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Abstract:

In this article we investigate the influence of the expansion on the properties and the dynamical evolution of a piece of unstable nuclear matter.

We discuss how the expansion modifies the spinodal decomposition of the system by deriving analytical results, which are confirmed and illustrated by numerical simulations in two dimensions. Moreover, the expansion is introducing a short freeze-out time which allows to unambiguously define the formed fragments. An important result of this study is that for a wide range of expansion velocities the fragmentation is dominated by the same unstable wave lengths, which happen to be weakly dependent on density and temperature provided that the system is lying well inside the spinodal instability region.

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1 Introduction

In the past decade heavy ion reactions have been the subject of intense studies. In particular it is now commonly believed that during some violent collisions a composite system can be formed and can subsequently expand either because of its probable initial compression or under the action of the thermal pressure. It has been suggested that a copious fragment production can be related to the occurrence of instabilities during the dynamical expansion of the nuclear system. In particular a widely discussed sources of instabilities are the volume (also called spinodal) instabilities related to a possible liquid-gas phase transition like in Van der Waals systems [1, 2, 3, 4].

However, since the system keeps expanding while it fragments, one may worry about the influence of the dynamical expansion on the instability properties of nuclear matter. This will be the subject of the present article. In order to straighten analytical derivations and the analysis of numerical results, we will consider a piece of infinite nuclear matter, for which the eigenmodes are plane waves.

However not only the occurrence of instabilities is crucial when one is considering the possibilities of fragmentation of nuclear systems, but also the amplitude of fluctuations, time scales and final state interaction are of paramount importance. In this article we will also try to shade some light on these important issues by simulating the fragmentation of expanding systems, using the recently developed stochastic mean field approaches [5, 6, 7, 8].

These approaches, and in particular the so-called Boltzmann-Langevin theory [5, 6], have been extensively investigated during the past five years. It has been shown that these methods are able to take into account the thermal fluctuations and that they may provide a well-founded description of the onset of fragmentation. This has been illustrated explicitly in the case of spinodal decomposition of classical systems, for which the comparison with the predictions of exact molecular dynamics calculations is easily feasible [9, 10]. In particular, it has been found that the fragmentation occurring well inside the spinodal region (i.e. far from the critical point and from the region of metastability) is dominated by a few unstable wave lengths [4, 11, 12]. These wave lengths are characteristic of the mean-field instabilities which correspond to an imaginary zero-sound velocity.

In this article we will discuss the fact that the expansion of the nuclear system does not strongly affect the typical length scales of the instabilities. Indeed, it can be shown that these scales are essentially related to the range of the nuclear interaction and to quantum effects. Therefore, an important result of this study is that for a wide range of expansion velocities the fragmentation is dominated by the same unstable wave lengths which happen to be nearly independent on density and temperature, provided that the system is well inside the spinodal region. These modes have a wave length around 10 fm. However, the number of produced fragments is increasing (and their mass is correspondingly decreasing) with the expansion velocity mostly because the fragmentation occurs in a more and more diluted system. Since the expansion is introducing a short freeze-out time, we can unambiguously define the fragments and thus discuss quantities such as fragment multiplicities and the associated fluctuations. We will see that the fragment multiplicity strictly reflects the number of nodes of the

most unstable wave length the system can afford.

2 Equation of motion and properties of expanding systems

In this section we will first give a simple example of a physical system in expansion: a gas of particles on an expanding ring, i.e. a system associated with only one coordinate. We will then generalize the concept to several expanding coordinates in order to describe systems in any dimension.

2.1 Simple example: the motion of a point on an expanding ring.

Let us start with the simple case of the expansion of a one dimensional system and let us take the example of the motion of a point bound to an expanding circle of radius $R(t)$. In such a case we can introduce, as a variable, the angle of the particle on the circle and the angular momentum associated with its motion :

$$J = m R^2(t) \dot{\theta} \equiv \mathcal{I}(t) \dot{\theta} \quad (1)$$

If we assume the force, which bounds the particle on the ring, to be radial and all the other forces, F , such as forces between particles to be tangential to the ring, the dynamics of J reads: $\dot{J} = F R(t)$.

In the case the force F derives from a potential $F R(t) = -\partial U / \partial \theta$, the above dynamics can be associated with the Hamiltonian:

$$H = \frac{J^2}{2\mathcal{I}(t)} + U(\theta, t) \quad (2)$$

If we now consider a gas of independent particles moving on the circle in the one body field U , their density in phase space evolves according to the Vlasov equation:

$$\frac{\partial f(\theta, J, t)}{\partial t} = \frac{\partial U(\theta, t)}{\partial \theta} \frac{\partial f(\theta, J, t)}{\partial J} - \frac{J}{\mathcal{I}(t)} \frac{\partial f(\theta, J, t)}{\partial \theta} \quad (3)$$

