FEEDBACK DAMPING OF HORIZONTAL BEAM TRANSFER ERRORS

E. Keil, W. Schnell and P. Strolin
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References
1. **Fundamental Principle**

During the transfer from the PS into the ISR, the proton beam passes among other deflecting magnets — two fast kicker magnets. Their errors induce a horizontal coherent dipole type betatron oscillation of the circulating beam in the ISR with a substantial amplitude. If nothing is done about this oscillation while it is still coherent, it will just cause an increase of the radius of the injected beam by the amplitude of the coherent oscillation.

However, the coherent betatron oscillation of the bunched injected beam can be detected with pick-up stations and the error signal can be fed back into the beam via a suitable amplifier and correction kicker. Since the errors of the kicker magnets are a few percent of their total strengths, and since the forced betatron oscillation remains coherent for some hundred revolutions, the strength of the correction kicker turns out to be about one per mille of the strength of an injection kicker magnet.

In this paper we shall provide a quantitative basis for the discussion of a feed back damping system for horizontal dipole mode injection errors in the ISR. We shall consider the improvements in operation and performance of the ISR to be expected, and indicate a method for the practical realisation of such a system.

The beneficial effect of a substantial reduction of the vertical beam size on ISR performance is much more obvious, but since we expect very small vertical coherent dipole oscillations, vertical feedback damping is not effective.

2. **Beam Transfer Errors between the PS and the ISR**

A detailed review of the errors occurring during the transfer of the PS beam into the ISR is given in the ISR Design Study Report 1). Only those errors which cause errors in position and/or angle of the injected beam are of interest to the present investigation. Current estimates for these errors and their contribution to the coherent betatron amplitude at the maximum horizontal $\beta$ value are listed in Table I.

<p>| TABLE I |
| BEAM TRANSFER ERRORS |</p>
<table>
<thead>
<tr>
<th>Assumed error</th>
<th>ISR amplitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>PS magnetic field</td>
<td>$\pm 10^{-4}$</td>
</tr>
<tr>
<td>PS radial orbit position</td>
<td>$\pm 1$ mm</td>
</tr>
<tr>
<td>PS ejection kicker magnet</td>
<td>$\pm 2%$</td>
</tr>
<tr>
<td>ISR injection kicker magnet</td>
<td>$\pm 1.5%$</td>
</tr>
<tr>
<td>ISR magnetic field</td>
<td>$\pm 10^{-4}$</td>
</tr>
<tr>
<td>Beam radius in the ISR without dipole errors</td>
<td>9.5 mm</td>
</tr>
<tr>
<td>Beam radius in the ISR with dipole errors added linearly</td>
<td>14.0 mm</td>
</tr>
</tbody>
</table>
3. **Estimate of the Coherence Time**

Consider an ensemble of oscillators all starting at the same amplitude and phase. Each oscillator has an error $\delta Q$ in the $Q$ value, and the distribution of the $Q$'s is Gaussian with variance $\Delta Q$. It was shown by Hübner \(^2\) that the $Q$ spread is the biggest contribution to the frequency spread. The centre of gravity $\bar{x}$ of the ensemble is then given by:

$$
\bar{x}(t) = \frac{1}{\Delta Q \sqrt{2\pi}} \int_{-\infty}^{\infty} \exp(-\frac{\delta Q^2}{2\Delta Q^2}) \sin\left[(Q + \delta Q)\omega t\right] d(\delta Q)
$$

(1)

$\omega$ is the angular revolution frequency. Evaluating the integral, (1) becomes:

$$
\bar{x}(t) = \sin Q \omega t \exp(-\omega^2 t^2 \Delta Q^2/2)
$$

(2)

Hence, the coherent amplitude decays like a Gaussian.

This analytical result may be compared to the result of a computer programme which was written to simulate the damping process. It differs from the analytical formula by assuming a distribution in amplitude and phase in addition to the distribution in $Q$ values. Still, the answers obtained by the two methods are in good agreement as is shown in Fig. 1.

