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Continuous Measurement and Localization in the Unruh effect

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Abstract

We consider the influence of continuous decoherence on the excitation of an accelerated detector, an effect which must be taken into account in physically realistic realizations of the Unruh effect. As a model we use a quantum two-level system (as detector) coupled to a massless scalar quantum field. This composite quantum system is treated as an open system which interacts with an environment so that continuous decoherence results. We develop a formalism which models the interaction with an environment approximately as continuous measurement, represented by a sequence of state reductions. This leads to a description in terms of 'histories' of the detector. Evolution operators corresponding to different histories of the detector can then be considered separately. The density matrix for the mixed state which results after a sequence of non-selective measurements is worked out explicitly. Based on the same formalism, it is also possible to correlate the time of the transition of the detector with the trajectories of the Minkowski particles which are emitted during this process, thus localizing the space-time region where they are created.

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1 Introduction

In the well-known Unruh effect [1], a uniformly accelerated particle detector which is sensitive to detect real massless particles becomes excited when moving through the Minkowski vacuum. It behaves as if it were immersed in a thermal heat bath. Unruh [1] recognized that the related spontaneous excitation is accompanied by the emission of a particle. This process was discussed in more detail by Unruh and Wald [2] (see also [3]). Audretsch and Müller [4] treated the stimulated excitation and de-excitation of the accelerated detector and discussed a localization using wave

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packets. In [5], they gave a quantum-optical illustration of the effect based on the influences of radiation reaction and vacuum fluctuations.

Following the work of Unruh and Wald [2], there has been a discussion concerning the reaction of the quantum field to the process of detector excitation. Grove [6] and Audretsch and Müller [7] pointed out the importance of the state reduction (associated with the observation in which state the particle detector is) for the interpretation of the structure of the particle emission process. Using a model consisting of a quantum two-level system as detector coupled to a scalar quantum field, it was shown in [7] that conclusions about field observables like energy density depend crucially on whether the two-level system has been reduced by measurement to one of its eigenstates in the out region or not. If no state reduction is performed, the total system is in a superposition of different states, which contribute to the values of field observables. There is no such superposition if state reduction is taken into account, and the conclusions about the reaction of the field are essentially different.

All these treatments of the Unruh effect were based on an in-out approach. The initial state of the two-level system and the quantum field was assumed to be prepared at past infinity and the final state was evaluated at future infinity. In the meantime an undisturbed coherent evolution in time according to the respective Hamilton operator including an internal interaction was supposed. Accordingly, the two-level system and the field were both regarded as quantum systems interacting only with each other during all their history, thus forming together one *composite system* which is closed.

With regard to experimental tests and other possible realistic applications, this is of course an oversimplified description. The main objection is that the composite system under consideration cannot be considered to be isolated from its *environment*. In physically realistic situations, for example in the electron storage ring experiments proposed to measure the Unruh effect [8], a large number of degrees of freedom external to the system are always present and interacting with the system. They may belong to the setup in which an experiment is performed or to the device which is accelerated together with the two-level system and performs the measurements. Accordingly, the composite system (consisting of detector and field) must be described as an *open system* and there is necessarily a loss of coherence to its environment (disappearance of certain phase relations). This process is called *decoherence*. Decoherence takes place particularly during any measurement of a quantum system whereby the measuring apparatus represents a particular environment. In this case, decoherence is responsible for the state reduction (see below) which is often called collapse of the wave function. For a review of several aspects of the quantum theory of measurement see [9].

In many cases, decoherence arises not because of specially arranged measurements but because of the unavoidable interaction of the composite quantum system with its natural environment. Therefore, if no special arrangements are made, this decoherence is a process which is continuously taking place. In this paper, we want to give a description of the Unruh effect based on continuous decoherence, thus taking the permanent interaction with the environment into account.

