HIGH-ENERGY NEUTRINO INTERACTIONS

Enoch C.M. Young
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Experimental studies of high-energy neutrino interactions were carried out at CERN during 1963-65, using the CERN heavy-liquid bubble chamber filled with freon C_{2}F_{2}Br (in all the runs), and the spark chamber (in the 1963/64 runs). An analysis of the results obtained from the bubble chamber will be presented in this report.

During the neutrino runs, $7.40 \times 10^{17}$ protons were extracted from the CERN Proton Synchrotron to strike a copper target to produce pions and kaons which then decayed to give neutrino beams. Five hundred and twenty-one events were attributed to neutrino interactions. During the antineutrino runs, $2.25 \times 10^{17}$ protons were extracted, and a total of 59 antineutrino events was observed. Based on the analysis of these events, three problems will be investigated; namely, the elastic neutrino reactions, the hyperon production by antineutrinos, and the single pion production.

This report consists of three parts. In Part I (Chapter 1), a brief theoretical review on the weak interactions involving the neutrinos will be given. The basic assumptions on which the theory of weak interactions is based will first be reviewed briefly. The theoretical considerations on the elastic neutrino reactions, the hyperon production, and the single pion production will then be presented.

Part II (Chapters 2 and 3) deals with the experimental matters. In Chapter 2, after the experimental arrangements are described, the methods by which the neutrino or the antineutrino events were analysed will be discussed. Chapter 3 is concerned with the estimation of the background events. Two types of background will be studied in detail; namely, those due to neutron-induced interactions and those due to incoming charged particles.

Part III (Chapters 4, 5 and 6) contains the discussions and interpretations of the experimental results. Chapter 4 deals with the elastic neutrino reactions. The axial vector form factor has been determined for different assumptions on its form. The experimental total elastic cross-section will be compared with the theoretical values. An account of the experimental study of hyperons production by antineutrinos will be the subject of Chapter 5. During the antineutrino runs, no hyperon candidates have been observed in the bubble chamber. As a result, the upper limit of the total cross-section for the production of all types of hyperons has been determined to be $1.98 \times 10^{-40}$ cm$^2$/nucleon, when averaged over the antineutrino spectrum. This result is in disagreement with the prediction based on the SU$_3$ model of weak interactions, and on the assumption of the vector form factors of the hyperon-nucleon current being dominated by the K$^-$ exchange. In Chapter 6, the single-pion production will be discussed. The experimental results are compatible with the predominance of $N_3^*$ production in single-pion production processes.
cross-sections for single-pion production have been determined and compared with the theoretical predictions. It seems that the assumption on the equality of the vector and the axial vector form factor is favoured by the experimental results.
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PART I

THEORY
CHAPTER 1

THEORETICAL REVIEW

1. INTRODUCTION

1.1 Historical background

The existence of a neutral particle, called the neutrino (Italian for "little neutral one"), was first postulated by Pauli \(^1\)\(^2\) in 1931 in order to save the energy-momentum conservation laws in face of the evidence presented by nuclear beta decay. This idea was soon utilized by Fermi \(^3\) to formulate his celebrated theory of weak interactions by which the shape of the electron spectrum of beta decay was successfully explained. In the late 1940's, after the discoveries of other weak reactions, such as \(\mu\) decay, and \(\mu\) capture, it became clear that all these reactions have approximately the same strength, well described by the universal Fermi constant \(G\). The form of Fermi's theory remained almost intact for several decades, except for some minor modifications. In 1956, the possibility of parity non-conservation in weak interactions was suggested by Lee and Yang \(^4\), and was immediately confirmed experimentally by Wu et al. \(^5\), from the observation of asymmetry in the beta decay of polarized \(^{60}\)Co, and shortly afterwards, by Garwin et al. \(^6\), in the study of the \(\pi,\mu\) decays.

After the discovery of the non-conservation of parity, the theory of weak interactions underwent rapid development. The two-component neutrino theory and a reformation of the universal Fermi interaction in terms of vector and axial vector coupling (the V-A theory) were proposed independently by Lee and Yang \(^7\), Feynman and Gell-Mann \(^8\), and Marshak and Sudarshan \(^9\) in 1957. This theory could explain essentially all low-energy weak processes which conserved strangeness. In particular, the branching ratio of \(\pi \rightarrow e + \nu/\pi \rightarrow \mu + \nu\) was correctly predicted.

The weak interactions so far extensively studied are the beta decay in which the momentum transfer is of the order of a few MeV, and the \(\mu\)-decay and \(\mu\)-capture processes which involve momentum transfer of the order of 100 MeV. These low-energy neutrino interactions are now well described by the simple V-A theory which assumes that the interaction occurs at the same space-time point. It is obviously desirable to investigate the question whether this theoretical approach leads to correct description of interactions involving higher momentum transfers. The neutrino, which has only a weak interaction with all other particles, is clearly an ideal probe for this investigation.

The feasibility of studying weak interactions with high-energy neutrino beams obtained from high-energy accelerators was first put forward independently by Pontecorvo \(^10\) (1959) and by Schwartz \(^11\) (1960). It is well known that pions produced in high-energy accelerators and the neutrinos from the decay of high-energy pions are highly collimated. Furthermore, neutrinos, with only weak interactions with matter, can penetrate a shield thick enough to screen all other particles produced by an accelerator.
The first neutrino experiment with high-energy accelerators was performed in 1962 by the Columbia University-Brookhaven National Laboratory group\(^{12,13}\). A 10-ton spark chamber was used as detector. In the 500-hour exposure, some 50 events were observed. The notable result of this experiment was the confirmation of the existence of two types of neutrinos, the electron neutrino \(\nu_e\) and the muon neutrino \(\nu_\mu\). However, the shortcomings of the Columbia-BNL experiment were the low intensity of the neutrino beam and the inherent poor resolution of spark chambers.

Over the years from 1963 to 1965, experiments with neutrino beams were performed in CERN, using the CERN 28 GeV Proton Synchrotron. Several new experimental techniques were put into use for the first time. In particular, the proton beam was extracted before it hit a target to produce high-energy pions and kaons, which were then focused with a "magnetic horn", a horn-shaped focusing device. A very large heavy-liquid bubble chamber filled with freon, \(\text{CF}_3\text{Br}\), was used as detector, in addition to spark chambers. In this report, we shall be concerned with the results obtained from the bubble chamber.

1.2 Basic properties of neutrinos\(^{14,15}\)

Before we go into the discussion of the theory of weak interactions involving neutrinos in the succeeding sections, we shall briefly recapitulate some of the basic properties of neutrinos.

1.2.1 Types of neutrinos. It has been conclusively demonstrated\(^{17}\) that there are at least two distinct types of neutrinos; namely, the electron neutrino \(\nu_e\) and the muon neutrino \(\nu_\mu\). The electron neutrino is found in reactions associated with the electron, e.g. the beta decay of the neutron,

\[
n \rightarrow p + e^- + \bar{\nu}_e, \tag{1.1}
\]

and the muon neutrino is produced in reactions involving the muon, e.g. the \(\pi\) decay

\[
\pi^- \rightarrow \mu^- + \bar{\nu}_\mu. \tag{1.2}
\]

Up to the present, our knowledge is consistent with the assumption that apart from muon number, all the properties of these two particles are identical.

1.2.2 Electric charge. The charge of a neutrino can be measured by the use of charge conservation. It has been established experimentally\(^{18}\) that the difference in charge between a neutron and a hydrogen atom is \(< 10^{-19}\) e. From the conservation of charge and the neutron beta decay Eq. (1.1), it follows that

\[
|e\nu_e| < 10^{-19} \text{ e}. \tag{1.1}
\]

The limit obtained for \(\nu_\mu\) is not so precise, because the charge of \(\mu\) is not measured so accurately. However, it has been found that the deviation of the muon charge from the electron is \(< 5 \times 10^{-6}\) e. It follows from charge conservation in \(\mu\) decay,

\[
\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu, \tag{1.3}
\]

that \(e\nu_\mu < 5 \times 10^{-6}\) e.
1.2.3 Mass. The best information about the mass of $\nu_e$ comes from a measurement of the end point of the Kurie plot in the tritium $\beta$ decay\(^{17}\). This measurement, whose interpretation is sensitive to the form of the Fermi coupling, gives a limit of $m_{\nu_e} < 0.7$ keV, which is also consistent with zero. Assuming the two-component theory, the limit is $m_{\nu_e} < 0.2$ keV.

The mass of $\nu_\mu$ is perhaps the least well-known parameter associated with the $\nu_\mu$. Our best information comes from a measurement of the muon momentum in the pion decay Eq. (1.2), and the limit is $m_{\nu_\mu} < 3$ MeV\(^{18}\).

If both the two-component theory and the law of conservation of lepton number are valid, the masses of $\nu_e$ and $\nu_\mu$ should be zero\(^{19}\).

1.2.4 Spin and helicity. The application of angular momentum conservation to weak decay processes involving neutrinos, e.g. $\pi \rightarrow \mu \nu_\mu$ and $\pi \rightarrow e \nu_e$, shows that they have spin $\frac{1}{2}$.

According to the two-component neutrino theory, the neutrino occurs in nature in only one helicity state, whereas the antineutrino occurs in the opposite helicity state.

The helicity of $\nu_e$ was determined by Goldhaber, Grodzins, and Suryan\(^{20}\), who used the process of electron K-capture in a $0^- \rightarrow 1^-$ transition. The result shows that the helicity of $\nu_e$ is left-handed and has a magnitude greater than 0.97, with an uncertainty of \(\pm 2\%\).

The helicity of $\nu_\mu$ has been measured through a determination of the muon helicity in the pion decay\(^{21}\) [Eq. (1.2)]. It is found that the helicity of $\bar{\nu}_\mu$ is +1, with a possible error of 20\%.

The above properties of the neutrinos can be summarized in Table 1.1.

### Table 1.1

Basic properties of neutrinos

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<th>$\nu_\mu$</th>
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<td></td>
<td>Experimental value</td>
<td>Assumed value</td>
<td>Experimental value</td>
</tr>
<tr>
<td>Charge</td>
<td>$&lt; 10^{-19}$ e</td>
<td>0</td>
<td>$&lt; 5 \times 10^{-6}$ e</td>
</tr>
<tr>
<td>Mass</td>
<td>$&lt; 0.2$ keV</td>
<td>0</td>
<td>$&lt; 3$ MeV</td>
</tr>
<tr>
<td>Spin</td>
<td>$\frac{1}{2}$ (m = 0)</td>
<td>$\frac{1}{2}$</td>
<td>$\frac{1}{2}$ (m = 0)</td>
</tr>
<tr>
<td>Helicity</td>
<td>&gt; 0.95</td>
<td>1</td>
<td>&gt; 0.80</td>
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2. **Some Basic Assumptions\(^{19,22}\)**

Besides being invariant under the Lorentz transformations, and satisfying the conservation laws of energy, momentum, charge, and baryon number, the weak interactions possess other properties. In the following, we shall attempt to outline the general theoretical aspects, which are satisfied by all known results of weak interactions.
2.1 Conservation of lepton numbers

At present, there are four different kinds of leptons: $\mu^-, e^-, \nu_{\mu}, \bar{\nu}_{\mu}$, and their anti- particles, $\mu^+, e^+, \bar{\nu}_{\mu}, \nu_{\mu}$. Leptons are assigned a leptonic number $L = 1$, and their anti- particles $L = -1$. For all other particles, $L = 0$. In addition, we can assign a muon number $L_\mu = 1$ for $\mu^-$ and $\nu_\mu$, $L_\mu = -1$ for $\mu^+$ and $\bar{\nu}_\mu$, and $L = 0$ for all other particles. Similarly, we define an electron number $L_e = 1$ for $e^-$ and $\nu_e$, $L_e = -1$ for $e^+$ and $\bar{\nu}_e$, and $L = 0$ for all other particles.

The conservation laws of lepton numbers then state that in all reactions the algebraic sums of $L_\mu$ and $L_e$ are separately conserved. The evidence\cite{19}\ comes from a number of experiments such as the lack of neutrinoless double $\beta$ decay,

$$Z \rightarrow (Z+2) + 2e^-,$$

and the absence of reaction

$$\nu_\mu + n \rightarrow p + e^-$$

in high-energy neutrino experiments\cite{12,13,24}.

2.2 The muon-electron universality\cite{25}

The principle of $\mu$-$e$ universality states that if any weak interaction process involving the $e, \nu_e$ occurs, then the same process with $e$ replaced by $\mu$ and $\nu_e$ replaced by $\nu_\mu$ also occurs. The two processes will differ only by virtue of the muon-electron mass difference. The strong evidence of muon-electron universality comes from the correct prediction of the branching ratio of $\pi^+ \rightarrow \mu^+ + \nu_\mu (\bar{\nu}_\mu)$ and $\pi^+ \rightarrow e^+ + \nu_e (\bar{\nu}_e)$\cite{19,26}.

2.3 Current-current hypothesis\cite{3,8,9}

It is assumed that the weak interaction Lagrangian density can be written as a product of two currents, i.e.

$$\frac{G}{\sqrt{2}} f_a(x) \tilde{f}_a^*(x)$$

where $\tilde{f}_a(x)$ is the Hermitian conjugate of $f_a(x)$, and $G$ is the weak interaction constant, which has been determined from the $\beta$ decay of $^{14}C$ and from the $\mu$ decay. In the system of natural units, $\hbar = c = 1$, which we shall use throughout this chapter, it has been found that

$$G = (1.01 \pm 0.01) \times 10^{-5} \frac{M_p^2}{m},$$

where $M_p$ is the proton mass.

Furthermore, the weak interaction is assumed to be a very short-range force so that the two currents interact only at the same space-time point $x$. The current $f_a(x)$ can be resolved into the sum of $j_a(x)$, the current operator for the leptons, and $J_a(x)$, the current operator for the strongly interacting particles (hadrons), i.e.

$$f_a(x) = j_a(x) + J_a(x).$$
For example, in the $\beta$ decay of the neutron Eq. (1.1), the lepton current $j_\alpha (\nu_e \rightarrow e^-)$ interacts with the nucleon current $j_\alpha (n \rightarrow p)$, and the interaction is described by

$$\frac{G}{\sqrt{2}} (j_\alpha \gamma^+ + j_\alpha \gamma^-). \quad (1.6)$$

Experiments have so far established the existence of the following currents:

i) lepton currents: $(\mu, \nu)$, $(e, \nu_e)$

ii) hadron currents: $(n,p)$, $(A^\alpha, p)$, $(\Xi^-, n)$, $(\pi^+, \pi^-)$, etc.

2.4 The V-A theory \(^{7-9}\)

The most general local four-fermion Lorentz invariant leptonic interaction is of the form

$$\frac{1}{\sqrt{2}} \sum_i G_i (\bar{\psi}_2 0_4 \phi_i) (\bar{\psi}_4 0_4 \phi_\nu) + h.c., \quad (1.7)$$

where $0_4$ are the Dirac covariant operators given by:

$$\begin{align*}
0_S &= 1 & \text{Scalar} \\
0_V &= \gamma_\alpha & \text{Vector} \\
0_T &= -\frac{\gamma_\alpha \gamma_\beta - \gamma_\beta \gamma_\alpha}{2} = -i \sigma_\alpha \beta & \text{Tensor} \\
0_A &= i \gamma_\alpha \gamma_5 & \text{Axial vector} \\
0_P &= \gamma_5 & \text{Pseudoscalar}
\end{align*} \quad (1.8)$$

where $\gamma_\varsigma$ are the $(4 \times 4)$ anticommuting Hermitian matrices satisfying

$$\gamma_\alpha \gamma_\beta + \gamma_\beta \gamma_\alpha = 2 \delta_\alpha \beta. \quad (1.9)$$

Under the parity operation, this interaction transforms as a scalar. However, a parity non-conserving interaction should be a mixture of scalar and pseudoscalar terms. The latter can be constructed by introducing $0^+_4 = 0_4 \gamma_5$ in the lepton bracket. Therefore, a parity-violating interaction is of the form

$$\frac{1}{\sqrt{2}} \sum_i (\bar{\psi}_2 0_4 \phi_i) [\bar{\psi}_4 0_4 (G_i + \bar{G}_i \gamma_5) \phi_\nu] + h.c. \quad (1.10)$$

The experimental result \(^{21}\) that only left-handed leptons and right-handed antileptons exist, indicates that lepton operators enter into the interaction in the form $(1 - \gamma_5) \rho$ or $\bar{\psi}(1 - \gamma_5)$ only, i.e. the two-component theory \(^{7-29}\). The lepton bracket in formula (1.10) should then have the form
Furthermore,

\[ (G_1 + G_1' \gamma_5) \psi_\nu = \frac{1}{2} (G_1 + G_1') (1 + \gamma_5) \psi_\nu + \frac{1}{2} (G_1 - G_1') (1 - \gamma_5) \psi_\nu . \]

(1.12)

Since only \((1 + \gamma_5) \psi_\nu\) can occur, it follows that

\[ G_1 = G_1' \]

which implies the maximum violation of parity.

The lepton bracket of formula (1.10) can now be rewritten as

\[ \bar{\nu}_\ell D_4 (1 + \gamma_5) \psi_\nu = \bar{\nu}_\ell (1 + \gamma_5) D_4 \psi_\nu \quad \text{for} \quad i = S, T, P \]

\[ = \bar{\nu}_\ell (1 - \gamma_5) D_4 \psi_\nu \quad \text{for} \quad i = V, A . \]

(1.13)

That leptons exist in the states \((1 + \gamma_5) \psi_\nu\) or \(\bar{\psi}(1 - \gamma_5)\) indicates that only \(V\) and \(A\) interactions can appear. The interaction (1.10) then becomes

\[ \frac{1}{\sqrt{2}} [\bar{\psi}_\ell \gamma_\alpha (G_V - G_A \gamma_5) \psi_\nu] [\bar{\psi}_\ell \gamma_\alpha (1 + \gamma_5) \psi_\nu] + \text{h.c.} \]

(1.14)

It has been found that for interactions involving only leptons, e.g. \(\mu \rightarrow e + \nu + \bar{\nu}\), \(G_V = -G_A\). In beta decay, \(G_A = -\lambda G_V\) with \(\lambda = 1.18\). The deviation of \(\lambda\) from unity is attributed to the renormalization effects of the strong interactions of the nucleons.

This interaction, containing only the vector and axial vector couplings and with \(G_V = -G_A\), is called the \(V-A\) interaction.

2.5 CPT invariance and \(G\) parity

It has been shown that in the framework of local field theory if a theory is invariant under proper Lorentz transformations, then the theory is also invariant under the joint operator of CPT, although it may not be invariant under each separate operator \(C\), \(P\), or \(T\).

The weak interactions with four-momentum transfer in the range \(\lesssim 1\text{--}2\) GeV, being well described phenomenologically by an effective local Lagrangian, are expected to be invariant under the combined operation of CPT. Under this assumption, CP invariance and T invariance are equivalent.

Time reversal invariance is generally assumed valid in weak interactions. This implies that the form factors in the hadron currents should be real.

The \(G\) parity is defined as the combined operation of charge conjugation \(C\), and rotation of \(180^\circ\) about the second axis of isospace, namely,

\[ G = C \exp (i\pi I_3) . \]

(1.15)
G parity invariance implies the vanishing of certain terms in the matrix element, as we shall see later in this chapter.

2.6 Conserved vector current hypothesis (CVC$^8,9$)

The strangeness-conserving ($\Delta S = 0$) hadron current $J_\alpha$ of the weak interaction can be decomposed into a vector part $V_\alpha$ and an axial vector part $A_\alpha$:

$$ J_\alpha = V_\alpha + A_\alpha. \tag{1.16} $$

It can be shown$^{32}$) that if the strong interactions are neglected, under G, the currents $V_\alpha$ and $A_\alpha$ transform as:

$$ V_\alpha \xrightarrow{G} V_\alpha, \tag{1.17} $$

$$ A_\alpha \xrightarrow{G} - A_\alpha. \tag{1.18} $$

They are called currents of the first class, whereas those that transform with an opposite sign are called currents of the second class.

The CVC hypothesis, proposed by Feynman and Gell-Mann, states that for the vector current $V_\alpha$,

a) \[ \frac{\delta V_\alpha}{\delta x_\alpha} = 0 \text{ for } \Delta S = 0 \tag{1.19} \]

and

b) the three current operators

$$ V_{\alpha', \beta} \left( \frac{G}{e} \right) \left( J_{\alpha}^{el} \right)_{I=1}, \text{ and } V_{\alpha}^{\ast} \tag{1.20} $$

transform, respectively, as the $I_3 = -1, 0, +1$ members of a single $I = 1$ triplet, where $(J_{\alpha}^{el})_{I=1}$ is the isovector part of the non-leptonic electromagnetic vector current.

This hypothesis relates the weak vector form factors to the corresponding isovector electromagnetic form factors. Let us consider this in more detail. In isotopic spin notation, the electromagnetic vector current can be written as:

$$ \overline{\psi}_p \gamma_\alpha \psi_p \text{ + pion current} = \overline{\psi} \gamma_\alpha \left( \frac{1 + I_3}{2} \right) \psi + \text{(pion currents)}, \tag{1.21} $$

where

$$ \psi = \left( \begin{array}{c} \psi_p \\ \psi_n \end{array} \right), \quad \overline{\psi} = \left( \overline{\psi}_p, \overline{\psi}_n \right). $$

The isovector component of Eq. (1.21) is then

$$ J^{\text{el}}_\alpha = \overline{\psi} \gamma_\alpha \left( \frac{I_3}{2} \right) \psi + \text{(pion current)}, \tag{1.22} $$
Similarly, the weak vector current is

\[ V_\alpha = \bar{\psi} \gamma_\alpha \gamma_\uparrow \phi + \text{(pion current)} \]

where

\[ I_+ = \frac{1}{2}(I_1 + iI_2) \].

The CVC hypothesis states that \( j^{\text{el}}_\alpha \) and \( V_\alpha \) are merely different components of the same isovector:

\[ \bar{\psi} \gamma_\alpha I_\uparrow \phi + \text{(pion current)} \].

Since it is known that the electromagnetic current \( j^{\text{el}}_\alpha \) is conserved, it follows that if isospin-breaking interactions, such as electromagnetic interactions, are neglected, the current (1.24) is also conserved. Hence \( V_\alpha \) in Eq. (1.23) is also conserved. Thus, the CVC hypothesis predicts unique relations between the matrix element of the weak hadron vector current and that of the electromagnetic current. This we shall discuss further, later in this chapter.

