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ON THE BIN-WISE AND STRIP INTEGRALS
OF DENSITY FUNCTION
IN HIGH ENERGY MULTIPARTICLE PRODUCTION

Zhang Yang

Liu Lianshou

Institute of Particle Physics, Hua-Zhong Normal University, Wuhan, China

ABSTRACT

It is shown through theoretical analysis and Monte Carlo simulation that when the translational invariance of semi-inclusive density function is destroyed the bin-wise and strip integrals of this function are inequivalent. The different experimental results about the anomalous scaling behaviour of factorial moment in the variables $(y, p_{\perp}, \phi)$ and that of strip integral in the variable Q^2 is discussed briefly.

INSTITUTE OF PARTICLE PHYSICS
HUA-ZHONG NORMAL UNIVERSITY
WUHAN CHINA

50 3505

Recent years have witnessed a remarkably intense experimental and theoretical activity in searching for the scale-invariance and fractality in multihadron production processes, for short also called "intermittency". Using the dynamical variables — rapidity y (or pseudorapidity η), transverse momentum $p_\perp$ and azimuthal angle ϕ , the experimentalists^[1] do find approximate anomalous scaling of factorial moments in various kinds of collisions ranging from e^+e^- annihilation to nucleus-nucleus collisions up to the highest attainable energies.

Theoretically, some author^[7a,8] argued that the method of factorial moments (FM) proposed firstly by Bialas and Peschanski^[6], which corresponds to bin-wise (BW) integral of density function, has the disadvantage of being unable to account for all the particle pairs with $\Delta x < \delta$, where x stands for the variable used and δ is the scale in consideration. Therefore, they propose to count all the particle pairs with $\Delta x < \delta$ instead of to divide the phase space into cells with width δ . This corresponds to the strip (ST) integral of the density function.

Extending this idea to 3-dimensional phase space, it is natural to use the invariant 4-momentum difference square $Q^2 = (p_1 - p_2)^2$ as the appropriate variable and to investigate the scale invariance in this variable. This has been done by UA1 Collaboration^[5] and anomalous scaling has really been found.

In view of this development the following questions naturally arise: Are the BW and ST integration of density function equivalent? What is the relation between the anomalous scaling in the variables $(y, p_\perp, \phi)$ and that in the variable Q^2 ? This paper is aimed to study these questions in some detail.

Firstly, let us notice that the anomalous scaling in Q^2 variable is evidently related to the effect of Bose-Einstein interference (HBT effect). As is well known, there is symmetry between the quantum wave functions of two identical bosons, which leads up to more possibility for them to appear nearer in the momentum space. If this B-E effect was the main source of intermittency, the intermittency index for identical particles would be much larger than that of unidentical particles. In particular, for the system whose size of interacting source is small ($R \leq 4\text{fm}$), e.g. in e^+e^- and πp collisions, the intermittency index φ_i^{ls} of negative pions is expected to be roughly twice as large as that of all the charged pions, i.e. $\varphi_i^{ls} \approx 2\varphi_i^{all}$ ^[3]. The factorial moments of identical and unidentical particles in 1-dimensional rapidity space^[4] and 3-dimensional (y, p_T, ϕ) space^[12] have been studied experimentally. The results, however, show no difference between φ_i^{all} and φ_i^{ls} . A little later, the same data as Ref.[4] have been analysed in the variable Q^2 ^[5]. It turns out that in this case the relation $\varphi_i^{ls} \approx 2\varphi_i^{all}$ is fulfilled for all orders i . This controversy is an experimental indication that the anomalous scaling in the variables (y, p_T, ϕ) and that in the variable Q^2 are probably inequivalent.

In order to check this assertion more carefully, let us turn to the comparison of BW and ST integrals of density function.

The BW integral is equivalent to the factorial moment defined as

$$F_i = \frac{1}{M} \sum_{m=1}^M \frac{\langle n_m(n_m - 1) \dots (n_m - i + 1) \rangle}{\langle n_m \rangle^i} = C_i, \quad (1)$$

where the rapidity space[†] is divided into M sub-bins, and n_m is the multiplicity in the

[†] For simplicity, we shall consider in the following the 1-dimensional rapidity space as an example.

m 'th sub-bin. As has been shown in Ref.[6], the factorial moments F_i can eliminate the statistical noise owing to limited multiplicity and is equal to the i 'th moment C_i of the probability distribution $P(p_1, p_2, \dots, p_M)$. (Here p_m is the probability for finding particle in the m 'th sub-bin)^[6]. Since the integrals of density function $\rho(y_1, y_2, \dots, y_i)$ are simply related to counting the number of points lying in the integration volume^[7], the factorial moment F_i can be expressed by the integral of density function in small rapidity intervals

$$F_i = I_i^{\text{BW}} = \frac{1}{M} \sum_{m=1}^M \frac{\int_{\Omega_m} \prod_k dy_k \rho_i(y_1, \dots, y_i)}{\int_{\Omega_m} \prod_k dy_k \rho_1(y_1) \dots \rho_1(y_i)}, \quad (2)$$

where the integration domain of y_k , $\Omega_{\text{BW}} = \sum_{m=1}^M \Omega_m$, consists of M i -dimensional boxes Ω_m with edge length equal to δy . It is, therefore, called bin-wise integration.

