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PROPERTIES OF A MULTICHANNEL RESONANT-LEVEL MODEL

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Abstract

By use of Tomonaga-Luttinger bosonization we find that a multichannel resonant-level model can be transformed into a single-channel Kondo model with antiferro- or ferromagnetic interaction depending on the parameter γ given in terms of the phase shifts. The specific heat and the dynamics of the local impurity have been studied which show a Fermi-liquid vs non-Fermi-liquid transition at $\gamma_c = 1$.

A phenomenological marginal-Fermi-liquid (FL) model, proposed by Varma *et al.*,¹ describes successfully the normal state of the copper-oxide based high-Tc superconductors. Thus, it is of great interest to study those models whose properties can be obtained either exactly or by well-controlled approximations and which exhibit a transition from FL to non-FL behavior as some parameters of the model are varied.

At the present stage, there are no exact results for lattice models (in more than one dimension) showing such a transition. Therefore, it is desirable to study the related impurity models and a better understanding of these models can provide useful information for the lattice problem. It is known that there are some impurity models exhibiting FL or non-FL behavior, respectively. For example, the single-channel resonant-level or Anderson model show a local FL behavior,^{2,3} while the x-ray edge⁴ or the multichannel Kondo problem^{5,6} display a local non-FL behavior. Recently, the FL-non-FL transitions have been explored in the generalized Anderson model^{7,8} and its spinless version^{9,10}: multichannel resonant-level (MCRL) model. It has been demonstrated that the partition function of the MCRL model is the same as that of the single-channel (SC) Kondo model,¹⁰ this result implicates that the essential properties of the MCRL model might be obtained from a much simpler model, e.g., the SC Kondo, model.

In this paper, we present an exact derivation, based on the bosonization technique, showing that the MCRL model can be transformed into the SC Kondo model at the operator level. Thus from the known results of the SC Kondo model and other related models, such as the dissipative two-state model,^{11,12} we can calculate the properties of the MCRL model, e.g., the specific heat and the dynamics of the local impurity.

The MCRL model is given by^{9,10}

$$H = \sum_{k,l} \epsilon_k C_{k,l}^\dagger C_{k,l} + \frac{j}{\sqrt{L}} \sum_k (C_{k,o}^\dagger d + d^\dagger C_{k,o})$$

$$+\frac{1}{L} \sum_{k,k',l} V_l C_{k,l}^\dagger C_{k',l} (d^\dagger d - \frac{1}{2}), \quad (1)$$

where $C_{k,l}$ ($C_{k,l}^\dagger$) are spinless conduction electrons with orbital momentum or channel index l , and d ($d^\dagger$) operators correspond to a localized orbital. As usual the transverse interaction involves only one channel $l = 0$. The chemical potential is set to zero and the model has particle-hole symmetry.

The conduction electrons in each channel can be reduced to a one-dimensional problem with only left-moving electrons if only s-wave scattering is concerned. Then we can represent the left-moving electrons in terms of the boson fields ϕ_l and their conjugate momenta Π_l .^{13,14} The bosonic form of the Hamiltonian (1) is

$$\begin{aligned} H_b = & (4\pi\rho)^{-1} \sum_l \int [\Pi_l^2(x) + (\nabla\phi_l)^2] dx \\ & + \frac{j}{\sqrt{2\pi a}} [e^{i\sqrt{4\pi}\Phi_{0,L}(0)} d + d^\dagger e^{-i\sqrt{4\pi}\Phi_{0,L}(0)}] \\ & + \frac{1}{\sqrt{\pi}} \sum_l V_l \frac{\partial}{\partial x} \Phi_{l,L}(0) (d^\dagger d - \frac{1}{2}), \end{aligned} \quad (2)$$

where $\rho = (2\pi\hbar v_F)^{-1}$ is the density of states at the Fermi energy, a^{-1} is the momentum cutoff, and

$$\Phi_{l,L}(x) = \frac{1}{2} [\phi_l(x) + \int_{-\infty}^x \Pi_l(x') dx'].$$

For convenience, we have included the right-moving electrons in the first term of the Hamiltonian (2) with the understanding that they have no effect on the dynamics of the original Hamiltonian. Employing the unitary transformation

$$U_1 = \exp[i\sqrt{4\pi}\rho \sum_l V_l \Phi_{l,L}(0) (d^\dagger d - \frac{1}{2})]$$

and rotating the boson fields $\Phi_{l,L}(0)$ to their principal axes as

$$\begin{pmatrix} \tilde{\Phi}_{0,L} \\ \tilde{\Phi}_{1,L} \\ \vdots \\ \vdots \end{pmatrix} = \bar{\bar{W}} \begin{pmatrix} \Phi_{0,L} \\ \Phi_{1,L} \\ \vdots \\ \vdots \end{pmatrix},$$

we find that the boson fields $\tilde{\Phi}_{l,L}(0)$ ($l > 0$) decouple from the local orbital, when the first line elements of the unitary matrix W are chosen as

$$\begin{aligned} W_{00} &= (1 - \delta_0/\pi)(1 + \gamma)^{-1/2}, \\ W_{0l} &= -(\delta_l/\pi)(1 + \gamma)^{-1/2} \quad (l > 0), \end{aligned}$$

with

$$\gamma = -2\frac{\delta_0}{\pi} + \sum_l \left(\frac{\delta_l}{\pi}\right)^2. \quad (3)$$