Let us now introduce the curvilinear coordinates of the particles on the ring $s = R(t)\theta$ and $p = m\dot{s}$ which are related to the angular variable through :

$$\theta = s / R(t) \quad (4)$$

$$J = (p - A(s, t)) R(t) \quad (5)$$

where $A(s, t) = m\dot{R}(t)s / R(t)$ is the collective momentum associated with the expansion in the curvilinear coordinate s . Within these coordinates the Hamiltonian now reads:

$$H = \frac{(p - A(s, t))^2}{2m} + U(s, t) \quad (6)$$

It should be noticed that the two boundaries, corresponding to $\theta = \pm\pi$, are associated with the coordinate $s = \pm\pi R(t)$ which expands at the velocity $v_s = \pm\pi\dot{R}(t)$. We have seen that the variable θ was more convenient for the expression of the hamiltonian and of the dynamics. Angular variables are also very useful to express boundary conditions since in the considered closed geometry they simply correspond to a periodic condition between $-\pi$ and π :

$$f(\pi, J) = f(-\pi, J) \quad (7)$$

In the curvilinear coordinates the periodic boundary conditions read:

$$f(\pi R(t), p + m\pi\dot{R}(t)) = f(-\pi R(t), p - m\pi\dot{R}(t)) , \quad (8)$$

in order to take care of the displacement of the boundaries in time and to compensate the momentum distribution for the change in collective velocity.

2.2 Generalization to any dimension

The above formalism can be generalized to any dimension by simply associating to any coordinate s_i and p_i , expanding at the rate $R_i(t)$, the generalized reduced variables Q_i and P_i which are analogous to the angular variables of the previous simple ring model and which are given by :

$$Q_i = s_i / R_i(t) \quad (9)$$

$$P_i = (p_i - A_i(s_i, t)) R_i(t) \quad (10)$$

where A_i is defined as the momentum of the collective expansion $A_i(s_i, t) = m\dot{R}_i(t)s_i / R_i(t)$. In such a case the hamiltonian can be written as:

$$H = \sum_i \frac{P_i^2}{2\mathcal{I}_i(t)} + U(\mathbf{Q}, t) . \quad (11)$$

where the inertia parameters $\mathcal{I}_i(t)$ are given by $\mathcal{I}_i(t) = mR_i^2(t)$. In terms of the coordinate s_i and p_i the hamiltonian thus reads:

$$H = \sum_i \frac{(p_i - A_i(s_i, t))^2}{2m} + U(\mathbf{s}, t) . \quad (12)$$

If we are now considering a gas of particles the Vlasov equation is given by:

$$\frac{\partial f(\mathbf{Q}, \mathbf{P}, t)}{\partial t} = \frac{\partial U(\mathbf{Q}, t)}{\partial \mathbf{Q}} \cdot \frac{\partial f(\mathbf{Q}, \mathbf{P}, t)}{\partial \mathbf{P}} - \frac{\mathbf{P}}{\mathcal{I}(t)} \cdot \frac{\partial f(\mathbf{Q}, \mathbf{P}, t)}{\partial \mathbf{Q}} \quad (13)$$

where we have assumed for simplicity that the moments of inertia were identical in all directions. It is interesting to notice that in the case of a uniform expansion (i.e. at constant velocity) this equation can be related to an usual Vlasov equation for the density in the $\mathbf{s}$ and $\mathbf{p}$ space:

$$\frac{\partial f(\mathbf{s}, \mathbf{p}, t)}{\partial t} = \frac{\partial U(\mathbf{s}, t)}{\partial \mathbf{s}} \cdot \frac{\partial f(\mathbf{s}, \mathbf{p}, t)}{\partial \mathbf{p}} - \frac{\mathbf{p}}{m} \cdot \frac{\partial f(\mathbf{s}, \mathbf{p}, t)}{\partial \mathbf{s}} \quad (14)$$

However this equation will get more complex in the case of an expansion which is not uniform.

As far as boundary conditions are concerned, they can easily be expressed using the reduced variables:

$$f(..., Q_i = \pi, ...; \mathbf{P}) = f(..., Q_i = -\pi, ...; \mathbf{P}) \quad (15)$$

In the curvilinear coordinates the periodic boundary conditions reads:

$$f(..., s_i = \pi R_i(t), ...; ..., p_i + m\pi \dot{R}_i(t), ...) = f(..., s_i = -\pi R_i(t), ...; ..., p_i - m\pi \dot{R}_i(t), ...) . \quad (16)$$

If we want to consider the case of a density dependent mean field, it will depend upon the density in $\mathbf{s}$:

$$\rho(\mathbf{s}) = \varrho(\mathbf{Q})/R^D(t) \quad (17)$$

where the density in $\mathbf{Q}$ is defined by

$$\varrho(\mathbf{Q}) = \int \frac{d\mathbf{P}}{h^D/g} f(\mathbf{Q}, \mathbf{P}) . \quad (18)$$

where D is the dimension of the considered space and g stands for a possible spin-isospin degeneracy. If the mean field is created by a force with a constant range σ_s in the coordinate s it will correspond to a time dependent range:

$$\sigma_Q = \sigma_s/R(t) \quad (19)$$

in the reduced variables.

It should be noticed that within this formalism it is easy to introduce a Boltzmann-like collision term which will take the usual form in the $\mathbf{s}$ and $\mathbf{p}$ coordinates.

