UZU - A PROGRAM WHICH SIMULATES

BUBBLE CHAMBER TRACKS

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The program UZU\textsuperscript{c}) monitors the geometrical reconstruction programs in the bubble chamber analysis by producing simulated tracks with given initial parameters. It can be applied to any bubble chamber equipped with a homogenous magnetic field, a parallel plate window, and three cameras whose optical axes are perpendicular to it. By means of control cards, it can simulate the tracks i) of pure helix, ii) with measurement errors, iii) with single or multiple scattering, iv) with ionization loss, v) with bremsstrahlung, or vi) any other combinations of these.

It can test the reconstruction programs in various ways, such as i) mathematical correctness of the program, ii) the mean values by the reconstruction program, iii) their variances, iv) the adequateness of various criteria (for the scanning or the measurement) and of corrections.

For those who are only interested in what UZU does, it will suffice to read Section 4, "Application of UZU", and for those who wish to run UZU, to read Appendix 1.

\textsuperscript{c}) UZU means "vortex" in Japanese.
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REFERENCES
1. **INTRODUCTION**

There exist two ways of looking at the bubble chambers. At early stages of a newly opened branch of physics, a bubble chamber serves as an excellent tool to pick up phenomena so far completely unknown to us. It fixes those instantaneous events on the photographic material, and then makes them subject to a thorough, permanent analysis. It visualizes in space the actual paths of the charged particles, their ionizing power, and their decay modes, thus enabling us to identify each of the multi-produced particles in \(4\pi\) geometry. A good example of this kind of use of the bubble chamber is the discovery of \(\Omega\) particle. The other way of looking at it, a more important and inevitable consequence of the first case, is the use of the bubble chamber as an accumulator of a number of same physical events. However, in some regions of physics, where the interpretation is unequivocal, this role of bubble chambers is sometimes challenged by the counters or spark chambers, since the accumulation of events with those detectors is much more efficient, and the reconstruction of the events in space is comparatively simple. Nevertheless, the bubble chamber still maintains its big advantage where the use of a magnetic field is indispensable in order to determine the momentum of the particle, due to its enormous solid angle. Not only this, but the visibility of vertex has a great advantage over other instruments, and particle identification is much more readily done in a bubble chamber by examining the ionization density\(^1\), decay modes, scattering, multiple-scattering, interaction, delta rays\(^2\), etc. In particular, these advantages are well demonstrated in rather low-yield experiments, such as the neutrino experiment, and this fact is confirmed by the building of larger and larger bubble chambers in order to increase the yield. A very simple question arises at this point: apart from increasing the size of bubble chambers, what is the development status of the chamber as a precise instrument. Determining the kinematical quantities in the photographed picture inevitably leads us to measure the points on the tracks, reconstruct them geometrically in space, and calculate the desired quantities according to various computational proscriptions. At this point it would be instructive
to recall those factors that may affect the estimation of kinematical quantities with good accuracies.

For the sake of illustration, the factors could be divided into two groups: 1) pre-photographic stage; 2) post-photographic stage; and their sub-groups. The pre-photographic stage can then be classified into two groups, one based on pure physical phenomena [1a]), the other being more technical, with the possibility of improvement through the development of the machine [1b]).

1a) i) Momentum loss by ionization. This makes the particle orbit deviate from the circle in the magnetic field to the spiral. At present, most of the reconstruction programs use the helix or circular fit. In such cases, the correction must be done accordingly.

ii) Multiple scattering. This is a purely statistical effect. Therefore the systematic correction cannot be carried out basically, as in the case of (i). One can only be satisfied with putting certain error limits to the results obtained. However, from the degree of multiple scattering, it is also possible to improve the estimate of the momentum (ELM)\textsuperscript{3}).

iii) Single scattering. If the large-angle scattering is overlooked and the circle is fitted, the error is introduced. To avoid this, the tracks are dissected and circles are fitted to the dissected parts independently, and later on the results are merged.

iv) When the particle is light, bremsstrahlung becomes an important effect. This again causes the track to deviate from the circle.

The combined effect of (ii) and (iv) leads to an optimum length measurement [Behr-Mittner\textsuperscript{4}].

1b) i) Magnetic fields and track length. If the sagitta of the curved track is small, it is obvious that the inaccuracy of the curvature measurement increases. In Fig. 1 the relation between
curvature and sagitta is given with the track length as a parameter. For instance, in the magnetic field of 20 kg, a 12 cm long track of 10 GeV/c has the sagitta of 100 microns. At present, this is about the limit of measurement. In designing any measurement rules, Fig. 1 is useful in this respect.

For a given track length and momentum, the sagitta increase is directly proportional to the magnetic field.

Non-uniformity of the magnetic field is also a source of error for the helix fitting.

ii) Distortion. The mechanical disturbance is imposed upon the tracks due to the turbulence of the liquid during the expansion. This effect increases with the light-flash delay\(^5\). The thermal effect, such as inhomogeneity in temperature over the chamber, causes anisotropy in the refractive index of the liquid, which may well bring about the error of tens of microns\(^6\). The inhomogeneity in illumination often leads the measurer to misjudge the real centre of the bubble, causing an error of several microns.

In contrast to those pre-photographic factors, the post-photographic factors are mainly concerned with the measurement [2a]] and the data-handling\(^7\) [2b]]. The measured track data are mathematically processed, and are made available to the physicists in an accessible form.

2a) i) Pre-measurement stage (i.e., scanning). The physicists decide what length should be measured, kinks or non-kinks, density of measurement points (cell length), and so on, based on their self-made measurement criteria.

ii) Measurements errors. Errors caused by the inaccuracy of measurement origin (jumps, drift, etc.), by the distortion of film. Errors caused by the measurer.
iii) Post measurement stage (i.e. data handling). Although this stage is only concerned with mathematical processing, and probably just because of it, the most unexplored part of the whole chain as the cause of error must lie in this stage.

Let us study point 2a), iii), in more detail. In recent years, the fast development of computers allowed the design of larger, more sophisticated, and more complex reconstruction programs. The number of these programs would easily amount to ten$^8,9,13$), and, if one includes the modified ones, probably to hundreds. The programs are usually written (and rewritten) by a minority of physicists or applied mathematicians, and their structures are such that it is sometimes beyond the users' (a majority of physicists) time and effort to comprehend them. This situation is not completely unrelated to the programmer as well. True, he himself has written a program, but often he encounters a predicament in that he questions whether the program really functions exactly as he would have expected. There is a geometrical concept of, for instance, a "corresponding point"; but, with the existence of various errors, should the apex still be treated as a corresponding point to be a constraint in fitting curves$^10$)? Or, what is the cut-off for rejecting a reconstructed point which is off in space from other points? Is it not really due to a scattering, and should not the track be measured in two sections and joined together later on? There exist hundreds of such problematic factors involved in the program; but the physicist's main concern is to decide whether those factors, chosen by the programmers for definite reasons, may affect and bias the final results in which he is interested. Also, he must be well acquainted with the effect of his own scanning and measuring criteria [2a), i)].

The physicist can test this in various ways. For instance, he sends the momentum-analysed beam into the chamber, takes the photograph, and then measures and checks the computer estimate against the original beam momentum. Or, he measures the two-body decay product from a stopping
particle and compares the measured momentum with the kinematically
calculated value (e.g. for $K^+ \rightarrow \mu^+ + \nu$ the muon momentum should be
236 MeV/c for stopping kaon). In fact, these are ways which are
often used to test the consistency of the measurement. However,
this method has certain disadvantages:

i) It is a quite expensive and time-consuming operation, since
the machines must be operated.

ii) The error involved in the final result is the over-all error.
Its reduction into its elementary factors is not entirely
impossible, but it can be very difficult.

iii) The mass of the particle, the momenta, the azimuthal angles,
the dip angles, the vertices, etc., cannot be varied simply
in the wide range.

At this point, it is conceivable to write a program which simulates
the measurement of the bubble chamber tracks. It would produce an
input tape to any reconstruction program with any parameters which
can be controlled by the tester (e.g. mass, curvature, etc.).

2. **PROGRAM UZU**

A brief description of such a program appeared in a CERN report
in 1962\(^{11}\). This was called artificial track construction (ATC) and
was used to test the reconstruction program at that time. Also there
is Johnson's work\(^{12}\). The writing of UZU began independently of ATC,
in the autumn of 1964 in FORTRAN IV, in order to monitor the continually
developing and improving reconstruction programs then being used for the
CERN neutrino experiment. Since then, UZU has continued to develop,
and it is thought that now is the time to report on the status thus far
achieved. The program UZU is capable of testing the geometrical recon-
struction programs, e.g. THRESH\(^{13}\), PHT\(^{8}\), etc., in various ways such as:
i) the mathematical correctness of the reconstruction program, by setting
all the error parameters in UZU to zero; ii) the mean values given by
the reconstruction program, by introducing proper errors; iii) the compari-
son between the quoted error by the reconstruction program and the actual
standard deviation of the reconstructed values of a number of tracks produced in UZU, by the Monte Carlo method; iv) the adequacy of various criteria and corrections.

The program UZU can be applied to any bubble chamber equipped with a homogenous magnetic field, parallel glass-plate window, and three cameras whose optical axes are perpendicular to the window surface. With these conditions provided, the program UZU is able to produce simulated tracks with or without scattering and/or ionization loss, project them onto the three photographic planes (films), and record a series of numbers in BCD mode, whose format is that required by REAP or any other input programs.

The size of UZU is ~ 25,000 words and has the following three main branches, of which the third is separable from the others when needed (e.g. the whole UZU is too large for a small computer).

UZU1 generates a helix or series of helices with fixed momentum and labels it, for example, AA(--------)AI, disregarding the occurrence of the single scattering. The scattering in UZU is the Rutherford scattering from the nuclei in the liquid [nuclear effect neglected]. The frequency of the single scattering is controlled by cut-off (max and min) angles in the deck (called TMAX and TMIN). (For the moment, the maximum number of scatterings on a track is chosen to be 496, otherwise a diagnostic message occurs and the computation goes over to the next track.)

