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Diagrammatic Approach to the Vacuum Energy Density:

The Gross-Neveu Model at Large N

HIROFUMI YAMADA

Department of Physics, Ochanomizu University

1-1 Ohtsuka 2, Bunkyo-ku, Tokyo 112, Japan

Abstract

An approach to the vacuum energy density based on a direct diagrammatic expansion in the generalized Hartree-Fock scheme is studied in the Gross-Neveu model as an example. To keep the coherence with the calculation of the chiral condensate, the vacuum energy density, $\mathcal{E}$, is expanded in terms of the effective charge associated with the condensate. Then $\mathcal{E}$ is calculated by substituting the optimal value of the effective charge which specifies the solution for the condensate. The result agrees well with the exact energy density to several higher orders of the expansion, showing that our procedure works good.

1 Introduction

In a recent study [1], it was examined that whether a direct diagrammatic expansion approach in the generalized Hartree-Fock (GHF) scheme [2-12] works in the calculation of the chiral condensate in the Gross-Neveu model at large N [13]. It was found that the approach yields a non-trivial power series in the renormalized coupling constant, $g^2(\mu)$, and, when the series is improved, gives us broken and un-broken solutions for the discrete γ_5 symmetry under the optimization due to the PMS idea [14]¹. The same method was also applied to QCD in the chiral limit and dynamical quark mass was obtained at the lowest non-trivial order [12]. We like to consider then how to calculate the vacuum energy density in general by the use of similar approach. The purpose of this paper is to provide an explicit procedure of calculating the vacuum energy density, $\mathcal{E}$, via direct diagrammatic expansion in GHF scheme. As an example we take the Gross-Neveu model at large N [13]. In particular, recalling that the chiral condensate is expressed in ref.[1] in terms of the effective charge [15] associated with the condensate, we expand $\mathcal{E}$ in terms of the same effective charge and simplify the calculation of $\mathcal{E}$ corresponding to the obtained solution for the condensate. For the sake of simplicity, we omit N which denotes the number of fermion fields.

We should like to note why it is so non-trivial to calculate the vacuum energy density diagrammatically. One apparent reason is that $\mathcal{E}$ is a non-perturbative quantity in the massless case since it is of the form $\sim \text{const.} \Lambda^2 = \text{const.} \left\{ \mu \exp(-\pi/g^2(\mu)) \right\}^2$ due to the renormalization group invariance, and the conventional massless expansion gives zero order by order in the dimensional regularization. In addition there is a problem characteristic with the diagrammatic approach in GHF scheme. Suppose we apply an expansion scheme used in the calculation of the dynamical mass [1]. Then at a large order, n , the vacuum energy density $\mathcal{E}$ turns out to behave as (see section 3)

$$\mathcal{E} \sim \frac{m^2}{2\pi n^2} \left\{ 1 - \alpha + O(\alpha^2 \log(m/n\mu)) \right\}, \quad \alpha = \frac{g^2(\mu)}{\pi}. \quad (1)$$

Thus the factor, $1/n^2$, strongly forces $\mathcal{E}$ to vanish. Therefore it is a subtle point how

¹Note that the conventional massless expansion gives zero order by order in the dimensional regularization. This is reasonable since the condensate depends on g^2 as $\mu \exp(-\pi/g^2(\mu))$ and does not allow one to obtain a power series in g^2 . On the other hand, by introducing a redundant mass parameter associated with the massive perturbative vacuum, one can have a power series and then, eliminating the redundant parameter, recover the correct non-analyticity. This is an upshot of GHF scheme as pointed out in ref.[9].

we can obtain non-zero energy density to all orders in a diagrammatic approach. This problem is not restricted with the model under consideration. For the quark condensate, for example, it would depend as $(m/n)^3$ according to the lowest order calculation in ref.[12]. It may be a universal feature for fermions that Ω with q mass dimension and no space-time dependence would be given, at large n , as $(m/n)^q$ times a series of the coupling constant with logarithmic corrections. Therefore the calculation of $\mathcal{E}$ in the Gross-Neveu model provides a good testing ground for the calculation of quantities with large mass dimension in general field theories. We start our study with the discussion of the renormalization problem.