Let us define the coherence time $T_c$ as the time when the coherent amplitude has decayed to 1/e of its initial value:

$$
T_c = \frac{T_2}{\omega \Delta Q}
$$

(3)

Changing to the number of revolutions $N_c = T_c f$, where $f$ is the revolution frequency, we have:

$$
N_c = \frac{1}{\pi \Delta Q \sqrt{2}}
$$

(4)

The $Q$ spread in the beam is mainly determined by the variation of $Q$ with momentum

$$
\Delta Q = \frac{dQ}{dp/p} \cdot \frac{\Delta p}{p}
$$

(5)

Let us evaluate separately the momentum spread $\Delta p/p$ and the $Q$ variation $dQ/dp/p$. Since the damping process will take place immediately after injection into the ISR, when the bunch shape has not yet been modified by the RF system in the ISR, the momentum spread $\Delta p/p$ is the same as in the PS, where the half momentum spread is $b = 6.7$ MeV/c at 25 GeV; this value contains, say, 90% of the beam. Assuming the density distribution to be Gaussian we want the following equation to hold:
The momentum dependence of the Q value \( dq/dp/p \) is determined by what spread is required to Landau damp transverse coherent instabilities in the ISR. The following figure was given by Hübner 3):

\[
\frac{dq}{dp/p} = 2.8
\]

Combining all this yields, including a safety factor \( F > 1 \).

\[
\Delta Q = 4.5 \times 10^{-4} \times F
\]

\[
N_c = \frac{1}{\pi\sqrt{2} \times 2.8 F \times 1.6 \times 10^{-4}} = \frac{500}{F}
\]

Thus, even with some safety factor \( F > 1 \); the decay of the coherent betatron oscillations requires a few hundred turns.

We have verified, in the manner used in 4), that the Q variation with betatron amplitude due to the octupole component of the guide field is indeed smaller than the figure (10).

5. Estimate of Damping Constant

The damping mechanism is best explained by a diagram in the normalized phase plane shown in Fig. 2 in which coherent betatron oscillations are represented by circles. The Q spread is disregarded for the time being, and hence coherent oscillations persist indefinitely, in the absence of damping.

Let the centre of gravity of a bunch be at the point \( P \) when passing the pick-up station. Its elongation there is a \( \cos \phi \). The bunch will then move on a circle in the phase plane by an angle \( 2\pi q \) - the electronic phase angle between the pick-up station and the kicker - until it gets into the kicker magnet. There it will receive a kick \( \epsilon \cos \phi \), assuming that the kicker is driven by the pick-up signal and a linear amplifier. From the geometrical construction in Fig. 2 follows that the relative change in amplitude is

\[
\beta = \frac{\Delta a}{a} = \epsilon \cos \phi \sin \psi
\]
where
\[ \phi + \phi = 2\pi q \quad (12) \]

The average damping is obtained by averaging over all initial phases \( \phi \). We find
\[ \bar{\beta} = \frac{\epsilon}{2\pi} \int_0^{2\pi} \cos\phi \sin(2\pi q - \phi) \, d\phi = \frac{\epsilon}{2} \sin 2\pi q \quad (13) \]

The average damping is half as fast as expected from the kicker strength \( \epsilon \). The phase angle between pick-up station and kicker magnet should be close to \( \pi/2 \), the precise phase angle is not very critical. One would thus expect the amplitude \( a \) to obey the differential equation:
\[ \frac{da}{dn} = -a \bar{\beta} \quad (14) \]

which has the solution
\[ a = a_0 e^{-\bar{\beta}n} \quad (15) \]

where \( a_0 \) is the initial amplitude. It is convenient to solve (15) for the number of turns required to achieve a given ratio \( a/a_0 \):
\[ \bar{\beta} n = \ln(a/a_0) \quad (16) \]

A few values are given in Table II.

To get a qualitative feeling for the influence of the coherence time \( T_c \), we may put \( n = N_c \) and calculate \( \bar{\beta} \). We should then remember that, when \( q = 1/4 \), the kicker strength \( \epsilon = 2\bar{\beta} \). The values of \( \bar{\beta} \) and \( \epsilon \) thus obtained are also given in Table II.