2.7 Possible existence of intermediate vector boson \( W^{31-33} \)

In analogy with the exchange of a photon in electromagnetic interactions, it is suggested that weak interactions are mediated by the exchange of a charged vector boson \( W \). For example, the elastic neutrino and antineutrino interactions [see Eqs. (1.31) and (1.32)] can be described by the following diagrams (Fig. 1.1):

![Feynman diagram](image)

**Fig. 1.1**
Feynman diagram for the exchange of vector boson \( W \) in weak interactions

The interaction will then have the form

\[ g(W_\alpha \phi_\alpha + \bar{W}_\alpha \phi_\alpha) + \text{h.c.} \]

where \( W_\alpha \) is the boson field, and

\[ g = \frac{GM_W^2}{\sqrt{2}} \]

where \( M_W \) is the boson mass.
If the vector boson $W$ indeed exists, it can be produced by high-energy neutrinos or anti-neutrinos in reactions:

$$\nu_\mu + Z \to W^+ + \mu^- + Z' \quad (1.26)$$

and

$$\bar{\nu}_\mu + Z \to W^- + \mu^+ + Z' \quad (1.27)$$

where $Z$ represents a target proton or nucleus which provides the electromagnetic field in which the $(W, \mu)$ pair is created, and $Z'$ is the recoil nucleon or nucleus. The boson would then decay\cite{14} within $\sim 10^{-10}$ seconds through various channels such as

$$W^+ \to \mu^+ + \nu_\mu (\bar{\nu}_\mu) \quad (1.28)$$

$$W^+ \to e^+ + \nu_e (\bar{\nu}_e) \quad (1.29)$$

and

$$W^+ \to \text{mesons, etc.} \quad (1.30)$$

The search for such a vector boson was one of the primary objectives of the BNL-Columbia\cite{34} and CERN\cite{35} neutrino experiments. No experimental evidence has so far been established for its existence. However, it has been found that if it exists at all, its mass must be larger than $\sim 2 \text{ GeV}\cite{38}$.

3. THE ELASTIC NEUTRINO INTERACTIONS\cite{36-38}

In this section, we shall discuss the form of the matrix element and the calculation of the cross-sections for the following two elastic interactions due to neutrinos and anti-neutrinos:

$$\nu_\mu + n \to p + \mu^- \quad (1.31)$$

$$\bar{\nu}_\mu + p \to n + \mu^+. \quad (1.32)$$

The first reaction (1.31), for example, can be described by Fig. 1.2, the four-momenta and masses of the various particles being represented in the diagram on the right:

![Feynman diagram for elastic neutrino reactions](image-url)
As has been seen earlier, the Lagrangian density for this reaction has the form

\[ \frac{\mathcal{L}}{\sqrt{2}} = j^a j_a \]

where

\[ j_a = \bar{\psi}_\mu \gamma_a (1 + \gamma_5) \psi_\nu. \]

The matrix element \( M \), in which all measurable quantities of the reaction are contained, can then be derived if \( j_a \) is known.

### 3.1 The matrix element

The matrix element \( M \) consists of two factors, namely, the leptonic factor \( \ell_a \) and the hadronic factor \( L_a \). As a result of the V-A theory, the leptonic factor is

\[ \ell_a = \bar{u}(p_\mu) \gamma_a (1 + \gamma_5) u(p_\nu) \]  \hspace{1cm} (1.33)

where \( u(p) \) are Dirac spinors.

The hadronic factor, however, is complicated by strong interactions of the nucleons, and it is expected that it will not have the same form as \( \ell_a \). What one can do is to write down the most general form of \( L_a \) and discuss each term with the available theoretical and experimental information.

The most general form of \( L_a \) is

\[ L_a = \langle p | J_a(x) | n \rangle = \bar{u}(k') \left[ G_a \gamma_a + \frac{\lambda}{4m} \left( \gamma_a \gamma_\mu (1 + \gamma_5) \gamma_\nu \right) q_\mu q_\nu + \frac{\lambda a}{4m} \gamma_a q_\mu \right] \]

\[ + \frac{\lambda b}{4m} \left( \gamma_a \gamma_\mu \gamma_\nu \gamma_5 \right) q_\mu q_\nu \]

\[ + u(k) \]  \hspace{1cm} (1.34)

where \( q = p_\nu - p_\mu = \) four-momentum transfer

\( M = \) nucleon mass

\( m_\mu = \) muon mass.

The coefficients \( G_a, \ldots, b \) are functions of the squared four-momentum transfer \( q^2 \).

If time reversal invariance in weak interactions is assumed, all the coefficients should be real. Moreover, G-parity symmetry implies that terms with coefficients \( A \) and \( B \) should vanish.

The matrix element for reaction (1.31) is then

\[ M = \frac{\mathcal{L}}{\sqrt{2}} \bar{u}(k') \left[ F_1(q^2) \gamma_a - \frac{\mu}{2M} F_2(q^2) \sigma_{\mu\nu} q_\mu q_\nu + \lambda \mathcal{F}_a(q^2) q_\mu q_\nu \right] u(k) \times \bar{u}(p_\mu) \gamma_a (1 + \gamma_5) u(p_\nu) \]  \hspace{1cm} (1.35)
where \( \lambda = g_A(0)/g_V(0) \), and the small contribution of the induced pseudoscalar term (with coefficient \( b \) and depending on the muon mass \( m_\mu \)) has been neglected. The form factors \( F_s(q^2) \), \( F_z(q^2) \), and \( F_A(q^2) \) are normalized to unity at \( q^2 = 0 \), e.g.,

\[
F_1(q^2) = \frac{g_V(q^2)}{g_V(0)} .
\]

They are named in accordance with the multiplying \( \gamma \) matrices:

- \( F_1 \) = vector form factor
- \( F_z \) = magnetic form factor
- \( F_A \) = axial vector form factor

### 3.2 The cross-sections for elastic processes

The elastic cross-section for reaction (1.31) can be calculated from the matrix element (1.35). It has been shown \(^{37-41}\) that the differential cross-section for neutrino energy \( E_\nu \) is (muon mass \( m_\mu \) is neglected)

\[
\frac{d\sigma}{dq^2} = \frac{G^2}{\lambda^2 E_\nu^2} \left[ A + B(s-u) + C(s-u)^2 \right]
\]

where

\[
A = q^2(\lambda^2 F_A^2 - 4F_z^2) + q^4 \left( F_1^2 + \frac{\mu^2}{M^2} F_z^2 + \frac{\lambda^2}{M^2} F_A^2 + \lambda^2 F_A^2 \right) - q^6 \frac{\mu^2 F_z^2}{4M^2}
\]

\[
B = 4q^2 \left( F_1 - \frac{\mu}{M} F_z \right) \lambda F_A
\]

\[
C = F_1^2 + \lambda^2 F_A^2 + q^2 \frac{\mu^2 F_z^2}{4M^2}
\]

and where we have used the Mandelstam variables defined by:

\[
s = -(p_\nu + k)^2 = -(p_\mu + k')^2 = \text{total energy in c.m. system}
\]

\[
t = -(p_\nu - p_\mu)^2 = -(k' - k)^2 = -q^2
\]

\[
u = -(p_\nu - k')^2 = -(k - p_\mu)^2 .
\]

In the laboratory system, we have

\[
s - u = 2E_\nu M - q^2 - m_\mu^2.
\]

Since no complete theory of strong interactions exists, the form factors cannot be calculated from theoretical principles. However, the CVC theory relates the weak vector form factor \( F_1(q^2) \) and the weak magnetic form factor \( F_z(q^2) \) to the isovector electromagnetic form factor \( F_V(q^2) \) so that
\[ F_1(q^2) = F_2(q^2) = F_Y(q^2) \] (1.40)

and \( \mu = \mu_p - \mu_n = 3.71 \) Bohr nucleon magnetons.

The form factor \( F_Y(q^2) \) has been determined from electron-proton scattering experiments. The only unknown is then the weak axial vector form factor \( F_A(q^2) \).

The total elastic cross-section can be calculated by integrating \( d\sigma/dq^2 \) over \( q^2 \) for an assumed \( F_A \). Various authors have made calculations on the elastic cross-sections for different types of form factors.

The cross-section for the antineutrino process (1.32) has the same form as that of the neutrino (1.37), except with the sign in front of the term \( B(s-u) \) changed, i.e. for reaction (1.32) the differential cross-section is

\[ \frac{d\sigma}{dq^2} = \frac{e^2}{32\pi E^2} \left[ A - B(s-u) + C(s-u)^2 \right]. \] (1.41)

It has been noted\(^9\) that the difference between the cross-sections of neutrino and antineutrino reactions is particularly sensitive to \( F_A(q^2) \):

\[ \frac{d\sigma}{dq^2} = \frac{d\sigma}{dq^2} - \frac{e^2}{16\pi E^2} B(s-u) = \frac{e^2}{4\pi E^2} q^2 \left( F_1 + \frac{F_2}{M^2} \right) \lambda F_A. \] (1.42)

The axial vector form factor \( F_A(q^2) \) can thus be determined, provided that sufficient number of events of both types is obtained.

In the above discussion of the cross-sections, we have neglected the existence of the intermediate vector boson \( W \). If weak interactions are indeed mediated by \( W \), then the form factors should be multiplied by the vector boson propagator, i.e. \( F_i(q^2) \) should be replaced by

\[ F_i'(q^2) = F_i(q^2) \frac{1}{1 + (q^2/M_w^2)} \quad \text{for } i = 1, 2, A \] (1.43)

where \( M_w \) is the boson mass. As has been seen before, the lower limit of \( M_w \) has been set at \( \sim 2 \text{ GeV} \).

4. ELASTIC PRODUCTION OF HYPERONS\(^{42,43}\)

In the last section, we discussed the elastic neutrino reactions (1.31) and (1.32). These reactions involve no change in strangeness between the initial and the final states, i.e. \( \Delta S = 0 \). In this section, we shall discuss another type of "elastic" reactions in which the strangeness changes by unity, i.e. \( \Delta S = 1 \).

Among the \( \Delta S = 1 \) reactions, the \( \Delta S = \Delta q \) rule allows only three "elastic" reactions, all arising from \( \bar{\nu} \) interactions:
\[ \bar{\nu}_\mu + p \rightarrow \mu^+ + \Lambda \]  
(1.44)

\[ \bar{\nu}_\mu + n \rightarrow \mu^+ + \Xi^- \]  
(1.45)

\[ \bar{\nu}_\mu + p \rightarrow \mu^+ + \Xi^0 . \]  
(1.46)

The theoretical predictions of hyperon production by antineutrinos are based on the SU3 model of weak interactions proposed by Cabibbo. In this section, this model will first be discussed briefly. Then the calculations of cross-sections in the framework of this model will be explained.

4.1 The SU3 model of weak interactions

The SU3 scheme for the strongly interacting particles was proposed by Gell-Mann and Ne'eman in 1962. In this scheme, the eight \( J^P = \frac{1}{2}^+ \) baryons (\( \Xi^0, \Xi^- , \Lambda , p , n , \Sigma^- , \Sigma^0 \)) can be identified as the linear combinations of an eight representation of the group SU3. The generators \( F_i \) of the group must obey commutation relations of the form

\[ [F_i, F_j] = C_{i j k} F_k \]  
(1.47)

where \( C_{i j k} \) are constants which define the structure of the group. It can be shown that with appropriate choice of the representation, the generators can be identified with certain physical quantities, e.g.

**isospin:**

\[ I_1 = F_1 \]
\[ I_2 = F_2 \]
\[ I_3 = F_3 \]  
(1.48)

**and**

**hypercharge:**

\[ Y = \frac{2}{\sqrt{3}} F_8 . \]  
(1.49)

Furthermore, to each of the quantities \( F_i \), a current may be assigned:

\[ F_i \rightarrow j_i^{(a)} \]  
(1.50)

such that

\[ F_i = \int j_i^{(a)}(x) d^3x \quad \text{with} \quad j_4 = ij_0 . \]

For example, we may write:

\[ I_3 \rightarrow j_3^{(3)} , \quad Y \rightarrow j_3^{(Y)} = \frac{2}{\sqrt{3}} j_3^{(s)} \]  
(1.51)

where

\[ I_3 = \int j_3^{(3)}(x) d^3x , \quad Y = \int \frac{2}{\sqrt{3}} j_3^{(s)}(x) d^3x . \]
The electric charge \( Q \) is the integral of the fourth Lorentz component of the electromagnetic current. Therefore, from the relation

\[
Q = I_3 + \frac{1}{\sqrt{2}} Y
\]

or

\[
Q = F_3 + \frac{1}{\sqrt{2}} F_8 ,
\]

we can write

\[
j_{\alpha}^{(e.m.)} = j_{\alpha}^{(i)} + \frac{1}{\sqrt{2}} j_{\alpha}^{(s)}. \tag{1.52}
\]

The electromagnetic current \( j_{\alpha}^{(e.m.)} \) is known to be conserved. It follows that all the currents in formula (1.50) are conserved in the limit of SU\(_3\) symmetry since they can be transformed into one another by SU\(_3\) rotations.

The SU\(_3\) symmetry can now be applied to the weak currents. The weak hadronic current \( J_{\alpha} \) may be divided into vector \( (V_{\alpha}) \) and axial vector \( (A_{\alpha}) \) parts:

\[
J_{\alpha} = V_{\alpha} + A_{\alpha} \tag{1.53}
\]

which can be further subdivided into parts according to the change of strangeness, i.e.

\[
V_{\alpha} = V_{\alpha}(\Delta S = 0) + V_{\alpha}(\Delta S = 1) \tag{1.54}
\]

\[
A_{\alpha} = A_{\alpha}(\Delta S = 0) + A_{\alpha}(\Delta S = 1). \tag{1.55}
\]

The success of SU\(_3\) in strong interactions in connecting strange and non-strange particles suggests the possibility that \( \Delta S = 0 \) and \( \Delta S = 1 \) currents might be similar. We recall that the CVC theory states that \( V_{\alpha}(\Delta S = 0) \) is a conserved current and is proportional to the I\(^{+}\) current of the isotopic spin, i.e.

\[
V_{\alpha}(\Delta S = 0) = \frac{G}{G^{V}} j_{\alpha}^{(i)} = \frac{G}{G^{V}} (j_{\alpha}^{(1)} + j_{\alpha}^{(2)}) \tag{1.56}
\]

where \( G^{V} \) and \( G \) are the coupling constants in beta decay and muon decay, respectively.

The Goldberger-Treiman relations for the axial current can be summarized in the statement:

\[
\delta A_{\alpha}(\Delta S = 0) \propto \varphi_{\pi} \tag{1.57}
\]

where \( \varphi_{\pi} \) is the pion field.

In constructing the SU\(_3\) model for weak interactions, Cabibbo made the following assumptions concerning the SU\(_3\) structure of the weak currents \( V_{\alpha} \) and \( A_{\alpha} \).

1) The vector current \( V_{\alpha} \) is a member of the octet \( j_{\alpha}^{(1)} \) to which the I-spin current (1.51) and the electromagnetic current (1.52) belong. Also, the axial current \( A_{\alpha} \) is a member of an octet of axial currents \( g_{\alpha}^{(1)} \), such that

\[
\delta g_{\alpha}^{(1)} \propto \varphi^{(i)} \tag{1.58}
\]

where \( \varphi^{(i)} \) is a pseudoscalar octet. We can write then:
\[ V_a = a j_a^{(0)} + b j_a^{(1)} \] 
\[ A_a = \sigma j_a^{(0)} + b \sigma j_a^{(1)} \]

where

\[ j_a^{(0)} = j_a^{(1)} + i j_a^{(2)} \]
\[ j_a^{(0)} = j_a^{(1)} + i j_a^{(2)} \]
\[ \sigma j_a^{(0)} = \sigma j_a^{(1)} + i \sigma j_a^{(2)} \]
\[ \sigma j_a^{(0)} = \sigma j_a^{(1)} + i \sigma j_a^{(2)} \]

The selection rules for these currents can be obtained from the commutation relations. The results show that currents with \( \Delta S = 0 \) have the properties \( \Delta Q = 1 \) and \( \Delta I = 1 \), and those with \( \Delta S = 1 \) have the properties \( \Delta Q = 1 \) and \( \Delta I = \frac{1}{2} \). Thus, these selection rules as a consequence of the first assumption are in agreement with the present experimental evidence.

ii) The second assumption is concerned with the "strength" of the current \( J_a \). In order to ensure that, whatever the direction of \( J_a \), its "length" remains constant, the following condition is set:

\[ a^2 + b^2 = 1 \]  

This implies a certain degree of universality in weak interactions.

From the above two assumptions, the weak current of the strongly interacting particles \( J_a \) can be written as

\[ J_a = \cos \theta \left[ j_a^{(0)} + \sigma j_a^{(0)} \right] + \sin \theta \left[ j_a^{(1)} + \sigma j_a^{(1)} \right] \]

where \( \theta = \tan^{-1}(b/a) \) is called the Cabibbo angle which expresses the relative strengths of the \( \Delta S = 0 \) and \( \Delta S = 1 \) currents. The angle \( \theta \) has been determined experimentally by comparing the rates of \( K_L^0 \) and \( \pi_L^0 \), and independently by comparing \( K_{\mu 2} \) and \( \pi_{\mu 2} \). The result is \( \theta = 0.257 \).

4.2 The cross-sections for hyperon production

The SU3 model of weak interactions discussed in the preceding section has been used by Cabibbo and Chilton to determine the cross-sections for reactions (1.44) and (1.45). Because of the \( \Delta I = \frac{1}{2} \) selection rule for the \( \Delta S = 1 \) weak current, reaction (1.46) is related to (1.45) by

\[ d\sigma(\Xi^0) = \frac{1}{2} d\sigma(\Xi^-) \]
As has been seen earlier in this chapter, the matrix elements of a mixed vector and axial vector current among two baryon states $B_1$ and $B_2$ can be expressed in terms of six independent form factors. If we assume invariance under $T$ and $G$, and ignore the pseudoscalar term which is proportional to the square of the lepton mass, $m_\mu^2$ and is thus very small, then only three independent and real form factors are left, i.e.

$$
\langle B_2|\mathcal{J}_a|B_1\rangle = \bar{u}_2(p_2) \left[ \gamma_\mu (G_V + G_A Y_3) + \frac{\sigma_{\mu\rho} q_\rho}{M_1 + M_2} F_V \right] u_1(p_1).
$$

(1.64)

For the particular cases of reactions (1.44) and (1.45), it has been shown that in the limit of exact $SU_3$ symmetry, the matrix elements are given by:

$$
\langle p|\mathcal{J}_a|A\rangle = -\frac{G}{\sqrt{2}} \sin \theta \left[ \sqrt{\frac{3}{2}} P_1(q^2) Y_3 + P_2(q^2) \frac{\sigma_{\mu\rho} q_\rho}{2M} + \frac{1 + 2x(q^2)}{3} g_A(q^2) Y_3 \gamma_5 \right]
$$

(1.65)

and

$$
\langle n|\mathcal{J}_a|\Sigma^-\rangle = \frac{G}{\sqrt{2}} \sin \theta \left[ P_1(q^2) + 2P_2(q^2) \right] Y_3 + \frac{1}{2N} \left[ P_1(q^2) + 2P_2(q^2) \right] \sigma_{\mu\rho} q_\rho
$$

\[
- \left[1 - 2x(q^2)\right] g_A(q^2) Y_3 Y_5
\]

(1.66)

where $P_1$, $P_2$, $F_V$, and $F_E$ are all normalized form factors so that at zero momentum transfer,

$$
P_1(0) = 1, \quad P_2(0) = 0, \quad F_V(0) = \mu_p, \quad F_E(0) = \mu_n,
$$

and $x(q^2) = 0.25$ and can be regarded as a constant. The form factors in Eq. (1.64) can then be written in the following form:

for $\langle p|\mathcal{J}_a|A\rangle$,

$$
G_V = -G \sin \theta \left(\sqrt{\frac{3}{2}}\right) F(q^2)
$$

$$
G_A = -1.18 G \sin \theta \left(\sqrt{\frac{3}{2}}\right) \left(\frac{1}{\sqrt{2}}\right) F(q^2)
$$

(1.67)

$$
F_V = -G \sin \left(\sqrt{\frac{3}{2}}\right) \mu_p F(q^2)
$$
and for \( |J_\mu| \Sigma^-\),

\[
G_V = G \sin \theta \cdot F(q^2) \\
G_A = 1.18 G \sin \theta \left( \frac{\Lambda}{\Lambda'} \right) \cdot F(q^2) \\
F_V = G \sin \theta \left( \mu_p + 2\mu_n \right) \cdot F(q^2)
\]

(1.68)

where \( G \) is the coupling constant for muon decay, and the ratio \( G_A/G_V \) has been taken to be 1.18. In the above, the axial vector form factor has been assumed to have the same form as the vector form factor,

\[
F(q^2) = \left( \frac{1}{1 + \left( \frac{q^2}{M^2} \right)} \right)^n
\]

(1.69)

where \( n = 1 \) or 2. With the assumed values of \( n \) and \( M \) in Eq. (1.69), the cross-sections for reactions (1.44) and (1.45) can be evaluated (see Chapter 5).

5. SINGLE-PION PRODUCTION PROCESS

The general form of the cross-sections for inelastic neutrino reactions has been discussed by Lee and Yang\(^{39}\). Consider the reaction

\[
\nu + Z \to \ell^- + A
\]

(1.70)

where \( Z \) represents the target nucleus and \( A \) is an assembly of any number of hadrons. It has been shown that as a consequence of the locality of the leptonic current, the cross-section for reaction (1.70) can be expressed as the sum of three terms which are functions of the momentum transfer \( q^2 \) and the invariant mass \( M_A^{*2} \) of \( A \) only, i.e.

\[
\frac{d^2\sigma}{dk_\ell d\Omega} = \frac{k_\ell}{8\pi^2k_\nu} \left[ \left( k_\nu + k_\ell \right)^2 - P^2 \right] \left[ x A_1 \left( q^2, M_A^{*2} \right) + x^{-1} A_2 \left( q^2, M_A^{*2} \right) + B \left( q^2, M_A^{*2} \right) \right]
\]

(1.71)

where \( k_\ell, k_\nu, \) and \( P \) are the momenta of the lepton, the neutrino, and the assembly of hadrons, and where

\[
x = \frac{k_\nu + k_\ell - P}{k_\nu + k_\ell + P},
\]

(1.72)

and \( A_1, B \) are positive real functions of \( q^2 \) and \( M_A^{*2} \).