For this purpose, let us briefly sketch the environment approach to quantum measurement [10]-[18], from which we can read off how an environment works in general. The quantized scalar field and the quantum two-level system (called the *detector*) form the composite system with state vector $|\psi\rangle$ which is an element of the corresponding product space. Because of the internal interaction with Hamiltonian H_I , the unitary time evolution will lead to an entangled state vector $|\psi\rangle$ of the composite system. This composite system is described with reference to a basis $|\psi_i\rangle$ and the environment by a particular corresponding basis $|\phi_i\rangle$. The interaction between composite system and environment is assumed to be such that the respective states become correlated: $|\psi_i\rangle|\phi_i\rangle$. We have therefore the following evolution of the composite system coupled to the environment if the initial state of the latter is $|\phi_0\rangle$:

$$|\psi\rangle|\phi_0\rangle = \left(\sum_i c_i |\psi_i\rangle \right) |\phi_0\rangle \xrightarrow{t} \sum_i c_i |\psi_i\rangle |\phi_i\rangle. \quad (1)$$

If the environment has many degrees of freedom, the environment states become approximately orthogonal. Neglecting the practically unobserved degrees of freedom of the environment amounts to tracing over these states, resulting in a reduced density matrix. This represents the physical

process of decoherence. Its effect is that the composite system (detector and field) is not in a superposition of states but in a mixed state described by the density matrix

$$\rho \approx \sum_i |c_i|^2 |\psi_i\rangle\langle\psi_i|. \quad (2)$$

Coherence is thereby not destroyed but dislocalized into the many degrees of freedom of the environment where it is physically out of reach. It is evident that this decoherence takes some time [16]. Its duration will be called *decoherence time* Δt_d . *Continuous decoherence* can be considered as a succession of the processes described above.

The following observation will be important: ρ of (2) agrees exactly with the density matrix of the mixed state which is obtained if one describes the measurement process phenomenologically with von Neumann's 'ad hoc' postulate where the resulting reduction of the state is formulated with the projection operators $|\psi_i\rangle\langle\psi_i|$. The density matrix (2) describes the output of a *non-selective measurement* where the composite system (field and detector) is afterwards in a definite but unknown state so that for the subsequent time evolution all possible outputs are to be taken into account according to their probabilities (which are now classical probabilities by nature).

The process of environment-induced decoherence has been worked out explicitly in some cases. The details of such a calculation depend on the physically realized situation and are therefore specific for the respective environment. See [14, 17, 19] for the corresponding physical models. The decoherence time can be calculated from these models. The calculations in this article will not be based on such a particular model for an environment.

We prefer instead an *approximate approach* based on the observation above in which the continuous decoherence is modeled in a *phenomenological way* by a succession of von Neumann projections that are separated by periods of unitary evolution of the coupled detector-field system. In this way, the permanent interaction of the coupled system (detector and field) with the environment and the related decoherence is represented by continuous measurements. The times $t_1, t_2, \dots, t_N$ when the projections are performed must be chosen to describe the real situation of the interaction between the composite system and the environment. For the continuous decoherence in question the intervals $\Delta t = t_n - t_{n-1}$ must therefore be equal to the decoherence time Δt_d :

$$\Delta t = \Delta t_d. \quad (3)$$

Unitary evolution over Δt_d together with projection represent one decoherence process. Accordingly, although the particular projection refers to a definite moment t_n , the influence of the environment modeled this way is not assumed to be an instantaneous one. Rather, the duration of the influence equals the decoherence time. This phenomenological approach has the advantage of being rather general. It contains parameters like Δt_d which remain to be specified. To a certain extent this can be done without modeling the environment in detail. The approach should be a good approximation to an actual decoherence process because the von Neumann projection is successfully applied in ordinary quantum theory to represent a measurement.

To be definite with regard to Δt_d , in our approach the interaction of the composite system with the environment can be interpreted as a measurement which allows to answer on which of the two levels the quantum mechanical two-level system is. Accordingly, if the energy difference between the two levels is E , then a measurement of energy is supposed to answer the question which of the two levels the system is on with the error $\Delta E < E$. Taking into account the uncertainty relation ($\hbar = 1$)

$$\Delta E \Delta t \gtrsim 1, \quad (4)$$

one sees that such a measurement cannot be performed during a time interval Δt less than $1/E$. This gives a lower limit for the decoherence time Δt_d . In the case of an accelerated detector, one should refer to its proper time. Based on these considerations, we will discuss in the first part of the paper the question how the traditional calculation of the Unruh effect (treating the coherent in-out process of an isolated composite system) is modified if continuous interaction with the environment is taken into account.