<table>
<thead>
<tr>
<th>( a/a_0 )</th>
<th>( \bar{\beta} n )</th>
<th>( \bar{\beta} )</th>
<th>( \epsilon )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.10</td>
<td>2.3</td>
<td>0.004</td>
<td>0.009</td>
</tr>
<tr>
<td>0.05</td>
<td>3.0</td>
<td>0.006</td>
<td>0.011</td>
</tr>
<tr>
<td>0.02</td>
<td>3.9</td>
<td>0.007</td>
<td>0.015</td>
</tr>
<tr>
<td>0.01</td>
<td>4.6</td>
<td>0.009</td>
<td>0.017</td>
</tr>
</tbody>
</table>
Quantitatively, the combined effect of damping and Q spread was studied by a computer programme disregarding the effect of the electronic filters to be described below. It works as follows:

**Initialization**

1) It generates 1000 particles at random with distribution functions in x and x' proportional to \( \exp(-x^2) \) and \( \exp(-x'^2) \) truncated at \( x = x' = 3.5/\sqrt{2} \). This results in a circular beam in normalized phase space with a radial distribution \( \exp(-r^2) \). 90% of all particles are contained within a beam radius \( R = 1.517\hat{u} \).

2) It displaces the whole beam by a distance \( \Delta \).

3) Each particle is given a private \( Q \) value \( Q = Q_0 + \Delta Q \), where the distribution function of \( \Delta Q \) is proportional to \( \exp(-\Delta Q^2/2\Delta Q^2) \), truncated at \( \Delta Q/\Delta Q = 3.5 \).

**One each revolution**

4) The particles are transformed through a phase advance

\[
2\pi(Q_1 - Q_0) = 2\pi \left[ Q_0 + \Delta Q - (Q_0 + \Delta Q) \frac{Q_0}{Q_0} \right]
\]

\[
= 2\pi(Q_0 + \Delta Q_0) \left( 1 - \frac{Q_0}{Q_0} \right) = 2\pi(1 + \Delta Q/Q_0)(Q_0 - Q_0)
\]

5) The position of the centre of gravity \( \bar{x} \) is calculated.

6) The particles are transformed through the phase advance \( 2\pi q_1 \) between the pick-up station and the correction kicker, where \( q_1/Q_0 = Q_1/Q_0 \).

7) The slope of all particles is changed by \( e\bar{x} \).

The parameters of the programme are thus \( a, Q_0, q_0, \Delta Q, \epsilon \). However, one can change to the time scale \( \epsilon n \) where \( n \) is the number of turns. Then the damping and coherence are expressed by one parameter \( \epsilon/\Delta Q \). This leaves only \( a, Q_0, q_0, \epsilon/\Delta Q \) as parameters. An example with \( a = 1, Q_0 = 8.75, q_0 = 0.25 \) is shown in Fig. 3 where the size of the beam containing 90% of the particles is plotted as a function of \( \epsilon n \), with \( \epsilon/\Delta Q \) as a parameter.

We derive from this figure, that good damping is achieved by having \( \epsilon/\Delta Q > 40 \), or, using the \( Q \) spread given in (10):

\[
\epsilon > 40 \Delta Q = 1.8 \times 10^{-2} \text{ F (17)}
\]
The computer programme was also used to check that $q_0$ has no influence on the damping process, and that the effect of $q_0$ is given by (13).

We conclude that a kicker strength $\epsilon = 0.02$ is sufficient.