The majority of the inelastic neutrino events recorded in the CERN neutrino experiments\(^{40}\) are events of single-pion production, i.e.

\[
\nu_\mu + N \to N' + \pi + \mu^-
\]

(1.73)

where \( N \) and \( N' \) are nucleons.
The kinematics of reaction (1.73) is represented in Fig. 1.3 where \( p_\nu \), \( p_\ell \), \( p_\pi \), \( P_1 \), and \( P_2 \) are the four-momenta of the primary neutrino, the produced lepton, the pion, the initial and final nucleons, respectively. The N\( N \pi \) interaction then depends on the following quantities:

\[
\begin{align*}
M^{*2} &= -(P_2 + P_\pi)^2 \\
t &= -(P_1 - P_2)^2 \\
q^2 &= (p_\nu - p_\ell)^2 = -m_\ell^2 + 2E_\nu E_\ell (1 - \beta_\ell \cos \theta)
\end{align*}
\tag{1.74}
\]

where \( \beta_\ell = |p_\ell|/E_\ell \) is the velocity of the lepton and \( \theta \) is the angle between the neutrino and the produced lepton.

![Fig. 1.3 Kinematics of single-pion production by neutrinos](image)

This reaction is, however, analogous to the inelastic electron scattering, and the cross-section can be represented by a Rosenbluth-type expression. The vector part of the nucleonic current can then be derived from the electromagnetic form factors on the basis of the conserved vector current hypothesis. The general form of the cross-section is then given by

\[
\frac{d^2\sigma}{dM^{*2}dq^2} = \frac{1}{E_\nu E_\ell} \left[ q^2 R(q^2, M^{*2}) + 2 \left( E_\ell E_\nu - \frac{1}{2} q^2 \right) S(q^2, M^{*2}) + (E_\ell + E_\nu) T(q^2, M^{*2}, \frac{2q^2}{|q^2|}) \right].
\tag{1.75}
\]

Many authors have discussed the single-pion production reaction (1.73), using different models\(^{19-33}\). However, most of the theoretical considerations are concerned with the dominance of the N\( N(\frac{3}{2}^-, \frac{1}{2}^-) \) nucleon isobar production\(^{31}\). In this model, \( N^*_3 \) production is considered in the framework of the Barita-Schwinger scheme\(^{54}\), analogous to the isobar model\(^{55}\) used in photoproduction of N\( N^* \). The single-pion process (1.73) is then visualized as a two-step reaction:

\[
\nu_\mu + N \rightarrow \mu^- + N^*_3 \rightarrow N' + \pi.
\tag{1.76}
\]
Since it is believed that the $N_{2\pi}^*$ production is the dominant contribution to the single-pion process, the main part of the cross-section comes from the region of small $q^2$ and $M_{2\pi}^2 \sim M_{2\pi}^2$, i.e. $(1236 \text{ MeV})^2$. The cross-section will then tend to a constant for increasing values of neutrino energy $E_\nu$,

$$\sigma_T(1\pi) \stackrel{E_\nu \to \infty}{\longrightarrow} 2 \int dq^2 S(q^2, M_{2\pi}^2), \quad (1.77)$$

The structure functions $R, S,$ and $T$ in Eq. (1.75) are of course contained in the hadronic current $J_a^\pi$. In analogy with photoproduction and electroproduction processes\(^{55}\), the hadronic current $J_a^\pi$ can be well approximated by summing the four graphs in Fig. 1.4. Graph (a) is the $N_{2\pi}^*$ production model of Berman and Veltman\(^{51}\), which gives the dominant contribution. Graph (b) represents the pion exchange. Graphs (c) and (d) are nucleon exchange graphs. Contributions from other graphs, such as the peripheral model\(^{56}\) in which vector mesons $\rho$ and $\omega$ are exchanged, are known to be very small.

![Feynman diagrams for single-pion production by neutrinos](image)

Fig. 1.4

Feynman diagrams for single-pion production by neutrinos

As in the case of elastic processes, the axial vector form factor $F_A(q^2)$ is left as the only unknown, which can be determined by comparing with the experimental results. The following forms of parameterization for the $F_A(q^2)$ can be used:

$$F_A(q^2) = \left(1 + \frac{q^2}{M_A^2}\right)^{-n} \quad (1.78)$$

where $n = 1$ or 2.

* * *

* * *
PART II

EXPERIMENTAL MATTERS
CHAPTER 2

EXPERIMENTAL APPARATUS AND METHODS OF ANALYSIS

1. THE EXPERIMENTAL APPARATUS

Since neutrinos interact with matter only by weak interactions, experiments on neutrino interactions are possible only if very intense neutrino beams can be produced and massive detectors are used. Neutrino fluxes are obtained from the decay of pions and kaons:

\[ \pi^+ \rightarrow \mu^+ + (\nu_\mu + \bar{\nu}_\mu) \quad \text{(lifetime } 2.55 \times 10^{-8} \text{ sec)} \quad (2.1) \]

\[ K^+ \rightarrow \mu^+ + (\nu_\mu + \bar{\nu}_\mu) \quad \text{(lifetime } 1.23 \times 10^{-8} \text{ sec)} \, . \quad (2.2) \]

In the rest frame of the parent particle, the most energetic neutrino comes from the decay of kaons [process (2.2)], with an energy of 235 MeV. Consequently, neutrinos of higher energies can only be obtained from the decay of fast K's and π's.

From the Lorentz transformation, the energy \( E_\nu' \), and the direction \( \theta \) of the neutrino in the laboratory (lab.) system and in the centre-of-mass (c.m.) system are related by

\[ E_\nu = \gamma (E'_\nu + \beta p'_\nu \cos \theta') \, , \quad (2.3) \]

and

\[ \tan \theta = \frac{1}{\gamma} \frac{\sin \theta'}{\cos \theta' + \beta' \beta} \, , \quad (2.4) \]

where the primed quantities refer to the c.m. system and \( p'_\nu \) is the neutrino momentum, and \( \gamma, \beta \) are the Lorentz factor and velocity of the parent particle, respectively.

From the above relations, it can easily be seen that the decay products, which are isotropic in the c.m. system, will be concentrated more in the forward direction in the lab. system with increasing parent energy. The basic problem in obtaining an intense beam of energetic neutrinos is then to produce a beam of fast pions and kaons of high intensity, and to allow them to decay into neutrinos. The experimental layout of the neutrino experiment is shown schematically in Fig. 2.1.

1.1 The neutrino beam

Energetic π and K mesons are produced by the bombardment of protons from the CERN 28 GeV Proton Synchrotron (PS)\(^{\text{(8)}}\) on a target. Pions constitute about nine-tenths of the secondary mesons, and kaons about one-tenth. Because of the high velocity of the primary protons, most secondary mesons are produced in the forward direction near the line-of-flight of the protons. In order to direct the energetic secondaries to the accessible part of the laboratory, the primary proton beam has to be extracted before it hits the target.
Fig. 2.1 Experimental layout of the neutrino experiment
The protons in the CERN PS are accelerated in a circular path of 100-metre mean radius. The proton beam is divided into 20 bunches spaced in time about 0.1 microseconds apart. For an acceleration of 25 GeV, the cycle is repeated every 3 seconds, and the intensity is about $7 \times 10^{11}$ protons per pulse. To extract the circulating proton beam, a kicker magnet is used to deflect the protons magnetically to make them leave the stable region of the synchrotron magnet field. After a quarter wavelength of the oscillation, protons are sufficiently distant from their stable trajectory and can be accepted by a bending magnet which introduces a deflection of about 25 mrad. This deflection is sufficient to bring the protons out of the PS and to focus them on a spot less than two millimetres in diameter on a target about 25 metres away. The target is a copper rod 25 cm long and 0.4 cm in diameter.

In order to get a maximum intensity of the neutrino beam, the secondary mesons ($\pi$'s and $K$'s) produced at the target are focused by a 'magnetic horn', which consists of two coaxial conical conductors. At the moment the extracted proton beam strikes the target, a strong current of about 300 kA is passed through the conductors and the intense magnetic field thus produced will deflect neutrino parents of one sign nearer to the axis and deviate the other away from it. At this current the field is strong enough to accept practically all $\pi$'s and $K$'s of one sign produced on the target. By reversing the current in the magnetic horn, either positive mesons or negative mesons can be focused. Thus the neutrino beam can be made predominantly of either neutrinos or antineutrinos.

After the magnetic horn, the $\pi$ and $K$ mesons are allowed to pass a decay tunnel of 25 m long with a half-angle of 2° in front of a shielding of iron. The decay length $l$ of a particle of lifetime $\tau_\circ$ is given by

$$ l = \frac{E}{m} \tau_\circ c, $$

(2.5)

where $p$ and $m$ are the momentum and mass of the particle, respectively.

The shielding is designed to absorb, as far as possible, all particles except neutrinos, so that after the shielding a pure beam of neutrinos can be obtained. Strongly interacting particles are attenuated roughly exponentially with an interaction length of about 140 g/cm$^2$ for iron. Muons from pion decay, however, lose energy almost entirely by ionization. Therefore, the chief determining factor of the shielding has been to choose a sufficient axial thickness to absorb muons from pion decay. In order to absorb the most energetic muons from the decay of $\pi$ or $K$ mesons produced by a 25 GeV beam, a thickness of iron of about 20 m is required. However, there remains a flux of cosmic-ray particles which has to be attenuated by vertical shielding. An equivalent of 2 m of iron shielding is therefore placed above the chamber. With this arrangement, about one cosmic-ray muon crosses the bubble chamber per picture, but this presents no difficulty in the interpretation of events.

1.2 The detector -- The bubble chamber

The neutrino experiment was carried out in CERN in three separate periods from 1963 to 1965. Two very large heavy-liquid bubble chambers were used as the detectors, one in the 1963 and 1964 runs (CERN HLBC) and an enlarged one in the 1965 runs (enlarged CERN
HLBC). In the 1963 and 1964 runs, spark chambers\textsuperscript{62) were also used in conjunction with the bubble chamber. In this report, we only confine ourselves to the analysis of the results obtained from the bubble chamber. A schematic diagram of the bubble chamber is shown in Fig. 2.2. Its relevant details are summarized in Table 2.1. The figures in brackets refer to the enlarged CERN HLBC.

Table 2.1
Bubble chamber parameters

<table>
<thead>
<tr>
<th>Shape of chamber</th>
<th>circular cylinder</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diameter</td>
<td>1.15 m (1.2 m)</td>
</tr>
<tr>
<td>Depth</td>
<td>0.5 m (1.1 m)</td>
</tr>
<tr>
<td>Total volume</td>
<td>300 l (1,180 l)</td>
</tr>
<tr>
<td>Fiducial volume</td>
<td>200 l (638 l)</td>
</tr>
<tr>
<td>Filling liquid:</td>
<td>freon CF$_2$Br</td>
</tr>
<tr>
<td>Density</td>
<td>1.5 g/cm$^3$</td>
</tr>
<tr>
<td>Radiation length</td>
<td>11 cm</td>
</tr>
<tr>
<td>Collision length</td>
<td>58 cm</td>
</tr>
<tr>
<td>Magnetic field</td>
<td>27 kG</td>
</tr>
</tbody>
</table>

The chamber was situated at the end of the iron shielding, with its axis horizontal and approximately perpendicular to the direction of the neutrino beam. Liquid freon CF$_2$Br (tri-fluoro-bromo-methane) was contained in the cylinder between an expansion membrane and a piece of glass of thickness 33.9 cm and refractive index 1.522. On the glass surface adjacent to the liquid are five fiducial marks which give references to the geometric reconstruction of events. The chamber is photographed by three cameras situated symmetrically about the chamber axis and at a distance of 220.3 cm from the glass-liquid surface which will give a stereoscopic view of tracks in the chamber.

In the reconstruction of events, the coordinate system of the chamber was chosen in such a way that the origin is at the centre of the glass-liquid surface and +x is the neutrino beam direction and +z is towards the cameras. The system XYZ is right-handed.

Freon CF$_2$Br was used as the chamber filling because it has a high density which gives high probability for interactions, and short radiation length which facilitates the detection of $\gamma$ rays. However, the analysis of the events will be complicated by the fact that interactions occur inside the complex nuclei. In subsequent sections, we shall explain how this can be overcome.

1.3 Summary of the experimental runs\textsuperscript{63,64)

During all the runs of the experiment, more than 1,500 hours of proton synchrotron time were used to give neutrino and antineutrino fluxes. A summary of the experimental runs is given in Table 2.2.
(a) Cross-sectional view.

(b) Side view with the pressure system on the left and the safety tank on the right.

(c) Plan view.

Fig. 2.2 Schematic diagram of the CERN heavy-liquid bubble chamber
Table 2.2
Summary of data on neutrino and antineutrino runs

<table>
<thead>
<tr>
<th>Neutrino runs</th>
<th>1963</th>
<th>1964</th>
<th>1965</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of ejected protons</td>
<td>$3.40 \times 10^{17}$</td>
<td>$3.70 \times 10^{17}$</td>
<td>$0.30 \times 10^{17}$</td>
<td>$7.40 \times 10^{17}$</td>
</tr>
<tr>
<td>No. of pictures taken</td>
<td>$2.88 \times 10^5$</td>
<td>$3.55 \times 10^5$</td>
<td>$0.44 \times 10^5$</td>
<td>$6.87 \times 10^5$</td>
</tr>
<tr>
<td>No. of machine hours</td>
<td>518</td>
<td>581</td>
<td>43</td>
<td>1142</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Antineutrino runs</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of ejected protons</td>
<td>$0.46 \times 10^{17}$</td>
<td>-</td>
<td>$1.79 \times 10^{17}$</td>
<td>$2.25 \times 10^{17}$</td>
</tr>
<tr>
<td>No. of pictures taken</td>
<td>$0.57 \times 10^5$</td>
<td>-</td>
<td>$3.23 \times 10^5$</td>
<td>$3.80 \times 10^5$</td>
</tr>
<tr>
<td>No. of machine hours</td>
<td>106</td>
<td>-</td>
<td>289</td>
<td>395</td>
</tr>
</tbody>
</table>

2. THE NEUTRINO SPECTRUM

Most of the results obtained in the neutrino experiment presupposes the knowledge of the neutrino spectrum. Any uncertainty in the estimate of the spectrum will induce uncertainty in the final results. It is therefore essential to obtain the most accurate estimate of the spectrum that can be achieved.

Since neutrinos are obtained from the decay of pions and kaons produced at the copper target by the bombardment of the extracted primary proton beam, it is necessary to acquire a detailed knowledge of the momentum and angular distributions of the secondary pions and kaons in order to calculate the neutrino spectrum. Up to the end of 1965, the calculation of the CERN neutrino spectrum was based essentially on the pion and kaon distributions obtained from the counter experiments $^{69-77}$ of protons incident on various target materials, in particular, beryllium and lead. Measurements were only made at discrete angles. As a result of the incomplete knowledge of the distribution of the secondaries, the neutrino spectrum so obtained was accurate to within only $\pm 30\%$ $^{69}$.

Owing to this uncertainty, a bubble chamber experiment $^{69}$ was carried out with the purpose of determining the secondary pion and kaon spectra to a higher accuracy. A copper target of 30 cm long and 0.4 cm in diameter placed in the Boole Polytechnique heavy-liquid bubble chamber was exposed to a beam of protons of 17.8 GeV/c. Secondaries produced from the target were measured at all angles. From this, the momentum and angular distributions of pions and kaons were obtained $^{69,70}$.
The neutrino spectrum can be calculated $^{63}$ from the properties of the magnetic horn and the spectra of the secondaries. Secondaries produced from the target inside the horn were first traced through the horn. These were then combined with the production spectra and with the probability of production at various parts of the target to give the resultant momentum and angular spectra of pions and kaons after the horn. By making use of the decay kinematics, the neutrino spectrum at the bubble chamber was then obtained.

The neutrino fluxes obtained by using the production information from the bubble chamber experiment are higher than the previous estimates by a factor of 1.2 to 1.6, depending on the neutrino energy. The best estimate of neutrino and antineutrino fluxes comes from the combined production data of the counter experiments and the bubble chamber experiment.

During the whole period of the neutrino experiment, two different horns were used, one in 1963 and the other in 1964 and 1965. The averaged neutrino and antineutrino spectra, properly weighted in proportion to the number of primary protons and the number of target nucleons in the fiducial volumes of the chambers, are shown in Fig. 2.3.

3. Identification of Particles

The analysis of neutrino and antineutrino events depends a great deal on the correct identification of the nature of the tracks produced in the interactions. It involves the determination of the momentum, the charge, and the mass, i.e., whether the track is due to a proton, $K^+$, $\pi^+$, or $\mu^+$. Electrons can be readily identified by their characteristic appearance (i.e., fast spiral). Neutral mesons decay into two $\gamma$'s, and their identification can be achieved by the dynamical analysis of the materialized $\gamma$'s in the chamber. Neutrons are recognized by their interactions to produce neutron stars or recoil protons. In general, the identification of a charged particle track in the freon chamber can be done by the following considerations:

a) radius of curvature,
b) ionization loss and range,
c) decay and interaction modes, and
d) $\delta$ rays.

Multiple scattering in principle gives the value of $p\theta$, but in freon this method has not yet been investigated in detail. Therefore, it has not been used in our analysis. Most of the methods mentioned above have been well established as standard methods used to identify particle tracks in bubble chambers. In the following, we shall summarize only the essential points.

3.1 The methods

3.1.1 The determination of momentum $^{71}$ from the radius of curvature. The momentum $p$ of a charged particle giving a track of radius of curvature $\rho$ in a magnetic field $H$ is well known to be

$$p(\text{GeV/c}) = \frac{0.3 \times 10^{-3} \rho H}{\cos \lambda},$$

(2.6)
Fig. 2.3  Differential neutrino and antineutrino spectra (weighted average over all runs)
where the magnetic field $H$ is measured in kG, the curvature $\rho$ in cm, and $\lambda$ is the dip angle, defined as the angle between the track and the plane perpendicular to the magnetic field $H$. However, the error owing to the multiple scattering in freon could be appreciable. The uncertainty in momentum measurement due to multiple scattering is given by:

$$\frac{\Delta p}{p} = K \frac{1}{\beta \nu L}$$  \hspace{1cm} (2.7)

where $K = \text{constant}$, 
$\beta = \text{velocity of the particle}$, 
$L = \text{length of the track}$, and 
$X_0 = \text{radiation length of the medium}$.

For freon $\text{CF}_3\text{Br}$, with $X_0 = 11 \text{ cm}$, and for a magnetic field of 27 kG, the uncertainty is

$$\frac{\Delta p}{p} = \frac{0.585}{\beta \cos \lambda \sqrt{L}}$$  \hspace{1cm} (2.7')

Thus, for a relativistic track of length 50 cm, the uncertainty amounts to about 8%. In the majority of tracks obtained in the experiment, the typical value is 15% to 20%.

3.1.2 Ionization loss and range. The rate of ionization energy loss of a singly charged particle during its passage through matter of atomic number $Z$ and atomic weight $A$ is well known to be:

$$\frac{dE}{dx} = (4\pi N r_e^2) \frac{Z}{A} \frac{m_e c^2}{\beta^2} \left[ \log \frac{2m_e c^2 \beta^2}{(1-\beta^2)^2} \right]$$  \hspace{1cm} (2.8)

where $N = \text{Avogadro's number} = 6.0249 \times 10^{23}$, 
$r_e = e^2/m_e c^2 = \text{classical radius of the electron} = 2.8178 \times 10^{-13} \text{ cm}$, 
$m_e c^2 = 0.5109 \text{ MeV}$,

$I = \text{ionization potential of the medium}$.

From this relation, a set of range-momentum curves for various particles in freon can be constructed. This provides some possibility of distinguishing light particles from the heavy ones in a limited momentum range. If a track stops inside the chamber, its range can be determined with great accuracy, and thus its momentum can be obtained. This gives a much more accurate momentum measurement than by the curvature method. Indeed, the majority of protons which stop inside the chamber have been measured in this way. For non-stopping tracks, ionization measurement gives an estimate of the velocity $\beta$, and this combined with momentum can in many cases permit the discrimination of particles of different masses.

3.1.3 Decay and interaction modes. If a particle stops in the chamber, its decay mode can often provide information for its identification. For example, the decay of $\pi^+$:

$$\pi^+ \rightarrow \nu_\mu + \mu^+ \rightarrow e^+ + \nu_e + \bar{\nu}_\mu$$  \hspace{1cm} (2.9)
should show a $\mu^+$ track before an electron. However, $\mu^+$ from the decay of $\pi^+$ at rest has very low momentum ($\sim 30$ MeV/c) and therefore its track is not always identifiable. In fact, it has been found that only about half of the $\pi^+$'s give a visible $\mu^+$ when decaying at rest.

A $\pi^-$ stopping in the chamber will be captured by the nucleus of the medium, quite frequently giving a star with evaporation protons, whereas a $\mu^-$ stopping in the chamber will decay or be absorbed, leaving no trace.

3.1.4 Measurement of $\delta$ rays. $\delta$ rays are knock-on electrons produced during the passage of a charged particle in a medium. For a charged particle of mass $m$ and momentum $p$, the maximum energy of a $\delta$ ray is given by:

$$E_{\text{max}} = 2m_e \frac{c^2}{\beta} \left( \frac{p}{mc} \right)^2$$

(2.10)

assuming $m \gg m_e$. It is seen that for a certain momentum, the maximum energy is inversely proportional to the square of the mass of the particle. For momentum greater than about 0.7 GeV/c, ionization energy loss and curvature measurement are mostly insufficient to distinguish between a proton and a positive pion. In this case, identification can still be possible if a $\delta$ ray of energy greater than that allowed for a proton is produced. However, $\delta$ rays of sufficiently great energy are so rare that individual identification of tracks by this method is almost unpracticable. It can only be used to establish statistically the relative proportion of protons and pions.

3.2 The identification$^{63,74)}$

We shall summarize in the following how charged particles and neutral particles are identified by using the methods described in the previous section, and the limitations of these methods.

3.2.1 Charged particles. In the neutrino runs, non-interacting negative particles were assumed to be muons, whereas light interacting negative particles were interpreted as due to negative pions. An interaction has been defined as one in which the momentum transfer to the target nucleus is greater than 100 MeV/c. This choice is based on Paty and Yoshiki's calculation$^{75)}$ of the mean free path for Coulomb scattering of muons in Freon. For a muon of momentum $\geq 0.2$ GeV/c, the mean free path is of the order of about 300 m, but for pions, it is about 0.7 m.