2 Renormalizability

We decompose the given massless Lagrangian as

$$\begin{aligned} L &= L_{free} + L_{int}, \\ L_{free} &= \bar{\psi}(i\partial - m)\psi - \frac{1}{2}\sigma^2, \\ L_{int} &= m\bar{\psi}\psi - g\sigma\bar{\psi}\psi + L_{counter-term}. \end{aligned} \quad (2)$$

We perform diagrammatic expansion of the vacuum energy density starting with the shifted massive perturbative vacuum defined by L_{free} . Due to the mass term in L_{free} any Feynman diagram is free from the infrared divergence. On the other hand the ultraviolet structure in this shifted expansion needs a consideration.

To begin with let us denote the r -point 1-P-I vertex function of σ as $\Gamma^{(r)}$. We also denote the series of one-loop diagrams shown in Fig.1 as $\Gamma^{(0)}$. $\Gamma^{(r)}$ is given by the sum of the r -point vertex function with k mass insertions, which we denote as $\Gamma_k^{(r)}$,

$$\Gamma^{(r)} = \sum_{k=0}^{\infty} \Gamma_k^{(r)}. \quad (3)$$

Fig.2 shows the diagrammatic representation of (3) for $r = 4$. At a finite order of expansion of $\mathcal{E}$ which we define in the next section, we deal with the truncated series such that

$$[\Gamma^{(r)}]_N = \sum_{k=0}^N \Gamma_k^{(r)}. \quad (4)$$

As shown in Fig.2 the above truncated series is generated from $\Gamma_0^{(r)}$ by the mass insertion up to N times. To obtain $[\Gamma^{(r)}]_N$ the following technique due to Neveu [9] is very useful.

First, shift the mass parameter, m , to $m(1 - \delta)$ in $\Gamma_0^{(r)}$. Then we expand $\Gamma_0^{(r)}(m(1 - \delta))$ in powers of δ up to δ^N . Since the coefficient function of δ^k agrees with $\Gamma_k^{(r)}$ we obtain $[\Gamma^{(r)}]_N$ by setting $\delta = 1$ in the expanded result. Let us note that the power of δ represents the number of the mass vertex.

Now it is easy to find that $\Gamma^{(r)}$ for $r > 2$ is finite. The divergence in $\Gamma^{(2)}$ is a renormalizable type and cancelled by the ordinary counter diagram. For $\Gamma^{(1)}$, it has been found [11] that the divergences in the sum cancel among themselves: The divergence in the first diagram, Fig.3-a, is cancelled by the divergence in Fig.3-b and the rest diagrams are all finite. Therefore only $\Gamma^{(0)}$ requires a consideration. Using dimensional regularization, the contribution of the first one-loop diagram shown in Fig.1 was calculated in ref.[16];

$$\Gamma_0^{(0)} = \frac{\Gamma(\epsilon/2)}{1 - \epsilon/2} \left(\frac{m^2}{4\pi} \right)^{1-\epsilon/2} = \frac{m^2}{4\pi} \left(\frac{2}{\epsilon} - \gamma - \log \frac{m^2}{4\pi} + 1 \right), \quad (5)$$

where $D = 2 - \epsilon$. By using the δ -expansion technique one can easily confirm that the divergence in (5) disappears when one expands (5) up to δ^2 and sets $\delta = 1$ [16]. In fact the sum of the first three diagrams is given by $m^2/4\pi$. Thus we find that $\Gamma^{(0)}$ is finite as long as the first three diagrams are kept in pairs. The expansion of (5) to the order δ^N gives

$$[\Gamma^{(0)}]_N = \frac{m^2}{2\pi} \left\{ \frac{1}{2}(1 - 2\delta + \delta^2) \left(\frac{2}{\epsilon} - \gamma - \log \frac{m^2}{4\pi} + 1 \right) - \delta + \frac{3}{2}\delta^2 - \sum_{r=3}^N \left(\frac{1}{r} - \frac{2}{r-1} + \frac{1}{r-2} \right) \delta^r \right\}. \quad (6)$$

By setting $\delta = 1$, we obtain

$$\frac{m^2}{2\pi} \frac{1}{N(N-1)}. \quad (7)$$

We note that for large N , $[\Gamma^{(0)}]_N$ decreases as $1/N^2$ and this signals the problem we mentioned in the introduction.