2.3 Evolution of thermalized solutions

In next sections we will assume for simplicity that the expansion rate is the same in all the directions. We are now looking for the possible existence of uniform solutions. Since the dynamics in the Q and P coordinates corresponds to a regular Vlasov equation it is easy to show that the Fermi Dirac distribution:

$$f^{(0)}(P) = \frac{1}{1 + \exp(-\beta(P^2/2\mathcal{I}(0) - \mu))} \quad (20)$$

is a stationary solution. However, since the inertia parameter $\mathcal{I}$ is not constant in time one can write the density at a time t as:

$$f^{(0)}(P, t) = \frac{1}{1 + \exp(-\beta(t)(P^2/2\mathcal{I}(t) - \mu(t)))} \quad (21)$$

where we have introduced a time-dependent temperature ($\beta(t) = 1/T(t)$):

$$T(t) = \frac{T(0)}{\alpha^2(t)} \quad (22)$$

and a time dependent chemical potential :

$$\mu(t) = \frac{\mu(0)}{\alpha^2(t)} \quad (23)$$

In these equations we have introduced the dilatation factor $\alpha(t) = R(t)/R(0)$. Using these variables the density can be also given in the $\mathbf{s}$ and $\mathbf{p}$ phase space:

$$f^{(0)}(\mathbf{s}, \mathbf{p}, t) = \frac{1}{1 + \exp(-\beta(t)(|\mathbf{p} - \mathbf{A}|^2/2m - \mu(t)))} \quad (24)$$

This expression can be easily understood because it corresponds to a Fermi distribution shifted by the local momentum of the expansion. The Fermi energy is changing in time reflecting the modification of the density. A positive expansion ($\dot{R} \geq 0$) cools down the system and thus reduce the temperature. It shall be noticed that $f^{(0)}(\mathbf{s}, \mathbf{p}, t)$ depends explicitly on time while the density in the Q and P variables is stationary.

2.4 Linearized evolution around equilibrium

After this discussion of the equilibrium solutions in expanding geometries, we will use the reduced variables because they allow to express the dynamics as an usual Vlasov equation and also the boundary conditions can be easily accounted for (see eq. (15)).

If we now expand f around the previously-discussed uniform solution $f^{(0)}$ we get the RPA equation for the small deviation $f^{(1)}$:

$$\frac{\partial f^{(1)}}{\partial t} + \frac{\partial f^{(1)}}{\partial \mathbf{Q}} \cdot \frac{\mathbf{P}}{\mathcal{I}} - \frac{\partial f^{(0)}}{\partial \mathbf{P}} \cdot \frac{\partial U[\varrho(\mathbf{Q})](t)}{\partial \varrho(\mathbf{Q})} \frac{\partial \varrho^{(1)}(\mathbf{Q})}{\partial \mathbf{Q}} = 0. \quad (25)$$

where we have explicitly introduced the density $\varrho(\mathbf{Q})$ (see eq. (18)). We may now perform a Fourier transform on the $\mathbf{Q}$ variable because f is periodic in $\mathbf{Q}$ with the period 2π :

$$f_{\mathbf{K}}^{(1)}(\mathbf{P}, t) = \int \frac{d\mathbf{Q}}{2\pi} e^{i\mathbf{K}\mathbf{Q}} f(\mathbf{Q}, \mathbf{P}, t), \quad (26)$$

Then the RPA equation reads:

$$-i \frac{\partial f_{\mathbf{K}}^{(1)}(\mathbf{P}, t)}{\partial t} + \frac{\mathbf{K} \cdot \mathbf{P}}{\mathcal{I}(t)} f_{\mathbf{K}}^{(1)}(\mathbf{P}, t) - \frac{\mathbf{K} \cdot \mathbf{P}}{\mathcal{I}(t)} \frac{\partial f^{(0)}}{\partial \varepsilon(t)} \frac{\partial U_{\mathbf{K}}(t)}{\partial \varrho} \varrho_{\mathbf{K}}^{(1)}(t) = 0. \quad (27)$$

where we have introduced the energy $\varepsilon(t) = P^2/2\mathcal{I}(t)$. The RPA equation is thus equivalent to:

$$i \frac{\partial f^{(1)}(\mathbf{K}, \mathbf{P}, t)}{\partial t} = \int \frac{d\mathbf{P}'}{h^D/g} \mathcal{K}(\mathbf{K}, \mathbf{P}, \mathbf{K}', \mathbf{P}', t) f^{(1)}(\mathbf{K}', \mathbf{P}', t) \quad (28)$$

where the RPA kernel $\mathcal{K}$ is diagonal in $\mathbf{K}$ and is equal to:

$$\mathcal{K}(\mathbf{K}, \mathbf{P}, \mathbf{K}', \mathbf{P}', t) = \delta(\mathbf{K} - \mathbf{K}') \left(\delta(\mathbf{P} - \mathbf{P}') \frac{h^D}{g} \frac{\mathbf{K} \cdot \mathbf{P}}{\mathcal{I}(t)} - \frac{\mathbf{K} \cdot \mathbf{P}}{\mathcal{I}(t)} \frac{\partial f^{(0)}}{\partial \varepsilon(t)} \frac{\partial U_{\mathbf{K}}(t)}{\partial \varrho} \right) \quad (29)$$