UZU2 generates a helix or series of helices with fixed momentum like UZU1, but whenever a single scattering occurs it labels as a new vertex. Consequently, the scattering point is treated as a corresponding point. Labelling follows AA, HH, II, JJ, and the maximum number of scatterings per track is 3. If more, a diagnostic message occurs. To avoid this, one has to choose the proper values of TMAX and TMIN. UZU2 furnishes the information about whether the joining of the dissected tracks is adequate enough to give a reasonable curvature with a reasonable error.
The reason that the single scattering is treated separately from the multiple scattering in UZU, is that by subjecting the less complicated tracks (i.e. with only single scattering or, say, model tracks) to the analysis, one has a better chance of obtaining a clear, over-all picture of the adequacy of the reconstruction procedure underway. If, however, one is interested in the realistic tracks (with multiple scattering), this is immediately taken into account by inserting a control card (No. 14) with an answer "YES" asking UZU to work on the multiple scattering plus single scattering. (See Appendix 1 regarding the control card, Appendix 2 for the multiple scattering.)

UZU3 takes into account the ionization loss and bremsstrahlung, as well as the single and/or multiple scattering. The generation of this sort of track is done not analytically, as with UZU1 and UZU2, but numerically; consequently the range-momentum list is loaded into the program. (For the procedure, see Appendix 1.)

In order to obtain a general view of UZU, the reader is referred to the flow charts in Fig. 2. The flows for UZU1 and 2 are almost alike, whereas for UZU3, a subroutine called SHELL does the main part of the job. The flow chart inside SHELL is shown in Fig. 3. In both figures, the names in capital letters are subroutine names which are used in that part of the flow.

The typical example of the full outputs of UZU1 and 2 are shown in Figs. 4, 5 and 6. Although those listings can be suppressed partly by data cards, for the sake of illustration the full outputs are printed here. Figure 4 illustrates the case where the pure helix is produced. The heading is the track card number and its data. It shows that the track is specified to leave a point (0.0, 0.0, -50.0) in space of the chamber with a radius of curvature (RHO) of 30.0 cm, whose tangential direction (PHI) is making an angle of zero radians to the X-axis and the dip angle (DIP) 1.0772 rad. The maximum length (PRL) is specified as 35 cm. When the produced helix reaches this length, the program automatically goes to the next track. THMIN and THMAX are the cut-off angles for the scattering. In this case, in order to make a pure helix, both
numbers are taken rather large (86 and 115 degrees) so as not to yield scatterings on the track. \( \text{FMAX}, \text{FMIN}, \text{SCHN} \) are only for the matter of monitoring. This is followed by a sentence beginning with NEW CHAPTER, etc., specifying the measurement errors introduced to this job. From the next line, the description of the produced track begins. Except for the heading of each track and the number of scatterings, the printing is optional from here onward. The Roman characters I, II, III, IV signify the four fiducial marks, whose projected positions in centimetres on the film are listed. From left to right, the first two blocks are \( X,Y \) positions without error (therefore those numbers stay the same throughout); the next two blocks are with errors. The origin of the \( X,Y \) coordinates is the optical centre of the camera. The fiducial mark listing is followed by the track description. Under given initial conditions of a track, the program looks for the centre of the helix, and each point on the track is represented by an azimuthal angle between the \( X \)-axis and the line combining this point and the centre of the helix: this azimuthal angle is called \( \text{THETA} \). Therefore, if the numbers of \( \text{THETA} \) in three views are the same, this signifies a corresponding point. In the listing, \( \text{THETA} \) and the actual depth of the point (\( Z \)) are printed as well as the projected coordinates on the film without and with error (\( X_X, Y_Y; X_3, Y_3 \)). Finally the content of the output tape is printed up to 200 lines if so desired.

In Fig. 5, the same UZU1 produced a track with three scatterings. Note the change in \( \text{THIN} \) and \( \text{TMAX} \) (1.1 and 28.6 degrees this time) which brought about the occurrence of the scattering. In the listing of the track, the data for the new branches appear. The \( X, Y \) and \( Z \) coordinates indicate the scattering position whose scattering angle is represented as \( \text{SCAT} \). \( \text{RHO}, \text{PHI}, \text{DIP}, \) and \( \text{PRL} \) are the radius, azimuthal angle, dip angle, and the length of the new branch, respectively. In the listing of measured point coordinates, the heading of \( \text{THETA}, \text{ZZ}, \text{XX}, \) etc., repeats itself as the new branch appears. As explained in the previous section, since UZU1 ignores the scattering (except for the first branch), the first point of each branch is not necessarily a corresponding point, as one sees from the difference in \( \text{THETA} \) in different views.
This computation is carried out in exactly the same way in UZU2 mode. Its output is shown in Fig. 6. In this case, the first point of each branch is measured as a corresponding point and labelled accordingly, as seen at the foot of the page.

For certain special tests, UZU is also able to perform the following computations: i) all measured points are corresponding points; ii) a fixed length to the first scattering point; iii) a fixed scattering angle at the first scattering point. These are the special features added to the program UZU and they are controlled by the data cards (Appendix 1).

3. REMARKS ON THE COMPUTATIONAL METHOD IN UZU

The plane made by the X- and Y-axes comprises the inner surface (the liquid side) of the glass window. The positive direction of the Z-axis is towards the camera, and all the points in the chamber have negative Z-values. The orientation of the X- and Y-axes is chosen in such a way that the X-, Y-, and Z-axes make a right-handed system. This is the convention of our coordinate system (Fig. 7).

For the projection of a space point on the film, the polar coordinate around the camera axis is used in UZU1 and UZU2, where the helices are produced. In this case, it has an advantage over a Cartesian system, since the space point is represented by a single parameter THETA, the azimuthal angle measured from the centre of the helix. For instance, the track length is simply computed by \((\text{THETA}) \times (\text{radius of curvature})/ \cos(\text{dip angle})\), thus facilitating the determination of scattering points or measured points which are given only in the real lengths along the track, and since the tangential angle of the track is always given by \(\text{THETA} + \left(\frac{\pi}{2}\right)\), the mathematical complexity of designating the initial parameters of the next branch in case of scattering is very much reduced.

In order to determine the image of a point on the film, it is sufficient to give the distance from the point to the camera axis and the depth of the point. The three pieces of the light-line, PA, AB, BH are coplanar, and are computed by iteration (Fig. 7). Figure 8 shows the relation of the variables that appear frequently in this part of computation.
With the computer of 36 bits/word (IBM 7090, for instance), the accuracy of a floating point number is limited to nine digits. When the centre of the helix O, the point P, and the camera axis C lie almost on a straight line, one has to be cautious about the error resulting from this limitation. Let us suppose, for instance, that the distances OP and OC are 20 and 30 cm, respectively. If CP becomes 10 cm, the points OPC should be theoretically on a straight line. However, in a 36-bits computer, 30.0 cm can be equivalent to 29.999 9998 cm. Supposing that other figures are exact, then if one calculates RC (Fig. 9), it will be about 25 microns. This figure is rather large when we speak of a 100 microns measurement accuracy (in space), although the inaccuracy quoted here is not along the sagitta but along the track. Therefore, in UZU we adopted a double precision computation of space-point coordinates. If a machine of more than 36 bits/word is used, this precaution is unnecessary.

In UZU3 the tracks are generated in a way which is slightly different to that described above (see flow chart, Fig. 3). Starting out as usual from a set of initial values which determine a helix, one extends the track until its sagitta becomes a predetermined value. This value is optional; at present it is taken to be one micron. The extended track length ΔL is therefore curvature dependent. Then the program asks whether a scattering or bremsstrahlung occurred in this segment (ΔL). If neither occurred, it then computes the new momentum caused by the ionization loss passing through this segment, and this new momentum determines the radius of curvature of the next segment. This continues, keeping the sagitta always constant; but the segment becomes obviously smaller as the momentum becomes smaller. The total number of segments per track at present allowed is 1500. If a scattering took place, its new direction is determined. If bremsstrahlung occurred, its new momentum and new direction are determined (one has to specify a cut-off momentum of bremsstrahlung PMIN in the data deck). Also, a new range is determined according to the new momentum. Those numbers can be all printed out if necessary. Unfortunately, the effect of the chosen value of the sagitta is not very small on the generated tracks. This effect is illustrated in Fig. 10. The value should be chosen according to the purpose.
4. APPLICATION OF UZU

If not otherwise stated, the reconstruction program used in this analysis is DRAW\textsuperscript{6)}, updated in May 1966. The chamber is the CERN HLBC whose constants are summarized below:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>radius of the chamber</td>
<td>45 cm</td>
</tr>
<tr>
<td>depth of the chamber</td>
<td>95 cm</td>
</tr>
<tr>
<td>magnetic field strength</td>
<td>18.95 KG</td>
</tr>
<tr>
<td>refractive index of the glass</td>
<td>1.522</td>
</tr>
<tr>
<td>thickness of the glass</td>
<td>33.94</td>
</tr>
<tr>
<td>distance to camera from front of glass</td>
<td>186.37</td>
</tr>
</tbody>
</table>

coordinates of camera 1: \( X = -24.26, \ Y = -42.00 \)
coordinates of camera 2: \( X = 48.50, \ Y = 0.00 \)
coordinates of camera 3: \( X = -24.25, \ Y = 42.00 \)
coordinates of fid. mark 1: \( X = -20.63, \ Y = 20.33 \)
coordinates of fid. mark 2: \( X = 20.39, \ Y = 20.47 \)
coordinates of fid. mark 3: \( X = 20.47, \ Y = -20.53 \)
coordinates of fid. mark 4: \( X = -20.46, \ Y = -20.66 \)

with a right-handed coordinate system with \( Z \) along the magnetic field and origin at the centre of the window interface to liquid.