To summarize this section we find that, as long as we keep divergent diagrams in pairs, the diagrammatic expansion of the vacuum energy is compatible with renormalization. While the authors of ref.[16] claimed that the direct diagrammatic approach fails due to the renormalization problem, we like to show in the next section an explicit expansion scheme which keeps the renormalizability.

3 Diagrammatic expansion of the vacuum energy density

We will define a systematic expansion scheme which is compatible not only with renormalization but also with the renormalization group invariance of the vacuum energy density.

The expansion is same as that defined in ref.[1] and it goes as follows: We first define the order of $\Gamma_k^{(r)}$ by $r + k$. In the diagrammatic expansion of the vacuum energy density, any contributing diagram with l loops is a product of l 1-P-I functions of σ . When the order of each function is given by s_p ($p = 1, 2, \dots, l$), we define the order of the full Feynman diagram by $\max(l, s_1, s_2, \dots, s_l)$. In simple words, we can obtain this expansion by first performing the conventional loop expansion and then replacing every $\Gamma^{(r)}(m=0)$ by $[\Gamma^{(r)}]_{n=r}$. Let us note that the ultraviolet divergences are all removed at and above the second order since the number of the mass insertion reaches two for $\Gamma^{(0)}$ and one for $\Gamma^{(1)}$.

In this expansion scheme we obtain the following results at the second and the third orders

$$n=2, \quad \mathcal{E} = \frac{m^2}{2\pi} \left\{ \frac{1}{2} - \alpha + \alpha^2 \left(\log \frac{m}{\mu} + 1 \right) \right\}, \quad (8)$$

$$n=3, \quad \mathcal{E} = \frac{m^2}{2\pi} \left\{ \frac{1}{6} - \frac{1}{4} \left(\alpha - \alpha^2 \log \frac{m}{\mu} + \alpha^3 \log^2 \frac{m}{\mu} \right) \right\}, \quad (9)$$

where $\overline{MS}$ subtraction scheme [17] was used. Note that $\mathcal{E}$ in (8) and (9) is renormalization group invariant up to the maximal orders of α as it should be on physical grounds. It is worth noting that whether the renormalization group invariance is respected or not is crucially dependent on the expansion procedure applied. In our case, renormalization group invariance is due to the recipe that, in the two and three loop contributions, the numbers of mass insertions in all one-loop tad poles, $\Gamma^{(1)}$ s, agree exactly with each other.

Let us remark the structure of the series of $\mathcal{E}$ at higher orders. At n -th order we obtain

$$\mathcal{E} = \frac{m^2}{2\pi} \left\{ \frac{1}{n(n-1)} - \frac{\alpha}{(n-1)^2} \left(1 - \alpha \left(\log \frac{m}{\mu} - c_1 \right) + \dots \right) \right\}, \quad c_1 = -1 + \sum_{r=1}^{n-2} \frac{1}{r} \quad (n \geq 2), \quad (10)$$

where the last term in (10) is of order α^{n-1} . For large n , the asymptotic form of the series is given by

$$\mathcal{E} = \frac{m^2}{2\pi} \frac{1}{n^2} \left\{ 1 - \alpha \left(1 - \alpha \left(\log \frac{m}{n\mu} + 1 - \gamma \right) + \dots \right) \right\}, \quad (11)$$

where we used

$$c_1 \rightarrow -1 + \gamma + \log(n) + O(1/n). \quad (12)$$

We recall that the 1-P-I diagrams of σ , which are the basic building blocks of Feynman diagrams, become functions of m/n when n is large enough [1]. Therefore the effective expansion parameter becomes $\alpha \log(m/n\mu)$ for large n . Hence the series involves large

logarithms and it needs an improvement. In addition, the series, (8), (9) and (10) are not suitable for the evaluation of $\mathcal{E}$ since it involves $g(\mu)$ and $\log(\mu)$ which are both renormalization group dependent. We therefore rewrite $\mathcal{E}$ in terms of manifestly renormalization group independent quantity in the next section.