Protons and positive pions can be separated from one another by means of ionization and curvature measurements up to momentum of about 0.7 GeV/c. For sufficiently long tracks, identification is sometimes possible up to 1 GeV/c. Above this momentum, pions can be identified by $\delta$-ray analysis, i.e. if they produce a $\delta$ ray of energy greater than that allowed for a proton of the same momentum. They have been used to establish the relative proportion of protons and positive pions. By counting the signature $\delta$ rays observed in the neutrino events, it has been established that most of the non-interacting positive particles of momentum above 1 GeV/c are protons. Therefore, in this analysis, they have been classified as protons. Light, interacting, positive particles were interpreted as pions.
We have neglected the presence of charged kaons, since they occur mostly in multi-
pionic events produced by very high-energy neutrinos. In our analysis of the neutrino
events, these events have so far not been very well understood. In any case, the emission
of the few kaons expected will not affect our conclusions.

In the antineutrinos runs, light, non-interacting, positive particles were interpreted
as muons.

3.2.2 Measurement of γ rays, electrons, and neutral pions. Owing to its short radia-
tion length (11 cm), freon C2FBr is a very good liquid for the detection of γ rays, which
materialize to give a pair of negative and positive electrons. In the bubble chamber,
electrons can readily be recognized by their characteristic spiral. Neutral pions can be
identified in the bubble chamber by the dynamical analysis of the electron pairs produced
by the materialization of the γ's. It is therefore important to be able to measure the
energy of the electrons as accurately as possible.

Electrons, being light particles, lose energy rapidly by radiation. Thus the usual
method of determining the momentum of a charged particle in a magnetic field by measuring
the curvature has to be corrected for bremsstrahlung losses. There are two methods by
which the energy of the electron is measured:

1) Behr-Mittler method (the BM method)

This method developed by Behr and Mittner allows the measurement of the energy
of electrons which do not lose more than a certain amount of the primary energy
in a single bremsstrahlung quantum emission. The emission of small quanta on
the measured track can be corrected for by taking an average energy loss. The
track must not show any kick over 4 cm which is the optimal length. By this
method, the momentum of electrons can be determined from a to an accuracy of
about 40% under optimum conditions. Better accuracy can sometimes be achieved
for electron pairs if the two electrons have roughly equal energies.

1) The total track length method (the TTL method)

This method consists of measuring the total track length (TTL) of the shower
generated by an electron and of establishing the relation between the electron
energy E and the TTL. To make the calibration, the heavy-liquid bubble chamber
was exposed to a beam of electrons of energy 600 MeV, and showers were selected
and measured. In order to get the E-TTL relation, similar measurements have
been made for several energies of the electrons. Another possible approach is
to simulate electron showers under similar experimental conditions by the Monte-
Carlo method. A set of E-TTL curves for various potential lengths of the
electrons have thus been constructed.

In our neutrino experiment, both methods were used in a supplementary manner. In
those cases where both methods could be used, agreement between the two was usually good,
although the TTL method generally gave better results. In the cases when the BM method
was not applicable, the TTL method was found particularly useful.
In our analysis of the neutrino events, it has been assumed that all the \( \gamma \)'s come from the decay of neutral pions. Since freon is a good detector of \( \gamma \) rays, in most cases, two \( \gamma \) rays can be detected at the same time. The mass of the neutral pion \( M \) should then agree with that calculated by the relation

\[
M = 2\sqrt{p_1 p_2 \sin^2 \frac{\gamma}{2}},
\]

(2.11)

where \( p_1 \) and \( p_2 \) are the momenta of the two photons, respectively, and \( \gamma \) is the angle between them.

3.2.3 Neutrons. Neutrons can be detected in the chamber only if they produce recoil protons. In freon, the collision mean free path is 58 cm, which gives a fair chance of detection. It is difficult to measure the neutron energy very accurately, because of Fermi motion of the nucleons in the target nucleus, and other nuclear processes such as secondary reactions and the emission of undetected neutral particles. The sum of the energies of the visible tracks gives the lower limit to the neutron energy. When the neutron is associated with an interaction in the chamber, its momentum is assumed to be along the line joining the two interaction points with a magnitude calculated from the measured energy. For neutrons coming from outside the chamber, it is difficult to estimate their individual directions, but it would be reasonable to assume that it is more likely to lie along the resultant momentum of the visible tracks, especially for the more energetic ones. The possible sources of these neutrons will be discussed in detail in the next chapter.

3.3 The processing of events

The pictures were scanned on the scanning tables with a magnification of about 0.7.

The following events were selected:

i) Events with no visible incoming track and at least one possible lepton candidate, i.e. non-interacting particle. These were classified as possible neutrino or anti-neutrino events.

ii) Events with identified pions and protons but without lepton candidate. These were attributed to incoming neutrons.

iii) Events due to interactions of identified incoming charged particles.

These events were then measured on the digitized coordinateographs, and all the measured coordinates and relevant information punched on paper tapes. A special version of THRESH-GRIND\(^*\) programs were then used for the geometrical and kinematical reconstruction of the events. At this stage, the events were analysed by using the methods described earlier in order to determine the nature of the tracks in each event. Another computer program, 'The Neutrino Analysis Program', was then used to calculate all the relevant quantities related to the event, such as the total visible energy \( E_{\text{vis}} \) the neutrino-lepton momentum transfer \( q^2 \), the invariant mass \( M^* \) of the recoil system, and so on.
4. KINEMATICS

The interpretation of neutrino events in the bubble chamber is based to some extent on simple kinematics. In this section, we shall examine the kinematics of two types of reactions:

\[ \begin{align*}
\text{elastic:} & \quad \nu + n \rightarrow \ell^- + p \\ 
\bar{\nu} + p & \rightarrow \ell^+ + n \\
\text{inelastic:} & \quad \nu + N \rightarrow \ell^- + N' \\ 
\bar{\nu} + N & \rightarrow \ell^+ + N'
\end{align*} \]  

(2.12a) (2.12b) (2.12c) (2.12d)

where \( N \) is the target nucleon,
\( N' \) is the system of all strongly interacting particles produced in the collision, i.e. apart from the lepton.

For the moment, we neglect the Fermi motion of the target nucleon and confine ourselves to the consideration in the laboratory system.

Let the four-momentum vectors of the incoming neutrino, the outgoing lepton, the target nucleon and the non-leptonic part of the final state be, respectively:

\[ \begin{align*}
\nu & = (p_{\nu}, iE_{\nu}) \\
\ell & = (p_{\ell}, iE_{\ell}) \\
N & = (0, iM) \\
N' & = (p_{N'}, iE_{N'})
\end{align*} \]  

(2.13)

where \( M \) is the nucleon mass (assume \( M = M_n = M_p \)).

Defining the four-momentum transfer of the lepton system as

\[ q = \nu - \ell , \]

we get

\[ q^2 = (\nu - \ell)^2 = \nu^2 + \ell^2 - 2\nu \ell . \]  

(2.14)

We also have

\[ \begin{align*}
\nu^2 &= -m_{\nu}^2 = 0 \\
\ell^2 &= -m_{\ell}^2 \\
\nu \ell &= \vec{p}_{\nu} \cdot \vec{p}_{\ell} - E_{\nu} E_{\ell} \quad \text{and} \quad |\vec{p}_{\nu}| = E_{\nu}
\end{align*} \]

where \( m_{\nu} \) and \( m_{\ell} \) are the masses of the neutrino and the lepton \( \ell \), respectively.
Considering the reaction in Fig. 2.4 where $\theta$ is the angle of emission of the lepton, we obtain immediately

$$q^2 = -m^2 + 2E_\nu(E_\ell - p_\ell \cos \theta) .$$  \hfill (2.15)

The maximum value of $q^2$ occurs when the lepton is in the opposite direction to the neutrino, i.e.

$$q_{\text{max}}^2 = -m^2 + 2E_\nu(E_\ell + p_\ell)$$

$$= -m^2 + 2E_\nu E_\ell (1 + \beta_\ell^2)$$  \hfill (2.16)

where $\beta_\ell = p_\ell/E_\ell$, the velocity of the outgoing lepton.

The invariant mass of the non-leptonic part is given by

$$M^2 = -(\nu - \ell + N)^2$$

$$= -q^2 + M^2 + 2M(E_\nu - E_\ell) .$$  \hfill (2.17)

From Eqs. (2.15) and (2.17) we get

$$E_\nu = \frac{M^2 - M^2 + 2ME_\ell - m^2}{2(M - E_\ell + p_\ell \cos \theta)} .$$  \hfill (2.18)

However, the above consideration is complicated by the following three factors.

i) Fermi motion of the target nucleon

Since the target nucleon is bound to a nucleus, it is associated with Fermi momentum. If we take the ideal Fermi gas model for the nucleus, corresponding to an infinitely deep potential well, and assume that all states are filled up to the maximum $p_F$, then for $r_0 = 1.12 \times 10^{-13}$ cm, we get

$$p_F = 267 \text{ MeV/c}$$

which corresponds to a Fermi kinetic energy $E_F$ of about 30 MeV. For a scattering event to take place, the Pauli principle requires that the three-momentum of the final nucleon be outside the Fermi sphere, i.e.

$$p_N > p_F .$$
ii) **Scattering or interaction of the outgoing nucleon or N' inside the nucleus**

Since leptons do not interact with matter by strong interactions, an outgoing lepton will leave the nucleus without appreciable change in momentum and direction. But this is not so for strongly interacting particles. However, the loss of energy owing to nuclear interaction in the nucleus is usually small (about 10 - 20% of the original energy), while the change of momentum direction may be considerable.

iii) **Undetected neutral particles**

Energy is carried off by undetected neutrons and heavy nuclear fragments. From experiments with charged beams, it is known that this energy constitutes only about 10% of the beam energy, in contrast to the more appreciable momentum imbalance. Thus, as a measure of the neutrino energy $E_\nu$, we take the energy $E_{\text{vis}}$ of all visible tracks, which is the sum of the total energy of particles created ($e$, $\mu$, $\pi$, $K$) and the kinetic energy of the nucleons.

4.1 **Elastic reactions**

For elastic events $M^* = M$, and from Eqs. (2.17) and (2.18), we get

$$q_{el}^2 = 2M(E_\nu - E_f) \quad (2.19)$$

$$E_{\nu}^{el} = \frac{E_f - m_t/2M}{1-\frac{E_f}{M}(1-\beta_f \cos \theta)} \quad (2.20)$$

From the above relations, we can see that for elastic events, the momentum and direction of the outgoing lepton will give sufficient information for the calculation of the neutrino energy and the squared four-momentum transfer.

We now take into account the Fermi motion of the target nucleon\(^{80}\). The kinematical situation is shown in Fig. 2.5.

![Fig. 2.5 Kinematics of neutrino reactions with Fermi momentum of the target nucleon](image)

If the lepton mass is neglected ($m_\ell \ll E_\ell$), Eqs. (2.15) and (2.20) will become

$$q^2 = 4E_\nu E_f \sin^2 \frac{\theta}{2} \quad (2.15')$$

and
\[
q_{\text{el}}^2 = \frac{E_{\nu}(1 - \beta_{\mu} \cos \eta) - E_{\nu}}{1 - \beta_{\mu} \cos \alpha + 2 \frac{E_{\nu}}{m_{\nu} \sin^2 \frac{\theta}{2}}}, \tag{2.20'}
\]

respectively, where \( \beta_{\mu} \) is the Fermi velocity of the nucleon. Equation (2.20') will tend to Eq. (2.20) for \( \beta_{\mu} \to 0 \) and \( \beta_{\mu} \to 1 \).

By substituting \( E_{\nu} \) by \( E_{\text{vis}} \), Eqs. (2.17) and (2.19) will become

\[
M^2 = -q^2 + M^2 + 2M(E_{\text{vis}} - E_{\nu}) \tag{2.17'}
\]

\[
q_{\text{el}}^2 = 2M(E_{\text{vis}} - E_{\nu}) \tag{2.19'}
\]

In principle the above relations will provide a method for analysing elastic events.

4.2 Inelastic reactions

The analysis of inelastic events is more complicated because strongly interacting particles are scattered or absorbed on their passage through the production nucleus. Here we only consider briefly the single-pion production. It is now believed that the single-pion production is mainly through the isobaric state \( N^* \left( \frac{7}{2}^+, \frac{5}{2}^+ \right) \) with mass 1236 MeV. The invariant mass distribution is therefore expected to spread about this value, the spread being mainly due to Fermi motion.

Apart from Eq. (2.15), the invariant mass of the final state can be calculated independently, e.g. in the \( (\pi^+, p) \) final state:

\[
M^2 = m_{\pi}^2 + m_p^2 + 2(E_{\pi} \vec{p}_p - E_p \vec{p}_{\pi}), \tag{2.21}
\]

where \( \vec{p}_{\pi}, \vec{p}_p \) are the momenta of the pion and proton, respectively.

4.3 Kinematical incompatibility -- the \( q^2 \) test

Equation (2.16) shows that for the target nucleon at rest, the squared four-momentum transfer \( q^2 \) cannot exceed the value \( q_{\text{max}}^2 \), which corresponds to the backward lepton emission along the neutrino direction. If Fermi motion of the target nucleon is taken into account and the lepton mass is assumed to be very small compared with \( q^2 \) (\( m_{\nu} \ll q^2 \)), it can be shown that

\[
q_{\text{max}}^2 = \frac{4E_{\nu}^2}{1 + \frac{M^2}{p_{\nu}^2}}. \tag{2.16'}
\]

Incompatible events with \( q^2 \) value above \( q_{\text{max}}^2 \) can thus be excluded from the sample.

As we shall see in the following chapter, these events are mostly incoming charged particle interactions in which the incoming particle is taken as an outgoing muon. It is, however, possible that a small proportion of the excluded events could be genuine neutrino interactions, in which \( E_{\text{vis}} \) is appreciably underestimated owing to undetected
neutrons, and thus $q^2_{\text{max}}$ gives too low a value. Since $E_{\text{vis}}$ is in general a good estimate of the total neutrino energy $E_{\nu}$, this proportion is negligible.

5. **SUMMARY OF $\nu$ AND $\bar{\nu}$ EVENTS**

Table 2.3 is a summary of all the events observed in the bubble chamber during all the neutrino and antineutrino runs. The incoming neutrons which produce pions and the identified incoming charged particles are also included. It can be seen that about 10% of the neutrino and antineutrino candidates can be excluded by the $q^2$ test.

**Table 2.3**

A summary of events observed in $\nu$ and $\bar{\nu}$ runs

<table>
<thead>
<tr>
<th>Event Type</th>
<th>$\nu$ runs</th>
<th>$\bar{\nu}$ runs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total number of events inside fiducial volume</td>
<td>571</td>
<td>63</td>
</tr>
<tr>
<td>Number of events excluded by $q^2$ test</td>
<td>50</td>
<td>4</td>
</tr>
<tr>
<td>Non-pionic events</td>
<td>276</td>
<td>30</td>
</tr>
<tr>
<td>Pionic events</td>
<td>245</td>
<td>29</td>
</tr>
<tr>
<td>Neutrons which produce pions</td>
<td>43</td>
<td>8</td>
</tr>
<tr>
<td>Incoming charged particles</td>
<td>71</td>
<td>42</td>
</tr>
</tbody>
</table>
CHAPTER 3

BACKGROUND ESTIMATION

1. INTRODUCTION

In the last chapter, we presented a phenomenological classification of all possible \( \nu \) and \( \bar{\nu} \) interactions, the incompatible events having been excluded by the kinematics tests. This sample of \( \nu \) and \( \bar{\nu} \) candidates may be contaminated by background events which are not genuine \( \nu \) or \( \bar{\nu} \) interactions, but which are misidentified as such. It is therefore essential to separate the background events from the genuine ones. In this chapter, we shall first discuss the possible sources of background, and then make some estimates of the background event rates. We shall next make a comparison of the estimates with the experimental results.

There are three main sources of background events which are detected in the chamber together with \( \nu/\bar{\nu} \) interactions:

i) Cosmic rays
Cosmic rays do not give rise to events which constitute recognizable background. The hadronic part is absorbed completely by the top shielding which is equivalent to about two metres of iron. Only muons can pass into the chamber, and they traverse almost invariably through the chamber. They interact only rarely, and their interactions can be easily recognized. In the cosmic-ray run in early 1963, no events have been found which can be confused with neutrino events\(^{63}\). Since the cosmic-ray background is negligible, we shall not discuss it any further.

ii) Neutron interactions
Neutron interactions in which a non-interacting charged secondary is produced can be misidentified as \( \nu \) or \( \bar{\nu} \) events, i.e. the secondary being taken for the lepton from \( \nu \) or \( \bar{\nu} \) interactions. For example, a non-interacting secondary proton or \( \pi^+ \) can be mistaken for a \( \mu^+ \) from \( \bar{\nu} \) interaction, and \( \pi^- \) for \( \mu^- \) from \( \nu \) interaction.

iii) Incoming charged particles
Incoming charged particles (\( \pi, K, p \)) which interact inside the chamber can appear as outgoing leptons from \( \nu \) or \( \bar{\nu} \) interactions, if the incoming direction is not recognized. Thus, incoming \( \pi^+ \) and \( p \) will simulate \( \mu^- \) from \( \nu \) interaction and \( \pi^- \) will simulate \( \mu^+ \) from \( \bar{\nu} \) interaction.

2. NEUTRON BACKGROUND ESTIMATION

In the phenomenological classification of events recorded in the chamber, neutral events with at least one lepton candidate (i.e. non-interacting particle) are classified as \( \nu \) or \( \bar{\nu} \) candidates. Neutral events without lepton candidates are taken as neutron-induced interaction. This is true only when neutrino interactions of the neutral current type are neglected. For example, the interactions
\[ \nu + p \rightarrow \nu + p, \]
\[ \nu + N \rightarrow \nu + N' \]  \( \text{e.g. pion-nucleon system} \)

will appear as a neutron star, the first reaction producing a proton recoil, and the second possibly a proton and a pion. The cross-section of these reactions has been shown\(^{15}\) to be \(< 10^{-40} \text{ cm}^2\) (i.e. \(< 3\%\) of the charge cross-section). It is therefore particularly important to obtain some estimate of the incoming neutron flux if the study of the neutral-current-type neutrino interactions is to be attempted. There are two possible sources of neutron background in the chamber:

i) Neutrons which are produced in the nucleonic cascade initiated by the primary protons and which leak around or through the shielding and finally come to the chamber. This source includes those due to the circulating proton beam which might not be extracted, thus hitting the PS magnet near the weak regions of the shielding.

ii) Neutrons produced in\( \nu \) interactions in the shielding. Some of the neutrinos interact with the massive shielding in front of the chamber, and neutrons thus produced may come into the chamber.

2.1 **Expected background based on recognized neutron stars**

The neutron background contamination in\( \nu \) and\( \bar{\nu} \) events can be estimated on the basis of the clearly identified neutron stars. In all the runs with the magnetic horn focusing positive particles (predominantly\( \nu \) beam), about 1,400 neutron stars have been detected. These include 43 events in which one or more pions are produced:

i) 20 events with \( \pi^0(s) \) and \( p(s) \)

ii) 10 events with \( \pi^- \) and \( p(s) \)

iii) 7 events with \( \pi^+ \) and \( p(s) \)

iv) 6 events with \( \pi^- \), \( \pi^+ \) and/or \( \pi^0 \) and \( p(s) \).

The visible energy distribution of these pionic neutron stars is shown in Fig. 3.1. The majority of the non-pionic neutron stars are small recoil protons with a typical visible energy of around 70 MeV. Their visible energy distribution is shown in Fig. 3.2. As we can see, only 16 events contain a \( \pi^- \) which, if not interacting, would have been taken as a \( \mu^- \) and the events would thus be included in the\( \nu \) sample. All 16 events pass the kinematics test when treated as\( \nu \) interactions. Two in category (ii), however, have a pion of low momentum. Range-momentum relation indicates that they should stop in the liquid and thus no background of this type should be included. For each \( \pi^- \) track, the interaction probability can be calculated. For free, the mean interaction length for pions is \( \lambda_{\pi} = 58 \text{ cm} \). If \( t \) is the potential length that a non-interacting pion can travel inside the chamber, the interaction probability is:

\[ P = 1 - \exp \left( -t/\lambda_{\pi} \right). \]
Fig. 3.1 Energy distribution of pionic neutron stars in $\nu$ runs

Fig. 3.2 Energy distribution of non-pionic neutron stars in $\nu$ runs
a) **Events simulating non-pionic \( \bar{\nu} \) events**

Thus the eight events of type (ii) give us the basis on which to estimate the background in non-pionic \( \bar{\nu} \) events. The calculation of interaction probability shows that a background of eight events is contaminated in this class. The average detection probability is about 50%.

b) **Events simulating pionic \( \bar{\nu} \) events**

The six events in point (iv) have a \( \pi^- \) and another one or two pions. They all pass the kinematics test. The same calculation shows the expected background as follows:

\[
\begin{align*}
1 \pi^- \bar{\nu} & \quad \text{events: 1} \\
1 \pi^+ 1 \pi^0 \bar{\nu} & \quad \text{events: 5}.
\end{align*}
\]

The energy distribution of the events (a) and (b) with the expected background is shown in Fig. 3.3

In all the runs with the horn focusing negative particles (\( \bar{\nu} \) beam), the following neutron stars have been recorded:

- 740 events containing recoil protons only, with total projected track length < 10 cm (i.e. with visible kinetic energy less than about 150 MeV),
- 38 events containing recoil protons only, with projected track length > 10 cm,
- 9 pionic neutron stars.

These include events found in the first scanning of all the films and in the second scanning of 65% of the films. Taking into account the detection efficiency of neutron stars, which has been estimated to be 69%, we expect a total of 890 events with projected track length smaller than 10 cm. The visible energy distribution of these events is shown in Figs. 3.4a and 3.4b. All nine pionic events pass the kinematics test, but three of them have a low-momentum relation. Using the same method of calculation, the following background is expected:

- background of elastic \( \bar{\nu} \) interaction: none
- background of pionic \( \bar{\nu} \) interaction: 5 events.