Properties of liquids:

<table>
<thead>
<tr>
<th>Chemical</th>
<th>Density</th>
<th>Refractive index</th>
<th>Radiation length</th>
</tr>
</thead>
<tbody>
<tr>
<td>Freon</td>
<td>1.52</td>
<td>1.23</td>
<td>10 cm</td>
</tr>
<tr>
<td>Light Freon</td>
<td>1.22</td>
<td>1.20</td>
<td>24 cm</td>
</tr>
<tr>
<td>Propane</td>
<td>0.44</td>
<td>( \sim 1.25 )</td>
<td>109 cm</td>
</tr>
</tbody>
</table>

4.1 Production of the circles, Internal (truncation) error.

We will start with the simplest case of a circle of fixed radius \( (r = 180 \text{ cm, } \lambda = 0.0, \varphi = 0.0) \) changing the measured length of \( L \) from 3 cm to 30 cm. Only the vertex (starting point) is the corresponding point,
and the measurement errors are set to zero. The measurement density (cell length) is $1.0 \pm 0.1$ cm. The measurements are repeated 30 times. The average of the fitted radii of those 30 tracks and the standard deviation from this average are computed. The following table and Fig. 11 give the summary of this run.

<table>
<thead>
<tr>
<th>Length measured</th>
<th>Average of fitted rad.</th>
<th>r.m.s. of fitted rad.</th>
<th>Average of quoted error by DEAT</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.0</td>
<td>107.2</td>
<td>23.7</td>
<td>114.2</td>
</tr>
<tr>
<td>5.0</td>
<td>167.3</td>
<td>24.4</td>
<td>78.6</td>
</tr>
<tr>
<td>7.0</td>
<td>172.4</td>
<td>11.1</td>
<td>35.1</td>
</tr>
<tr>
<td>9.0</td>
<td>173.0</td>
<td>6.5</td>
<td>18.9</td>
</tr>
<tr>
<td>11.0</td>
<td>178.0</td>
<td>5.0</td>
<td>12.3</td>
</tr>
<tr>
<td>13.0</td>
<td>177.3</td>
<td>3.0</td>
<td>7.8</td>
</tr>
<tr>
<td>15.0</td>
<td>178.4</td>
<td>2.0</td>
<td>5.5</td>
</tr>
<tr>
<td>18.0</td>
<td>179.28</td>
<td>1.31</td>
<td>3.6</td>
</tr>
<tr>
<td>22.0</td>
<td>179.52</td>
<td>1.07</td>
<td>2.1</td>
</tr>
<tr>
<td>26.0</td>
<td>179.93</td>
<td>0.63</td>
<td>1.46</td>
</tr>
<tr>
<td>30.0</td>
<td>179.56</td>
<td>0.38</td>
<td>0.98</td>
</tr>
<tr>
<td>35.0</td>
<td>179.79</td>
<td>0.32</td>
<td>0.66</td>
</tr>
</tbody>
</table>

In spite of the fact that we have not chosen any measurement errors, the apparent large flags in Fig. 11 in the lower $L$ are not due to the errors caused by the interpolation in the reconstruction, but to the insufficient number of digits transmitted from the measurements to the reconstruction programs. To prove this, UZU is able to produce the "over-all" corresponding measurement. That is to say, each randomly chosen point in the first view is correspondingly repeated in views 2 and 3, and the three light rays meet each other exactly in space; thus the procedure eliminates the interpolation in the reconstruction program. By so doing, the mathematical equivalence between UZU and the reconstruction program is realized except for the numerical truncation. The result is
shown in Fig. 12, and indicates no improvement whatsoever in errors after the normal measurements are changed to the corresponding one. This points to the fact that the large flags in Fig. 11 are only due to the truncation of numbers. In the case of the CERN HLBC, 4 + 4 decimal digits are transmitted to the reconstruction program as $\xi, \eta$ coordinates. Therefore, the number carries the error of five parts in every hundred thousand as an intrinsic uncertainty. Supposing the diameter of the chamber is 100 cm, this amounts to about 50 microns in space. In fact, if one computes the error $\Delta(1/\rho)$, fixing the external (i.e., measurement) error to zero, the error from truncation comes out to about 25 $\mu$ (see subsection 4.2). Therefore, throughout the following discussion of our whole computation, one has to keep in mind that this amount of intrinsic error is always present, apart from the normal measurement error.

In Fig. 13, the results of reconstruction by two typical reconstruction programs, DRAT$^8$) and THRESH$^9$), are presented. The same parameters are used for both cases ($\rho = 200$ cm, $\lambda = \varphi = 0$, no measurement error, the measurement density 1.0 ± 0.2 cm/measurement. Thirty tracks for each track length). The doubled uncertainty in the measurement density from 0.1 cm (Fig. 11) to 0.2 cm, now affects the size of the flags at less than 5 cm measured length almost quadratically.

4.2 Measurement density. Estimation of internal error.

The effect on the errors of the measurement density on a track is investigated. According to the calculations$^{16}$), when the measurements are equally spaced, the errors for the curvature and the direction are given as

\[
\left( \frac{\Delta \rho}{\rho} \right)^2 = \frac{(8 \cdot \Delta \xi)^2 \cdot \alpha_n}{L^4}
\]

(1)

\[
\left( \Delta \varphi \right)^2 = \frac{(4 \cdot \Delta \xi)^2 \cdot \beta_n}{L^2}
\]

(2)
where \( n \) is the number of cells measured on a track, \( \Delta \) the measurement error (either internal or external), and \( \alpha_n \) and \( \beta_n \) are given by

\[
\alpha_n = \frac{11.25 n^3}{(n-1)(n+1)(n+2)(n+3)}
\]

\[
\beta_n = \frac{0.75(2n+1)(8n-3)n}{(n-1)(n+1)(n+2)(n+3)}.
\]

When the effect of \( n \)'s is not taken into account, \( \alpha_n \) and \( \beta_n \) are usually set to unity in the error matrices. To start with, we make the following assumptions:

i) The length of track to be measured is set to 15 cm.

ii) There is no external measurement error, therefore the error is due solely to internal (truncation) error.

iii) Except for the initial measurement (apex), the measurements do not correspond, thus the cell length varies randomly by 10% (or 30%) of the present average values. This is also the case in the actual measurement. The cell lengths are chosen from 0.5 cm (shortest cell length) to 5.0 cm, with 0.5 cm step.

iv) The radius of the curvature is changed in the range of 10, 50, 100, 150 and 200 cm to provide any systematic effect.

v) The number of tracks produced in each category is 30.

vi) In presenting the data, the reconstructed track lengths are averaged in each category, and the averaged values are divided by the cell length to give the number of cells \( n \) which is the horizontal coordinate in Figs. 14 to 19.

The vertical coordinate in Figs. 14 to 19 represents the values of the error normalized to \( L = 15 \) cm.

The result indicates the following:

i) There is no systematic difference observed between 10% and 30% cell-length variation (therefore in Figs. 14 to 19, only the case of 10% is presented).
ii) There is no systematic effect observed due to the differences in radius of curvature with the present statistics.

iii) In the lower values of $n$ ($n < 4$), the deviation from theoretical curve is observed. Except for this region, the fit is quite good to give $\Delta \varepsilon_{\text{int}} = 27$ microns and 23 microns for $1/\rho$ and $\varphi$. When $\alpha_n$ and $\beta_n$ are set to 1, the appropriate values for $\Delta \varepsilon$'s are indicated with broken lines. Thus the internal error of 25 microns (± a few microns) would be a reasonable choice. This is the error that one cannot avoid, and it is intrinsic to the numerical handling of our measuring and recording system.

The deviation from the theoretical prediction in lower $n$-values still exists even if one has an external measurement error (of, say, 180 microns, the points with vertical flags in Fig. 14), confirming that the deviation is not intrinsic to the truncation.

As the explanation, the following is pointed out by Soop. When the fit to $\rho$ is asked to yield a) the best estimate of $\rho$, and b) the best estimate of $\Delta \rho$, it cannot be expected to get a significant quoted error on $\rho$ for, say, $n < 8-9$. In fact, for $n < 5$ one can never use a $\Delta \rho$ estimated purely from the sample. It must be put in externally as minimum constant, as is done in D R A T. In fact, there is only one circle possible for three points, which always gives $\Delta \rho = 0$, unless one provides an external error.

iv) Most general form for $\Delta \lambda$ takes the form of

$$\Delta \lambda = [F(\lambda, G) \cdot \Delta \varepsilon] \frac{\gamma_n}{L^2} \tag{5}$$

where $F$ is a function depending on the dip angle $\lambda$ and a set of parameters related to the geometry. We will try to determine $F$ empirically. $\gamma_n$ is given by

$$\gamma_n = \frac{12/0 n}{(n+1)(n+2)} \cdot \tag{6}$$
From this run, if $\Delta \varepsilon_{\text{int}} = 25$ microns is assumed, the estimate for $F$ is

$$F(0, G) = 9.5.$$ 

A further study of the behaviour of $F$ for different $\lambda$'s is under way.

\textit{v)} The covariances are calculated to be

$$\left(\frac{\Delta}{\rho}\right)\left(\Delta \varphi\right) = -\frac{(8 \cdot \Delta t)^2 \cdot \alpha_n}{2L} \tag{7}$$

$$\left(\frac{\Delta}{\rho}\right)\left(\Delta \lambda\right) = 0 \tag{8}$$

$$(\Delta \varphi)(\Delta \lambda) = 0 \tag{9}$$

In Fig. 17, the case of Eq. (7) is presented. From Eq. (7), $\Delta \varepsilon_{\text{int}}$ is again found to be 27 microns, which is very close to the estimates given by $(\Delta 1/\rho)^2$ and $(\Delta \varphi)^2$. On the other hand, for Eqs. (8) and (9), they are scattered about symmetrically around zero, their magnitudes converging to zero rapidly before the cell number reaches 10. (Figs. 18 and 19.)

\textit{vi)} Figure 20 shows the distribution of $\rho$, $\lambda$, and $\varphi$, respectively. The $\alpha_n$, $\beta_n$, and $\gamma_n$ distribution against $n$ is shown in Fig. 21.