4 Expansion of the vacuum energy density in terms of α_M

Our goal is to set up the procedure of evaluating $\mathcal{E}$ corresponding to the optimal chiral condensate obtained in ref.[1]. For the purpose we recall that the optimal $\langle g\sigma \rangle$ is specified by the effective charge [15], α_M , which also gives optimal m , or, equivalently the shifted perturbative vacuum. Therefore we try to calculate $\mathcal{E}$ by carefully following the calculational procedure which we applied in the calculation of $\langle g\sigma \rangle$, and starting with the same perturbative vacuum. More explicitly, we expand $\mathcal{E}$ in terms of α_M , rather than the conventional coupling constant, $\alpha(\mu)$, and then substitute the optimal values of m and α_M to evaluate the corresponding $\mathcal{E}$. By rewriting $\mathcal{E}$ in terms of α_M , the examination that which solution of $\langle g\sigma \rangle$ corresponds to the vacuum becomes an easy task.

As a first step we improve the expansion defined in the previous section as to respect the rigorous renormalization group invariance at every order, as done in ref.[1]. For the purpose we sum up the chain series of $\Gamma^{(2)}$ and use it as a corrected σ propagator as represented in Fig.4. Note that, in order to avoid double counting, we include only the one-loop self-energy with possible mass insertions; We do not include many-loop diagrams with the tad-pole structure. In this improved expansion scheme Feynman diagrams which we should deal with are reduced to those without $\Gamma^{(2)}$ as long as the corrected σ propagator is used to connect $\Gamma^{(r)}$. Then results for the improved order n from 2 to 7 are given as follows;

$$\begin{aligned} \mathcal{E} &= \frac{m^2}{2\pi} \left\{ \frac{1}{2} - x \right\} \\ \mathcal{E} &= \frac{m^2}{2\pi} \left\{ \frac{1}{6} - \frac{1}{4} \left(x - \frac{1}{6} x^3 \right) \right\} \\ \mathcal{E} &= \frac{m^2}{2\pi} \left\{ \frac{1}{12} - \frac{1}{9} \left(x - \frac{2}{9} x^3 + \frac{1}{108} x^4 \right) \right\} \end{aligned} \quad (13)$$

²We use the terms, optimal m or α_M , in the meaning that, at the value, $\langle g\sigma \rangle$ is stationary.

$$\begin{aligned}\mathcal{E} &= \frac{m^2}{2\pi} \left\{ \frac{1}{20} - \frac{1}{16} \left(x - \frac{1}{4}x^3 + \frac{1}{64}x^4 + \frac{269}{1920}x^5 \right) \right\} \\ \mathcal{E} &= \frac{m^2}{2\pi} \left\{ \frac{1}{30} - \frac{1}{25} \left(x - \frac{4}{15}x^3 + \frac{1}{50}x^4 + \frac{298}{1875}x^5 - \frac{1199}{37500}x^6 \right) \right\} \\ \mathcal{E} &= \frac{m^2}{2\pi} \left\{ \frac{1}{42} - \frac{1}{36} \left(x - \frac{5}{18}x^3 + \frac{5}{216}x^4 + \frac{223}{1296}x^5 - \frac{599}{15552}x^6 + \frac{15605}{279936}x^7 \right) \right\},\end{aligned}$$

where x is given by

$$x = \frac{1}{\log(m/\Lambda_{MS}) - c_1}. \quad (14)$$

Note that x defined in (14) agrees exactly with x defined in ref.[1]. The above result is summarized as