This kernel is time dependent since the inertia, the potential, the temperature and the chemical potential are evolving with time. However it can be easily seen that at a given time it corresponds to the linear response kernel of the static expanded system. Therefore, the instantaneous eigen frequencies of this kernel are not affected by the expansion. However, in order to study properties such as the stability of the system one needs to follow the propagation of $f^{(1)}$ over a finite time t , which formally reads:

$$f_K^{(1)}(t) = T \left(\exp(-i \int_0^t dt' \mathcal{K}(t')) \right) f_K^{(1)}(0) \equiv \mathcal{M}_K(t) f_K^{(1)}(0) \quad (30)$$

where $\mathcal{M}$ is the monodromy matrix and T is the time ordering operator. The solution of eq. (30) gives the time evolution of the perturbation $f_K^{(1)}$. This time evolution can be studied by introducing eigenstates and eigenfunctions of the monodromy matrix:

$$f_{K,\nu}^{(1)}(t) = \mathcal{M}_K(t) f_{K,\nu}^{(1)}(0) = \exp(i\omega_{K,\nu}t) f_{K,\nu}^{(1)}(0) \quad (31)$$

As it is well know, in the case of a non expanding system, since the kernel $\mathcal{K}$ is no more time dependent, the solutions of eq.(31) are the eigenstates of the RPA kernel and the eigenfrequencies ω_K are obtained from the associated dispersion relation:

$$1 = \int \frac{d\mathbf{P}}{h^D/g} \frac{\frac{\partial f^{(0)}}{\partial \epsilon} \frac{\partial U}{\partial \rho} \frac{\mathbf{K} \cdot \mathbf{P}}{T}}{\frac{\mathbf{K} \cdot \mathbf{P}}{T} + \omega_{K,\nu}} \quad (32)$$

In the case of an expanding system, applying the Magnus transformation, it is possible to rewrite eq.(30) as follows:

$$f_K^{(1)}(t) = \exp(-i \int_0^t dt' \mathcal{K}(t') + \mathcal{B}(t)) f_K^{(1)}(0) \quad (33)$$

where $\mathcal{B}$ is a sum of integrals which depends upon the commutators of the *RPA* kernel taken at two different times. For relatively slow expansion velocities or for a very short time interval t , one can consider just the first term of the expression displayed above and neglect the term $\mathcal{B}$. Starting from this assumption it is possible to express the time dependence of $f_K^{(1)}(t)$ as in eq.(31) where $\omega_K(t)$ is now given from the solution of the equation:

$$1 = \int \frac{d\mathbf{P}}{h^D/g} \frac{\frac{\partial f^{(0)}}{\partial \epsilon(t)} \frac{\partial \bar{U}}{\partial \rho} \frac{\mathbf{K}(t) \cdot \mathbf{P}}{\bar{I}(t)}}{\frac{\mathbf{K}(t) \cdot \mathbf{P}}{\bar{I}(t)} + \omega_K(t)}. \quad (34)$$

where we have introduced the time averaged inertia parameter:

$$\bar{I}^{-1}(t) = \int_0^t \frac{dt'}{t} I^{-1}(t') \quad (35)$$

and the time dependent potential:

$$\bar{U}_{\mathbf{K}}(t) = \frac{\bar{I}(t)}{I(t)} \int_0^t \frac{dt'}{t} U_{\mathbf{K}}(t') \quad (36)$$

In such a case we can also introduce a renormalized Landau parameter:

$$\bar{F}_0^K(t) = -\frac{\partial \bar{U}_{\mathbf{K}}(t)}{\partial \rho} \int \frac{d\mathbf{P}}{h^D/g} \frac{\partial f^{(0)}}{\partial \varepsilon(t)} \quad (37)$$

and a renormalized free-response:

$$\bar{\xi}(\omega_K, t) = \int \frac{d\mathbf{P}}{h^D/g} \frac{\frac{\partial f^{(0)}}{\partial \varepsilon(t)} \frac{\mathbf{K} \cdot \mathbf{P}}{\mathcal{I}(t)}}{\frac{\mathbf{K} \cdot \mathbf{P}}{\mathcal{I}(t)} + \omega_K} \left(\int \frac{d\mathbf{P}}{h^D/g} \frac{\partial f^{(0)}}{\partial \varepsilon(t)} \right)^{-1}. \quad (38)$$

so that the dispersion relation can be recasted under the standard form:

$$\bar{\xi}(\omega_K(t), t) = -\frac{1}{\bar{F}_0^K(t)} \quad (39)$$

This analysis shows that during a given time, over which the commutator of the RPA kernel, taken at the time t , with the initial one is small compared with the eigenvalues of any of these two kernels, the system follows the standard phenomenology and so will present instabilities, provided that $\bar{F}_0^K(t) \leq -1$. In such a case the expansion of Fermi Liquids will lead to a modification of the sound velocity associated with long wave lengths.