4.3 \textbf{External (measurement) error}

After having estimated the internal error, we proceed to the case where the external error (measurement error) is additionally present. Since we have seen that formulae (1) and (2) are justified if $n > 4$, we use these formulae to correct the resulting values in the analysis. We now fix the radius of curvature and vary the length of the track to see if the variances are proportional to $(\Delta \varepsilon)^2/L^4$. The cell length is set to $1.0 \pm 0.1$ cm; therefore, if $L > 4$ cm, $n$ becomes larger than 4, and the correction by $\alpha_n$ in formula (1) is justified.
In Fig. 22, a comparison is made between the 5 and 10 micron errors (on the films, the errors are external).

In Figs. 23 to 25, the results for $\Delta(1/\rho)$, $\Delta\lambda$, and $\Delta\phi$ are presented with their square roots of variances against the length of tracks.

Since there is no correlation between the internal error $\Delta \varepsilon_{\text{int}}$ and the external error $\Delta \varepsilon_{\text{ext}}$, the over-all error $\Delta \varepsilon$ must be expressed as

$$\Delta \varepsilon = \sqrt{(\Delta \varepsilon_{\text{int}})^2 + (\Delta \varepsilon_{\text{ext}})^2}.$$  \hspace{1cm} (10)

As indicated in the discussion in Section 4.2, it is now well established that the magnitude of $\Delta \varepsilon_{\text{int}}$ is 25(±5) microns. Here we chose $\Delta \varepsilon_{\text{ext}} = 0$, 90, and 180 microns. [These numbers correspond to 0, 5, and 10 microns on the film (the magnification factor is 17.9 at this depth).] Thus we expect $\Delta \varepsilon = 25$, 93.5, and 182 microns in the three cases.

In Fig. 23, $\Delta(1/\rho)$ is plotted against the track length L. The proportionality to $L^{-2}$ is well demonstrated for three $\Delta \varepsilon$'s.

Glasser calculated the variance matrices for a space point: according to him, for a corresponding point, if N views are available, $\Delta \varepsilon$ must be multiplied by a factor $[(1/N) + (r^2/R^2)]^{1/2}$ where N is the number of views available, $r^2$ is the square of the projected distance of the space point to the origin ($= x^2 + y^2$), $R^2$ is the sum of the square of the projected distances of the cameras. The origin of the coordinates is set to the centre of gravity of the camera positions. In the present test case, $r^2$ is $\sim 100$ cm$^2$, whereas $R^2$ is $\sim 7500$ cm$^2$; therefore, we can neglect the second term of the correction factor. Thus, assuming $\Delta \varepsilon_{\text{ext}}$ can be replaced by $\Delta \varepsilon_{\text{ext}}$ times this correction factor in Eq. (10), we expect $\Delta \varepsilon = 25$, 57.8, and 107 microns in three cases.

As for the proportionality between $\Delta \varepsilon$ and the variance, the results for the last two $\Delta \varepsilon$'s are somewhat larger than expected. This is probably due to the bad correspondence of the chosen points due to the truncation which leads to modification of the factor $[(1/N) + (r^2/R^2)]^{1/2}$ in the bad direction.
More or less similar are the situations for \( \Delta \lambda \), \( \Delta \varphi \), and the covariance \( \Delta(1/\rho)\Delta \varphi \). (In Figs. 23 to 26, the \( \Delta \) \( \epsilon \) \text{int} are estimated to be 23, 2, 22, and 25 microns, respectively.)

As for the covariances \( \Delta(1/\rho)\Delta \lambda \) and \( \Delta \varphi \Delta \lambda \), the former has a good convergence to zero (Fig. 27), whereas for the latter, it appears to tend to stay on the negative side when the measurement error \( \Delta \epsilon \) \text{ext} amounts to 180 microns (Fig. 28).

4.4 Single and multiple scattering

So far we have investigated the effects of the errors \( \Delta \epsilon \), the measurement density \( n \), and the track lengths \( L \) on the qualities of the reconstructed and fitted quantities. In this section we will show how the UZU program, in its producing the tracks, will switch on the Rutherford scattering.

The radiation length \( \chi_0 \) (in g cm\(^{-2}\)) is defined as

\[
\frac{1}{\chi_0} = 4\alpha \frac{N}{A} 2^2 r_e^2 \ln(1832^{-1/2}) .
\] (11)

This quantity is related to the mean square angle of multiple scattering as

\[
\vartheta_{\vartheta}^2 = \left( \frac{0.021}{\beta p} \right)^2 \frac{1}{\chi_0} \quad (p \text{ in GeV/\(c\)})
\] (12a)

when the lower and upper limits of scattering \( \vartheta_1 \) and \( \vartheta_2 \) are taken as

\[
\vartheta_1 = z^{1/3} \frac{m_e c}{p}
\] (12b)

\[
\vartheta_2 = 280A^{1/3} \frac{m_e c}{p}
\] (12c)

respectively (Rossi). This is a general remark on the relation (12a).
In computing the multiple scattering contribution in UZU however, although the screening angle \( \delta \), of expression (123) is consistently used for the lower limit, \( \Theta_{\text{MIN}} \) is used for the upper limit instead of \( \theta_2 \), and the mean square angle of multiple scattering for this limit is computed. If the scattering angle is larger than \( \Theta_{\text{MIN}} \), it is treated as a single scattering. In other words, Rutherford scattering works both for the single and multiple scattering, whose boundary is specified by an angle \( \Theta_{\text{MIN}} \) (Appendix 2).

Setting the radius of curvature at 180 cm in a magnetic field of 18.95 kG, and the dip angle at zero, a hundred tracks are produced for each of the four different track lengths, 10, 16, 26 and 41 cm. \( \Theta_{\text{MAX}} \) is set to 3 degrees, which is sufficiently close to \( \theta_2 \) in Eq. (12c), and also from the practical point of view it is easily recognisable as a kink in the scanning. On the other hand, \( \Theta_{\text{MIN}} \) is at the moment set to 0.017 degree, which makes the average distance of single scatterings for muon about 1 mm. The corresponding cell lengths for the above four track lengths are set in such a way that the number of cells \( n \) becomes about 10, regardless of the lengths. A 5% fluctuation of the cell length is allowed. No external error is introduced \( (\Delta c_{\text{ext}} = 0.0) \). The results are summarized in the following table and in Fig. 29. The liquid is the light freon.

<table>
<thead>
<tr>
<th>L</th>
<th>(&lt; \rho &gt;)</th>
<th>(\text{Expected st. deviation} \times 10^3)</th>
<th>(\text{Average of quoted error} \times 10^3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>176.1</td>
<td>23.6</td>
<td>22.0</td>
</tr>
<tr>
<td>16</td>
<td>180.3</td>
<td>22.2</td>
<td>11.7</td>
</tr>
<tr>
<td>26</td>
<td>182.4</td>
<td>19.2</td>
<td>5.7</td>
</tr>
<tr>
<td>41</td>
<td>179.5</td>
<td>12.7</td>
<td>2.8</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>(&lt; \lambda &gt; \times 10^3)</th>
<th>(\text{Expected st. deviation} \times 10^3)</th>
<th>(\text{Average of quoted error} \times 10^3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>2.9</td>
<td>8.2</td>
</tr>
<tr>
<td>16</td>
<td>2.5</td>
<td>6.5</td>
</tr>
<tr>
<td>26</td>
<td>0.3</td>
<td>4.8</td>
</tr>
<tr>
<td>41</td>
<td>1.4</td>
<td>3.9</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>(&lt; \varphi &gt; \times 10^3)</th>
<th>(\text{Expected st. deviation} \times 10^3)</th>
<th>(\text{Average of quoted error} \times 10^3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>1.2</td>
<td>5.1</td>
</tr>
<tr>
<td>16</td>
<td>0.8</td>
<td>3.1</td>
</tr>
<tr>
<td>26</td>
<td>1.0</td>
<td>2.4</td>
</tr>
<tr>
<td>41</td>
<td>-0.7</td>
<td>2.0</td>
</tr>
</tbody>
</table>
As this example shows, the result of the reconstruction and the fitting is very satisfactory. However, let us study more closely the behaviour of the variances with respect to the track lengths. In addition to the measurement error $\Delta \varepsilon$, the presence of multiple scattering adds new terms to the error matrices, which are represented in each of the first terms in the following expressions ($\lambda = 0$):

\[
(\Delta \frac{1}{\rho})^2 = \frac{C_{1/\rho}^2 (K/\beta)^2}{L} + \frac{(8 \cdot \Delta \varepsilon)^2 \beta_n}{L^4 n} \quad (13)
\]

\[
(\Delta \lambda)^2 = \frac{C_{\lambda}^2 (K/\beta)^2 L}{2} + \frac{[\Phi(0, \varepsilon) \Delta \varepsilon]^2 \gamma_n}{L^2} \quad (14)
\]

\[
(\Delta \phi)^2 = \frac{C_{\phi}^2 (K/\beta)^2 L}{2} + \frac{(\lambda \cdot \Delta \varepsilon)^2 \beta_n}{L^3 n} \quad (15)
\]

\[
(\Delta \frac{1}{\rho}, \Delta \rho) = -C_{c}^2 (K/\beta)^2 L^2 + \frac{(8 \cdot \Delta \varepsilon)^2 \alpha_n}{2L^3} \quad (16)
\]

where $K$ is related to the radiation length $X_0$ as $K = 0.21/\sqrt{X_0}$; for the light foom, $K = 0.0042$.