$$\begin{aligned}\mathcal{E} &= \frac{m^2}{2\pi} \left\{ \frac{1}{n(n-1)} - \frac{x}{(n-1)^2} \left(1 - \frac{n-2}{3(n-1)}x^2 + \frac{(n-2)(n-3)}{24(n-1)^2}x^3 \right. \right. \\ &+ \left. \left(-\frac{(n-2)(n-3)(n-4)}{180(n-1)^3} + \frac{(n-2)^2}{4(n-1)^2} \right) x^4 \right. \\ &- \left. \left(-\frac{(n-2)(n-3)(n-4)(n-5)}{1440(n-1)^4} + \frac{(n-2)^2(n-3)}{12(n-1)^3} \right) x^5 \right. \\ &+ \left. \left(-\frac{(n-2)(n-3)(n-4)(n-5)(n-6)}{12600(n-1)^4} \right. \right. \\ &+ \left. \left. \frac{(n-2)^2(n-3)^2}{144(n-1)^4} + \frac{(n-2)^2(n-3)(n-4)}{72(n-1)^4} - \frac{(n-2)^3}{4(n-1)^3} \right) x^6 + O(x^7) \right\}. \quad (15)\end{aligned}$$

The next step is to expand $\mathcal{E}$ in terms of α_M . This is performed by expanding x in powers of α_M . The effective charge α_M is defined at order n by $\langle g\sigma \rangle = m/(n-1) \cdot \alpha_M$ [1]. In our improved expansion scheme α_M is given as a series of x and its behavior is governed by $m\partial\alpha_M/\partial m = 1/\theta(\alpha_M)$ where θ is calculated in ref.[1] as

$$\theta = \frac{-1}{\alpha_M^2} \left[\alpha_M + \left(1 - \frac{\alpha_M}{n-1} \right)^{n-1} \right] = \frac{-1}{\alpha_M^2} + \theta_{reg}. \quad (16)$$

Formally solving (16) for m we have

$$\frac{m}{\Lambda_{MS}} = \exp \left(c_1 + \int^{\alpha_M} dx \theta \right) = \exp \left(c_1 + \frac{1}{\alpha_M} + \int_0^{\alpha_M} dx \theta_{reg}(x) \right), \quad (17)$$

and then from (14) and (17) we express x in terms of α_M as

$$x = \frac{1}{\int^{\alpha_M} dx \theta(x)} = \frac{\alpha_M}{1 - \sum_{r=2}^{n-1} \frac{n-1}{(r-1)(n-1)^r} C_r}, \quad n-1 C_r = \frac{(n-1)!}{r!(n-1-r)!}. \quad (18)$$

And from (18) x is expanded as

$$x = \alpha_M + \frac{n-2}{2(n-1)}\alpha_M^3 - \frac{(n-2)(n-3)}{12(n-1)^2}\alpha_M^4 + \dots \quad (19)$$

Here we truncate the above infinite series according to the maximal power of x in $\mathcal{E}$. The explicit truncated series of (19) is given below for n from 2 to 7;

$$\begin{aligned}x &= \alpha_M \\ x &= \alpha_M \left(1 + \frac{1}{4}\alpha_M^2 \right) \\ x &= \alpha_M \left(1 + \frac{1}{3}\alpha_M^2 - \frac{1}{54}\alpha_M^3 \right) \\ x &= \alpha_M \left(1 + \frac{3}{8}\alpha_M^2 - \frac{1}{32}\alpha_M^3 + \frac{109}{768}\alpha_M^4 \right) \\ x &= \alpha_M \left(1 + \frac{2}{5}\alpha_M^2 - \frac{1}{25}\alpha_M^3 + \frac{61}{375}\alpha_M^4 - \frac{401}{12500}\alpha_M^5 \right) \\ x &= \alpha_M \left(1 + \frac{5}{12}\alpha_M^2 - \frac{5}{108}\alpha_M^3 + \frac{115}{648}\alpha_M^4 - \frac{67}{1728}\alpha_M^5 + \frac{1007}{12960}\alpha_M^6 \right).\end{aligned} \quad (20)$$