For short wave lengths we know that the range of the force associated with the potential U is reducing or even suppressing the instability by introducing an ultraviolet cut-off. The simplest way to take into account the effects of a range is to expend the Fourier transform of the potential in powers of K and to consider only the lowest orders as follows:

$$U_K(t) = U(t)(1 - K^2 \sigma_Q^2(t)/2) \quad (40)$$

where σ_Q is related with the actual range of the interaction in Q and therefore is changing in time according to $\alpha(t)$ (see eq. (19)). In such a case $\bar{U}$ also reads

$$\bar{U}_K(t) = \bar{U}(t)(1 - K^2 \bar{\sigma}_Q^2(t)/2) \quad (41)$$

where the renormalized range is given by :

$$\bar{\sigma}_Q^2(t) = \sigma_Q^2(t) \frac{\int_0^t dt' U(t') \frac{\alpha^2(t')}{\alpha^2(t)}}{\int_0^t dt' U(t')}. \quad (42)$$

If we consider only the first order terms in $\beta = \alpha - 1$, it results:

$$\bar{\sigma}_Q^2(t) \approx \sigma_Q^2(t) \alpha(t) \quad (43)$$

The associated interaction range in the s variable therefore evolves according to the same equation:

$$\bar{\sigma}_s^2(t) \approx \sigma_s^2(t) \alpha(t) \quad (44)$$

Therefore, it is clear that as soon as the dilatation coefficient is increasing the range σ_Q is enhanced. This means that the expansion mimics a force with a longer range.

This can be understood by considering that the particles which are now too far to interact were closer in the past and so may have been in interaction at that time. Therefore, the memory of this interaction enlarges the effective range, as described in eq. (44).

However, one must notice that as the system expands it gets more and more diluted. Indeed the average density evolves according to α^{-D} . Therefore, it exists a maximum time interval over which the previous considerations hold because either the system will be out of the spinodal region or the density will get so low that the clusterization mechanism will rather be analogous to the nucleation in a diluted gas. This density value can be estimated to be around 1/10 of the saturation density by requiring that the number of particles included in the typical density fluctuations associated with the most unstable wavelengths is small (lower than 10). This gives an upper limit to the dilatation coefficient: $\alpha^D \approx 5$. One can see that the values of α are limited to rather small numbers, so that the simplified solution discussed above can be considered to give a qualitative description of the processes involved in spinodal decomposition. The time that the system will spend inside the spinodal region is related to the inverse of the expansion velocity, $t \approx 1/\dot{\alpha}$. Therefore, the system will break according to the instable mean field modes if this time is large enough to allow a good amplification of the fluctuations, *i.e.* when $1/\dot{\alpha} > \tau_K$, leading to the relation:

$$\dot{\alpha} < 1/\tau_K, \quad (45)$$

where τ_K is the characteristic time responsible for the growing of fluctuations in unstable systems. We will see in the following that, in the case of unstable nuclear matter, the instability times are around 50 fm/c (so that $\dot{\alpha} < 0.02$) and, indeed, we will see in the simulations that the considered systems, which fulfill the relation (45), do break according to the most unstable modes of the mean field. For real nuclei the relation (45) corresponds to a maximum velocity at the time of breaking around 0.2 - 0.4 c, depending on the actual size of the system and of the amount of initial fluctuations. For larger expansion velocity the system will directly be vaporized.

3 Testing the dispersion relation in expanding 2D nuclear matter

In order to better illustrate the previous section, we have studied a gas of Fermions situated in a two-dimensional periodic box in expansion. The mean field potential is calculated using a *Skyrme*-like form $U(\rho) = A(\rho/\rho_0) + B(\rho/\rho_0)^2$. If we require, as in the standard three-dimensional matter, a Fermi momentum at saturation of $p_F = 260$ MeV/c, the saturation density ρ_0 corresponds to $\rho_0 = 0.55 \text{ fm}^{-2}$. Moreover, we can require the binding energy per particle to be about 16 MeV per nucleon. This leads to the following parameterization of the *Skyrme* potential: $A = -100.3 \text{ MeV}$, $B = 48 \text{ MeV}$. The compressibility results such that a density doubling leads to approximately zero total energy $E/A(\rho)$ (corresponding to a compressibility modulus of $K \approx 300 \text{ MeV}$ for a calculation in three dimensions). Finally the potential is folded with a

gaussian function of width $\sigma_s = 0.9 \text{ fm}$, which accounts for the range of nuclear forces and for surface effects. The used width mimics the correct value of the nuclear surface energy in 3D.

3.1 Analytical results.

Let us consider, for instance, the simple case of unstable nuclear matter, without expansion, at a density ρ , inside the spinodal region, and at temperature $T = 0$. In such a case, we obtain two opposite imaginary solutions for the frequency ω_k . Fluctuations are exponentially amplified or either depressed with a characteristic time τ_k given from the solution of the dispersion relation [2, 11]:

$$\frac{1}{\tau_k} = k \left(\frac{\epsilon}{\rho} + \frac{\partial U_k}{\partial \rho} \right) \left[-\frac{m}{\rho} \left(\frac{\epsilon}{2\rho} + \frac{\partial U_k}{\partial \rho} \right) \right]^{-\frac{1}{2}}, \quad (46)$$

where k is the wave number of the unstable mode (in fm^{-1}) and ϵ is the Fermi energy associated with the specified initial density ρ . In this equation:

$$\frac{\partial U_k}{\partial \rho} = \frac{\partial U}{\partial \rho} e^{-k^2 \sigma_s^2 / 2} \quad (47)$$

where the partial derivatives of the mean field U are calculated at the actual density.