The phase shift of the electrons in the l th channel is given by $\delta_l = \pi\rho V_l$. Then, the Hamiltonian (2) becomes

$$\begin{aligned} \tilde{H}_b &= U_1 H_b U_1^\dagger \\ &= (4\pi\rho)^{-1} \int [\tilde{\Pi}_0^2(x) + (\nabla \tilde{\phi}_0(x))^2] dx \\ &\quad + \frac{j}{\sqrt{2\pi a}} (e^{i\sqrt{4\pi}(1+\gamma)^{1/2}} \tilde{\Phi}_{0,L}(0) d + d^\dagger e^{-i\sqrt{4\pi}(1+\gamma)^{1/2}} \tilde{\Phi}_{0,L}(0)), \end{aligned} \quad (4)$$

in which the free scalar fields $\tilde{\phi}_l$ ($l > 0$) have been omitted. Through a further unitary transformation

$$U_2 = \exp[-i\sqrt{4\pi}[\sqrt{2} - (1+\gamma)^{1/2}] \tilde{\Phi}_{0,L}(0)(d^\dagger d - 1)]$$

we get

$$\begin{aligned} \tilde{\tilde{H}}_b &= U_2 \tilde{H}_b U_2^\dagger \\ &= (4\pi\rho)^{-1} \int [\tilde{\Pi}_0^2 + (\nabla \tilde{\phi}_0)^2] dx \\ &\quad + \frac{j}{\sqrt{2\pi a}} [e^{i\sqrt{8\pi}} \tilde{\Phi}_{0,L}(0) d^\dagger + d^\dagger e^{-i\sqrt{8\pi}} \tilde{\Phi}_{0,L}(0)] \\ &\quad + \left(\frac{2}{\pi}\right)^{1/2} \rho^{-1} \left[1 - \frac{1}{\sqrt{2}}(1+\gamma)^{1/2}\right] \frac{\partial}{\partial x} \tilde{\Phi}_{0,L}(0) (d^\dagger d - \frac{1}{2}). \end{aligned} \quad (5)$$

Here the spinless fermion operators $d, d^\dagger$ should be written by the Pauli matrices as $\sigma^z = 2d^\dagger d - 1$, $\sigma^+ = d^\dagger$, $\sigma^- = d$ to keep the correct signature. We find thus the Hamiltonian (5) in fact describes a SC Kondo model, or more precisely its spin degree of freedom after the same bosonization procedures, with the following Hamiltonian

$$H_k = \sum_{\sigma, k>0} \hbar v_F k a_{k,\sigma}^\dagger a_{k,\sigma} + J_{//} S^z s^z(0) + \frac{1}{2} J_{\perp} [S^+ s^-(0) + s^+(0) S^-], \quad (6)$$

where

$$\begin{aligned} s^z(0) &= \frac{1}{2} \sum_{\sigma} \sigma a_{\sigma}^\dagger(0) a_{\sigma}(0), \\ a_{\sigma}^\dagger(0) &= N^{-1/2} \sum_k a_{k,\sigma}^\dagger \\ &= (2\pi a)^{-1/2} \exp[i\sqrt{4\pi}\phi_{\sigma}(0)]. \end{aligned}$$

The effective exchange constants are defined as

$$J_{//} = 2\rho^{-1} \left[1 - \frac{1}{\sqrt{2}}(1+\gamma)^{1/2}\right], \quad J_{\perp} = 2(2\pi/a)^{1/2} j. \quad (7)$$

The pseudo-spin operators $S^{\pm,z}$ and $s^{\pm,z}$ represent charge rather than spin excitations of the local orbital and conduction band, respectively. It is noted that the MCRL model has been transform into a multichannel Kondo problem by the bosonization method.⁹

The Hamiltonian (6) is our main result. Before going further, we would like to discuss the validity of the approaches leading to the Hamiltonian (6). One of the key steps in the bosonization treatment is the linearization of the dispersion ϵ_k for the conduction electrons in the Hamiltonian (1) and it is a good approximation only for the weak coupling MCRL model, e.g., the interaction parameters satisfy $V_l \ll 1$. In fact this can also be seen by comparing the phase shifts $\delta_l = \pi\rho V_l$ we obtained with those of the exact calculation¹¹ $\delta_l^{ex} = 2 \tan^{-1}(\frac{\pi}{2}\rho V_l)$. For weak coupling we have $\delta_l \simeq \delta_l^{ex}$. It is expected that the effective Hamiltonian (6) might be extended to the strong coupling case $V_l \gg 1$ by rectifying the phase shifts δ_l to the exact expressions δ_l^{ex} .^{8,10}