Under the condition of the multiple scattering and the three-point measurement ($n = 2$), the theory predicts that C's are independent of $L$ and

\[
C_{1/\rho} = \sqrt{2/3} \sim 0.82
\]

\[
C_{\lambda} = \sqrt{1/6} \sim 0.41
\]

\[
C_{\phi} = \sqrt{1/12} \sim 0.29
\]

\[
C_{c} = \sqrt{1/12} \sim 0.29
\]

From our knowledge of the $\Delta \varepsilon$-dependent terms, as discussed so far, one is able to subtract the second terms in Eqs. (13) to (16), and to check the validity of the $L$ dependence of the first terms as well as to determine...
the coefficients C's. In Figs. 30 to 33, the uncorrected variances for \(1/\rho\), \(\lambda\), \(\varphi\), and the expected covariance \(V(1/\rho, \varphi)\) are shown, as well as their corrected (\(\Delta\varepsilon\)-depending parts subtracted) values. In subtraction, the numbers found in the previous sections are used consistently (i.e. \(\Delta\varepsilon = 25\) microns, \(F(0,G) = 9.5\), \(\alpha_m\), etc., are set to \(n = 10\)). The result of the correction exhibits very well the predicted L-dependence of Eqs. (13) to (16), determining the values for the C's as follows.

\[
\begin{align*}
C_{1/\rho} &= 0.64 \\
C_{\lambda} &= 0.29 \\
C_{\varphi} &= 0.21 \\
C_{\Delta} &= 0.21 .
\end{align*}
\]

(18)

If one compares these numbers with the predicted C's (multiple scattering and three-point measurement [Eq. (17)]):

\[
\frac{C}{C_{\text{theory}}} = \begin{pmatrix}
(1/\rho)^2 & (\lambda)^2 & (\varphi)^2 & 1/\rho \\
0.78 & 0.71 & 0.73 & 0.73
\end{pmatrix}
\]

(19)

The consistency between the ratios suggests that there may exist a common factor of about 0.75, which could be accounted for by the difference of the conditions between theory and the present case. The conjecture is also supported by the following. Theory predicts

\[
\frac{(\Delta 1/\rho \Delta \varphi)^2}{(\Delta 1/\rho)^2 (\Delta \varphi)^2} = - \sqrt{2/4} = - 0.354 .
\]

(20)

On the other hand, our computed value for this ratio, taken from the simulated events, is -0.329, which is close to the theoretical prediction. This was also pointed out earlier\(^{11}\).
The situation does not change much even if TMIN is altered. In the figures, the cases where TMIN is set to 0.086 degree are shown with crosses (L at 26 cm). The average distance between single scatterings is about 23 mm in this case.

The other covariances \((\Delta \rho \Delta \lambda)\) and \((\Delta \phi \Delta \lambda)\) are computed to be

\[
\left( \Delta \frac{1}{\rho} \Delta \lambda \right)^2 = (3.4 \pm 24.8) \times 10^{-8}
\]

\[
(\Delta \phi \Delta \lambda)^2 = (-1.08 \pm 3.03) \times 10^{-6}
\]

(21)

and show no correlations among those variables, as expected.

As can be seen from Eqs. (14) and (15), there are the optimum lengths \(L_{\text{opt}}\) for the \(\lambda\) and \(\phi\) measurements. \(L_{\text{opt}}\) is given by

\[
L_{\text{opt}} = \left( \frac{2B}{A} \right)^{\frac{1}{3}}
\]

(22)

where \(A\) and \(B\) are coefficients of \(L\) and \(L^{-2}\) terms in Eqs. (14) and (15).

If one accepts \(C_{\lambda}\) and \(C_{\phi}\) of Eq. (18), \(L_{\text{opt}}'s\) for \(\lambda\) and \(\phi\) are 9.1 and 4.3 cm, respectively. Theoretical predictions for those values are 7.1 and 3.7 cm, respectively.

4.5 Effect of ionization on the curvature and azimuthal angle

This section, and the following one, deal with the electron in media having a short radiation length. The study is especially important for the heavy-liquid chamber where, for instance, one wishes to measure the energy of converted gamma rays, and it is most important to know as much as possible about such measurements.

UZU3 provides us with a lot of information in this respect, where any combination of the scattering, the ionization loss, the bremsstrahlung, and the measurement error -- or just one of those -- can be incorporated in producing tracks.
In Figs. 34 and 35 are shown typical examples of electron tracks produced by UZU3 in the light freon and the freon, respectively, projected on the x-y plane. The lightly drawn, large, Ionic spirals are the electron tracks with only ionization, whereas the dark, short tracks (five for each figure) are the ones with scattering and bremsstrahlung. Since the stopping power of the light freon and the freon is about the same, the curves for ionization are almost identical in both figures, whereas the presence of bremsstrahlung and scattering brings about the obvious differences between the two. The radiation lengths for light freon and freon are 24 and 10 cm, respectively.

One of the virtues of UZU is that there is the possibility of separating the contributions from different sources of errors. We should start out with the case where only the ionization exists. The angular and radiation cut-off values chosen are high enough for the scattering and bremsstrahlung to be eliminated.

With the cell length of 1.8 ± 0.02 cm, tracks of four different initial momenta (20, 40, 60, and 80 MeV/c) with various measurement lengths in the light freon are produced. There are no external measurement errors, no scattering, no bremsstrahlung. One object of this run is to see how the correction must be done to the curvature, the initial direction, and the dip angle obtained by fitting a circle to the measured points. $L_p$, which is defined as the length along the track between the starting point and the point at which the particle has a momentum corresponding to the reconstructed curvature, is normally assumed to be half that of the total track length measured. The correction is made accordingly in order to get the curvature at the starting point from the reconstructed curvature. In Fig. 36, the $L_p/L$ are plotted against $p/L$, where $p$ and $L$ are "reconstructed and fitted" momentum and length. Regardless of initial momenta, they fall on a curve, and empirically this gives a relation

$$\frac{L_p}{L} = \frac{1}{15(p/L - 0.85)} + 0.488 \quad (p \text{ in } \text{MeV/c}, L \text{ in cm}). \quad (23)$$
Thus to take $L/p$ as always being half is a good correction, only one should be cautious when $p/L$ takes small values.

Let us, however, look into this problem a little more analytically. Since the energy loss of electrons is very well approximated by

$$\frac{dE}{dl} = -\alpha,$$

(24)

where $\alpha$ is const. of 2.41 MeV/c (light freon), there follows a linear relationship between the radius of curvature and the track length as

$$\rho = -cl + \rho_0,$$

(25)

where $c = \alpha/0.3B$ (kG), and $\rho_0$ is the initial radius. We average out the curvature $1/\rho$ over the measured length, and call the inverse of this quantity $\bar{\rho}$ the averaged radius of curvature. The ratio of $\bar{\rho}$ to $\rho_0$ is then

$$\frac{\bar{\rho}}{\rho_0} = \frac{cl}{\rho_0 \log \left(1 - \frac{cl}{\rho_0}\right)}.$$

(26)

Under the condition $(cl/\rho_0 < 1)$ Myatt obtained the following formulae, after fitting a circle to five points equally spaced along a track, represented by Eq. (25). This gives

$$\frac{\bar{\rho}}{\rho_0} = \left[1 + \frac{o}{2\rho_0} L + \frac{1}{3} \left(\frac{o}{\rho_0}\right)^2 \left(\frac{77}{80}\right) L^2\right]^{-1}.$$

(27)

Also, it was shown that for the angular difference between the tangent at the starting point of the original track and at that of the reconstructed one:

$$\Phi = \frac{1}{3} \frac{L^2}{\rho_0} \left[\frac{o}{2\rho_0} + \left(\frac{27}{20}\right) \frac{1}{3} \left(\frac{o}{\rho_0}\right)^2 L\right].$$

(28)
These formulae and the result of the UZU-DRAT chain are compared in Fig. 37. Equation (26) fits rather well to the observed points, although if one takes empirically a 10% smaller value in \( c \), the best fit can be obtained. Equation (27) also gives a good fit, especially at the short lengths where the measurements are usually practised, but as the measured length becomes greater it deviates slightly upwards as a consequence of the form of Eq. (27). As for the angular correction, the deviation of the reconstructed values from Eq. (28) becomes appreciably larger at longer lengths. Therefore, we can rewrite Eq. (28) to an empirical formulae with a parameter \( \omega \), i.e.

\[
\tilde{\gamma} = \frac{1}{6} \frac{L^2}{\rho_0} \left( \frac{c}{2 \rho_0} + \omega \frac{1}{3} \left( \frac{c}{\rho} \right)^2 L \right) \tag{29}
\]

and look for the best value of \( \omega \). It turns out that if \( \omega = 5.0 \), the over-all fit to any electron energy becomes excellent.

It must be emphasized that in determining \( \rho \) -- since only the track under the influence of ionization can be represented by analytical formula such as Eq. (25), and there are no statistical fluctuations such as those due to bremsstrahlung along its path -- if one knows the correction to be made to the fit, then it is advantageous to measure the longer length according to the laws discussed in the preceding sections.

4.6 Effect of bremsstrahlung and multiple scattering, and also of ionization

In this section, we will discuss the cases where i) the bremsstrahlung and the scattering are present but there is no ionization, ii) all three are present. In the same way as for \( T_{\text{MAX}} \) and \( T_{\text{MIN}} \) in the case of scattering, we have to deal with the bremsstrahlung with upper cut-off and lower cut-off in energy in UZU. The lower cut-off, in order to avoid the infra-red catastrophe, is set to 0.2 MeV in the present run. For the upper cut-off we take the value of 50% of the incident electron momentum, as for the CERN \( K_\mu^3 \) experiment. If bremsstrahlung occurred
with more than this energy limit, the measurement is automatically turned off at that point. The initial momenta of the electron are taken at 244.0 and 43.9 MeV/c.