Let us turn to the second problem we want to address in this article. The in-out approach showed that excitation of the accelerated detector is accompanied by the emission of Minkowski particles.

Taking decoherence into account by means of the approach sketched above, we may localize the excitation of the detector in space-time with a precision given by Δt_d . Is then the creation of the emitted particle as well localized in space and time and how are the two space-time regions correlated?

In our phenomenological picture the continuous decoherence is based on a series of von Neumann type measurements of the detector state. Each measurement gives the result $-$ or $+$ corresponding to a decoherence of the detector to the ground or excited state, respectively. Accordingly, the history of decoherence is represented by a series of pluses and minuses. Two successive observations giving opposite results fix the change of the detector state (for example its excitation). The time instants of these measurements together with the trajectory of the detector allow to localize the event of excitation or deexcitation, respectively.

It turns out that the excitation of the accelerated detector is accompanied by the creation of Minkowski photons (excitation of the field). Trajectories of the created photons and therefore the region of their creation are correlated with the region of the detector excitation. The analysis of this correlation is the second main goal of the present paper.

The article is organized as follows: In Sec. 2 we discuss the general scheme of sequential measurements in the spirit of the restricted-path-integral approach to the quantum theory of continuous measurements following Mensky [20]-[22]. Each sequence of measurement outputs represents a particular decoherence history. The related evolution operators for an Unruh detector and massless scalar particles are worked out in Sec. 3 for different orders of the coupling constant. A discussion of the decoherence time for this type of composite system with internal interaction follows in Sec. 4. Based on this, the continuous decoherence resulting in a mixed state density matrix is given in Sec. 5 and compared with the pure state result of the coherent in-out process. Finally, the localization of the detector excitation and the creation region of the emitted scalar particles is treated in Sec. 6.

2 Amplitude operators depending on detector histories

In the previous section we have sketched a general scheme which takes into account in a phenomenological way the effects of decoherence on an accelerated detector. Let us now make this formalism more concrete. We consider a quantum two-level system called *detector* which is coupled to the quantized particle field via an interaction Hamiltonian H_I . Let us fix some value of the proper time of the detector τ and denote by $P_-(\tau)$ (correspondingly $P_+(\tau)$) a projector on the ground (excited) state of the two-level system at the time τ . It is evident that

$$P_-(\tau) + P_+(\tau) = 1. \quad (5)$$

Consider now the sequence of time instants $\tau_1, \tau_2, \dots, \tau_N$ to characterize the detector history. Denoting $i_k = +$ or $-$, one has the 'path'

$$p = (i_1, i_2, \dots, i_N)$$

which describes the *history* of the detector. It represents the information about the state of the detector in the selected time instants. Let us denote the product of the corresponding projectors as

$$P(p) \equiv P_{i_N}(\tau_N) \dots P_{i_2}(\tau_2) P_{i_1}(\tau_1).$$

It is clear, as a direct consequence of Eq. (5), that

$$\sum_p P(p) = 1. \quad (6)$$

It has been shown in [7], that the operator describing a coherent evolution of the composite closed system from the in to the out region can be written in the interaction picture in the form of a time-ordered exponent

$$A = \mathcal{P} \exp \left(-i \int H_I(\tau) d\tau \right) \quad (7)$$

where H_I is the interaction Hamiltonian and $\mathcal{P}$ is Dyson's time-ordered product. In [7], amplitudes of the form $\langle f|A|i\rangle$ between initial and final states were calculated in an in-out approach. This did not allow to determine at which time the detector is excited. Now we shall develop a description allowing this.

