the derivation is the observation that, apart from the Coulomb energy, the ratio K/K_0 should be approximately proportional to E/E_0 , where E is the binding energy per particle of a finite nucleus and E_0 the binding energy per particle of nuclear matter. The reason for this expectation is that if the density distribution of a finite nucleus is imagined stretched out radially by a scaling factor $\lambda = R/R_0$, say, the binding energy $E(\lambda)$ will tend to vanish when the typical inter-nucleon distance has exceeded by a few times the range of nuclear forces. Since this implies a characteristic scaling λ_c approximately independent of A , the appearance of a plot of $E(\lambda)$ vs. λ (for $\lambda \geq 1$) will be a series of (inverted) bell-shaped curves of *fixed range* but different initial depths, the depths being proportional to $E(1)$, the equilibrium binding energies of the nuclei in question. Insofar as the bell-shaped curves can be assumed to have the same *intrinsic shape* for different nuclei, the curvatures at $\lambda = 1$, which are the stiffnesses K against scaling, will be proportional to the binding energies $E(1)$. It follows then that, for uncharged finite nuclei, we might expect

$$\frac{K(A,Z)}{K_0} \approx \frac{E_n(A,Z)}{E_0} \quad , \quad (1)$$

or

$$K(A,Z) \approx \frac{E_n(A,Z)}{-a_1} K_0, \quad (2)$$

where we have written $E_n(A,Z)$ for the nuclear binding energy per particle in the absence of the Coulomb energy, and a_1 for the magnitude of the binding energy per particle in standard nuclear matter: $a_1 \approx 16$ MeV.

This expectation is confirmed by the results of Ref. 4 where, for a series of model nuclei calculated according to the Thomas-Fermi method (Ref. 5), the binding energy per particle and the effective stiffness K_{eff} against radial scaling were determined (in the case of $N = Z$ and no Coulomb energy). In the range $A > 8$ the results could be accurately represented by

$$\begin{aligned} -E &= 16.527 - 20.268 x - 8.290 x^2 + 15.951 x^3 \\ &= 16.527(1 - 1.2264 x - 0.5016 x^2 + 0.9651 x^3) \end{aligned} \quad (3)$$

$$\begin{aligned} K_{\text{eff}} &= 301.27 - 406.55 x - 214.02 x^2 + 473.12 x^3 \\ &= 301.27(1 - 1.3495 x - 0.7104 x^2 + 1.5704 x^3), \end{aligned} \quad (4)$$

where $x = A^{-1/3}$.

The ratio K_{eff}/E , listed in Table I, is remarkably constant (to within 10%) in a range where K_{eff} changes by a factor of 3.

Table I. Compressibilities and bindings of finite Thomas-Fermi nuclei (Ref. 4).

$A^{-1/3} = x$	0	.1	.2	.3	.4	.5
K_{eff}	301.27	258.95	215.18	172.82	134.69	103.63
$-E$	16.527	14.433	12.269	10.131	8.114	6.314
$-K_{\text{eff}}/E$	18.23	17.94	17.54	17.06	16.60	16.41

(Note: The Thomas-Fermi model used in Ref. 4 had not been optimally fitted to nuclear data and is used here for illustration only. See Section 6.)

3. Effect of the Coulomb Energy

In the presence of electrostatic forces the argument for the proportionality between $K(A,Z)$ and $E(A,Z)$ is no longer valid because the Coulomb energy, being produced by long-range forces, does not vanish at some finite characteristic scaling λ_c . Even so, the modification of eq. (2) caused by the Coulomb energy is readily derived in the following way. (Compare Ref. 3).

Let us express our assumption of a universal scaling dependence of the binding energy of finite nuclei by writing

$$\frac{E_n(A,Z;\Omega)}{E_n(A,Z)} = \frac{E_o(\Omega)}{E_o} \quad , \quad (5)$$

where E_n refers to the nuclear part of the binding energy per particle of a finite nucleus and E_o to the binding energy per particle of standard nuclear matter. In what follows we shall find it convenient to use $\Omega \equiv 1/\lambda$ as our scaling variable. The denominators in eq. (5) refer to $\Omega = 1$.