This dispersion relation has a maximum, corresponding to $\lambda \approx 8 - 10 \text{ fm}$, which has a very small dependence on density and temperature, provided that the system is well inside the spinodal region. The associated growing time τ_k is around 40 fm/c for $\rho = \rho_0/2$, but it is smaller at lower densities inside the spinodal region[11].

Let us give now an expansion velocity $\dot{\alpha}$ to the same system, as considered before. We still obtain, in the approximation of small values of α , where the simplified solution considered in the previous section is valid, two imaginary solutions for ω_k . We note that now the k variable is associated with the coordinate s taken at the final time t . The associated characteristic time τ_k is now:

$$\frac{1}{\tau_k(t)} = k \left(\frac{\epsilon(t)}{\rho(t)} + \frac{\partial \bar{U}_k(t)}{\partial \rho(t)} \right) \left[-\frac{\bar{m}}{\rho(t)} \left(\frac{\epsilon(t)}{2\rho(t)} + \frac{\partial \bar{U}_k(t)}{\partial \rho(t)} \right) \right]^{-\frac{1}{2}}, \quad (48)$$

where we have introduced the time averaged inertia parameter:

$$\bar{m}^{-1}(t) = m^{-1} \int_0^t dt' / t' \frac{\alpha^2(t)}{\alpha^2(t')} \approx m^{-1} \alpha \quad (49)$$

and the time dependent potential:

$$\bar{U}(t) \approx A(\rho(t)/\rho_0)(2\alpha - 1) + B(\rho(t)/\rho_0)^2(3\alpha - 2) \quad (50)$$

This potential is associated with a renormalized range, as defined in eq.(44).

Eq.(48) represents a time dependent correction with respect to the growing time τ_k calculated for static unstable matter. When the expansion velocity is small ($\dot{\alpha} <$

0.02), fluctuations will have time enough to develop and one can conclude that in such a case the dynamics of the unstable system is still dominated by the exponential growing of the most unstable wave lengths. Moreover, in this case, it is possible to observe that the correction terms in eq.(48) are not very important and therefore the most important modes and their associated growing time will essentially keep the same as in the static situation. Therefore the most important wave lengths will appear around $\lambda = 8 - 10 \text{ fm}$ and the associated growing times will be around $30 - 60 \text{ fm}/c$.

3.2 Numerical simulations

We have seen in the previous chapters that in the case of a uniform expansion we simply recover a normal Vlasov propagation except for the boundary conditions and the energy conservation.

In order to account for the time evolution of a piece of nuclear matter at given density and temperature, the Vlasov propagation has been complemented by a Boltzmann like collision term which takes into account the effects of the Pauli blocking. Therefore we solve numerically the transport equation, which describes the time behaviour of the one-body density function f , using a test-particle method [13]. We take explicitly into account the specific boundary conditions. In fact during each time step we look for the particles crossing the time dependent boundaries. At the time of their crossing, the particles are put on the other side of the system and their collective expansion velocity is inverted.

Again special care is taken of the expanding periodic conditions also when treating the collision integral. In fact collisions are implemented within the local expanding frame, i.e. we subtract the expansion velocity when performing the collisions.

We consider the infinite 2D system introduced before, at temperature $T = 3 \text{ Mev}$. Several expansion velocity have been taken: $\dot{\alpha} = 0.003 \text{ c/fm}$, $\dot{\alpha} = 0.006 \text{ c/fm}$, $\dot{\alpha} = 0.012 \text{ c/fm}$.

We notice that, for instance, expansion velocities of the order of $v/c = 0.1$ are obtained for a composite sources of radius about 10 fm formed in central reactions between heavy ions at intermediate energies [14]. This leads to $\dot{\alpha} \approx 0.01 \text{ c/fm}$ which is analogous to the fastest expansion we have considered. However this must be considered just as a qualitative comparison because of the uncertainties on the experimental values and because of the fact that the illustrative calculations we are performing are actually in 2D.

We have done calculations considering different values of the initial density, inside the spinodal region. Our purpose was to study the response of the system to small perturbations of the density associated to unstable wave lengths related to the time τ_k , characteristic of the exponential growing of these modes. This time, calculated for the case $\dot{\alpha} = 0.003 \text{ c/fm}$, is displayed in Fig.1, as a function of different initial densities (thin line), for the most important mode, after a propagation over $50 \text{ fm}/c$. The same growth times are also reported as a function of the final density (thick line). Values similar to the ones calculated in the case of a non expanding system (see ref.[11]) are obtained (dashed line), showing that the expansion, at low velocity, does not affect very much the properties of the dynamical instabilities. As shown in

the figure, the calculated times τ_k correspond approximately to the solutions of the static dispersion relation, associated with the value of the density at an intermediate time between 0 and 50 fm/c .