2.2 **The Monte-Carlo calculation on neutron flux**

In this section, we shall investigate the neutron flux from nucleonic cascades initiated by the primary proton beam in the shielding. The attenuation and absorption of neutrons is a complex process\(^{31}\) involving many different kinds of interactions with atoms in the absorber. A typical interaction of an energetic nucleon on hitting a heavy nucleus is the production of a nuclear star in which a number of heavy-particle fragments, protons, neutrons, and pions share the energy and momentum of the incident nucleon. Previous calculations on the development of the particle cascade initiated by a multi-GeV incident proton in shielding materials either treat the one-dimensional infinite slab problem\(^{32}\) or cover too short a penetration distance\(^{33}\) to be useful for
Fig. 3.3  Energy distribution of neutron stars simulating $\nu$ events
Fig. 3.4  Energy distribution of neutron stars in \( \bar{\nu} \) runs
our purpose. The analytic calculations of the three-dimensional development of a nuclear cascade with a specific geometry would involve considerable complexity. The only straightforward way is the use of the Monte-Carlo method. It also has the advantage that a complicated production spectrum can be readily incorporated into the calculation.

2.2.1 Basic assumptions. Owing to the irregularity of the configuration of the actual shielding, and incomplete knowledge of secondary particle production spectra, drastic simplifications have to be made in order to make the calculation manageable. In spite of this, it is hoped that some ideas about the neutron rates in the chamber can still be obtained. Experiments\textsuperscript{65,64} have established that the momentum spectrum of secondary pions is characterized by an exponential tail at the high-energy end, whereas that of the secondary protons shows a flat distribution. Also, the cross-section for a pion to produce a nucleon is very small. The meson cascade will be insignificant after a relatively short distance. The electromagnetic showers initiated by neutral mesons also die out fast in common shielding materials owing to their short radiation lengths. After a distance of a few metres, the nucleonic cascade will then be dominated by protons and neutrons\textsuperscript{84}. For our practical purposes, it will be sufficient to consider the nucleonic components only.

The shielding is approximated to a truncated iron cone immersed in concrete or in air. Its dimensions are shown in Fig. 3.5.

In the calculation, elastic scattering, multiple scattering, and ionization energy losses are taken into account.

Fig. 3.5
Approximated dimensions of the iron shielding in the neutrino experiment
2.2.2 Spectra of secondary nucleons. These spectra are not yet very well known. However, there is evidence \(^{34}\) that in the centre-of-mass system, the longitudinal momentum spectrum of protons is peaked towards the maximum possible momentum, and the transverse momenta are distributed approximately as a Boltzmann law. As a crude model, we use a rectangular momentum distribution for protons, and we take the same spectra for both protons and neutrons. To calculate the angular distribution, the following formulae for transverse momentum are used \(^{35}\):

\[
\frac{1}{P_0} \frac{d^2N}{dp_1 dp} = \frac{6}{P_0} \exp\left[-\frac{(p_1 - 0.3)^2}{0.2}\right] \quad p_1 > 0.3 \text{ GeV/c}
\]

where \(N\) = number of secondary protons,
\(P_0\) = momentum of the primary nucleon,
\(p\) = momentum of the secondary proton, and
\(p_1\) = transverse momentum of the secondary proton.

All the above momenta are measured in GeV/c. Integrating these expressions gives a multiplicity of secondary nucleons of 1.8 per collision, and the total energy of secondary nucleons of 80% of the primary energy. A comparison of the empirical formulae (3.1) with experimental data \(^{35-37}\) on particle production for protons incident on Be shows a reasonably good fit, as can be seen in Fig. 3.6. Furthermore, we shall use these formulae for the primary momentum down to 0.3 GeV/c, although at low momenta they tend to give a bias against secondaries of low momentum transfer.

2.2.3 Other data used in the calculation.

i) Elastic scattering

The total cross-section for elastic scattering \(\sigma_{el}\) used in the calculation is assumed to be half of that for absorption \(^{36}\). For the angular distribution of elastically scattered nucleons in the laboratory system, we use the formula \(^{37}\):

\[
\frac{d\sigma_{el}}{d\Omega_{lab}} = \frac{\sigma_{el} \cdot p^2}{3\pi} \exp\left(-\frac{1}{3} \frac{r^2}{R^2}\right)
\]

where \(r = \sqrt{E} \cdot (\hbar/m_p)\) is the nuclear radius,
\(p\) = momentum of the nucleon,
\(\theta\) = angle scattered in the lab. system.

This is in reasonably good agreement with experimental data \(^{38}\).

ii) Multiple scattering

For multiple scattering of protons, we take a Gaussian angular distribution. The following formula is used for the root-mean-square spatial angle of scattering \(\sigma_{3}^{\text{rms}}\):

\[
\]
Ref. 65 : $p_0 = 11.8$ GeV/c  •
  = 18.8 GeV/c  ○
  = 23.1 GeV/c  △
Ref. 67 : $p_0 = 30.0$ GeV/c  □

Fig. 3.6 Comparison of formula (3.1) with experimental production data
\[
\beta^2 = \left( \frac{E_{\text{b}m}}{E_{\text{b}c}} \right)^3 \frac{D}{X_0}
\]

where \( E_{\text{b}m} = 21 \text{ MeV} \),
\( E_{\text{b}c} \) is in MeV,
\( D \) = thickness of shielding material traversed, and
\( X_0 \) = radiation length = 1.77 cm for iron
3.58 cm for baryte concrete.

iii) The ionization energy loss

The ionization energy loss is calculated by using the following formula:

\[
- \frac{dE}{dx} = \frac{4\pi e^4 N Z}{m_e c^2 \beta^2} \left[ \ln \left( \frac{2m_e c^2 \beta^2}{I} \right) - \ln \left( 1 - \beta^2 \right) - \beta^2 \right]
\]

where \( N \) = number of target nuclei per cm\(^3\),
\( Z \) = atomic number of target nuclei, and
\( I \) = average ionization potential of the target nuclei.

By using \( \beta^2 = p^2 + m^2 \) and \( \beta = p/\sqrt{p^2 + m^2} \), this can be transformed to

\[
- \frac{dE}{dx} = \frac{4\pi e^4 N Z}{m_e c^2} \left[ \ln \left( \frac{2m_e c^2}{I} \cdot \frac{p^2}{m^2} \right) - \beta^2 \right].
\]

For iron, \( N = 8.41 \times 10^{22} \)
\( Z = 26 \)
\( I = 2430 \text{ eV} \).

\[
\therefore \frac{dE}{dx} = -1.132 \times 10^{-3} \frac{1}{\beta^3} \left[ \ln 4228 \left( \frac{p}{m} \right)^2 - \beta^2 \right] \text{ GeV/c per cm}.
\]

For concrete, \( N = 7.53 \times 10^{22} \)
\( I = 125 \text{ eV} \).

\[
\therefore \frac{dE}{dx} = -3.74 \times 10^{-4} \frac{1}{\beta^3} \left[ \ln 8180 \left( \frac{p}{m} \right)^2 - \beta^2 \right] \text{ GeV/c per cm}.
\]

For air, \( N = 1.976 \times 10^{20} \)
\( I = 94 \text{ eV} \).

\[
\therefore \frac{dE}{dx} = -1.007 \times 10^{-7} \frac{1}{\beta^3} \left[ \ln 10872 \left( \frac{p}{m} \right)^2 - \beta^2 \right] \text{ GeV/c per cm}.
\]
2.2.4 The method of calculation. The purpose of the calculation is to estimate the neutron flux at the chamber, i.e. about 22 metres from the beginning of the shielding or more than 120 nuclear mean free paths in iron. It is therefore extremely unlikely, with the Monte-Carlo method, to get any flux at this point even with a very large number of incident protons. In order to overcome this difficulty, and to save computer time, the shielding is divided into sections of 25 cm deep. We also use a momentum cut-off at 0.3 GeV/c which corresponds to a track of a few centimetres long in the heavy-liquid bubble chamber. At lower energies, the nuclear absorption cross-sections show strong energy dependence and therefore cannot be regarded as constant. It is known \(^1\) that these cross-sections remain approximately constant from very high momentum down to about 0.3 GeV/c, and thereafter increase rapidly with decreasing energies. For simplicity of calculation, we consider only particles of momentum higher than 0.3 GeV/c and use a constant collision length for nucleons to be 140 g/cm\(^2\). We start with 1000 particles incident on a point at the beginning of the shielding. Each of these particles is followed until it crosses to the next section or it loses energy to less than the cut-off. For each generation of inelastic scattering, a weight of 1.8 is given since only one secondary is followed in each collision, but the multiplicity used is assumed to be 1.8. The number of particles crossing into the succeeding section is then scaled up to 1000 by duplicating the particle parameters. In this way, the whole length of the shielding can be followed through with reasonable computer time.

The calculations have been performed on the CDC 6600 computer by using the Monte-Carlo method. This method consists of choosing at random the parameters of a process according to their probability distributions. Parameters of the shielding materials such as nuclear mean free paths, radiation lengths, and other constants are given in the input. A primary proton of momentum 25 GeV/c is then allowed to hit at \(x = y = z = 0\) (the primary proton beam is in the positive \(z\)-direction, and the beginning of the shielding is the \(z = 0\) plane). The collision point is then randomly determined. In the inelastic collision, it is assumed that there is an equal likelihood of the protons and neutrons being produced. The program then proceeds to determine the momentum, the polar and the azimuthal angles of the secondary, and the range to the next collision. If the momentum is less than 0.3 GeV/c, the particle is considered lost. For protons, multiple Coulomb scattering and ionization losses are taken into account. Before proceeding further, the collision point is tested to see whether it is inside the iron core of the shielding, and whether it has penetrated into the next section. Constants appropriate to the shielding materials are then used. For interactions within the same section, another generation of secondaries is followed. If the collision point is beyond the current section, the point of intersection of the particle track and the sectional plane is then determined. The particle parameters at this point, such as momentum, coordinates, direction angles, and weight are stored. The same procedure is repeated for 1000 primary protons. Momentum and radial distributions of all protons and neutrons crossing the plane are printed out in a tabulated form. Particles emanating from this plane are then scaled up to 1000 and regarded as input particles to the next section. This is equivalent to increasing the input primary protons in the same ratio.
This is repeated for the whole length of the shielding. The output gives us the radial density distributions and momentum distributions of proton and neutron tracks crossing planes perpendicular to the shield axis at various distances and for a primary proton beam hitting at X=Y=Z=0. In practice, the proton beam is diverged slightly owing to diffraction and Coulomb scattering in the copper target. But these effects are very small. Using the formula (3.3) and the experimental results on diffraction \(^9\), it has been estimated that practically all the full-energy protons hit the shield wall within a radius of 40 cm. Because of this divergence, the same calculation has been repeated for a number of incident points on the shield wall, assuming that all protons come from the target 25 m in front of the wall.

2.2.5 The results of the calculation. In the case of an iron core immersed in concrete, the neutron density attenuation as a function of depth in the shield at various radial distances is shown in Fig. 3.7. The integrated neutron density curve shows a slight bending at about 9 m, which separates the curve into two parts with different slopes. The attenuation length corresponding to the first part is roughly 23 cm, which coincides approximately with that of iron. This indicates that the nucleonic cascade develops mainly within the iron core up to a distance of about 5 m, and thereafter, the leakage of neutrons to concrete becomes apparent. The slopes of the density distribution curves remain approximately constant except at points of transition from concrete to iron. From these curves, a set of contours corresponding to equal attenuations can be constructed and is shown in Fig. 3.8. It can be seen that with this arrangement practically no neutrons can leak through the shielding to the chamber. A similar set of contours is also shown in Fig. 3.8 for the arrangement of iron core surrounded by air. From Fig. 3.7, the lateral attenuation of neutrons at different depths in the iron shield can also be constructed. The results are shown in Fig. 3.9.

The actual shielding arrangement is, however, much more complicated. If we assume that half of the iron core is surrounded by concrete and the other half by air, it is evident that neutrons can only come to the chamber by leaking to air and undergoing back-scattering either in air or by other materials in the experimental hall.

From Fig. 3.8, it is estimated that for each primary proton hitting the shield wall, about \(3 \times 10^9\) neutrons can leak through the iron core and come to the region of about 7-8 metres from the chamber. The average energy of these particles is of the order of 100 MeV to 200 MeV. Experimental and theoretical calculations on the energy spectra for large-angle scattering are still very scanty. In order to get some idea of the situation, we assume that the scattering is isotropic in the centre-of-mass system and the effective path \(l\) for neutrons to be scattered to the chamber is 3 metres. It is estimated\(^4\) that on the conservative side, not more than 5\% of the neutrons can undergo large-angle scattering > 75\(^\circ\). The probability for a neutron to be scattered to the chamber is

\[
P = \frac{dS}{4\pi R^2} \left[ 1 - \exp \left( - \frac{l}{R} \right) \right] \times \%\; of\; back-scattering \tag{3.5}
\]
Fig. 3.7 Neutron density in the shield for various radial distances
Fig. 3.8 Neutron attenuation contours in the shield
Fig. 3.9 Lateral attenuation of neutrons at various depths
where $dS =$ area of chamber receiving neutron flux = $\pi(0.55)^2$,

$R =$ mean distance from the scattering point to the chamber = 7 m,

$l =$ 3 m, and

$\lambda =$ mean free path for scattering in air $\approx 600$ m.

We then obtain

$$p \approx 4 \times 10^{-7}.$$  

From this crude estimate, it is seen that for $10^{18}$ incident primary protons, about one neutron can come to the chamber. Since scatterings over large angles give a large reduction in energy, it is expected that neutrons from this source are mostly of low energy. In the $\nu$ runs, $7.4 \times 10^{17}$ primary protons were extracted from the PS and directed on to the copper target. During these runs, some 1,400 neutrons were observed in the chamber. Most of them have energies below 100 MeV.

It should be pointed out here that, owing to the drastic simplifications, the above estimate can only serve to give the order of magnitude of the neutron background from this source. However, there is reasonably good agreement between the estimated neutron rate and the observed one, especially if we only consider neutrons in the low-energy region (< 100 MeV).

2.3 Estimation of neutron flux from $\nu$ interactions in the shielding

The neutron flux estimated in the last section cannot account for all the neutrons recorded in the chamber, especially for the more energetic ones. These neutrons and some of the low-energy ones must come from other sources. In this section, we shall estimate the effects of the neutrons from $\nu$-interactions in the shielding. To this end, we shall calculate the ratio of the number of neutrons recorded in the chamber from $\nu$-induced reactions in the shield to that associated with $\nu$ interactions in the chamber.

2.3.1 Neutron flux inside the iron shield. Let us first assume that the dimensions of the bubble chamber are very small compared to those of the iron shield, and that it is situated at the end of the shielding and surrounded by it (Fig. 3.10). We further assume that the neutrino flux is uniform over the whole length of the shield. At a sufficient depth along the shield, a state of equilibrium will be established, at which the number of neutrons absorbed in unit volume is equal to that produced by $\nu$ interactions. A constant flux of neutrons will then pass through any surface within this shield.

Fig. 3.10
Assumed position of the chamber relative to the iron shield
Consider now a volume element of unit area and thickness \( \text{dx} \). For simplicity, we assume that neutrons are produced in the forward direction. We shall use the following symbols:

\[ c = \text{No. of neutrons produced/sec g of matter}, \]
\[ \lambda_s = \text{attenuation length of neutrons in iron}, \]
\[ \lambda_L = \text{collision mean free path in freon}, \]
\[ \Phi_N = \text{neutron flux in iron in equilibrium}. \]

Then, we get immediately

\[ \Phi_N \approx \int_0^\infty c \exp \left( -\frac{\lambda_s}{\lambda_s} \right) \text{dx} = c \lambda_s. \]

(3.6)

This is also the flux crossing any surface in any direction. The number of neutrons \( N_s \) observed in the chamber from this incoming flux is then

\[ N_s = c \lambda_s A \left[ 1 - \exp \left( -\frac{t}{\lambda_L} \right) \right] \]

(3.7)

where \( A \) = the total area of the surface of the chamber which accepts the flux,

\( t \) = the average length of a track crossing the chamber.

Since the chamber is a cylinder of radius \( R = 57.5 \text{ cm} \) and depth \( d = 50 \text{ cm} \), and \( t = \pi R/2 \) for a track parallel to the cylinder base, we have

\[ N_s = c \lambda_s (2\pi R^2 + 2\pi Rd) \left[ 1 - \exp \left( -\frac{t}{\lambda_L} \right) \right]. \]

(3.8)

2.3.2 Number of neutrons \( N_L \) accompanied with \( \nu \) interactions observed in the chamber. This is given by:

\[ N_L = \int_0^l c \left[ 1 - \exp \left( -\frac{\lambda}{\lambda_L} \right) \right] S \text{dx} \]

(3.9)

where \( S = \text{cross-sectional area of the chamber perpendicular to the } \nu \text{ beam} \). Then, we get

\[ N_L = c \left[ 1 - \lambda_L + \lambda_L \exp \left( -\frac{1}{\lambda_L} \right) \right] S. \]

(3.10)

2.3.3 Comparison with experimental results. Now, we have

\[ \lambda_s = 140 \text{ g/cm}^2 \]
\[ \lambda_L = 90 \text{ g/cm}^2 \]
\[ t = 90 \text{ cm} = 135 \text{ g/cm}^2 \]
\[ R = 57.5 \text{ cm} \]
\[ d = 50 \text{ cm}, \text{ and} \]
\[ S = 2 \times R \times d = 5.75 \times 10^3 \text{ cm}^2 \]
With these values, we obtain immediately the ratio

\[
\frac{N_\beta}{N_L} = 11.
\]

For the enlarged chamber with radius of 60 cm and depth of 110 cm, the corresponding ratio is about 8. In the 1963/64 runs with the \( \nu \) beam, 67 neutrons accompanied with \( \nu \) interactions have been recorded in the chamber. Their energy distribution is shown in Fig. 3.11. It is very similar to the energy distribution of the incoming neutrons of energy higher than about 100 MeV (see Fig. 3.2). For lower energies, incoming neutrons show a predominance in proportion. To make a sensible comparison, we make an energy cut at 100 MeV. There are then 38 accompanied neutrons and 573 incoming ones. They give a ratio of \( 15 \pm 2.5 \). From this, it seems reasonable to conclude that about 70% of the incoming neutrons in the higher energy region come from the \( \nu \) interactions in the shielding when the horn is focusing positive particles (\( \nu \) beam). The rest is presumably due to leakage effects from the primary proton beam. In the 1965 runs, with the horn focusing negative particles (\( \bar{\nu} \) beam), only 10 neutrons accompanied with \( \bar{\nu} \) interactions have been found. Owing to the inadequacy of statistics, it is more difficult to make a sensible comparison. However, since the \( \bar{\nu} \) cross-section is smaller than that of the \( \nu \), it is expected that the proportion of incoming neutrons from \( \bar{\nu} \) interactions would be considerably reduced. Thus, the neutron background may be dominated by leakage effects.

It should be pointed out here that in the above estimates, the ratio \( \frac{N_\beta}{N_L} \) depends sensitively on the choice of the area \( S \) which is the total area of the chamber capable of accepting the neutron flux \( \frac{S}{N} \). We have chosen \( S \) to be the total surface area of the chamber. In other words, we have assumed that the neutron flux is uniform over the whole surface of the chamber. In practice, we expect the neutron flux in the forward neutrino direction to be more intense than in the backward direction. If we assumed the effective area \( S \) to be only half the total surface area, then the ratio would be reduced to half of its value. In consequence, the above estimate only gives the maximum value of the ratio.

3. BACKGROUND FROM INTERACTIONS OF INCOMING CHARGED PARTICLES

As was mentioned in Section 1 of this chapter, charged particles entering and interacting in the chamber can be misidentified as neutrino or antineutrino events, if their direction of motion is not recognized. In this section, we shall estimate the background events due to this source among the neutrino and antineutrino candidates on the basis of those events clearly identified as incoming particles by the direction of motion.

3.1 The probability of recognizing the direction of motion

The direction of motion of a charged particle can be recognized by two methods:

i) by the \( \delta \) ray produced along the track, and

ii) by the change of curvature of the track due to ionization energy loss.
Fig. 3.11 Energy distribution of neutrons accompanied with $\nu$ interactions in 1963/64 runs
The probability $P_\delta(p)$ of recognizing the direction of a track by its $\delta$ rays can be calculated from the knowledge of the mean free path $\lambda_\delta(p)$ for the production of a significant $\delta$ ray. This mean free path $\lambda_\delta(p)$ as a function of momentum has been determined by inspecting the muon tracks in neutrino events. The probability $P_\delta(p)$ for a track of length $l$ is then

$$P_\delta(p) = 1 - \exp\left(-\frac{l}{\lambda_\delta(p)}\right).$$  \hfill (3.11)

The probability $P_{\Delta\rho}(p)$ of recognizing the direction of motion of a particle by the change of curvature of its track has also been determined by Venus and Paty. A difference of 1 mm in the sagitta of the first and the last 10 cm of the measured track was assumed to be discernible. With this criterion, the variation of $P_{\Delta\rho}(p)$ as a function of the particle momentum for various track lengths was then computed.

The total probability $P(p)$ of recognizing the direction of motion of a pion either by its significant $\delta$ ray or by the change of curvature of its track is then given by:

$$1 - P(p) = [1 - P_\delta(p)] [1 - P_{\Delta\rho}(p)]$$
or

$$P(p) = P_\delta(p) + P_{\Delta\rho}(p) - P_\delta(p)P_{\Delta\rho}(p).$$  \hfill (3.12)

The probability $P(p)$ plotted against momentum for various track lengths is shown in Fig. 3.12.

3.2 Estimate of background events

By making use of the probability graph (Fig. 3.12), the background events due to incoming pions can be estimated on the basis of the clearly identified incoming events.