Then, by substituting (20) into $\mathcal{E}$ in (14) we obtain in respective orders,

$$\begin{aligned}\mathcal{E} &= \frac{m^2}{2\pi} \left\{ \frac{1}{2} - \alpha_M \right\} \\ \mathcal{E} &= \frac{m^2}{2\pi} \left\{ \frac{1}{6} - \frac{1}{4} \left(\alpha_M + \frac{1}{12}\alpha_M^3 \right) \right\} \\ \mathcal{E} &= \frac{m^2}{2\pi} \left\{ \frac{1}{12} - \frac{1}{9} \left(\alpha_M + \frac{1}{9}\alpha_M^3 - \frac{1}{108}\alpha_M^4 \right) \right\} \\ \mathcal{E} &= \frac{m^2}{2\pi} \left\{ \frac{1}{20} - \frac{1}{16} \left(\alpha_M + \frac{1}{8}\alpha_M^3 - \frac{1}{64}\alpha_M^4 + \frac{1}{1280}\alpha_M^5 \right) \right\} \\ \mathcal{E} &= \frac{m^2}{2\pi} \left\{ \frac{1}{30} - \frac{1}{25} \left(\alpha_M + \frac{2}{15}\alpha_M^3 - \frac{1}{50}\alpha_M^4 + \frac{1}{625}\alpha_M^5 - \frac{1}{18750}\alpha_M^6 \right) \right\} \\ \mathcal{E} &= \frac{m^2}{2\pi} \left\{ \frac{1}{42} - \frac{1}{36} \left(\alpha_M + \frac{5}{36}\alpha_M^3 - \frac{5}{216}\alpha_M^4 + \frac{1}{432}\alpha_M^5 - \frac{1}{7776}\alpha_M^6 + \frac{1}{326592}\alpha_M^7 \right) \right\}.\end{aligned} \quad (21)$$

The above results are summarized as follows:

$$\begin{aligned}\mathcal{E} &= \frac{m^2}{2\pi} \left\{ \frac{1}{n(n-1)} - \frac{1}{(n-1)^2} \left(\alpha_M + \sum_{r=3}^n \frac{(-1)^{r-1}(n-2)!}{r!(n-r)!(n-1)^{r-2}} \alpha_M^r \right) \right\} \\ &= \frac{m^2}{2\pi n(n-1)} \left(1 - \frac{\alpha_M}{n-1} \right)^n - \frac{m^2}{4\pi} \left(\frac{\alpha_M}{n-1} \right)^2, \quad (n=2, 3, \dots, 7).\end{aligned} \quad (22)$$

The energy density corresponding to the un-broken solution, $\alpha_M = 0^-$, is computed to give $\mathcal{E} = 0$. Now, for the broken solution, $\alpha_M = n-1$, we find the corresponding energy density up to 7-th order,

$$\mathcal{E} = -\frac{1}{4\pi} \Lambda_{MS}^2. \quad (23)$$

This is indeed the exact vacuum energy density obtained in the conventional approach in ref.[13].

5 Discussion

The result (22) strongly suggests that it will hold to an arbitrary higher order n . If this is the case we see that (22) leads to the exact vacuum energy density to all orders at $\alpha_M = n - 1$. We remark that, at just the optimal values of m and α_M , $\mathcal{E}(\alpha_M)$ in (21) is stationary. Namely, the optimal α_M for $\langle g\sigma \rangle$ also optimize $\mathcal{E}$. This agreement continues to all orders if (22) holds since the following result is derived from (16), (17) and (22),

$$\frac{d\mathcal{E}}{dm} = \frac{m}{\pi n(n-1)} \left(1 - \frac{\alpha_M}{n-1}\right)^n. \quad (24)$$

and (24) shows that $\mathcal{E}$ is locally flat at $\alpha_M = n - 1$.