Before going into the result of the UZU-DLAT test, it is worth discussing briefly the systematic and statistical error on the curvature in the presence of bremsstrahlung and ionization. We use the bremsstrahlung spectrum as used by Heitler. With \( p \) denoting the momentum of the electron at the track length \( t \), then

\[
\frac{d(1/p)}{dt} = \frac{a}{p^2} + b \int_{0}^{k_{\text{max}}} \frac{1}{p \log \left( \frac{p}{p-k} \right)} \frac{k}{p(p-k)} \, dk
\]  

(30)

represents the rate of change in curvature in the unit length, where the first term is the curvature change due to the ionization \( (a = 2.41 \text{ MeV/om}) \), and the second term represents the average of the curvature change due to the bremsstrahlung; \( k \) is the energy of the quantum emitted, and \( k_{\text{max}} \) is the cut-off level of the measurement and it is the half of the electron momentum in the present case; \( b \) is \( (X_0 \ln 2)^{-1} \) where \( X_0 \) is the radiation length of the liquid. Equation (30) is a good equation when the rate of change of the momentum by ionization is sufficiently smaller than that of bremsstrahlung, i.e., where the electron momentum is much greater than the critical energy \( E_c \) of the liquid. When \( E \leq E_c \), the analysis becomes more complicated. The integration leads to a function whose power series expansion has a form of

\[
A_1(y) = y - \frac{y^2}{2 \cdot 2!} + \frac{y^3}{3 \cdot 3!} - \cdots \]  

(31)

If one defines the quantity \( y_0 \) as \( k_{\text{max}} = p(1 - e^{-y_0}) \), Eq. (30) becomes

\[
\frac{d(1/p)}{dt} = \frac{a}{p^2} + b \ A_1(y_0) \frac{1}{p} .
\]  

(32)
This immediately gives the solution for $1/p$:

$$
\frac{1}{p} = \left[ (a + b \, A_4(y_0) p_0) \exp \left( -b \, A_4(y_0) L \right) - a \right]^{-1}
$$

(33)

where $p_0$ is the initial momentum of the electron. From this one is able to calculate $\bar{p}/p_0$, where $\bar{p}$ is the inverse of the average over measured length of the curvature:

$$
\frac{\bar{p}}{p_0} = \left\{ \frac{p_0}{aL} \log \left[ \frac{p_0}{bL} \frac{b \, A_4(y_0)}{e \, A_4(y_0)} \left\{ 1 - \exp \left[ b \, A_4(y_0) L \right] \right\} a \right] \right\}^{-1}
$$

(34)

If there is only the bremsstrahlung ($a \to 0$), this reduces to

$$
\left( \frac{\bar{p}}{p_0} \right)_{\text{brems}} = \frac{A_4(y_0) \, bL}{e \, A_4(y_0) \, bL - 1},
$$

(35)

and if only the ionization, it reduces to Eq. (26). In the absence of $a$, and if $y_0$ and $bL$ are both small, this equation reduces to the Behr-Mittner Eq. (3) [Ref. 4)]. Myatt has made a five-point fitting of a circle to the track expressed by Eq. (33), and also obtained

$$
\frac{\bar{p}}{p_0} = \left[ 1 + a_1 L + a_2 \left( \frac{77}{80} \right) L^2 \right]^{-1},
$$

(36)

and for the azimuthal correction

$$
\phi = \frac{1}{6} \frac{L^2}{\rho_0} \left( a_1 + \omega L \, a_2 \right),
$$

(37)

where

$$
a_1 = \frac{a}{2 \rho_0} + \frac{b \, A_4(y_0)}{2}, \quad a_2 = \frac{1}{6} \left[ \frac{a}{\rho_0} + b \, A_4(y_0) \right] \left[ \frac{2 \rho_0}{\rho_0} + b \, A_4(y_0) \right]
$$

and $\omega = 27/20$, respectively.
The variance of curvature due to the bremsstrahlung is calculated as follows. At a point \( \ell \), the average over the bremsstrahlung of the curvature is expressed by Eq. (33) (except for the numerical constant), whereas the average over bremsstrahlung of the square of curvature is calculated similarly from Eq. (30) to Eq. (33) to give

\[
\frac{1}{\rho^2} = \frac{b^2 A_2(y_0)^2}{\left(\alpha + p_0 \frac{A_2(y_0)}{2} \exp \left[-b \frac{A_2(y_0)}{2} \ell \right] - \alpha \right)^2}
\]

(38)

where \( A_2 \) is a function

\[
A_2(y) = 2 \left( y + \frac{y^3}{3.5!} + \frac{y^5}{5.5!} + \cdots \right)
\]

(39)

The variance, at a point \( \ell \), is therefore the difference between Eq. (38) and the square of Eq. (33). The variance averaged over the measured length \( L \) must be carried out considering the proper weight due to the change of the electron flux along the track. However, this weighting function is not so different from unity as to change the general aspect of the argument. In fact, this variance has the following general properties:

i) When there is no bremsstrahlung \((b \to 0)\), it tends to zero. This is what we expect since the track takes a well-defined spiral and will have no statistical fluctuations caused by the bremsstrahlung.

ii) When expanded into power series, the term proportional to \( \alpha \) (ionization loss per unit length) has a coefficient positive definite.

iii) When there is no ionization loss \((\alpha \to 0)\), it becomes

\[
\left( \frac{\Delta \frac{1}{\rho}}{1/\rho_0^2} \right)^2 = \frac{1}{A_2} \frac{1}{bL} \left[ \exp(A_2 b L) - 1 \right] - \frac{1}{A_1} \frac{1}{b^2 L} \left[ \exp(A_1 b L) - 1 \right]^2
\]

(40)

If \( bL \) and the argument of \( A_1 \) and \( A_2 \), \( y_0 \), are small, this reduces to
\[
\left( \Delta \frac{1}{\rho} \right)^2 = \frac{1}{4} b \ L \ y_0^2, \tag{41}
\]

which corresponds to Eq. (5) of Ref. 4) (except for the numerical factor 1.5).

The variance averaged over the total length of the track L, neglecting the change in flux, is written

\[
\left( \Delta \frac{1}{\rho} \right)^2 = (0.3 \ b)^2 \left[ f[A_z(y_0)/2] - f[A_z(y_0)] \right] \tag{42}
\]

where

\[
f(A) = \frac{b \ A \ L}{L} \left[ \frac{1}{\alpha^2} \log \left( 1 + \frac{1 - \exp(b \ A \ L)}{p_0 \ b \ A} \right) \right]
\]

\[
= \frac{1 - \exp(b \ A \ L)}{\left( 1 + \frac{1 - \exp(b \ A \ L)}{p_0 \ b \ A} \right) \cdot p_0 \ b \ A}
\]

In order to illustrate how an electron behaves in the light freon, its momentum versus the travelled distance is shown in Fig. 38. These examples are produced by UZU using the Monte Carlo method. Two straight lines and two smooth curves in the figure are the special cases of Eq. (33), the average momentum versus the length. They are, from the top of the figure: i) with no ionization, no bremsstrahlung (\(a = b = 0\)); ii) with no ionization but with bremsstrahlung (\(a = 0, b = 0.058\)); iii) with ionization only (\(a = 2.41, b = 0.0\)); and iv) with ionization and with bremsstrahlung (\(a = 2.41, b = 0.058\)). The lines broken by horizontal steps are the electrons produced in case (ii), and those broken by diagonal steps are the electrons produced in case (iv). For the case of no ionization, the distributions of electron momentum at certain lengths were compared with that theoretically obtained by Heitler [Chapter VII, Eq. (17)] and found in agreement in the statistical limit.
We first present the results of an experiment, where the initial electron momentum is set to 244 MeV/c, well above the critical energy ($E_c$ of light freon is about 70 MeV). Figure 39 shows the absolute variances of $1/\rho$ with the various measured lengths. The open circles represent the case with no ionization, whereas for the closed circles the ionization is taken into account. There is no statistically significant difference between the two cases, and they show a well-marked dip around 10 cm, the optimum length pointed out by Behr and Mittner.\(^4\) At a small length, the error due to multiple-scattering is significant according to the $1/l$-law [in the figure $C_{1/l}$ is that in Eq. (17)], whereas at a large length, the error due to bremsstrahlung becomes dominating. The absolute values of the variance are, however, lower than the theoretically predicted values [Eqs. (17) and (40)]. This could be partly corrected by considering the cell numbers used in the measurement, but it would remain as a minor effect.

It is interesting and also instructive to see how the fitted radius of curvature changes as one increases the measurement length, say, from one cell to the next. The summary of this experimentation is presented in Figs. 40 and 41, with and without ionization, respectively. The initial radius of curvature is 43.0 cm corresponding to the 244 MeV/c electron.

The result of reconstructed and fitted values of $1/\rho$, $\lambda$, and $\varphi$ in the UZU-DRAT chain for 48.9 MeV/c electrons without ionization are presented in Fig. 42. This momentum is especially worth noting, since the typical electron momentum of the converted gamma of $\pi^0$ in the $K^+\mu^3$ experiment is around this value. Also, it is smaller than the critical energy $E_c$, and can therefore be used to check the validity of Eq. (30).

In Fig. 42, a comparison is made between Eqs. (35), (36), and (37). Within the statistical error, the formulae check with the points satisfactorily. For Eqs. (37) and (36), the ionization coefficient $c$ in $a_1$, and $a_2$ is set to zero, and $\omega$ in Eq. (37) is again chosen to 5 instead of the original coefficient $27/20$. 

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In Fig. 43, the simulated variances for the curvatures and the theoretical prediction are presented. The simulated variances are rather widely scattered around the predicted values, but as far as the order of magnitude is concerned they are in agreement. Unfortunately, the attempt to observe the minimum was unsuccessful, probably because of the paucity of statistics regarding the present run. Figure 44 shows the other variances with respect to the measured lengths.

Switching on the ionization loss in addition to the multiple scattering and the bremsstrahlung, the fitted $\rho$, $\lambda$, and $\varphi$ are presented in Fig. 45 with predicted curves. The open and closed circles belong to two independent runs. For $\varphi$, Eq. (37), again with $\omega = 5$, is an appropriate correction within the statistical limit. For $\rho$, Eq. (36) fits better than Eq. (34). One of the most noticeable changes, however, from the case without ionization, is the apparent disappearance of the minimum in the variance for the curvature. As mentioned in point (ii), p. 29, the present theory predicts the increase of the variance with $\alpha$. As indicated in Fig. 46, however, by turning on $\alpha$, the variance appears to decrease with higher $L$. This effect could probably be attributed to the following three points.

i) The accuracy of UZUJJ could be insufficient at this energy. Many minute approximations which could be the cause of this inaccuracy should be discussed elsewhere.

ii) The incident electron energy is smaller than $E_0$, so that Eq. (30) no longer holds. The bremsstrahlung emission on the track becomes more unlikely in the presence of ionization loss. The theory is based on the average of quantities $1/\rho$ and $1/\rho^2$ over the infinitely short distance $dl$, without taking into account their degradation in momentum over this distance, which will introduce a change.

iii) By choosing a longer measurement length, we may automatically select such tracks as were not affected by the bremsstrahlung. (However, in order to formulate this, we have to discuss the momentum distribution of electrons at any point along the track, which reduces to
the identical problem mentioned in point (ii), and would be beyond the scope of the present report.) In this respect, the readers are referred to Heitler's book\textsuperscript{18}, Chapter VII. The other variances versus the measured length are shown in Fig. 46.