The strip integral, on the other hand, is defined in the similar manner as in Eq.(2) but the integration volume is changed from Ω_{BW} to Ω_{ST} , which is the strip region consisting of all the points whose distance to the axis $y_1 = y_2 = \dots = y_i$ in i -dimensional space is less than some scale δy . In Fig.1 are shown the integration regions Ω_{BW} (solid lines) and Ω_{ST} (dashed lines) respectively for the case of $i = 2$.

In order to compare FM and ST integrals, let us divide the string of squares Ω_{BW} into two sets of triangles Ω_a and Ω_b , a typical one of each set is labeled in Fig.1. (Here we only discuss the case of $i = 2$. The higher order cases are straightforward extension of it).

$$I_2^{\text{BW}} = \int_{\Omega_{\text{BW}}} \rho_2(y_1, y_2) dy_1 dy_2 = \int_{\Omega_a} \rho_2(y_1, y_2) dy_1 dy_2 + \int_{\Omega_b} \rho_2(y_1, y_2) dy_1 dy_2, \quad (3)$$

In the mean time, Ω_{ST} can also be divided into four parts:

$$\begin{aligned} I_2^{\text{ST}} &= \int_{\Omega_{\text{ST}}} \rho_2(y_1, y_2) dy_1 dy_2 \\ &= \int_{\Omega_a} \rho_2(y_1, y_2) dy_1 dy_2 + \int_{\Omega_b} \rho_2(y_1, y_2) dy_1 dy_2 \\ &\quad + \int_{\Omega_c} \rho_2(y_1, y_2) dy_1 dy_2 + \int_{\Omega_d} \rho_2(y_1, y_2) dy_1 dy_2, \end{aligned} \quad (4)$$

where Ω_c and Ω_d are two sets of triangles, a typical one of each set being labeled in Fig.1. It is evident that

$$\begin{cases} \int_{\Omega_c} \rho_2(y_1, y_2) dy_1 dy_2 = \int_{\Omega_a} \rho_2(y_1, y_2 + \delta y) dy_1 dy_2 \\ \int_{\Omega_d} \rho_2(y_1, y_2) dy_1 dy_2 = \int_{\Omega_b} \rho_2(y_1 + \delta y, y_2) dy_1 dy_2 \end{cases},$$

therefore, the comparison between I_2^{BW} and I_2^{ST} is meaningful only when there is translational symmetry in the two-particle density function $\rho_2(y_1, y_2)$ so that the following equations hold approximately

$$\begin{cases} \int_{\Omega_a} \rho_2(y_1, y_2 + \delta y) dy_1 dy_2 = \int_{\Omega_a} \rho_2(y_1, y_2) dy_1 dy_2 \\ \int_{\Omega_b} \rho_2(y_1 + \delta y, y_2) dy_1 dy_2 = \int_{\Omega_b} \rho_2(y_1, y_2) dy_1 dy_2 \end{cases}. \quad (5)$$

In the contrary case, when Eq's.(5) do not hold, I_2^{BW} and I_2^{ST} will not be simply related (also see Ref.[10]).

The two-particle density function $\rho_2(y_1, y_2)$ can be written as

$$\rho_2(y_1, y_2) = \rho_1(y_1)\rho_1(y_2)(1 + G_2(y_1, y_2)), \quad (6)$$

where $\rho_1(y)$ is the single-particle inclusive distribution, $G_2(y_1, y_2)$ is the normalized two-particle correlation function. Note that the latter is usually denoted by C_2 . Here we use another notation to avoid confusion with the probability moment C_i in Eq.(1). In this form it is clear that the anomalous scaling of F_2 may come from two origins. Firstly, the inclusive distribution function $\rho_1(y)$ may be fractal, and secondly the two-particle correlation function $G_2(y_1, y_2)$ may have power-like singularity

$$G_2(y_1, y_2) \propto \frac{1}{|y_1 - y_2|^{f_2}}. \quad (7)$$

In both of these cases the factorial moment will possess anomalous scaling property.