5. Estimate of the Kicker Strength

The initial coherent amplitude was estimated to be 4.5 mm at the maximum value of $B_H$. This corresponds to an angle $\phi_c$

$$\phi_c = \frac{4.5 \text{ mm}}{0.01 \text{ m}} = 0.113 \text{ mrad}$$ (18)

The kicker angle $\phi_k$ is then

$$\phi_k = \epsilon \phi_c = 2.3 \text{ mrad}$$ (19)

The strength of the kicker becomes

$$(B_k L)_k = (B_0)\phi_k = 0.23 \times 10^{-3} \text{ Tm}$$ (20)

for $B_0 = 100 \text{ Tm} (= 28 \text{ GeV/c})$, or expressed in length * Ampere-turns/cm

$$L \times N \times I = 1.8 \text{ length} * \text{ Ampere-turns/cm}$$ (21)

This strength is about 10% of the strength of the injection kicker magnet.

Assuming a gap height of 2 cm as for the injection kicker magnet and $L = 1 \text{ m}$ we find

$$N \times I = 3.6 \text{ Ampere-turns}$$ (22)

6. Expected Improvement in ISR Performance

There are essentially two extreme possibilities to exploit feedback damping of injection errors in the ISR. One can either improve their luminosity by stacking a larger momentum spread into the additional space made available by the reduction in betatron amplitude, or one can use damping to improve the operational reliability of the ISR.

6.1. Stacking more pulses in the ISR

Let us consider the first possibility, stacking more pulses into the ISR. Fig. 4 shows how the horizontal ISR aperture is used without and with damping. No changes to the size of the injected beam are possible because the damping requires a circulating beam over several hundred revolutions. However, the radius of the stacked beam is reduced
to 9.5 mm. The position of the edge of the stack towards the injection orbit is determined by the screen which protects the stacked beam from the stray field of the injection kicker. Hence, no space is gained between the inner edge of the vacuum chamber and the screen.

The gain in current is a factor $\frac{55}{46} = 1.20$, and the gain in luminosity is the square of this factor, viz. $1.43$. This is certainly quite a worthwhile improvement although it is achieved by increasing the momentum spread in the ISR beam, and, hence, may not be desirable for all ISR experiments.

6.2. Improvements in ISR operational reliability

The improvements in the operational reliability of the ISR are essentially due to the fact that with a damping system it is possible to obtain a good quality stack even when the beam transfer from the PS into the ISR is done fairly poorly, for one or several of the following reasons: unstable running of the PS and of the ISR, difficulties in achieving the required tight tolerances in the setting of the kicker magnets, etc.

The upper limit of the amplitude of coherent oscillation which one is able to tolerate in this scheme of ISR operation is given by the strength of the correction kicker magnet and by the apertures and good field regions of both the injection kicker and the correction kicker. It is possible to reach the situation shown in Fig. 5.

The field of the injection kicker remains within 1% of its design value up to 5.5 mm from the open edge of the kicker magnet 5). This leaves us with a usable good field region of 39.0 mm width in the injection kicker magnet. This value is, at the same time, the maximum permitted beam size. The maximum permissible coherent amplitude then becomes $(29-19)/2 = 10$ mm. This is 2.2 times the value considered above. If we fill the ISR aperture according to the same rules as before we find the stack width becomes 45.5 mm in this case.

Hence we conclude that with a correction kicker with two metres length, two centimetres gap height and 3 Ampere-turns, and with the present aperture of injection kicker magnet we can handle an injected beam with a coherent betatron oscillation of 10 mm amplitude. Despite this increase in the size of the injected beam, we still have practically the same stack width as in the undamped design case.

Another desirable consequence of feedback damping of horizontal injection errors is that the ratio between the horizontal and vertical emittances is closer to unity than in the undamped case. This would remove the harmful consequences of unwanted coupling between the horizontal and vertical betatron oscillations.
7. The Proposed Feedback System

Since the feedback system should affect each bunch individually, at least parts of the feedback loop must be made fast, i.e. with rise and fall times smaller than what corresponds to the free space between bunches. Making the entire loop fast would, however, make it difficult to satisfy the following conditions:

a) The kick applied to each bunch should only depend on the amplitude of the coherent oscillation and not on static errors due to closed orbit displacement or off-sets of the pick-up electrode.

b) The kick should remain approximately constant during the passage of a bunch, in spite of the fact that the pick-up electrode yields a signal proportional to the product of radial displacement and instantaneous longitudinal charge density.