\section*{Acknowledgements}

The author is very grateful to Dr. C.A. Ramm for his enlightening comments, his interest, and his support during the entire period when this work was being done. Without his encouragement and understanding the present work would have been interrupted long ago. He is greatly indebted to Dr. G.H. Trilling; during the discussions with him, many obscure points were clarified, and the author is thankful to him for the progress of the work that followed. For the entire period, the author owed very much to Mr. K. Soop, who was unsparing in his efforts to provide information regarding the computers and programming techniques. He is also indebted to Dr. G. Myatt for many useful discussions. Lastly, he would like to add that he is obliged to a great number of people in the Data-Handling and Track Chamber Divisions of CERN, whose help was, in fact, indispensable to the completion of the present work.

* * *
THE OPERATION OF THE PROGRAM UZU

Here is a brief description on how to prepare a data deck for UZU. The head numbers are sequence numbers, each of which usually corresponds to a card except for: i) range momentum load deck; ii) the measurement error specification cards; and, iii) the initial condition cards [the two groups of cards must be terminated in each case by a stop card which has 1 (for 3400) or T (for 6600) in column 80]. "C" is an abbreviation for "column".

1) Any 12 digits octal number from C1 to start the random number (012).

2) Answer 1, 2 or 3 at C73 according to your choice, UZU1, 2, or 3. As is well known, UZU1 generates a track with a single label AA, A1; UZU2, AA, AH, HH, etc., where the apexes are scattering points; UZU3 has taken into account: i) ionization loss; ii) scattering; iii) bremsstrahlung, and it is often applied for electrons.

3) Do you want the scattering list suppressed? If "YES" on C73 to C75, the list of information concerning scattering points (X,Y,Z-coordinates, new azimuthal and dip angle, the length of this branch, the scattering angle) is suppressed. If "NO" (or blank) the list is printed.

4) Do you want the measurement list suppressed? If "NO" (or blank), the list of measured points will be printed. The first column of the print output is THETA, which is the azimuthal angle specifying this point; the second, the Z-coordinate of the point in the chamber; the third and the fourth, the projected X,Y-coordinates of this point on the film; the fifth and the sixth, those with measurement errors being taken into account.

5) Do you want the output tape list suppressed? If "NO" (or blank), the final output of UZU written on a tape in BCD mode is printed out to a certain record. This serves to check whether the output is in the form acceptable by the program being tested. Tape unit 5.
6) Do you want corresponding measurements? If "YES", all the points measured in three views become corresponding points. This is for special tests.

7) Do you want a fixed length to the first scattering point? If "YES", the first scattering point is not generated randomly according to the Rutherford cross-section, but a fixed length to the point is given (see next question).

8) If "YES", how long? Answer in centimetres from C61 to C80 (F 20.8).

9) Do you want a fixed scattering angle at the first scattering point? If "YES", the scattering angle of the first scattering point is not generated randomly, but a fixed angle at the point is given (see next question).

10) If "YES", how much (in radians)? From C61 to C80 (F 20.8).

11) How many tracks are to be generated per input card? Answer in L4 on C73 to C76. If the number is n, with the identical initial conditions (given by a following data card), the program constructs n tracks with different scattering patterns.

12) How many repeating measurements per track? Answer in L4 on C73 to C76. If the number is m, for a single track created, the measurements are repeated m times according to a Gaussian error distribution, whose standard deviation is specified by a following card.

13) What is the sign of particle? Answer "NEGATIVE" or "POSITIVE" in C73 to C80. "NEGATIVE" is clockwise looking into chamber from the camera.

14) If "YES", the multiple scattering is taken into account.

15) Range-momentum load-deck. Used only for U2U3. If for U2U1 or 2, this deck can be substituted by a single card with three *** on C1, C2, C3. Otherwise, the first card of the deck starts with a single * on C1, followed by any comments. From the second card onwards, the usual GRIND load-deck is used.
16) (8 F 10).
   i) Refractive index of the glass.
   ii) Refractive index of the liquid.
   iii) Refractive index of the air.
   iv) Thickness of the glass.
   v) Thickness of the air (i.e. from front surface of the glass
to the centre of the lens).
   vi) Depth of the chamber (give in a negative value).
   vii) Radius of the chamber.
   viii) Lens - film distance.

17) (8 F 10).
   i) Mass of the particle of the track (in MeV).
   ii) Magnetic field strength (in kG).
   iii) Atomic number squared, averaged over molecule.
   iv) Mass number averaged over molecule.
   v) Density of the liquid.
   vi) Cut-off angle for Rutherford scattering (maximum) (radians).
   vii) Cut-off angle for Rutherford scattering (minimum) (radians).
   viii) Cut-off energy of bremsstrahlung (MeV).

18) (3 F 10).
   i) Cut-off energy of bremsstrahlung/particle energy.
   ii) The momentum where measurement to be stopped/initial momentum.
   iii) Size of sagitta (cm).

19) (20 X, 2 F 10). Specify the camera coordinates. Punch the X,Y-
20) coordinates of each camera axis.

22) (A5, 2 F 10). Specify the fiducial mark coordinates (4 marks).
23) Punch the X,Y-coordinates of fiducial marks from C6 to C25.
24) 25)

26) (3 F 10).
   i) Measurement error expressed in the standard deviation in the
   X,Y-direction on the film (in cm).
   ii) The projected average length of the cell length being measured
   [in real space (not on the scanning table, nor on the film) in cm].
iii) The standard deviation of (ii).

**Note:** This card can be followed by any number of similar cards, up to 50, specifying new sets of errors. The group should be terminated by a control card with "1" or "T" in C80 (see below).

27) A control card to terminate the error specifications: "1" (CDC 3400, 3600 FORTRAN), or "T" (FORTRAN IV, CERN FORTRAN) at C80.

28) (4 F 10, F 15, 2 F 10). Specifies a track by giving the initial values.

   i) X,Y,Z-coordinates of the initial point (cm).

   ii) Radius of curvature (cm).

   iv) Azimuthal angle of the tangent of the track at its initial point measured from the X-axis (radians).

   vi) Dip angle (radians).

   vii) Total length of the track (cm).

**Note:** This card can be followed by any number of similar cards, specifying new tracks. The group should be terminated by a control card with "1" or "T" at C80 (card 29).

29) A control card to terminate the track production: "1" (CDC 3400, 3600 FORTRAN), or "T" (FORTRAN IV, CERN FORTRAN) at C80.

30) END-OF-FILE.

* * *
MULTIPLE SCATTERING

The limits for the single scattering angles are named TMIN, TMAX (Section 2). When the multiple scattering is being suppressed in the operation of UZU, it produces a chain of fragmental helices, whose deflection points represent the single scattering. The deflection angles from one fragment to the other lie between TMIN and TMAX (Fig. A1). The average length of the fragments (i.e. the branches) depends on the kind of particle, its momentum, the kind of liquid, and TMIN, TMAX. For instance, for a 1 GeV/c muon in the light freon, it is about 1mm when TMAX and TMIN are set to 2.92 and 0.017 degrees, respectively. When, by means of a control card (No. 14), the multiple scattering is allowed to enter, UZU starts by computing the first branch of the helices exactly as if no multiple scattering were to be allowed. The end-point is the first single scattering point. Then, based on the usual multiple scattering theory, the position and the direction of the track at the end-point are replaced by new values. After adding the single scattering angle to the new direction, these values become the initial values for constructing the second branch of the track (Fig. A2). The mean square angle of multiple scattering $\theta_s^2$ is computed according to

$$\theta_s^2 = 3\pi N Z^2 r_e^2 \left( \frac{m_e c^2}{\beta p} \right)^2 \ln \frac{T_{\MIN}}{\theta_1}$$

where $\theta_1$ is the screening angle given by

$$\theta_1 = \alpha Z^{1/3} m_e c/p.$$  

The random numbers of spacial and directional displacement (these are correlated) are generated according to bivariate normal distribution. The procedure is repeated until the end of the whole track. The choice of TMIN has most probably very little influence on the result of reconstruction, as is pointed out in Section 4.4.

* * *

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A1. The continuous chain of helices where only single scatterings occur.

A2. The broken chain of helices with the multiple scattering.
LIST OF IMPORTANT SUBROUTINES IN UZU

READCX  reads initial parameters of a track.
READER  reads errors.
READPR  reads momentum-range deck.
RUTHER  computes Rutherford cross-section according to the
cut-off angle TMAX, TMIN.
SCATTR  determines scattering points in space.
RSCT    called by SCATTR; produces scattering angle.
RPRLL2  called by SCATTR; determines the length from one
scattering point to the next.
NEW     called by SCATTR; determines the new dip and azimuthal
angle.
START   computes the starting point of each branch (the part of a
track from one scattering point to the next). If UZU2,
THETA is always set to the beginning of the branch. If
UZU1, THETA is set to the point which is left over from
the previous branch (THISURplus).
ABANCE  advances the point to be measured on the track, and projects
the new point on the film.
UZUTST  controls the continuation of the computation on a track.
TRAX4   writes on the tape labels and the coordinates, 132 characters/
record.
DSCRPT  writes heading of an event on the output tape so that it will
be accepted by DRAT (or THRESH).
FIDCL   converts fiducial coordinates and writes them on the output
tape in required format.
FIX     converts the projected coordinates (in cm) to two sets (x and y)
of four-digit integers.
WRITE encodes the number given by FIX to 10 words BCD characters.