Without losing generality the overall averaged inclusive distribution may be assumed to be flat.[†] Then the first factor $\rho_1(y_1)\rho_1(y_2)$ in the RHS of Eq.(6) will have translational invariance in the average, so that its contribution to both I_2^{BW} and I_2^{ST} will be the same. On the contrary, the second term $G_2(y_1, y_2)$ in the RHS of Eq.(6) may violate the translational invariance and makes I_2^{BW} and I_2^{ST} differ.

As a demonstration, we have made Monte Carlo simulation of the random cascading α -model^[6,9]. Just as in Ref.[9], in order to keep the conservation of probability in each step of cascading we have choiced the independant random variables of cascading process with λ pieces in each splitting as

$$\omega_i = \frac{1 + \alpha r_i}{\lambda + \alpha \sum_{i=1}^{\lambda} r_i}, \quad (8)$$

where r_i is the even-distributed random number in $[-1, 1]$, and α denotes the fluctuation strength of the model. In this model the inclusive rapidity distribution $\rho_1(y)$ of the final particles is fractal and flat, i.e. the $\rho_i(y_1, \dots, y_i)$ is translational invariant. In order to see the contrary, we might draw some singular correlation in this cascading model as following. In the simulation of each event, the rapidity y_1 of a particle is produced at first by using the above α -model generator and the rapidity y_2 of the second particle is gotten randomly with the sampling function of Eq.(7); Then the third particle is produced just as the first one and the fourth as the second one, and so on until all of the N_{ch} particles are produced. It should be noticed that in this MC process the inclusive distribution lost its flat property because of the correlation introduced. In order to eliminate this extra factor, we have used the integral variable suggested in Ref.[11]

$$x(y) = \frac{\int_0^y \rho(y) dy}{\int_0^{y_{\text{max}}} \rho(y) dy}. \quad (9)$$

In the Monte Carlo simulation of this model, we have chosen for definiteness the parameter as: $\lambda = 2$, $\alpha = 0.8$, $f_2 = 0.8$ and $N_{\text{ch}} = 8$. In Fig's.2a,c is shown the comparison

[†] In the contrary case, the integral variable x ^[11] may be used to make the overall distribution flat.

between the results of second and third order BW and ST integrals of the density function of 1000 Monte Carlo events. The power-law equation $\ln I_q = \varphi_q \ln(1/dx) + b_q$ has been fitted to the MC data in least square method. The corresponding fitting parameters are collected in Table I. It can be seen from Fig.2a,c and Table I that when the translational invariance of the semi-inclusive density function is destroyed, the BW and ST integrals are indeed inequivalent in range of statistical errors; In Fig's.2b,d is shown the similar data of 10000 MC events, which confirms the same conclusion with much smaller statistical errors.

This model as it stands is unessential, but it confirms the assertion that the methods of strip integral and that of bin-wise integral are in general inequivalent. Therefore, the experimental findings of the anomalous power law for the factorial moments in the variables (y, p_T, ϕ) , related to BW, and that in the variable Q^2 , related to ST, are probably independent phenomena. From this point of view, the apparent controversy of $\varphi_i^{ls} \approx 2\varphi_i^{all}$ in the ST integral with variable Q^2 and $\varphi_i^{ls} \approx \varphi_i^{all}$ in the FM with variables (y, p_T, ϕ) is understandable. So we must take care when we try to conclude the physical origin of the intermittency from the experimental data in different methods.

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Figure Captions:

- Fig.1** The integral region of the correlation function $\rho(y_1, \dots, y_i)$ in case of $i = 2$. The solid line corresponds to the region for FM, the dashed one to that for STI. The dashed-dotted one is the line $y_1 = y_2$.
- Fig.2** The MC results of the distorted α -model with some rapidity correlation (see Eq.(7) of text) introduced. " Δ " denotes the results of Bin-Wise integrals and "o" denotes those of strip integrals of density function in 1000 (a,c) and 10000 (b,d) MC events respectively. The corresponding parameter of α -model is given in the text.

Table Caption:

Table I Results of the fit of the MC data presented in Fig.2a,c to the equation $\ln I_q = \varphi_q \ln(1/dx) + b_q$.

second order			
	φ_2	b_2	χ^2/NDF
BW	0.4432 ± 0.0030	-0.1235 ± 0.0036	96/28
ST	0.2796 ± 0.0014	-0.0778 ± 0.0005	581/29
Third order			
	φ_3	b_3	χ^2/NDF
BW	0.8587 ± 0.0087	-0.1933 ± 0.0088	14/28
ST	0.6302 ± 0.0039	-0.2188 ± 0.0011	99/29

Fig.1

Fig.2