4 Stochastic mean field approach and fragmentation processes.

An important issue of the study of a system in expansion is to understand the way a dense system is splitting apart. In particular, in the course of its expansion the system may enter a forbidden region of the phase diagram such as the coexistence zone between liquid and gas or a region of instabilities such as the spinodal zone where the system is unstable against density fluctuations.

4.1 Langevin dynamics

In the recent years the direct study of the dynamics of unstable nuclear systems has been made possible with the development of stochastic mean field approaches[5, 6, 7, 8]. Indeed, it is clear from the intuitive point of view that a given unstable system may lead to several different final states. Therefore deterministic approaches such as the one-body theories must be complemented by a source of fluctuations which will allow to test instabilities and spontaneous symmetry breaking.

In particular fully microscopic stochastic mean-field theories have been developed [5, 6]. They lead to the so-called Boltzmann-Langevin dynamics, in which the mean-field evolution is complemented by a fluctuating collision term. It has been shown that these approaches correctly describe the thermal fluctuations of a Fermi gas[7, 8, 15] and allow to describe the clusterization of unstable nuclear matter [2], [10].

Recently, simplified Langevin approaches have been developed; in particular it has been proposed to replace the complex description of the stochastic collisions by a simple random force[4, 11, 12]. It has been shown that, when the strength of the random term is correctly tuned, these simple Langevin approaches provide a good approximation to the complete treatment, even when the system presents instabilities. It has been also tested that the simplest way to introduce a white noise in the dynamics is to use a finite sampling of the phase space, i.e. a test-particle resolution of the mean field equations with a finite number of particles chosen to mimic the full Boltzmann-Langevin dynamics[4, 11, 12]. However, other methods are also possible such as the introduction of random terms in the Hamiltonian[16].

In this study, we will use the simplified Boltzmann-Langevin approach described in refs. [4, 11, 12] in order to illustrate the influence of the expansion on the fragmentation of the system.

4.2 Clusterization in presence of an expansion

Figure 2 presents the results of a simulation of the dynamics of a periodic expanding system inside the spinodal region (at $\rho = \rho_0/2$, $T = 3 MeV$) for four different expan-

sion velocities $\dot{\alpha}$, from 0 to 0.012 c/fm . In our calculations the mean-field $U(x, y)$ was averaged with respect to the transverse direction y so that it then depends on x only. Moreover the system is expanding only in the x direction. In such a case the instabilities will develop only in this direction making easier the analysis of the fragmentation process. The noise used in these simulations has been tuned as described in ref. [4, 11, 12]. These simulations are clearly showing that the fluctuations get amplified by the instabilities and then lead to the disassembly of the system. It can be observed that the faster the expansion the sooner the system is fragmenting. This can be related to the fact that as the system expands one needs less and less density variations to break it into pieces and it is consistent with the estimation of the time τ_k obtained from eq.(48) discussed in the previous section. We have also seen that the most unstable mode, defined as the mode for which the frequency $1/\tau_k$ has a maximum, is nearly independent upon the expansion velocity. Indeed we observe $\lambda \approx 8 - 10 fm$ in all situations, as one can see in Figure 2. It appears also that more expanded and so more diluted systems leads to the formation of more fragments of correspondingly reduced mass.

To get a deeper insight into the description of the fragmentation process it might be interesting to perform an analysis in terms of the instantaneous eigenmodes of the linearized evolution, i.e. to perform a Fourier transform of the density:

$$\rho(k, t) = \int_0^{L_x} dx e^{-ikx} \delta\rho(x, t), \quad (51)$$

where $\delta\rho(x) = \bar{\rho}(x) - \rho_0$ represents the fluctuating part of the density and L_x is the size of the system in the x direction. Moreover the behaviour of a particular event is not relevant; the common features of many of such evolution (generated by running the code many times) have to be considered. Therefore we can introduce the following ensemble averaged quantity:

$$\sigma_k^2(t) = \langle |\rho(k, t)|^2 \rangle = \int \int dx dx' e^{-ik(x-x')} \langle \delta\rho(x, t) \delta\rho(x', t) \rangle. \quad (52)$$

which is nothing but the Fourier transform of the spatial correlation function $\sigma(x - x') = \langle \delta\rho(x, t) \delta\rho(x', t) \rangle$ [6].

In Fig. 3 it is shown the correlation function σ_k^2 associated with the different expanding systems. On this figure we can notice that during the early amplification of the fluctuations of the system the correlation σ_k^2 remains peaked approximately at the same position, whatever the system. This is clearly shown in Figure 4 which presents the wave length $\lambda = 2\pi/k$ associated with the peak in the Fourier transform σ_k^2 , as a function of time. It is clear that until the fluctuations are sufficiently developed the wave length $\lambda \approx 8 - 10 fm$ is dominating the Fourier spectra. This wave length can be related to the most unstable wave length of a static diluted system and it is approximately independent of the actual density, provided the system is far from the boundaries of the spinodal zone (see ref. [11] for more details).

This result is in agreement with our analysis of the monodromy matrix.