Let us write

$$E_o(\Omega) = a_1 f(\Omega) \quad , \quad (6)$$

where $f(\Omega)$ is a dimensionless function with the property that

$$f(1) = -1 \quad , \quad \text{i.e., } E_o = -a_1 \quad (7)$$

$$f'(1) = 0 \quad (\text{equilibrium condition}) \quad (8)$$

$$\left. \frac{d^2 E_o(\Omega)}{d\Omega^2} \right|_{\Omega=1} = a_1 f''(1) = K_o \quad . \quad (9)$$

The scaling dependence of the binding energy per particle of an uncharged finite nucleus may now be written as

$$E_n(A,Z;\Omega) = -E_n f(\Omega) \quad , \quad (10)$$

where E_n , from now on, stands for $E_n(A,Z)$. Let us add to this the Coulomb energy per particle, which we write in the form $C\Omega$, where C is the usual Coulomb energy per particle before scaling (i.e., for $\Omega = 1$), and the proportionality to Ω (i.e., to $1/\lambda$) follows from

the definition of the electrostatic energy as an integral over $1/r_{12}$, the reciprocal of the distance between charge elements. For example, in the case of a uniform spherical charge distribution of radius R_0 we have

$$C = \frac{3}{5} \frac{(Ze)^2}{R_0 A} = \frac{3}{5} \frac{e^2}{r_0} \frac{Z^2}{A^{4/3}} , \quad (11)$$

where e is the elementary charge and $R_0 = r_0 A^{1/3}$. The total energy per particle is thus

$$E(A, Z; \Omega) = -E_n f(\Omega) + C\Omega . \quad (12)$$

Write $\Omega = 1 + \varepsilon$ and expand about $\Omega = 1$:

$$E(A, Z; \varepsilon) = -E_n \left(-1 + \frac{1}{2} f'' \varepsilon^2 + \frac{1}{6} f''' \varepsilon^3 + \dots \right) + C(1 + \varepsilon) , \quad (13)$$

where the derivatives f'' , f''' are evaluated at $\Omega = 1$. The equilibrium value of ε is given by

$$-E_n \left(f'' \varepsilon + \frac{1}{2} f''' \varepsilon^2 \right) + C = 0 , \quad (14)$$

which gives

$$\varepsilon_{eq} = C / E_n f'' \quad (15)$$

to lowest order in C .

The second derivative of $E(A, Z; \varepsilon)$ at this value of ε is the compressibility $K(A, Z)$:

$$\begin{aligned} K(A, Z) &= -E_n f'' - C f''' / f'' \\ &= -\frac{E_n}{a_1} K_0 - C f''' / f'' , \end{aligned} \quad (16)$$

to lowest order in C . (We have checked that going to the next order produces a negligible modification).

The first part of eq. (16) gives the previously derived correction to K_0 , and the second part the correction due to the Coulomb energy. The quantity f''' / f'' is a dimensionless number, characteristic of the functional form of $f(\Omega)$, which we shall now estimate.

4. The Value of f''' / f''

To estimate f''' / f'' we shall use for $f(\Omega)$ the family of functions which results from the Thomas-Fermi treatment of nuclei, as introduced by Seyler and Blanchard (Ref. 6) and generalized in Refs. 5,7–11. In both the original and the generalized models, $f(\Omega)$ is a quintic of the form

$$f(\Omega) = a\Omega^2 - b\Omega^3 + c\Omega^5, \quad (17)$$

where a, b, c are related to the adjustable parameters of the effective nucleon-nucleon interaction in the Thomas-Fermi model. Depending on the values of those parameters, the function $f(\Omega)$ can describe situations ranging from the incompressible liquid drop, to super-soft nuclear matter about to lose the saturation property (see Ref. 5, Fig. 1). For our purposes, however, we need not go into a discussion of the Thomas-Fermi interaction parameters in eq. (17): all that matters is the general functional form of that equation.