From the definition of $J_{//}$ we have $J_{//} > 0$ when $\gamma < 1$. This implies that the weak coupling MCRL model maps onto an antiferromagnetic SC Kondo problem which exhibits FL behavior. At the special case $\gamma = 0$, the Hamiltonian (4) can be written in terms of spinless fermion operators with the help of the inverse bosonization

$$H_T = \sum_k \hbar v_F k S_k^\dagger S_k + \frac{j}{\sqrt{L}} (\sum S_k^\dagger d + d^\dagger S_k) \quad (8)$$

with

$$N^{-1/2} \sum_k S_k^\dagger = (2\pi a)^{-1/2} \exp[-i\sqrt{4\pi} \tilde{\Phi}_{0,L}(0)].$$

This exactly solvable Hamiltonian is the Toulouse limit of the SC Kondo problem¹⁵ and it determines the strong-coupling fixed point of the problem. On the other hand, when $\gamma > 1$ the Hamiltonian (6) stands for a ferromagnetic SC Kondo problem and its properties are controlled by a weak- coupling fixed line $J_\perp = 0$ (or $j = 0$).¹⁶ The system exhibits a power-law decay of correlation functions with a nonuniversal exponent, displaying non-FL behavior as that of the mutichannel x-ray edge model. In fact, γ approaches $n - 2$ as the interactions V_l tend to infinity, where n is the channel number. Therefore, at least three channels of conduction electrons are needed to observe the non-FL behavior. The above analysis shows that, by the bosonization method we can map the weak-coupling MCRL model onto the strong-coupling SC Kondo problem, displaying FL behavior which is controlled mainly by Hamiltonian (8). On the other hand, the strong-coupling MCRL model is related to the weak-coupling SC Kondo problem, exhibiting non-FL behavior which can be calculated around the fixed line $J_\perp = 0$ (or $j = 0$) by a perturbation theory.

It is worth to point out that, by the bosonization method the SC Kondo model can be transformed into a dissipative two-state system described by the so-called spin-boson

Hamiltonian^{11,12}

$$H_{SB} = \frac{1}{4\pi a} \sigma_x + \left(\frac{J_{//}}{4\pi} - \hbar v_F \right) \sigma_z \sum_{k>0} e^{-ak/2} \left(\frac{\pi k}{L} \right)^{1/2} (b_k + b_k^\dagger) + \hbar v_F \sum_{k>0} k b_k^\dagger b_k \quad (9)$$

This Hamiltonian has been used to study the dissipative effects of environmental degree of freedom on the quantum tunneling systems occurring in various fields of physics and its properties depends strongly on the dimensionless coupling strength defined as $d = (1 - \frac{1}{2}\rho J_{//})^2 = \frac{1}{2}(\gamma + 1)$.¹² Therefore we can investigate the properties of the MCRL model by mapping to the known results of the SC Kondo model¹⁶⁻¹⁸ and hence of the dissipative two-state system.^{11,12,19} The following are the main conclusions for the specific heat C and the dynamics $P(t) = \langle d^\dagger(t)d(t) \rangle$ of the local impurity at low temperature. For $\gamma < 1$, $C = A(\gamma)T$ and $P(t)$ is described by incoherent relaxation. At $\gamma = 0$, the exact solution of the Hamiltonian (8) gives $C = (\pi \hbar v_F / 6 j^2) k_B T$ and $P(t) = \frac{1}{2}(1 + e^{-t/\tau})$ with $\tau^{-1} = j^2 / \hbar^2 v_F$, independent of temperature. While $C \sim T^{\gamma-1}$ for $2 > \gamma > 1$ and $P(t)$ follows exponential decay with the rate $\tau^{-1} \sim T^\gamma$ for $\gamma > 1$. The transition from FL to non-FL behavior at $\gamma_c = 1$ can be seen clearly from the above results. Moreover, the scaling behavior of such transitions is controlled by the renormalization group equations¹⁶

$$\begin{aligned} \frac{d\gamma}{d \ln \tau} &= -4(1 + \gamma) \tilde{j}^2 + O(\tilde{j}^4) \\ \frac{d\tilde{j}}{d \ln \tau} &= \frac{1}{2}(1 - \gamma) \tilde{j} + O(\tilde{j}^3) \end{aligned} \quad (10)$$

with $\tilde{j} = \rho(2\pi a)^{1/2} j$. These equations are the same as those obtained in Ref.[10].

In conclusion, we find that the MCRL model can be transformed, by the bosonization method, into the SC Kondo model at the operator level. The specific heat and the dynamics of the local impurity have been calculated by mapping to the known results of the SC Kondo model and they demonstrate the transition from FL to non-FL behavior in the MCRL model as the interactions increases.

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