After this first phase of amplification of the fluctuations, the fragments start to be formed and then they will fly one apart from each other as the system is continuing

to expand. This introduces a linear increase of the Fourier wavelength proportional to the variation of the mean distance between fragments. This linear increase is clearly seen, at large time, in Fig.4. In the intermediate time between the instability growth and the asymptotic linear increase of the mean distance between fragments the system presents, in the case of the smallest considered velocity ($\dot{\alpha} = 0.003 \text{ c/fm}$) a rapid augmentation of the peak wave length, partly connected with the occurrence of fragment coalescence due to the final state interaction.

One of the advantage of expanding geometries is that the expansion is introducing a clear freeze-out time and allows to define without ambiguities the different fragments. Let us then look to the mean multiplicity of fragments. The evolution of this quantity is presented in Fig. 5. It is clear from this figure that the fragments appear sooner in the system in faster expansion. Moreover the freeze-out time comes shortly after the full clusterization has been achieved. This is also clear from the figure showing the typical fluctuation of the cluster multiplicity: Fig. 6. As far as the magnitude of the multiplicity is concerned one can see that when the fragmentation occurs in a more expanded system more fragments are formed. This is in agreement with the finding that the fragmentation is dominated by a given wave length and that the number of fragments is simply related to the size of the system at the time of fragmentation divided by this typical length. The multiplicity fluctuations appear reduced from the expansion effects. In fact when fragments are formed the curves have a maximum; afterwards fragments separate and stop to interact and then their multiplicity is better defined, leading to reduced fluctuations. Eventually both quantities, the multiplicity and its variance, converge to their asymptotic value when all final state interactions are over.

5 Conclusions

In conclusion, we have studied the instability properties of expanding nuclear matter. We have investigated the small amplitude response of the system and we have derived an approximated dispersion relation valid for finite time propagation, provided that the dilatation of the system remains small. We have shown how the expansion only weakly modifies the most unstable modes. In particular, we have concluded that the expansion slightly increases the effective range which enters in the dispersion relation and consequently increases the wave lengths associated with the observed ultra-violet cut-off. Therefore the system presents approximately the same most unstable wave lengths, unrespectively of the actual expansion velocity. As far as small expansion velocities are concerned, unstable expanding nuclear systems fragment according to the same dominant wave lengths observed for a static piece of nuclear matter. On the other hand, for larger expansion velocities, the system may even not have the time to develop instabilities before reaching too small densities and getting vaporized. In this sense, a critical velocity can be calculated, depending on the characteristics of the considered system and on the amplitude of the introduced fluctuations.

These findings have been illustrated and confirmed by studying numerically the properties of infinite nuclear matter in 2D. Expansion velocities up to $\dot{\alpha} \approx 0.01 \text{ c/fm}$

have been considered. More fragments are formed for more diluted systems, while the dominating wave lengths keep the same. In the present case of volume instabilities, these wave lengths correspond to $\lambda \approx 8 - 10 \text{ fm}$. This occurrence of a fluctuation scale is directly related to the range of the nuclear interactions.

This represents a very important point: the occurrence of volume instabilities in fragmentation patterns can therefore be signed by the development of very specific modes and this characteristic does not depend strongly upon the expansion velocity. These instabilities will therefore lead to a primary partition of the system in approximately equal size fragments, their mass being given by the number of nucleons in a fluctuation of a typical size of $\lambda \approx 8 - 10 \text{ fm}$, at the time and density of the break-up.

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Figure 1:

The time characteristic of the exponential growing of the most unstable mode (thin line), calculated after a propagation over $50 \text{ fm}/c$, for an expanding 2D nuclear system, as a function of initial density inside the spinodal instability region. The temperature of the system is $T = 3 \text{ MeV}$. The expansion velocity is $\dot{\alpha} \approx 0.003c/\text{fm}$. The same times are displayed also as a function of the density value reached after $50 \text{ fm}/c$ (thick line). The dashed line represents the results obtained for a static piece of nuclear matter.

Figure 2:

Time evolution of the density distribution of a piece of infinite nuclear matter, at half normal density and temperature $T = 3 \text{ MeV}$, with boundary conditions. Four different expansion velocities ($\dot{\alpha} \approx 0., 0.003, 0.006, 0.012 \text{ c}/\text{fm}$) are considered. More fragments are formed when the expansion velocity is larger, since fragmentation occurs in a more expanded system while the typical length of the instabilities remains approximately constant. Please note the different times for the different calculations.

Figure 3:

The variance σ_k^2 as function of the node number k shown at different times for the same system of figure 2.

Figure 4:

Time evolution of the wave length λ , in the case of the expansion velocities $\dot{\alpha} \approx 0.003 \text{ c}/\text{fm}$ (thin solid line), $\dot{\alpha} \approx 0.006 \text{ c}/\text{fm}$ (dotted line), $\dot{\alpha} \approx 0.012 \text{ c}/\text{fm}$ (thick line).

Figure 5:

Time evolution of the mean fragment multiplicities obtained from the periodic unstable system, in the case of the expansion velocities: $\dot{\alpha} \approx 0.003c/\text{fm}$ (thin solid line), $0.006c/\text{fm}$ (dotted line), $0.012 \text{ c}/\text{fm}$ (thick line).

Figure 6:

Time evolution of the fragment multiplicity variance for the same system of Figure 5. Expansion velocities are the same as described in Figure 5.

Position x (fm)