Thus, using eqs. (7–8), we have

$$f(1) = a - b + c = -1 \quad (18)$$

$$f'(1) = 2a - 3b + 5c = 0 \quad (19)$$

$$f''(1) = 2a - 6b + 20c = K_0 / a_1, \quad (20)$$

and, in addition

$$f'''(1) = -6b + 60c. \quad (21)$$

Eliminating a, b, c between the above four equations we find

$$f''' / f'' = 7 - 30 \frac{a_1}{K_0}, \quad (22)$$

a relation that makes no explicit reference to the values of a, b and c ! Using the illustrative values $a_1 \approx 16$ MeV, $K_0 \approx 240$ MeV, we find $f''' / f'' \approx 7 - 2 = 5$, with a relatively weak dependence on the precise value of K_0 . (Compare the qualitatively similar result in Ref. 3.) The final formula relating $K(A, Z)$ to the compressibility of standard nuclear matter reads

$$K(A, Z) = \frac{E_n}{-a_1} K_0 - \left(7 - 30 \frac{a_1}{K_0} \right) C . \quad (23)$$

Note that the effect on K of a change in E_n is given by

$$\frac{\Delta K}{\Delta E_n} = \frac{K_0}{-a_1} \approx -15 \text{ MeV / MeV} , \quad (24)$$

and the effect of a change in C is given by

$$\frac{\Delta K}{\Delta C} = - \left(7 - 30 \frac{a_1}{K_0} \right) \approx -5 \text{ MeV / MeV} . \quad (25)$$

Writing $E(A, Z) = E_n + C =$ total binding energy per particle, we can rewrite eq. (23) as

$$K(A, Z) = \frac{E(A, Z)}{-a_1} K_0 + \left(30 \frac{a_1}{K_0} + \frac{K_0}{a_1} - 7 \right) C . \quad (26)$$

5. A Numerical Test

The triangles, circles and squares in Fig. 1 show the effective stiffness against scaling, K_{eff} , for finite nuclei calculated according to the Thomas-Fermi model of Ref. 5, as reported in Ref. 4. The three cases shown are for uncharged $N = Z$ nuclei, and for nuclei along the valley of stability given by $N - Z = 0.4 A^2 / (A + 200)$, without and with the Coulomb energy. The three curves in Fig. 1 illustrate eq. (23), written in the form

$$K(A, Z) = \left(1 - 1.2264 x - 0.5016 x^2 + 0.9651 x^3 - \frac{1.8984 I^2}{1 + 1.6902 x} \right) 301.27 - 4.0939 Z^2 / A^{4/3} \text{ MeV} , \quad (27)$$

where $I = (N - Z)/A$ and $x = A^{-1/3}$. The first line in eq. (27) represents the reduction of K_0 due to the reduced nuclear binding energy (cf. eq. (3)), the last term in the brackets representing the effect of the symmetry energy (with a surface symmetry energy correction in the denominator, Ref. 10). The second line is the Coulomb energy correction, eq. (11), with $r_0 = 1.13$ fm (the parameters appropriate to the Thomas-Fermi model of Ref. 5 were used consistently). Equation (27) is seen to give a fair representation of the effective compressibilities calculated by scaling individual Thomas-Fermi nuclei. Note

once again that the Thomas-Fermi model on which Fig. 1 is based has not been fitted optimally to nuclear data. In the next section we use an up to date version of the Thomas-Fermi model.