If one had only a single bunch one could satisfy these conditions easily, by inserting a filter transmitting only one of the observable components of the oscillation, e.g. the frequencies \((Q) f_r \) or \((1 - Q) f_r\), where \(Q\) is the fractional part of \(Q\) and \(f_r\) is the revolution frequency.

To adapt this solution to a situation with many bunches we propose the following arrangement (cf. Fig. 6).

The system contains as many independent filters as there are bunches, viz. twenty in our case. Each filter is made to interact with one, and only one, bunch by means of two sets of fast electronic switches. All switches are synchronized to the revolution frequency and each individual switch is closed for a time corresponding to the passage of one bunch. The timing is adjusted in such a way that each filter receives an excitation from only one bunch and applies a kick to that same bunch.

The condition of \(\pi/2\) phase shift with respect to the betatron oscillation can be achieved independently from the switch timing by means of separate phase shifters, one associated with each filter and located somewhere between the two fast switches. Alternatively, one can adjust the betatron phase-shift by means of a single wide-band delay line, situated in the fast part of the system, provided the timing of the switches is adjusted accordingly, so as to avoid interaction between different bunches. Both methods are indicated in Fig. 6.

These phase adjustments could be made fully automatic by deriving them from an automatic measurement \(^6\) of \(Q\). This measurement can be performed at each injection and used to correct the next cycle.

The filters can either be low-pass types with a cut-off at \(0.5 f_r\) and a lower cut-off, below \(0.05 f_r\), say, to reject spurious signals) or band-pass filters ranging from \(0.5 f_r\) to \(f_r\). Higher modes do not seem to offer any advantages.
8. Practical Considerations

The timing pulses for the fast electronic switches can be taken from the existing system for missing bunch phase-lock. The switches themselves, and their drivers, could be of the same type as the ones used in that system. The filters may be similar to the ones already used for automatic Q-measurement. The sensing electrode can be any of the normal beam-observation pick-up electrodes.

The final amplifier, energizing the kicker magnet, will probably have to take the form of a rather large distributed amplifier with several kilowatts input power.

The kicker itself could be a C-shaped ferrite magnet, as assumed in chapter 5 and 6. If sufficient straight section space can be made available one may also consider a vacuum-cored magnet. Such a magnet would be less efficient but one could partially compensate for this by making the magnet longer, taking advantage of the lower specific cost, greater simplicity and faster signal propagation. The requirements on field homogeneity do not seem to be very stringent and there are no high voltage problems.

The main injector kicker requires a mechanically movable screen to shield the stacked beam from the stray field. Whether such a screen is also required for the feedback kicker is still an open question. One may argue that one percent of stray field at the inner edge of the stacked beam, applied in the course of one hundred injection cycles would reproduce about as much blow up at the stack edge as has been removed from each injected beam, prior to stacking. In reality, several percent of stray field may be permissible, since few particles are likely to remain near the inner stack edge for as much as one hundred stacking cycles.

It is planned to build a kicker magnet without a removable screen at first. If the stray field turns out to be too high at the extreme inner edge of the stack (15 mm away from the outer edge of the injected beam), one could still apply damping at the beginning of the stacking cycle, gaining space near the outer wall of the vacuum chamber only. This would yield half of the improvement given in 6.1. A movable screen could then be added later.

Acknowledgements

Our thinking about feedback damping of coherent injection errors was stimulated by a discussion with M. Sands. We should like to thank Miss M. Hanney for writing the computer simulation programme.
References

1. CERN Report AR/int. SG/64-9 (1964)
3. K. Kühner, private communication
4. H.G. Hereward, CERN Report AR/Int. SG 64-8 (1964)
Fig. 1 Coherence time for a beam with Gaussian Q distribution and rms spread ΔQ
Fig. 3 Damping of coherent oscillations

$\alpha = 1 \quad Q_0 = 8.75 \quad q_0 = 0.25$
No horizontal damping

With horizontal damping

Fig. 4

Fig. 5
Fig. 6  Bunch-to-bunch feedback system