3.2.1 Selection of recognized pion interactions. Table 3.1 shows a summary of all the recognized pion interactions observed in the neutrino and antineutrino runs. All recognized pion interactions, excluding the non-contemporary ones, were first selected. They were then treated as $\nu$ (or $\bar{\nu}$) events, the incoming pion being taken to be the outgoing muon. Events which failed the kinematical $q^2$ test (i.e., $q^2 > q^2_{\text{max}}$) were then excluded from the sample. Finally, in the neutrino runs, events which would be interpreted as antineutrino events were also rejected. Similar procedure was repeated for events in the antineutrino runs. The background estimate was then based on the remaining events which would have been accepted as good $\nu$ (or $\bar{\nu}$) events, if their direction of motion had not been recognized. There are 34 such events in the neutrino runs and 21 in the antineutrino runs.
Fig. 3.12 Probability of identifying the direction of pions for given momentum and track length.
Table 3.1
Summary of recognized pion interactions

<table>
<thead>
<tr>
<th></th>
<th>$\nu$ runs</th>
<th>$\bar{\nu}$ runs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total number</td>
<td>71</td>
<td>42</td>
</tr>
<tr>
<td>Excluded by $q^2$ test</td>
<td>17</td>
<td>11</td>
</tr>
<tr>
<td>Events simulating $\bar{\nu}$ events in $\nu$ runs</td>
<td>20</td>
<td>10</td>
</tr>
<tr>
<td>Events simulating $\nu$ events in $\bar{\nu}$ runs</td>
<td></td>
<td></td>
</tr>
<tr>
<td>No. on which background estimate is based</td>
<td>34</td>
<td>21</td>
</tr>
</tbody>
</table>

3.2.2 Estimate of background in different event classes. From the momentum and the track length of each recognized event, and with the help of Fig. 3.12 a weight $W = 1/P$ can be found. The background estimate in each event class is obtained by subtracting the number of recognized events from the sum of their weights. The results thus obtained are given in Table 3.2, which shows an estimate of 38 background events among the neutrino candidates and 19 among the antineutrino candidates.

Table 3.2
Estimate of background events due to incoming pion interactions

<table>
<thead>
<tr>
<th>Event simulating</th>
<th>Non-pionic</th>
<th>$\mu^+\pi^-$</th>
<th>$\mu^-\pi^+$</th>
<th>Other one-pionic</th>
<th>$2\pi$</th>
<th>Total No.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\nu$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sum of weights</td>
<td>10</td>
<td>38</td>
<td>--</td>
<td>21</td>
<td>3</td>
<td>72</td>
</tr>
<tr>
<td>No. of recognized events</td>
<td>5</td>
<td>18</td>
<td>--</td>
<td>9</td>
<td>2</td>
<td>34</td>
</tr>
<tr>
<td>Background estimate</td>
<td>5</td>
<td>20</td>
<td>--</td>
<td>12</td>
<td>1</td>
<td>38</td>
</tr>
<tr>
<td>$\bar{\nu}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sum of weights</td>
<td>5</td>
<td>--</td>
<td>25</td>
<td>10</td>
<td>0</td>
<td>40</td>
</tr>
<tr>
<td>No. of recognized events</td>
<td>3</td>
<td>--</td>
<td>14</td>
<td>4</td>
<td>0</td>
<td>21</td>
</tr>
<tr>
<td>Background estimate</td>
<td>2</td>
<td>--</td>
<td>11</td>
<td>6</td>
<td>0</td>
<td>19</td>
</tr>
</tbody>
</table>

From the above estimates, it is seen that the background of pion interactions among the $\nu$ (or $\bar{\nu}$) events is quite appreciable. They constitute about 7% in the neutrino events and 30% in the antineutrino case. It is therefore desirable to be able to identify these background events among the $\nu/\bar{\nu}$ candidates. It will be seen that this can be achieved, to some extent at least, by considering the distribution of the longitudinal momentum $p_L$ versus the visible energy $E_{vis}$ of the events.
Fig. 3.13 $P_L - E_{vis}$ plot of recognized pion interactions
Fig. 3.14. The $P_L - E_{vis}$ distribution of non-pionic $\nu$ and $\bar{\nu}$ candidates
3.3 $P_L$ versus $E_{\text{vis}}$ distributions

It is expected that for true $\nu$ (or $\bar{\nu}$) events, the longitudinal momentum $P_L$ along the neutrino beam direction should be equal to the total energy $E_{\text{vis}}$ apart from energy losses due to undetected particles, whereas for pion interactions, when treated as neutrino events, $P_L$ should be zero for all values of $E_{\text{vis}}$. Thus, in the $P_L$ versus $E_{\text{vis}}$ plot, true neutrino events should lie on the line bisecting the two axes, while pion interactions should lie predominantly on the $P_L = 0$ line. However, effects of Fermi motion of the target nucleons, measurement errors, and the loss of visible energy due to undetected particles, will cause the points to spread about their corresponding lines. The $P_L$-$E_{\text{vis}}$ plots of all the recognized pion interactions in the neutrino and antineutrino runs are shown in Figs. 3.13a and 3.13b, respectively. It is evident that all incoming pion events have low values of $P_L$ (typically less than 0.3 GeV/c) for all values of $E_{\text{vis}}$, and they are approximately distributed symmetrically about the $P_L = 0$ line.

Let us now examine the $P_L$-$E_{\text{vis}}$ distributions of the $\nu$ and $\bar{\nu}$ candidates in different classes. Figures 3.14a and 3.14b show the distributions for non-pionic events. In the neutrino case, the expected background is five events. Apart from those excluded by the $q^2$ test, there are six events with low values of $P_L$ and lying far from the $P_L = E_{\text{vis}}$ line; they are probably pion interactions. It should be noticed, however, that for $E_{\text{vis}}$ greater than 1 GeV, possible background is limited to two events only.

In the antineutrino case, the expected background is two events. From Fig. 3.14b, it can probably be concluded that event 1112 being far from the $P_L$-$E_{\text{vis}}$ line should be regarded as a pion interaction. There may still be one or two background events among those with $E_{\text{vis}} < 0.5$ GeV. But it seems safe to conclude that except for event 1112 there will probably be no background events among those with $E_{\text{vis}} > 0.5$ GeV or $P_L > 0.3$ GeV/c.

The largest estimated background is, however, among the one-pion events. In the neutrino case, the background due to $\pi^+$ scattering among the $\mu^-\pi^+$ candidates is of the order 20, and that among other one-pion events is estimated to be 12. In the antineutrino case, they are estimated to be respectively eleven and six events. From the $P_L$-$E_{\text{vis}}$ scatter plots of one-pionic events shown in Figs. 3.15a and 3.15b, and by comparing them with the corresponding plots for recognized pion interactions (Figs. 3.13a and 3.13b), it can be seen that many of the $\mu^-\pi^+$ or $\mu^-\pi^-$ events with $P_L$ less than 0.5 GeV/c are probably background events. The same can also be applied to the other one-pion events.

3.4 Interactions of protons

In the bubble chamber, a proton of momentum below 0.5 GeV/c can be readily recognized by the heavy ionization of its track, and it produces only evaporation protons when interacting with nuclei. If its incoming direction is not recognized, it will be classified as a non-pionic neutron star. Therefore, for an incoming proton to be misidentified as an outgoing $\mu^-$, it must have a momentum greater than about 0.6 GeV/c.

It has been found that about 95% of all the protons of momentum above 0.6 GeV/c from neutrino interactions are emitted into the forward hemisphere with respect to the neutrino direction. Therefore, proton interactions which could be taken as non-pionic
neutrino events should fail the $q^2$ test, even if the incoming direction is not recognized. Also, these events should be easily recognizable on the $p_L - E_{\text{vis}}$ plot. By inspection of Fig. 3.14a and taking into account the good agreement between the estimated background due to incoming pions and the likely background on the $p_L - E_{\text{vis}}$ plot, it can be concluded that the background due to proton interactions is negligible.

A proton interaction can be misidentified as a one-pion event only if a charged pion is produced. The event will then have a visible energy $E_{\text{vis}}$ above 1 GeV. If the proton is taken as a $\mu^-$, the event will almost certainly be excluded by the $q^2$ test. By inspecting Figs. 3.15a and 3.15b, no event except one can be treated as a proton background suspect.

3.5 Conclusions

From the above considerations, the following conclusions on background due to incoming charged particles can be drawn:

3.5.1 Non-pionic events. For neutrino candidates, background should be very small (less than 2) for events with visible energy $E_{\text{vis}}$ above 1 GeV. For events with $E_{\text{vis}}$ below 1 GeV, there may exist a few background events among the non-pionic candidates. For antineutrino events, background should be negligible for events with $E_{\text{vis}}$ above 0.5 GeV or $p_L$ above 0.3 GeV/c.

3.5.2 One-pion events. For the longitudinal momentum $p_L$ below 0.3 GeV/c, most events of this type are probably background events. Others should be accepted as good neutrino (or antineutrino) events.

3.5.3 Events with more than one pion. Background of this type is very small and negligible.

* * *
Fig. 3.15 The $P_L$ versus $E_{VIS}$ distributions of one-pionic $\nu$ and $\bar{\nu}$ candidates
PART III

DISCUSSION OF EXPERIMENTAL RESULTS
CHAPTER 4

ELASTIC NEUTRINO INTERACTIONS

In this chapter, we shall study the particular type of neutrino interactions, namely, the elastic interactions:

\[ \nu_\mu + n \rightarrow \mu^- + p \quad (4.1) \]
and

\[ \bar{\nu}_\mu + p \rightarrow \mu^+ + n. \quad (4.2) \]

We shall first discuss how the separation of the elastic events from the \( \nu/\bar{\nu} \) sample can be achieved. Next, we shall evaluate the axial vector form factor and calculate the elastic cross-sections which can then be compared with the theoretical predictions.

1. THE ANALYSIS OF ELASTIC EVENTS

The \( \nu/\bar{\nu} \) events observed in the chamber are classified into two types phenomenologically: 'non-pionic' events which contain a muon (non-interacting particle) and one or more protons, and 'pionic events' which contain a muon and at least one identified pion. However, the non-pionic events, from which the elastic events are extracted, contain three types of background:

i) neutron stars in which a non-interacting pion is misidentified as a muon;
ii) interacting incoming charged particles which are misidentified as outgoing muons from neutrino interactions;
iii) pionic neutrino events in which the pions have been absorbed during their passage through the parent nucleus.

The first two types of background have been discussed in detail in the last chapter. We recall that these background events are mostly of low energy, and that above 1 GeV the background from these two sources is negligible. It is therefore important to study the pionic absorption effect, and hence to obtain an estimate of the contamination of inelastic events in the non-pionic sample. In the following, we shall discuss how this can be done by investigating the events with small four-momentum transfer \( q^2 \).

1.1 Events with small four-momentum transfer \( q^2 \)

As we have seen before, the Fermi motion of the target nucleon may present difficulties in the interpretation of events. The analysis of low \( q^2 \) events has the advantage that much of the effect of Fermi motion disappears as \( q^2 \rightarrow 0 \). This can be seen from the following kinematical considerations.

The squared four-momentum transfer \( q^2 \) is given by

\[ q^2 = -m_\mu^2 + 2E_\nu (E_\nu - p_{\mu z}) \quad (4.3) \]
where \( x \) is the direction of the incident neutrino. If the interaction is assumed to be elastic, i.e. \( M^* = M \), the neutrino energy can be calculated from the energy and the direction of the lepton. Taking into account the Fermi motion, the neutrino energy \( E_\nu \) is given by [cf. Eq. (2.18)]:

\[
E_\nu = \frac{-m_\mu^2 + p_\mu^2 + 2mE_\mu + 2p_\mu \cdot p_\nu}{2(M - E_\mu + p_{\mu x} + p_{\mu y})}.
\] (4.4)

On the other hand, if the neutrino energy is known, then the mass of the recoil system can be calculated and is given by [cf. Eq. (2.17)]:

\[
M^{*2} = -q^2 + M^2 + 2M(E_\nu - E_\mu) + 2p_{\mu x}(E_\nu - E_\mu) + 2(E_\nu p_{\mu x} - p_\mu \cdot p_\nu) - P_F^2.
\] (4.5)

For low \( q^2 \) events, \( E_\mu \approx p_{\mu x} \) and Eqs. (4.4) and (4.5) will become, respectively:

\[
E_\nu \approx E_\mu + \frac{P_F^2 - m_\mu^2}{2(M + P_{\mu x})},
\] (4.6)

\[
M^{*2} \approx -q^2 + M^2 + 2M(E_\nu - E_\mu) + 2p_{\mu x}(E_\nu - E_\mu) - P_F^2.
\] (4.7)

Taking \( p_{\mu x} = \pm 250 \text{ MeV}/c \) and \( p_F^2 = 0.07 \text{ (GeV/c)}^2 \), the following relations for elastic events can be obtained:

\[
E_\nu \approx E_\mu
\] (4.8)

\[
M^{*2} = M^2 \pm 0.7.
\] (4.9)

Among the 276 non-pionic events observed in the neutrino runs (see Table 4.1), 94 events with \( q^2 \) less than 0.2 (GeV/c)^2 have been selected, which contain 52 events with zero or one proton, and 42 with two or more protons. The \( M^* \) distribution of these non-pionic single-proton and multiproton events are shown in Figs. 4.1a and 4.1b. It is clearly seen that the single proton events are spread around the proton mass, whereas the multiproton events are distributed around a higher mass and more widely spread. This shows that most of the multiproton events are not elastic, but that absorption of a pion has taken place in the production nucleus.

Another comparison can also be made on the direction of the expected recoil for the elastic production hypothesis, and the vector sum of the observed proton momenta \( \vec{P}_P \).

All momenta are projected on to the neutrino-\( \mu \)-on plane (Fig. 4.2), and the recoil direction is calculated from:

\[
P_{\mu}^* = E_\nu - p_{\mu x}
\]

\[
P_{\mu}^* = -p_{\mu y}
\]

\[
P_{E}^* = -p_{\mu z}
\] (4.10)
Fig. 4.1 Invariant mass distribution of non-pionic $\nu$ events with $q^2 < 0.2$ (GeV/c)$^2$
where \( p_x \), \( p_y \), \( p_z \) are the momentum components of the recoil system, and where the Fermi motion of the target nucleon has been neglected.

![Fig. 4.2](image)

Kinematics of low \( q^2 \) non-pionic neutrino events.

The transverse momentum \( p_t \) (see Fig. 4.2) distribution for a sample of single-proton and multiproton events of \( q^2 < 0.2 \ (\text{GeV/c})^2 \) are shown in Fig. 4.3. It can be seen that the single-proton events are distributed symmetrically about the direction of the calculated recoil for elastic interactions, whereas the multiproton events show marked asymmetry.

It is expected that the effect of the Fermi motion and the scattering of the nucleon will smear this distribution about the original directions. From this, it is obvious that in the low \( q^2 \) sample the one-proton events are mostly composed of elastic events, whereas the multiproton events are predominantly inelastic.

1.2 Selection of elastic events

The above analysis can only apply to events with low \( q^2 \), since at high \( q^2 \) the recoil directions of elastic and inelastic events are similar. In elastic reactions the four-momentum transfer \( q^2 \) is related to the kinetic energy \( T_p \) of the proton by [cf. Eq. (2.19)]:

\[
q^2 = 2MT_p
\]

Therefore the high \( q^2 \) elastic events may have several protons as a result of the nuclear interaction of the more energetic recoil proton in the nucleus.

In view of this, a Monte-Carlo calculation has been performed by Myatt\(^{94}\) to determine the proportion of the elastic events which contain more than one proton track corresponding to kinetic energy greater than 30 MeV. This energy limit is applied to exclude evaporation protons which may be liberated from the target nucleus. In this calculation, the degenerate Fermi gas model of the nucleus was used, in which the radius of the nuclear potential was taken to be \((1.3 \times 10^{-13})\Lambda^{1/3} \text{ cm, where } \Lambda \text{ is the atomic number. The nuclear}

potential and the Fermi momentum were taken to be 37 MeV and 234 MeV/c, respectively. The result of the calculation is shown in Fig. 4.4, in which the fraction of elastic events with zero or one proton of energy greater than 30 MeV is plotted as a function of the kinetic energy of the recoil proton inside the nucleus.

The analysis of the last section has established that the one-proton non-pionic events of small \( q^2 \) are predominantly elastic, and that the major contamination of inelastic events in the non-pionic sample is due to the absorption of pions. Since there is no
Fig. 4.3 Transverse momentum distribution of non-pionic $\nu$ events with $q^2 < 0.2 \text{ (GeV/c)}^2$
Fig. 4.4  Fraction of elastic events with $41$ proton as function of recoil proton kinetic energy
reason to assume that the one-proton events of high $q^2$ contain a greater contamination of inelastic events than those of low $q^2$, we can take the one-proton non-pionic events as elastic and make corrections for those elastic events with more than one proton.

Our main objective in studying the elastic process is to estimate the axial vector form factor $F_A$, and to determine the experimental cross-section for the elastic process. To this end, we consider only elastic events with visible energy $E_{\text{vis}}$ greater than 1 GeV, since below 1 GeV the $q^2$ distribution, from which information on the $F_A$ can be obtained, is not sensitive to the form factor, and above 1 GeV the background due to neutrons and incoming charged particles is negligible. The classification into different categories of the non-pionic neutrino events is summarized in Table 4.1. Of the 276 non-pionic events obtained in all the neutrino runs, 145 have visible energy $E_{\text{vis}}$ greater than 1 GeV. Eighty-five of these have zero or one proton of kinetic energy greater than 30 MeV. To study these possible elastic candidates in greater detail, we consider the distribution of the invariant mass squared $M^2$ of the recoil system, which can be calculated from the equation:

$$M^2 = -q^2 + M^2 + 2M(E_\nu - E_\mu)$$  \hspace{1cm} (4.12)

[cf. Eq. (2.17)] in which the target nucleon has been assumed to be at rest initially. The distribution of the values of $M^2$ is shown in Fig. 4.5.

It is expected that owing to the Fermi motion of the target nucleons in the complex nucleus and to the energy loss of the recoil proton, the $M^2$ values are distributed around a value slightly less than $M^2$ (i.e. square of the proton mass). This can be clearly seen in Fig. 4.5. However, Fig. 4.5 also shows that some events have values of $M^2$ far removed from $M^2$. From the sharp peaking of the $M^2$ distribution around $M^2$ and the consideration of the Fermi motion causing the spread, it is reasonable to assume that these events are not elastic. Only events with $0.48 < M^2 < 1.28 \text{ (GeV)}^2$ are therefore accepted. The application of this criterion should eliminate only a few per cent of the true elastic events. Twenty-two events have thus been rejected.

**Table 4.1**
Summary of non-pionic $\nu$ events

<table>
<thead>
<tr>
<th>Energy</th>
<th>Total</th>
<th>One-proton</th>
<th>Multi-proton</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Total</td>
<td>0.48 &lt; $M^2$ &lt; 1.28</td>
</tr>
<tr>
<td>&gt; 1 GeV</td>
<td>145</td>
<td>85</td>
<td>63</td>
</tr>
<tr>
<td>&lt; 1 GeV</td>
<td>131</td>
<td>74</td>
<td>74</td>
</tr>
<tr>
<td>Total</td>
<td>276</td>
<td>159</td>
<td>137</td>
</tr>
</tbody>
</table>
Fig. 4.5 $M^2$ distribution of one-proton events with $E_{\text{vis}} > 1$ GeV
We are left with a sample of 63 elastic one-proton events. For each of these, the kinetic energy $T_p$ of the recoil proton can be obtained from the knowledge of the $q^2$ [Eq. (4.11)]. The fraction of multiproton events produced at this $q^2$ can then be obtained from Fig. 4.4. It has been found that 7.5 multiproton events must be added to the 63 one-proton events. They have been added to the energy distribution of elastic events in proportion to the number of events at a certain energy, because the fraction of multiproton events produced depends on the $q^2$ value (see Fig. 4.4), and for energies above 1 GeV the $q^2$ distributions are fairly similar. As is shown in Table 4.1, there are altogether 145 non-pionic neutrino events with visible energy greater than 1 GeV. By our method of selection, we have 76.5 true elastic events. This shows that about half of the non-pionic events, including most of the multiproton events, are in fact inelastic events in which the pion has been absorbed during its passage through the nucleus.

It should be pointed out here that there may still be a slight contamination of inelastic events in the selected one-proton sample. But this is very difficult to estimate. It can be assumed that any such possible contamination may be offset by the possible exclusion of true elastic one-proton events with $M^2$ outside the 0.48-1.28 (GeV)$^2$ range.

Table 4.2 shows a summary of all the non-pionic events collected during the antineutrino runs. There are 30 such events. We have already known that for the elastic neutrino reaction (4.1) in complex nuclei, the majority of events appear with only one proton track corresponding to a kinetic energy greater than 30 MeV. In the antineutrino case, we accept these events as elastic which contain a $\mu^+$ candidate and no proton of kinetic energy greater than 30 MeV.

<table>
<thead>
<tr>
<th>Single $\mu^+$ with no proton &gt; 30 MeV</th>
<th>Single $\mu^+$ with at least one proton &gt; 30 MeV</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_x &gt; 0.3$ GeV/c</td>
<td>$P_x &lt; 0.3$ GeV/c</td>
<td>Total</td>
</tr>
<tr>
<td>11</td>
<td>9</td>
<td>20</td>
</tr>
<tr>
<td>10</td>
<td>30</td>
<td></td>
</tr>
</tbody>
</table>

In order to eliminate possible background, we also require that the resultant momentum $P_x$ of all the particles in the antineutrino direction should be greater than 0.3 GeV/c. This test should eliminate essentially all background due to incoming particles. However, it also introduces discrimination against low-energy events. In order to make a more sensible comparison with theory, we have considered only events with total visible energy $E_{vis}$ greater than 0.5 GeV.

After applying these selection criteria, we are left with 11 elastic events. The percentage of events with one or more protons of kinetic energy greater than 30 MeV can be estimated from the $q^2$ distribution of the selected "elastic" events and from Fig. 4.4.
Of the total of $3.23 \times 10^8$ pictures taken during all the antineutrino runs in 1965, 65% has been scanned twice and 35% only once. It has been found that the combined efficiency of the two scans of this type of events is 97.5%, and that of the first scan 78%. Taking into account the correction for multiproton events and the scanning efficiency, we expect that 3.1 events should be added to our elastic sample to make a total of 14.1 events.

2. **DETERMINATION OF THE AXIAL VECTOR FORM FACTOR**

2.1 The form factors

As has been seen in Chapter 1, if the validity of the conserved vector current theory is assumed, the weak nucleon vector form factors are identical to the isovector form factors in electromagnetic interactions. The empirical formulas obtained by Hofstadter et al.\(^{95}\) from electron scattering experiments can be written as:

$$F_1(q^2) = F_2(q^2) = \left(1 + \frac{q^2}{M_{em}^2}\right)^{-2} \quad (4.13)$$

with $M_{em} = 0.84$ GeV.

The only unknown left in the determination of the elastic cross-section [Eq. (1.37)] is the axial vector form factor $F_A(q^2)$. Since there is no a priori form for $F_A(q^2)$, we can assume that it is of the same form as $F_1(q^2)$ or $F_2(q^2)$, and use the parametric form

$$F_A(q^2) = \left(1 + \frac{q^2}{M_A^2}\right)^{-2}. \quad (4.14)$$

The elastic cross-section as a function of the four-momentum transfer $q^2$ can then be calculated for various values of $M_A$. The best fitted value of $M_A$ is determined by comparing the theoretical $q^2$ distribution with the experimental one. This choice of form factors we shall call assumption "A".