SCTWRI prints the scattering data.

FIDWRI prints the projected coordinates of fiducial marks, also introduces measurement errors to the projected coordinates, and prints them.

TAPE dumps the output tape if asked.

SHELL exclusive for UZU3. For a small computer, this part is readily separated from the main program.

BRM in bremsstrahlung, computes a point where the emission occurs (DOD) and the momentum left to the electron after the emission (PRES).

ANGBMS determines the angle of the bremsstrahlung (PHI) and the final momentum (PFIN).

DIFUSE determines a point of scattering in a section of track.

COORD searches a point to be measured on the produced track in SHELL, called by SHELL.

VORTEX computes new parameters of the track after advancement of a certain length, unless there is no bremsstrahlung or scattering in that length.

BRANCH computes new parameters of the track at a point where bremsstrahlung or scattering occurs.

DSLDRO at a given point on a track, given a dip angle, computes the change of radius of curvature Δρ due to ionization for a length of track Δl, called by VORTEX.

PROJECT a point xyz is projected on the film giving XP, YP.

BEGIN solves a routine trigonometric problem.

ROOT finds a root in any monotonically descending functions.

CLM indefinite integral of Rutherford cross-section over the angle. Serves for random choice of a scattering angle.
DAM gives optical relation among the objects.
PTOR TC library.
PRINT TC library.
MXRNG given momentum, finds its range.

* * *
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* * *
Also it is useful to read the following:

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* * *
Fig. 1 The relation of the sagitta to the arc length with the radius of curvature as the parameter.
Subroutines

READ
read titles
project fiducial marks on the film

FIMARK
read amount of errors to be introduced

READCX
read \( \rho_0, \lambda_0, \varphi_0 \) (1 card) → if no more card → CALL EXIT

RUTHER
computes Rutherford c.s. for \( 0 < \theta < \theta_{\text{MAX}} \)
if UZU 1,2
if UZU 3

SCATTIR
compute scattering points and angles
No. of scattering points > max. allowed?

FIOCL
write heading on the auxiliary tape
write fiducials on the tape
if UZU
next view
if not last view

START
set \( \Theta \) to initial point of the branch

AVANCE
project the point (\( \Theta \)) on the film

ERROR
add two-dimensional error to the projected point

ERROR
advance \( \Theta \) to next point

UZUTST
track still shorter than requested?
point still in chamber?

UZUTST
is this point still in the same branch?
if UZU
if UZU

START,TRAX
go to next branch with surplus of \( \Theta \)
write this branch on tape
set \( \Theta \) to the next scattering point

START
next view
is the view last?
next track
No. of track enough as requested?
new track

Fig. 2 Flow chart of UZU1, 2, and 3.
Subroutine SHELL (I)

is it first view (I = 1)?

yes

read initial values (x0y0z0f0ρ0λ0)

set initial values (L_{track} = 0)

increase L_{track} by ΔL_{track}(P)

new L_{track} shorter than Range?

yes

new L_{track} shorter than track length requested?

yes

in this ΔL_{track} scattering occurred?

if yes, reset ΔL_{track} to scattering

compute ΔF/ΔL_{track}

compute hv

compute new XYZϕφλ

DIFUSE

BRM

VOXER

compute ΔF/ΔL_{track} new XYZϕφλ

compute new P_{track}, Range, ΔL_{track}

1500 cycle max.

store X_{track}Y_{track}Z_{track}

set ST = 0

COORD

measur point at ST along L_{track}

PROJECT

project the point on the film

ERROR

add two-dimensional error to the projected point

ERROR

advance ST to the next point

yes track still shorter than it ought to be?

100 cycle max.

point still in chamber?

TRAYL

write track on auxiliary tape with a finish (M)

return

Fig. 3 Flow chart of subroutine SHELL, which constitutes the main part of U2J3.
Fig. 4. An example of print-out of UZU (no scattering).
Fig. 6 An example of print-out of UZU2.
Fig. 7 The coordinate system.

Fig. 8 Frequently used variables names in UZU.

Fig. 9
Fig. 10 The effect of choice of sagitta in UZU3.
Fig. 11 Helix ($\rho = 180$ cm, $\lambda = 0$, $\varphi = 0$) produced by UZU at 50 cm depth in the chamber was reconstructed by DRAT with various measurement lengths ($L$). Each point is an average of 30 tracks, and the flag is its standard deviation. No external measurement error is introduced. The cell length is $1.0 \pm 0.1$ cm.
\[ \phi = 180 \text{ cm} \]

NORMAL MEASUR.  \( \times \) 1.0 \( \pm \) 0.1 cm / MEASUR.

CORRESPONDING
- 1.0 \( \pm \) 0.2 "
- 1.0 \( \pm \) 0.4 "
- 1.0 \( \pm \) 0.6 "

**Fig. 12** Comparison between the normal measurement and the corresponding measurement.
Fig. 13 Comparison between THRESH and DRAT ($\rho = 200$ cm, $\lambda = 0$, $\phi = 0$, no external measurement error, the cell length $1.0 \pm 0.2$ cm).
Fig. 14. The distribution of r.m.s. of $1/\rho$ against the number of cells in 15 cm track (the number of cells = number of measurement points - 1).
Fig. 15 The distribution of r.m.s. of $\lambda$ against the number of cells in 15 cm track (the number of cells = number of measurement points + 1).
Fig. 16 The distribution of r.m.s. of $\phi$ against the number of cells in 15 cm track (the number of cells = number of measurement points - 1).
Fig. 17  The distribution of r.m.s. of $|\Delta 1/\rho \Delta \phi|$ against the number of cells in 15 cm track (the number of cells = number of measurement points - 1).
Fig. 18 The distribution of r.m.s. of $\Delta 1/\rho \Delta \lambda$ against the number of cells in 15 cm track (the number of cells = number of measurement points -1).
Fig. 19 The distribution of r.m.s. of $\Delta \phi \Delta \lambda$ against the number of cells in 15 cm track (the number of cells = number of measurement points - 1).
Fig. 20 The reconstructed $\rho$, $\lambda$, $\varphi$-distribution against the number of cells on a 15 cm track.
Fig. 21 Function $\alpha_n$, $\beta_n$ and $\gamma_n$ in Eqs. (3), (4), and (6).
MEASUREMENT ERROR

○ 5 MICRONS
● 10 MICRONS

Fig. 22 The effect of external measurement errors (the figure are errors on the film).
Fig. 23 $\Delta 1/\rho$ versus $L$, with the external measurement errors 0, 5, and 10 microns on the film. Assuming $\Delta \epsilon_{\text{int}} = 25$ microns, the three straight lines represent the expected values.
Fig. 24. $\Delta \lambda$ versus $L$, with the external measurement errors 0, 5, and 10 microns on the film. Assuming $\Delta \varepsilon_{\text{int}} = 25$ microns, the three straight lines represent the expected values.
Fig. 25 $\Delta \phi$ versus $L$, with the external measurement errors 0, 5, and 10 microns on the film. Assuming $\Delta \epsilon_{\text{ext}} = 25$ microns, the three straight lines represent the expected values.
Fig. 26 $|\Delta \varphi \Delta 1/\rho|^{1/2}$ versus L, with the external measurement errors 0, 5, and 10 microns on the film. Assuming $\Delta \varepsilon_{\text{int}} = 25$ microns, the three straight lines represent the expected values.
Fig. 27 $\Delta \frac{1}{\rho} \Delta \lambda$ versus $L$, with the external measurement errors 0, 5, and 10 microns on the film. Assuming $\Delta \varepsilon_{\text{int}} = 25$ microns, the three straight lines represent the expected values.
Fig. 28 $\Delta \phi$ versus $L$, with the external measurement errors 0, 5, and 10 microns on the film. Assuming $\Delta \varepsilon_{\text{int}} = 25$ microns, the three straight lines represent the expected values.
Fig. 29 The reconstructed $\rho$, $\lambda$, $\phi$ for the tracks with scattering. (See text for details.)
Fig. 30 The variance of $1/\rho$ with scattering.
Fig. 31 The variance of $\lambda$ with scattering.
Fig. 32  The variance of $\varphi$ with scattering.
\[ \Delta \frac{1}{\rho} \Delta \varphi \text{(cm}^{-1}\text{)} \]

\[ 10^{-5}, 10^{-6}, 10^{-7}, 10^{-8} \]

\[ L \text{(cm)} \]

Fig. 33 The covariance of \(1/\rho\) and \(\varphi\) \textit{with} scattering.

- o CORRECTED FOR \(\Delta \varepsilon\)
Fig. 3. Produced electron tracks in light freon.
Fig. 35  Produced electron tracks in freon.
Fig. 36 The definition of $L_p$ should be referred to the text. The relation between $L_p/L$ and $p/L$ (p, reconstructed momentum; $L$, reconstructed length) is given.
Fig. 37 The tracks only under the influence of the ionization are reconstructed (the liquid is light Freon) and compared with some expected formulae.
Fig. 38 The momentum change of randomly produced electrons subject to the bremsstrahlung energy loss.
1) The assembly of lines broken by horizontal steps are the cases without ionization.
2) The lines broken by diagonal steps are the cases with ionization.
The smooth curves are an average of those cases theoretically expected [Eq. (33)].
Fig. 39 The variance of curvature for 244 MeV/c electron versus measured length. The open circles are for cases with no ionization; the closed, with ionization.
Fig. 41 The fitted radius of curvature for a dozen electrons with respect to the measured length (with ionization) for $\rho_0 = 4.5$ cm (244 MeV/\(c\)).
Fig. 42 Tracks only under the influence of the bremsstrahlung and scattering are reconstructed and compared with some expected formulae.
Fig. 44. The variances only under the influence of bremsstrahlung and scattering.
Fig. 45 The tracks under the influence of all effects (the ionization, the bremsstrahlung, and the scattering) are reconstructed and compared with some expected formulae.