6. Compressibilities of Finite Nuclei

In applying eqs. (23) or (26) in practice, one may use measured values of the binding energies per particle or suitable liquid drop, Droplet Model or other expressions for $E(A,Z)$ and C . The Thomas-Fermi model of Ref. 5 has now been fitted to a large number of nuclear data (Refs. 9–11). The resulting properties of nuclear matter are described by $r_0 = 1.14$ fm, $a_1 = 16.04$ MeV, $K_0 = 234$ MeV. The binding energy per particle of finite nuclei is given, to an approximation sufficient for our purposes, by

$$E(A,Z) \approx -16.04 + 18.5 A^{-1/3} + 9.1 A^{-2/3} - 11.6 A^{-1} + \frac{32 I^2}{1 + 1.87 A^{-1/3}} + \frac{3}{5} \frac{e^2}{1.14} \frac{Z^2}{A^{4/3}} \text{ MeV} . \quad (28)$$

This leads to

$$K(A,Z) \approx \left(1 - 1.153 x - 0.567 x^2 + 0.723 x^3 - \frac{1.995 I^2}{1 + 1.87 x} \right) 234 - 3.75 Z^2 / A^{4/3} \text{ MeV} . \quad (29)$$

Table II lists the resulting estimated compressibilities of nuclei with $N = Z$ and no Coulomb energy (first column), and of nuclei along the valley of stability without and with Coulomb energy (second and third columns).

Finally, we would like to reiterate the need for caution when interpreting the measured giant monopole frequency ω in terms of a scaling mode of oscillation. A scaling mode is only an approximation to the true normal mode, one that imposes a constraint on the vibration. As such it is bound to give for ω a value higher than the true frequency (Rayleigh's principle). How serious the resulting error might be can be illustrated by the example of a compressible liquid drop model. In the notation of Ref. 12, eq. (6A-50), the

true normal mode (for which the bulk density does *not* remain uniform, contrary to the scaling assumption) has a frequency

$$\omega = \frac{\pi}{R_0} \sqrt{\frac{b_{\text{comp}}}{M}} , \quad (30)$$

where $b_{\text{comp}} = K/9$, M is the nucleon mass (the mass per particle) and R_0 the radius of the drop. The constrained scaling mode has a higher frequency

$$\omega_{\text{sc}} = \sqrt{\frac{K}{M\langle r^2 \rangle}} = \sqrt{\frac{5}{3}} \frac{1}{R_0} \sqrt{\frac{K}{M}} . \quad (31)$$

The ratio $\omega_{\text{sc}}/\omega = \sqrt{15}/\pi = 1.233$ illustrates the very serious inaccuracy of the scaling assumption, at least in this simple model. (See Ref. 3 for a less pessimistic assessment of the inaccuracy of the scaling method).

Table II. Compressibilities of nuclei with $N = Z$, and along the valley of β -stability, without and with Coulomb energy. Based on the Thomas-Fermi model of Refs. 9–11.

A	N = Z	β	$\beta + \text{Coul}$
10	97.1	97.0	92.8
20	125.1	134.7	118.3
40	148.0	146.7	137.1
60	159.2	156.6	144.7
80	166.4	162.1	148.4
100	171.4	165.5	150.3
120	175.3	167.6	151.2
140	178.3	169.0	151.4
160	180.9	169.9	151.2
180	183.0	170.4	150.8
200	184.8	170.7	150.2
220	186.4	170.8	149.4
140	187.9	170.8	148.6
260	189.1	170.7	147.8

7. Summary

Using the assumption of a universal scaling dependence of the nuclear part of the binding energy of nuclei, as given by eq. (5), we derived a simple formula, eq. (23), for the stiffness against scaling of finite nuclei. The bottom line is, roughly speaking, that for each MeV of decreased non-Coulomb binding per particle of a finite nucleus the compressibility coefficient decreases by about 15 MeV, whereas for each MeV per particle of unbinding produced by the Coulomb energy the compressibility coefficient decreases by about 5 MeV.

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Figure Caption

Figure 1. The effective compressibilities of nuclei with $N = Z$, and of nuclei along the β -stability valley without and with Coulomb energy. The symbols are from Ref. 4 and the curves correspond to eq. (23) used with the illustrative parameters of that reference.

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Figure 1