The weak form factors can also be determined in a different way. On the basis of the equal time commutation relations proposed by Gell-Mann\(^{96,97}\), sum rules have been derived\(^{98,99}\) which relate the vector and axial vector weak form factors of the nucleons. Furlan et al.\(^{99}\) have derived the following relation which is particularly simple for experimental comparison:

$$F_A(q^2) = F_1(q^2) + \frac{q^2}{4M^2} F_2(q^2) \quad (4.15)$$

where $M$ is the proton mass. The axial vector form factor $F_A(q^2)$ is thus related to the Dirac and Pauli vector form factors $F_1(q^2)$ and $F_2(q^2)$. The elastic cross-section [Eq. (1.37)] is then completely determined, if the conserved vector current hypothesis is assumed and the induced pseudoscalar term neglected. However, by relaxing the constraint imposed by CVC, one can choose for $F_1(q^2)$ and $F_2(q^2)$ the expression:

$$F_1(q^2) = F_2(q^2) = \left(1 + \frac{q^2}{M_0^2}\right)^{-2} \quad (4.16)$$
and determine $M_N$ from the comparison of the calculated cross-section with the experimental data\textsuperscript{109}. We shall call this choice of form factors assumption "B".

2.2 Effects of the Fermi motion and the exclusion principle

Since interactions take place in complex nuclei, the target nucleon is associated with Fermi motion. In Section 4 of Chapter 2, we mentioned that for a scattering event to take place, the Pauli principle requires that the three-momentum of the final nucleon be outside the Fermi sphere, i.e.,

$$P_N > P_F.$$  \hfill (4.17)

The differential cross-section given in Eq. (4.37) is, however, calculated for a free target nucleon. In order to be able to compare with the observed results, the differential cross-section has to be corrected for the above two effects. The cross-section for a bound target nucleon is then

$$\left( \frac{d\sigma}{d\Omega} \right)_{\text{nucleus}} = \frac{3}{4\pi p_F^4} \int_0^{\pi} d\alpha \int_0^{2\pi} d\phi \int_0^{\pi} d\theta \sin \alpha \left( \frac{d\sigma}{d\Omega} \right)_{\text{free}},$$

where $\alpha$ is the angle between the incident neutrino momentum and the nucleon momentum (see Fig. 2.5), and $\phi$ is the azimuthal angle with respect to the neutrino direction.

The effect of the Fermi motion and the exclusion principle is to reduce the cross-section at low values of $q^2$. For neutrino interactions with energy greater than 1 GeV, and assuming $F_A(q^2) = F_V(q^2)$, the reductions have been estimated\textsuperscript{108} and are shown in Table 4.3.

<table>
<thead>
<tr>
<th>$q^2$ (GeV/c)$^2$</th>
<th>% reduction</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 to 0.1</td>
<td>48</td>
</tr>
<tr>
<td>0.1 to 0.2</td>
<td>16</td>
</tr>
<tr>
<td>0.2 to 0.3</td>
<td>3</td>
</tr>
<tr>
<td>&gt; 0.3</td>
<td>0</td>
</tr>
</tbody>
</table>

It can be noted in Table 4.3 that for $q^2 > 0.3$ (GeV/c)$^2$, there is no reduction in the cross-section due to these effects.

2.3 Estimates of the form factors

Since all form factors are functions of the four-momentum transfer $q^2$, they can be investigated from the study of the $q^2$ distributions. The four-momentum transfer $q^2$ of the $\nu-\mu$ system can be calculated by two methods:
i) from the muon momentum only, assuming that the interaction is elastic and that the
target nucleon is initially at rest [Eqs. (2.19) and (2.20)];
ii) from the muon momentum and the neutrino energy $E_\nu$ which is taken to be $E_{\text{vis}}$, the
visible energy of the event [Eq. (2.15)].

However, the distributions obtained from these two methods are essentially the same.
We shall, therefore, use the results of the second method for comparisons with theoretical
predictions.

The theoretical distribution of events $dN(q^2)$ as a function of $q^2$ can be obtained by
integrating the differential cross-section over the neutrino spectrum:

$$
\frac{dN}{dq^2} = N_P N_T \int \frac{d\sigma(E_\nu)}{dq^2} \varphi(E_\nu) dE_\nu
$$

(4.18)

where

$N_P = $ number of ejected protons hitting the copper target;

$N_T = $ number of target neutrons contained in the fiducial volume of the chamber;

and

$\varphi(E_\nu) = $ neutrino flux at energy $E_\nu$ per ejected proton.

A maximum likelihood method can then be used to obtain the value of $M_A$ which will give
the best fit of Eq. (4.18) to the experimental distribution. Assuming a Poisson distribution
of events, the likelihood function is

$$
\chi^2(N) = \frac{N_0 - N}{N_0} e^{-N},
$$

(4.19)

where $N$ and $N_0$ are the numbers of expected and observed events, respectively, in a certain
$q^2$ interval.

The value of $M_A$ so obtained depends sensitively on the shape of the experimental $q^2$
distribution and the magnitude of the neutrino fluxes. In order to minimize any possible
uncertainty in the determination of the neutrino fluxes, the theoretical $q^2$ distribution
can be normalised to the number of observed events. The value of $M_A$ thus obtained will
be determined by the shape of the $q^2$ distribution only (normalized $q^2$ fit).

The $q^2$ distribution of the elastic neutrino events of visible energy $E_{\text{vis}} > 1$ GeV
and that of the elastic antineutrino events of visible energy $E_{\text{vis}} > 0.5$ GeV are shown in
Figs. 4.6 and 4.7, together with the calculated distributions. The best fitted values of
$M_A$ are listed in Table 4.4.

<table>
<thead>
<tr>
<th>$F_1 = F_2$</th>
<th>$F_A$</th>
<th>$M_A$ or $M_V$ (GeV) from normalized $q^2$ fit</th>
<th>$M_A$ or $M_V$ (GeV) from unnormalized $q^2$ fit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\left(1 + \frac{q^2}{(0.84)^2}\right)^{-2}$</td>
<td>$\left(1 + \frac{q^2}{M_A^2}\right)^{-2}$</td>
<td>$0.81 \pm 0.13$</td>
<td>$0.50 \pm 0.10$</td>
</tr>
<tr>
<td>$\left(1 + \frac{q^2}{M_V^2}\right)^{-2}$</td>
<td>$F_1 = \frac{q^2}{1.26}$ $F_2$</td>
<td>$0.77 \pm 0.07$</td>
<td>$0.69 \pm 0.03$</td>
</tr>
</tbody>
</table>
Fig. 4.6 Comparison of theoretical and experimental $q^2$ distributions for elastic events under assumption "A"
Fig. 4.7 Comparison of theoretical and experimental $q^2$ distributions for elastic events under assumption "B"
The possible errors quoted in Table 4.4 are purely statistical and do not include the uncertainty in the determination of the neutrino fluxes. Variations of the neutrino fluxes can affect the values of $N_A$ obtained from the unnormalized $q^2$ fit.

It can be concluded that from the best fitted value of $N_A$ and with the choice of form factors under assumption "A", the axial vector form factor of the nucleon is very similar to the isovector form factors.

Another comparison of the theoretical and experimental $q^2$ distributions can also be made, which makes use of the energy distribution of the events and is independent of the neutrino spectrum. We note that

$$N_pN_T \phi(E_\nu) = \frac{1}{\sigma(E_\nu)} \frac{dN}{dE_\nu}. \quad (4.20)$$

Equation (4.18) can then be rewritten as:

$$\frac{dN}{dq^2} = \int \frac{d\sigma(E_\nu)}{dq^2} \frac{1}{\sigma(E_\nu)} dN$$

$$= \sum_{E_\nu} \frac{d\sigma(E_\nu)}{dq^2} \frac{\Delta N(E_\nu)}{\sigma(E_\nu)}, \quad (4.21)$$

where $\Delta N(E_\nu)$ is the number of observed events in the energy interval between $E_\nu$ and $E_\nu + \Delta E_\nu$. As we have seen before, $d\sigma(E_\nu)/dq^2$ can be calculated for various values of $N_A$ and $\sigma(E_\nu)$ can be obtained by integrating $d\sigma(E_\nu)/dq^2$ over all values of $q^2$. The visible energy distributions of the elastic neutrino and antineutrino events are shown in Figs. 4.8a and 4.8b, respectively.

The $q^2$ distributions calculated by this neutrino spectrum-independent method with the best fitted values of $N_A$ for assumption "A" and "B" are also shown in Figs. 4.6 and 4.7, respectively. It can be seen that they agree very well with both the experimental distribution and that calculated by the other method mentioned earlier.

3. THE TOTAL ELASTIC CROSS-SECTION

As mentioned previously, the theoretical total cross-section for elastic neutrino reactions as a function of the neutrino energy can be calculated by assuming the form of the axial vector form factor $F_A(q^2)$ and the value of $N_A$. On the other hand, the experimental total elastic cross-section can be estimated from the energy distribution of the elastic events (Fig. 4.8) and the neutrino fluxes, i.e.

$$\sigma(E_\nu) = \frac{1}{N_pN_T \phi(E_\nu)} \frac{\Delta N}{\Delta E_\nu}. \quad (4.22)$$
A comparison of the experimental and theoretical results for elastic neutrino events is shown in Figs. 4.9 and 4.10. In Fig. 4.9, the curves have been obtained for various values of $M_A$ under assumption "A". The curves in Fig. 4.10 have been obtained from the choice of form factors under assumption "B", and varying values of $M_V$. The uncertainties in the experimental total cross-section are statistical. It can be seen that the agreement between the experimental results and theoretical curves calculated with values of $M_A$ or $M_V$ as listed in Table 4.4 is reasonably good within statistical errors.

The same comparison for elastic antineutrino events is shown in Fig. 4.11.
Form factors: \( F_1 = F_2 = \left( 1 + \frac{q^2}{(0.8\omega)^2} \right)^{-2} \)

\[ \sigma \left( 10^{-38} \text{cm}^2 \right) \]

\( \omega \) (GeV)

Fig. 4.9 Experimental and theoretical total cross-sections for elastic neutrino reactions under assumption "A".

Form factors: \( F_A = \frac{F_2}{4M_V^2} F_2 \)

\( F_1 = F_2 = \left( 1 + \frac{q^2}{M_V^2} \right)^{-2} \)

\( M_V \) (GeV)

Fig. 4.10 Experimental and theoretical total cross-sections for elastic neutrino reactions under assumption "B".
Form factors:

\[ F_1 = F_2 = \left( 1 + \frac{q^2}{(0.84)^2} \right)^{-2} \]

\[ F_A = \left( 1 + \frac{q^2}{M_A^2} \right)^{-2} \]

\[ F_A = F_1 + \frac{q^2}{4M^2} F_2 \]

\[ F_1 = F_2 = \left( 1 + \frac{q^2}{M_V^2} \right)^{-2} \]

\[ M_A \text{ or } M_V \text{ (GeV)} \]

Fig. 4.11 Experimental and theoretical total cross-sections for elastic antineutrino reactions
CHAPTER 5

ELASTIC PRODUCTION OF HYPERONS

1. INTRODUCTION

In this chapter, we shall discuss the three 'elastic' reactions allowed by the $\Delta S = \Delta Q$ rule among the $\Delta S = 1$ reactions, all arising from antineutrino interactions:

1. $\bar{\nu}_\mu + p \rightarrow \mu^+ + A$  \hspace{1cm} (5.1)
2. $\bar{\nu}_\mu + n \rightarrow \mu^+ + \Sigma^-$ \hspace{1cm} (5.2)
3. $\bar{\nu}_\mu + p \rightarrow \mu^+ + \Sigma^0$ \hspace{1cm} (5.3)

Hyperon production is forbidden in neutrino reactions by the $\Delta S = \Delta Q$ rule. In neutrino reactions, a negatively charged lepton must be produced in order to conserve lepton number. Consequently, the electric charge of the hadronic system increases by unity, i.e. $\Delta Q = +1$. In hyperon production, strangeness of the hadronic system changes by -1, i.e. $\Delta S = -1$, and this change is therefore forbidden by the $\Delta S = \Delta Q$ rule.

The theoretical implications of these reactions were discussed in Chapter 1. The main objective of the antineutrino experiment\(^{101}\) performed in 1965 using the enlarged CERN heavy-liquid bubble chamber was to determine the elastic hyperon production cross-sections and to compare them with the theoretical predictions.

2. THEORETICAL PREDICTIONS

The cross-sections for reactions (5.1) and (5.2) have been calculated on the basis of the SU_3 model of weak interactions, and are shown in Figs. 5.1 and 5.2. The total cross-sections shown here are not the same as those given in Ref. 42). According to Cabibbo and Veltman, the interference term in Eq. (28) of Ref. 42) was given the wrong sign. The cross-sections shown here are the results of recalculation with the correct sign. As we saw earlier in Chapter 1, because of the $\Delta I = \frac{1}{2}$ selection rule for $\Delta S = 1$ weak currents, reaction (5.3) is related to reaction (5.2) by

$$d\sigma(\Sigma^0) = \frac{1}{2} d\sigma(\Sigma^-).$$ \hspace{1cm} (5.4)

All the form factors used in the calculations of the cross-sections are assumed to have the form

$$F(q^2) = \left(1 + \frac{q^2}{M^2}\right)^{-n}$$ \hspace{1cm} (5.5)

where $n = 1$ or 2. In analogy with the isovector nucleon electromagnetic form factor being dominated by the p-meson exchange, the above authors assume that the vector form factors of the hyperon-nucleon current are dominated by the exchange of $K^*$, and they therefore set

$$M = M_{K^*} = 891 \text{ MeV}$$
Fig. 5.1  Total cross-section for $\bar{\nu} + p \rightarrow A + \mu^+$ as function of antineutrino energy $E_{\bar{\nu}}$.
Fig. 5.2 Total cross-sections for $\bar{\nu} + n \rightarrow e^+ \mu^+$ as function of antineutrino energy $E_\bar{\nu}$.
with \( n = 1 \) in the form factor (5.5). However, this prediction is not in agreement with the experimental results (see next section). We have therefore calculated the cross-sections as shown in Figs. 5.1 and 5.2 for various assumed values of \( M \) and for \( n = 1 \) and 2.

From the theoretical cross-sections and the known antineutrino spectrum (Fig. 2.3), the total cross-sections \( \bar{\sigma} \) averaged over the spectrum for antineutrino energy \( E_\nu \geq 0.5 \) GeV can be obtained by:

\[
\bar{\sigma} = \frac{\sum \sigma(E_\nu) \Delta \varphi(E_\nu)}{\sum \varphi(E_\nu)}, \quad (5.6)
\]

where \( \Delta \varphi(E_\nu) \) is the antineutrino flux between energy \( E_\nu \) and \( E_\nu + \Delta E_\nu \). For \( E_\nu \leq 0.5 \) GeV, the cross-sections are negligibly small and the uncertainty in the spectrum is relatively large.

The expected numbers \( N_{th} \) of events of energy greater than 0.5 GeV for various values of \( M \) and \( n \) have also been calculated from:

\[
N_{th} = N_p N_T \sum \frac{\sigma(E_\nu) \Delta \varphi(E_\nu)}{E_\nu}, \quad (5.7)
\]

where \( N_p \) is the number of primary protons hitting the copper target during the antineutrino runs and \( N_T \) is the number of target nucleons contained in the fiducial volume of the bubble chamber, i.e., number of protons for reactions (5.1) and (5.3), and number of neutrons for reaction (5.2).

The averaged total cross-sections and the expected numbers of events for different form factors and various values of \( M \) are shown in Table 5.1.

### Table 5.1

Averaged total cross-sections and expected numbers of events in hyperon production by antineutrinos

(The cross-sections are expressed in units of \( 10^{-40} \) cm\(^2\)/nucleon)

<table>
<thead>
<tr>
<th>Form factor</th>
<th>( M ) (GeV)</th>
<th>( \bar{\sigma}_A )</th>
<th>( N_{th,A} )</th>
<th>( \bar{\sigma}_B )</th>
<th>( N_{th,B} )</th>
<th>( \bar{\sigma}_C )</th>
<th>( N_{th,C} )</th>
<th>( \bar{\sigma}_D )</th>
<th>( N_{th,D} )</th>
<th>( \bar{\sigma}_{\text{total}} )</th>
<th>( N_{\text{total}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_A = P_V = \left(1 + \frac{q^2}{M^2}\right)^{-1} )</td>
<td>1.0</td>
<td>1.77</td>
<td>4.06</td>
<td>1.18</td>
<td>2.71</td>
<td>0.50</td>
<td>1.14</td>
<td>3.45</td>
<td>7.91</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.89</td>
<td>1.55</td>
<td>3.58</td>
<td>1.02</td>
<td>2.34</td>
<td>0.43</td>
<td>0.98</td>
<td>2.64</td>
<td>6.05</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.8</td>
<td>1.37</td>
<td>3.15</td>
<td>0.89</td>
<td>2.04</td>
<td>0.38</td>
<td>0.86</td>
<td>2.64</td>
<td>6.05</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>0.7</td>
<td>1.16</td>
<td>2.67</td>
<td>0.74</td>
<td>1.69</td>
<td>0.33</td>
<td>0.72</td>
<td>2.21</td>
<td>5.08</td>
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<td>0.6</td>
<td>0.96</td>
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<td>0.59</td>
<td>1.36</td>
<td>0.25</td>
<td>0.56</td>
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<td>4.14</td>
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<tr>
<td>0.5</td>
<td>0.74</td>
<td>1.70</td>
<td>0.45</td>
<td>1.03</td>
<td>0.19</td>
<td>0.44</td>
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<td>3.17</td>
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<tr>
<td>( P_A = P_V = \left(1 + \frac{q^2}{M^2}\right)^{-2} )</td>
<td>1.6</td>
<td>1.89</td>
<td>4.34</td>
<td>1.24</td>
<td>2.86</td>
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<td>1.20</td>
<td>3.65</td>
<td>8.40</td>
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<td></td>
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<td>1.4</td>
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<td>1.71</td>
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<tr>
<td>0.89</td>
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<td>1.95</td>
<td>0.53</td>
<td>1.21</td>
<td>0.22</td>
<td>0.51</td>
<td>1.66</td>
<td>3.88</td>
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</tbody>
</table>
3. EXPERIMENTAL RESULTS

3.1 Absorption of hyperons in freon CF\textsubscript{2}Br

Since hyperons are produced inside the complex nuclei of freon in the bubble chamber, serious distortion of the basic weak interactions by the strong interactions of the nucleons will occur. As a result, hyperons will often be captured in the parent nuclei to form hyperfragments and cryptography. It is therefore important to estimate the absorption of hyperons in freon in order to calculate the total cross-section.

Information on the absorption of hyperons in freon at various energies is still very scanty. Nevertheless, the average trapping probability of A hyperons produced in the absorption of K\textsuperscript{−} mesons at rest in freon has been estimated\textsuperscript{102} to be (34 ± 8)\%. However, in this case, the hyperons have kinetic energies of the order of 30 MeV, whereas the hyperons produced by antineutrinos have typically an energy of 150 MeV.

The absorption of A hyperons produced in heavy nuclei has also been studied by Jastrow\textsuperscript{103} by a Monte-Carlo method. In this calculation, the A hyperon was assumed to be produced at a point in the nuclear interior and moving in a conventional nuclear potential well of some 25 MeV. It was then followed by a Monte-Carlo method through a number of collisions until it emerged from the nucleus or until its kinetic energy fell below 25 MeV when it was regarded as captured. The results give estimates of the fraction of A hyperons captured in nuclei of various sizes. The capture probability, interpolated for freon, as a function of the initial kinetic energy of the hyperon is shown in Fig. 5.3. It can be seen that from this calculation the capture probability is rather high for hyperons of relatively low energy. For example, at a kinetic energy of 100 MeV, the capture probability in freon is about 45%.

A similar calculation of the absorption of hyperons in freon has recently been carried out by Franzinetti\textsuperscript{104}, who used a more refined nuclear model and the recent values of nuclear cross-sections. The result of this calculation shows a similar shape for the capture probability curve as a function of the hyperon kinetic energy as that shown in Fig. 5.3, but the fraction of capture is higher by about 40%. In fact, it is almost identical to a curve drawn through the upper limits of Jastrow's calculation.

The kinetic energy T of the produced hyperons can be calculated from the relation:

\[ q^2 = -(M_h - M)^2 + 2MT, \]  

(5.8)

where \(M_h\) and \(M\) are the hyperon mass and the nucleon mass, respectively. The capture probability curve, combined with the expected \(q^2\) distribution\textsuperscript{42} for hyperons produced in reactions (5.1) to (5.3), will enable us to calculate the total absorption. By using the capture probability curve as shown in Fig. 5.3, it has been estimated that the total absorption amounts to 32%. However, if the upper limits of the capture probability (i.e. almost identical to Franzinetti's results) are used, the total absorption is estimated to be 45%.

It should be pointed out here that owing to the lack of experimental information on the absorption of hyperons in freon, it is very difficult to give an accurate estimate on the absorption. However, it can be safely assumed that the total absorption is unlikely to be more than 50%.
3.2 Experimental total cross-section

During all the antineutrino runs in this experiment, no possible hyperon candidates have been observed. The detection efficiency for the hyperons expected to be produced in the elastic reactions in the heavy-liquid bubble chamber is estimated to be greater than 90%. From Eqs. (5.6) and (5.7), i.e.,

$$N_{\text{th}} = \frac{N_N N_e}{P} \sum_{E_{\nu}} \delta \sigma(E_{\nu}),$$

it has been estimated that on averaging over the antineutrino spectrum for energies greater than 0.5 GeV, one hyperon event corresponds to a total cross-section of $0.435 \times 10^{-40}$ cm$^2$/nucleon for all types of hyperon production. Therefore, if the absorption of hyperons in freon is not taken into account, the experimental total cross-section for all types of hyperon production is less than $1.09 \times 10^{-40}$ cm$^2$/nucleon with 90% confidence.

If the absorption effect is taken into account, the experimental total cross-section can be estimated from the above figure and the assumed probability of absorption. By assuming a total absorption of hyperons in freon to be 32%, the experimental upper limit for the total cross-section is set at $1.60 \times 10^{-40}$ cm$^2$/nucleon. For a 45% absorption, the corresponding upper limit is $1.98 \times 10^{-40}$ cm$^2$/nucleon.