Now let us discuss different expansion scheme which is remarked in [1]. In this scheme we perform diagrammatic expansion by incorporating the mass insertion to all orders in each order of g^2 and simultaneously letting m large as to keep the ratio, m/n , finite. In this case the order of this expansion agrees with the highest number of loops, L , minus 1. For example, from (15), we find that, in the limit that $m, n \rightarrow \infty$ with m/n finite, $\mathcal{E}$ at 7-th order becomes

$$\mathcal{E} \rightarrow \frac{m^2}{2\pi n^2} \left\{ 1 - \left(x - \frac{1}{3}x^3 + \frac{1}{24}x^4 + \frac{11}{45}x^5 - \frac{199}{1440}x^6 - \frac{17331}{75600}x^7 \right) \right\}, \quad (25)$$

where

$$x \rightarrow \frac{1}{\log(m/n\Lambda_{\overline{MS}}) + 1 - \gamma}. \quad (26)$$

Then we rewrite $\mathcal{E}$ in terms of α_M in the same manner as we did in section 4. In this process, we use the fact that x can be expanded by α_M via (18) in which θ should be replaced by a series expansion of $\theta_\infty = -(\alpha_M + \exp(-\alpha_M))/\alpha_M^2$ up to the first $L - 1$ terms. In fact, the result is easily obtained by using (22) such that first one lets $n \rightarrow \infty$ in (22) which gives

$$\mathcal{E} \rightarrow \frac{m^2}{2\pi n^2} e^{-\alpha_M} - \frac{m^2}{4\pi n^2} \alpha_M^2, \quad (27)$$

and then expands $\mathcal{E}$ up to relevant orders in α_M . For example, at 7-th order, the result reads

$$\mathcal{E} = \frac{m^2}{2\pi n^2} \left(\sum_{k=0}^7 \frac{(-\alpha_M)^k}{k!} - \frac{1}{2} \alpha_M^2 \right). \quad (28)$$

³This is obtained from (17) by taking the $n \rightarrow \infty$ limit.

According to ref.[1], this expansion gives the optimal α_M , $\alpha_M = 1, 1.596 \dots, 2.1806 \dots$ at 2, 4 and 6 orders and the optimal $m/n = 1.781 \dots, 0.682 \dots, 0.469 \dots$, respectively. Then the substitution of the optimal values gives

$$\mathcal{E} = -\frac{\Lambda_{\overline{MS}}^2}{4\pi} (0, 1.186 \dots, 1.050 \dots). \quad (29)$$

Thus, except for the first case, the scheme yields good numerical results⁴. If we rely upon (22) to all orders, we can show that the exact energy density is recovered in the $n \rightarrow \infty$ limit. This expansion is interesting in the point that the definition is simpler and it may be easier to apply it to other general field theories.

Concluding this paper we have given a framework in which one can calculate the vacuum energy density diagrammatically and search the vacuum solution of the condensate. Our diagrammatic approach is improved as being capable of practical use by rewriting the energy density in terms of the effective charge, α_M , associated with the condensate. It was found that the substitution of the broken solution gives the correct vacuum energy density up to 7-th orders. This means that α_M is a good expansion parameter for the energy density. We hope that our procedure may work also for the vacuum energy density in QCD and for the quark condensate.

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⁴The reason that the first result fails to give negative energy density may be due to the fact that the optimal condensate is roughly the double of the exact value (see ref.[1]) at this order and not yet in good precision.

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Figure Captions

- Fig.1 One-loop vacuum graphs in the generalized Hartree-Fock framework. The filled blob represents the mass vertex due to the mass term in L_{int} .
- Fig.2 4-point 1-P-I function of σ expanded in terms of the mass insertions.
- Fig.3 One-loop tad pole diagrams. The diagram (b) cancels the ultraviolet divergence of the diagram (a).
- Fig.4 A corrected σ propagator represented by the thick line. No tad pole graph is included in the self energy of σ to avoid double counting.

Fig.1

(a)

(b)

Fig. 3

Fig.2

Fig. 4