3.3 Conclusions

The theory of Cabibbo and Chilton,\footnote{Cabibbo, G. and Chilton, A.} based on the SU$_3$ model of weak interactions, predicts a total cross-section of $3.0 \times 10^{-40}$ cm$^2$/nucleon, when averaged over the antineutrino spectrum, for hyperon production reactions (5.1) to (5.3). In this prediction, the vector form factors of the hyperon-nucleon current are assumed to be dominated by the $K^*$ exchange in the following form:

$$F_A(q^2) = F_V(q^2) = \left(1 + \frac{q^2}{M_{K^*}^2}\right)^{-1},$$

with $M_{K^*} = 891$ MeV. This prediction disagrees with our experimental results which give an upper limit of the total cross-section of $1.98 \times 10^{-40}$ cm$^2$/nucleon with 90% confidence.

If, however, the following parametric forms for the form factors are used:

$$F_A(q^2) = F_V(q^2) = \left(1 + \frac{q^2}{M_n^2}\right)^{-n}, \quad \text{with } n = 1 \text{ or } 2,$$

an upper limit of $M$ can be set for both cases of $n = 1$ or $n = 2$. By comparing the experimental total cross-section with those shown in Table 5.1 for various values of $M$, it can be concluded that with 90% confidence, for $n = 1$, the upper limit of $M$ is 0.65 GeV. Similarly, for $n = 2$, the corresponding upper limit is 1.0 GeV.

* * *
CHAPTER 6

SINGLE-PION PRODUCTION

In the previous two chapters, we have discussed the elastic neutrino and antineutrino reactions. In this chapter, we shall study a type of inelastic neutrino and antineutrino reactions, i.e., the single-pion production processes in which the final products consist of a lepton, a pion, and a nucleon:

\[ \nu_\mu + N \rightarrow N' + \pi + \mu^- \]  \hspace{1cm} (6.1)

and

\[ \bar{\nu}_\mu + N \rightarrow N' + \pi + \mu^+ \]  \hspace{1cm} (6.2)

where \( N \) and \( N' \) are the initial and final nucleons, respectively. We shall first explain how single-pion events can be selected from the neutrino and antineutrino candidates. Then we shall discuss the experimental results and make comparisons with theoretical predictions. In particular, we shall discuss the charge ratio of the pions, the cross-sections for the single-pion production processes, and the four-momentum transfer \( q^2 \) distributions from which an estimate of the form factors can be made.

1. SELECTION OF EVENTS

As has been described earlier in Chapter 1, the production of the nucleon isobar \( N^*(1/2^+,3/2^+) \) is predicted to be the dominant process \(^1\) of single-pion production in neutrino interactions. However, nuclear effects (such as the absorption of the pion in the nucleus, Fermi motion of the target nucleons, scattering of the secondary particles) may modify the appearance of the events to such an extent that the original process is completely concealed. In particular, many single-pion events will appear 'non-pionic', due to the absorption of the pion. An elastic collision will appear 'pionic' if the nucleon produces a pion in secondary collisions in its passage through the nucleus. Therefore, the true single-pion events are contained in both the 'non-pionic' as well as the 'pionic' events. In this section, we shall attempt to extract the single-pion events from these two categories.

1.1 The criteria of selection

As has been seen in Chapter 4, from the results of the analysis of the low-\( q^2 \) events and the Monte Carlo calculation on nuclear cascades induced by neutrinos, the majority of the non-pionic one-proton events were shown to be elastic events. Indeed, these events with the value of \( N^* \) within the 0.45-1.28 GeV\(^2 \) range were classified as elastic events. Events with \( N^* \) above 1.28 GeV\(^2 \) are unlikely to be elastic. In view of this, non-pionic one-proton events with 1.28 < \( N^* \) < 2.2 (GeV)\(^2 \) have been classified as single-pion events in which the pion is assumed to be absorbed in the production nucleus. The upper limit of \( N^* = 2.2 \) (GeV)\(^2 \) has been chosen to include practically all \( N \), events but
exclude higher resonances and double-pion events. Of the total of 159 non-pionic one-proton events observed in all the neutrino runs, nine have thus been attributed to single-pion processes in this way.

It has also been mentioned earlier that the non-pionic multiproton events are predominantly inelastic, in which the pion has been absorbed. To study this effect in more detail, we recall that much of the effect of Fermi motion of the nucleons disappears as \( q^2 \) approaches zero. There are 42 events with \( q^2 < 0.2 \text{ (GeV/c)}^2 \) in the category of non-pionic multiproton events. The invariant mass \( M^* \) distribution of these events was shown earlier in Fig. 4.1b. It can be seen that they are distributed around a mass of about 1.08 GeV, which is about 150 MeV above the proton mass. If these events are, in fact, single-pion events, the shift of \( M^* \) towards lower values than that of the \( N_3^* \) mass of 1.236 GeV can be explained qualitatively by the loss of visible energy due to undetected neutrons. From the relation

\[
M_{\text{inv}}^* = -q^2 + M^2 + 2M(E_\nu - E_\mu),
\]

(2.17)

it is clear that an underestimate of \( E_\nu \) will cause a decrease in the computed \( M^* \). On the other hand, if these events are considered elastic, no possible mechanism can be conceived to explain the systematic increase of \( M^* \) by 150 MeV. Thus, it seems reasonable to assume that most of the non-pionic multiproton events are indeed single-pion events in which the pion has been absorbed.

For the low-\( q^2 \) events in this category, another investigation of the direction of the expected recoil for the elastic and \( N_3^* \) production hypotheses has also been made. As before, all the momenta were projected on to the neutrino-muon plane (see Fig. 4.2), and the recoil directions for the two hypotheses were calculated. The distributions of \( p_\perp \), defined as the momentum component of all the observed hadrons transverse to the calculated direction, for a sample of these events, for the two hypotheses are shown in Figs. 4.3b and Fig. 6.1, respectively. It can be seen that the non-pionic multiproton events are distributed symmetrically about the recoil direction on the \( N_3^* \) production hypothesis, but show marked asymmetry when treated with the elastic production hypothesis. It can be concluded from this analysis that in the low-\( q^2 \) sample, most of the multiproton events are compatible with the \( N_3^* \) production assumption. To exclude possible two-pion events, we only consider events with \( M_{\text{inv}}^* \) below 2.2 (GeV)^2. One hundred and eight events have thus been classified in all the neutrino runs, and four in the antineutrino runs. The invariant mass \( M_{\text{inv}}^* \) distribution of the events in the non-pionic sample attributed to single-pion events is shown in Fig. 6.2a.

As was shown earlier in Chapter 3, most of the one-pion events with longitudinal momentum \( p_1 \) below 0.3 GeV/c are probably background events due to incoming pion interactions. Therefore, we only considered one-pion events with \( p_1 \) greater than 0.3 GeV/c. As in the case of the non-pionic multiproton sample, events with one identified pion were classified as single-pion events if their values of \( M_{\text{inv}}^* \) were below 2.2 (GeV)^2.

There are 111 such events in the neutrino runs, and 16 in the antineutrino case. The only possible contaminations in this sample are perhaps the elastic events in which a pion is produced by the secondary collision of the energetic nucleon, and the two-pion
Fig. 6.1 Transverse momentum distribution of multiproton non-pionic events for the $N^*_3$ production hypothesis.

Fig. 6.2 Invariant mass distribution of "single-pion events"
events in which a pion is absorbed during its passage through the nucleus and in which the loss of visible energy has caused an underestimate of the $M^2$ value to less than $2.2 \ (GeV)^2$. But these effects are expected to be very small. That this sample contains mostly true single-pion events can also be seen from their $M^2$ distribution (Fig. 6.2b) which shows a symmetrical spread around a value slightly less than $1.52 \ (GeV)^2$ expected for $N_3^*$ production. This slight shift towards a lower value can be attributed to the loss of visible energy.

To summarize, the following criteria have been used to select single-pion events compatible with $N_3^*$ production:

a) non-pionic one-proton events with $1.28 < M^2 < 2.2 \ (GeV)^2$;

b) non-pionic multiproton events with $M^2 < 2.2 \ (GeV)^2$;

and c) single-pion events with $M^2 < 2.2 \ (GeV)^2$.

By applying the above selection criteria, 228 events have been attributed to the single-pion production processes in all the neutrino runs. The visible energy distribution of these events is shown in Fig. 6.3.

Similar criteria have been applied to the antineutrino candidates for the selection of single-pion events. Four non-pionic and 16 pionic events were thus classified as due to the single-pion production processes. However, in the antineutrino runs, 65% of the pictures taken were scanned twice and 35% only once. It has been estimated that the combined efficiency of the two scans of the pionic events is 95.5%, and that of the first scan 75%. The scanning efficiency being taken into account, a total of 22.7 events is expected. Among the 16 pionic events, four are ambiguous between types $\mu^+\pi^-$ and $\mu^-\pi^+$ in which the pion does not interact in the bubble chamber. Some of these events may be actually due to the interactions of the background neutrinos in the antineutrino beam. From the background $\nu$ spectrum in the $\bar{\nu}$ runs (see Fig. 2.3) and the theoretical cross-section calculated by assuming the form factors

$$P_A = P_V = \left(1 + \frac{q^2}{(0.9)^2}\right)^{-2},$$

it has been estimated that 7.6 background neutrino single-pion events are expected. During the scanning of the antineutrino films, eight clearly identified neutrino single-pion events were observed. One of them is an electron-neutrino event. It therefore seems reasonable to assume that most of the background neutrino single-pion events were already observed. In view of this, we have also classified the four ambiguous events into the antineutrino single-pion sample. The invariant mass distribution and the visible energy distribution of the selected antineutrino single-pion events are shown in Figs. 6.4 and 6.5, respectively.

A summary of all the single-pion events is given in Table 6.1.
Fig. 6.3  Energy distribution of neutrino single-pion events
Fig. 6.4  Invariant mass distribution of antineutrino single-pion events

Fig. 6.5  Visible energy distribution of antineutrino single-pion events
Table 6.1

Summary of single-pion events

<table>
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<tr>
<th></th>
<th>Neutrino 128 &lt; M^2 &lt; 2.2</th>
<th>Neutrino M^2 &lt; 2.2 Total</th>
<th>Antineutrino M^2 &lt; 2.2</th>
<th>Antineutrino Total</th>
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<tr>
<td>Total</td>
<td></td>
<td></td>
<td>228</td>
<td>20</td>
</tr>
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</table>

1.2 The efficiency of selection

We shall now estimate the possible percentage of true single-pion events which might have been rejected by our selection criteria. First, we want to find the proportion of pions which are produced and reabsorbed in the nucleus. The average absorption probability of a pion of energy $E_p$ created at random inside the CF$_2$Br nucleus has been computed by Venus$^{[60]}$). It was assumed that scattering and charge exchange inside the nucleus involves only single nucleons, and that the cross-sections correspond to those in free space. The result of this calculation is shown in Fig. 6.6. The absorption probability, coupled with the expected spectrum of the pion and the neutrino spectrum, enabled us to obtain the following ratio$^{[60]}$:

$$r = \frac{\text{No. of absorbed pions}}{\text{Total No. of pions}} = 0.45 \pm 0.07$$  \hspace{1cm} (6.3)

where the error specifies the accuracy of the calculation. Assuming that all the non-pionic events in our single-pion sample are due to the absorption of the pion in the freon nucleus, we have estimated the experimental ratio to be

$$r = 0.51 \pm 0.08,$$

which is in good agreement with the calculated ratio. However, from the existing data on pion absorption$^{[60]}$, it has been estimated that in CF$_2$Br not more than 20% of the absorbed pions will produce single protons, whereas the other 80% will produce two or more nucleons. Therefore, the non-pionic one-proton sample should constitute less than 9% of the true single-pion events. In view of the fact that experimentally there are only 4% of non-pionic one-proton events in our sample, we can conclude that not more than 5% should have been excluded by our criteria.
Fig. 6.6 The absorption probability of pions in the production nucleus as a function of pion total energy
Some of the single-pion events will appear with two or more pions due to the pion production by pions in the nucleus. From the theoretical pion spectrum and the experimental data on pion interaction, this effect has been estimated to be less than $6\%$.

The effect of secondary production of pions by nucleons is negligible, since only nucleons of momentum above $1.5$ GeV/c can give any effect, and in our case they are very rare.

To sum up, we can safely state that at least $99\%$ of the single-pion events are contained in our selected sample.

2. DISCUSSION OF THE EXPERIMENTAL RESULTS

2.1 The charge distribution

The three possible neutrino reactions in which a single pion is produced are as follows:

\begin{align*}
\text{i) } \nu_\mu + p &\to \mu^- + p + \pi^+ (I_3 = \frac{3}{4}) \\
\text{ii) } \nu_\mu + n &\to \mu^- + n + \pi^+ (I_3 = \frac{1}{2}) \\
\text{iii) } \nu_\mu + n &\to \mu^- + p + \pi^0 (I_3 = \frac{3}{2})
\end{align*}

where the third component of the isospin $I_3$ for the final state is indicated in the brackets.

The amplitudes for the above reactions can be expressed by isospin expansion and they are, respectively,

\begin{align*}
T^1 &= \frac{1}{2} \\
T^2 &= \frac{1}{8} (\frac{1}{8} + \frac{1}{8}) \\
T^3 &= \frac{\sqrt{2}}{3} (\frac{1}{8} - \frac{1}{8})
\end{align*}

Pure $N^*(\frac{3}{2}, \frac{3}{2})$ production implies a ratio of final states $\pi^+ : n\pi^+ : p\pi^0 = 9 : 1 : 2$, or an over-all ratio $\pi^+/\pi^0 = 5/1$. For pure $I = \frac{1}{2}$ interaction, the corresponding ratio should be $2/1$. If we take into account the fact that in freon CF$_3$Br there are more neutrons than protons (81 neutrons and 68 protons), we should expect a ratio of

\begin{equation}
\frac{\text{No. of } \pi^+}{\text{No. of } \pi^0} = 4.3.
\end{equation}

Considering only the single-pion events in which the pion was identified (see Table 6.1), we obtain the ratio

\begin{equation}
\frac{\text{No. of } \pi^+}{\text{No. of } \pi^0} = 1.9 \pm 0.6.
\end{equation}

However, it is expected that the charge distribution is severely distorted by interactions of the final products in the nucleus and by charge exchange scatterings. The presence of $\pi^-$ indicates that charge exchange process has taken place. The exact distortion of the charge distribution due to secondary interactions of the pions in the nucleus is not
yet well known, but it is clear that the ratio of the identified $\pi^+$ (i.e. $p_\pi < 0.7$ GeV/c) to $\pi^0$ would be very much reduced. Energetic $\pi^+$'s could be classified as protons, and the events are included in the 'non-pionic' category. This effect being taken into account, the ratio is estimated to be about $2.2^{+0.8}_{-0.7}$, which is in good agreement with the observed ratio. Thus, the experimental result is not incompatible with a dominant contribution of $N^*$ production in the single-pion process. It may be pointed out here that the above result is obtained from events with $M < 2.2$ (GeV)$^2$. For events with higher invariant mass, second and third resonances may take place, which are in $I = \frac{3}{2}$ states. As a result, the ratio $N(\pi^+)/N(\pi^0)$ will be smaller.

Similarly, the three possible single-pion production processes initiated by antineutrinos are

1) $\bar{\nu}_\mu n \rightarrow \mu^+ + n + \pi^-$ (I$_3 = -\frac{3}{2}$)  
2) $\bar{\nu}_\mu p \rightarrow \mu^+ + p + \pi^-$ (I$_3 = \frac{1}{2}$)  
3) $\bar{\nu}_\mu p \rightarrow \mu^+ + n + \pi^0$ (I$_3 = -\frac{1}{2}$).  

For the pure production of $N^*$, isospin considerations give the ratio of final states $n\pi^- : p\pi^+ : n\pi^0 = 9 : 1 : 2$, or an over-all ratio of $\pi^/\pi^0 = \frac{3}{7}$. When the neutron excess in freon nuclei is taken into account, the expected ratio is

$$\frac{\text{No. of } \pi^-}{\text{No. of } \pi^0} = 5.8.$$  \hspace{1cm} (6.15)

The experimental ratio is

$$\frac{\text{No. of } \pi^-}{\text{No. of } \pi^0} = 1.1 \pm 0.9.$$ \hspace{1cm} (6.16)

Owing to the poor statistics in the antineutrino events, it is more difficult to make a sensible comparison. However, it is expected that the charge distribution will be even more distorted by the interactions of the $\pi^-$ with the nucleons in the freon nuclei, and by the charge exchange scatterings than in the neutrino case. It is therefore not surprising to find that the experimental ratio of $\pi^-/\pi^0$ is reduced.

2.2 Cross-sections for single-pion production

From the energy distributions of the observed single-pion events (Figs. 6.3 and 6.5) and the neutrino spectra (Fig. 2.3), the experimental cross-sections for the single-pion production processes by neutrinos and antineutrinos can be calculated from the relation:

$$\sigma_{\pi}(E_\nu) = \frac{1}{N_N N_\nu E_\nu} \frac{dN}{dE_\nu},$$  \hspace{1cm} (6.17)

where $dN$ is the number of single-pion events in the energy interval between $E_\nu$ and $E_\nu + dE_\nu$.  

The theoretical cross-sections for single-pion production as a function of the neutrino energy can be obtained by assuming the vector form factor \( F_V(q^2) \) and the axial vector form factor \( F_A(q^2) \). As we have seen before, the \( N_3 \) production is the dominant process of single-pion production by neutrinos and antineutrinos. Following Berman and Veltman,\(^{3+}\), we have used the following form factors:

\[
F_A(q^2) = F_V(q^2) = \left(1 + \frac{q^2}{M_X^2}\right)^n,
\]

where \( n = 1 \) or \( 2 \), and where \( M_X \) is a parameter to be determined by comparison with experimental results. The theoretical cross-sections so obtained for neutrino and antineutrino processes are shown in Figs. 6.7 and 6.8, respectively, together with the experimental cross-sections. As can be seen from Figs. 6.7 and 6.8, the cross-sections are influenced greatly by the choice of form factors and of the value of \( M_X \). With the choice of \( n = 1 \), the cross-sections increase rapidly with the neutrino energy, indicating that large values of \( q^2 \) contribute appreciably to the cross-sections.

By comparison with the theoretical curves for \( n = 2 \), the experimental points are included between the two curves corresponding to \( M_X = 0.7 \) GeV and \( M_X = 0.9 \) GeV for the neutrino case, and show a reasonably good fit with the theoretical curve with \( M_X = 0.8 \) GeV. For the antineutrino case, the best fit is for \( M_X = 0.7 \) GeV.

Similarly, for \( n = 1 \), the experimental points seem to exclude \( M_X > 0.5 \) GeV and \( M_X < 0.55 \) GeV. A comparison with the curve corresponding to \( M_X = 0.42 \) GeV show a very good fit for both the neutrino and the antineutrino cases.

It should be pointed out here that the possible errors in the determination of the experimental cross-sections are only statistical. They do not take into account the possible uncertainty in the neutrino spectra. Another method of estimating the form factors, which is less dependent on the neutrino spectra, is the study of the \( q^2 \) distribution, as will be discussed in the following section.

2.3 The \( q^2 \) distributions

The form factors for single-pion production can be investigated from the study of their \( q^2 \) distributions. By assuming the form factors, the differential cross-section \( \frac{d\sigma}{dq^2} \) as a function of the four-momentum transfer \( q^2 \) can be calculated. The theoretical distribution of the single-pion events \( dN(q^2) \) as a function of \( q^2 \) can be obtained by integrating the differential cross-section \( \frac{d\sigma}{dq^2} \) over the neutrino spectrum:

\[
\frac{dN}{dq^2} = N_p N_\nu \int \frac{d\sigma(p)}{dq^2} \varphi(E_\nu) dE_\nu.
\]

As in the case of elastic reactions, a maximum likelihood method can be used to estimate the value of \( M_X \) for the best fit with the experimental results. The experimental and theoretical \( q^2 \) distributions for neutrino and antineutrino single-pion events and for \( n = 1 \) and \( 2 \) are shown in Fig. 6.9.
Fig. 6.7  Experimental and theoretical cross-sections for single-pion production by neutrinos
**Fig. 6.8** Experimental and theoretical cross-sections for single-pion production by antineutrinos.
Fig. 6.9 Comparison of experimental and theoretical $q^2$ distributions for single-pion events
In all the cases, the theoretical distributions are normalized to the number of the observed single-pion events in the interval \( 0 < q^2 < 2 \text{ (GeV/c)}^2 \). The results obtained for different energy cut-offs are very similar. The best fitted values of \( M_x \) are as follows:

\[
M_x = 0.75 \pm 0.15 \quad \text{for } n = 2 \quad (6.20)
\]
\[
M_x = 0.35 \pm 0.11 \quad \text{for } n = 1 \quad (6.21)
\]

for the choice of form factors given in Eq. (6.18). Figure 6.9a shows the experimental and theoretical \( q^2 \) distributions for single-pion neutrino events with energy greater than 0.3 GeV. Figure 6.9b shows the same distributions for events with energy greater than 1 GeV. The same comparison for antineutrino events with energy greater than 0.5 GeV is given in Fig. 6.9c.

2.4 Conclusions

All the above analysis of single-pion processes involves systematic errors due to the fact that reactions occur inside complex nuclei. In view of this and poor statistics, very precise conclusions cannot be drawn at the present time. In the next phase of the neutrino programme, which is scheduled to be run in early 1967, single-pion production will be investigated by using free protons contained in propane as targets. It is hoped that sufficient data will be accumulated for a more detailed study of the single-pion processes.

However, with the analysis of the present experimental data, it may be appropriate to draw the following conclusions:

i) Our experimental results on the single-pion production process are compatible with the \( \pi^+ \) production being dominant.

ii) The assumption for the equality of vector and axial vector form factors is in good agreement with the experimental results. By assuming

\[
P_V(q^2) = P_A(q^2) = \left( 1 + \frac{q^2}{M_x^2} \right)^{-n} \quad (6.18')
\]

with \( n = 1 \) or 2, the values of \( M_x \) which fit the experimental data best are

\[
M_x = 0.75 \pm 0.15 \quad \text{for } n = 2
\]

and

\[
M_x = 0.35 \pm 0.11 \quad \text{for } n = 1.
\]
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