Introducing the unity presented in the form (6) into the formula (7), one may expand the amplitude operator A with regard to 'partial' amplitudes as in the path integral approach as "sum over paths":

$$A = \sum_p A(p), \quad (8)$$

where we have defined

$$A(p) \equiv \mathcal{P}P(p) \exp\left(-i \int H_I(\tau) d\tau\right). \quad (9)$$

The operator $\mathcal{P}$ here acts on time-dependent operators both in the exponent and in the factor $P(p)$.

An amplitude operator (7) expressed by the sum (8) describes the process with no decoherence taken into account. Detector and quantum field form a closed system. Separate terms (9) in this sum represent quantum (interfering) alternatives of the process. Their square moduli have no physical sense in this case.

If however we take decoherence into account (treating detector and quantum field as an open system interacting with the environment) in the phenomenological scheme described above, each of the partial amplitudes (9) acquires physical sense on its own. It describes the evolution of the system in the case when the history of the detector is represented by the particular path p . The square modulus of the amplitude gives the probability of the corresponding detector history.

The amplitude operator $A(p)$ describes (apart from normalization) the evolution of the initial state $|\psi_0\rangle$ according to

$$|\psi_0\rangle \rightarrow |\psi_p\rangle = A(p)|\psi_0\rangle \quad (10)$$

in the case where the detector history is p . The same evolution can be represented by density matrices:

$$\rho_0 \rightarrow \rho'_p = A(p)\rho_0 A^\dagger(p). \quad (11)$$

If we do not know the concrete history of the detector or are not interested in it, we have to sum up over all alternative histories. The evolution of the system is described in this case as follows:

$$\rho_0 \rightarrow \rho' = \sum_p A(p)\rho_0 A^\dagger(p). \quad (12)$$

This situation is referred to as a *non-selective measurement*, leading to mixed states ρ even for a pure initial state ρ_0 . The condition that the final density matrix should be normalized (provided an initial matrix is normalized) leads to a generalized unitarity condition [22]

$$\sum_p A^\dagger(p)A(p) = 1.$$

3 Calculation of amplitudes in a perturbative scheme

After having established in a general way the scheme for calculating transition amplitudes with continuous decoherence taken into account, we now specify the details of our model. The detector is represented by a two-level system. The energies of its levels $|-\rangle$ and $|+\rangle$ are supposed to be 0 and E , respectively. Its free Hamiltonian can be written

$$H_0 = E d^\dagger(\tau) d(\tau), \quad (13)$$

where $d(\tau) = |-\rangle\langle+|$. τ is the proper time on the trajectory of the two-level system. The free evolution of d is given by

$$d(\tau) = d(0) \exp(-iE\tau). \quad (14)$$

The projection operators

$$P_-(\tau) = |-\rangle\langle-| \quad \text{and} \quad P_+(\tau) = |+\rangle\langle+| \quad (15)$$

are ascribed to definite instants of time (to deal with in the chronological product), though their form does not depend explicitly on time.

The two-level system is coupled to a real massless scalar field (which represents particles) by the interaction

$$H_I = \lambda m(\tau) \phi(x(\tau)). \quad (16)$$

Here λ is the coupling constant to which we will refer in the following perturbative scheme. The field operator $\phi(x(\tau))$ is evaluated along the world line of the two-level system. The detector monopole moment is given by

$$m(\tau) = \mu(d(\tau) + d^\dagger(\tau)),$$

where μ is a real number.

In order to investigate the problem in a perturbation theory approach in the interaction picture, we shall rewrite the formula (7) as

$$A = \sum_{n=0}^{\infty} \frac{(-i)^n}{n!} \int d\tau'_1 \dots d\tau'_n \mathcal{P}(H_I(\tau'_1) \dots H_I(\tau'_n)). \quad (17)$$

Then Eq. (9) takes the corresponding form

$$A(p) = \sum_{n=0}^{\infty} A_n(p) = \sum_{n=0}^{\infty} \frac{(-i)^n}{n!} \int d\tau'_1 \dots d\tau'_n \mathcal{P}(P(p) H_I(\tau'_1) \dots H_I(\tau'_n)). \quad (18)$$

where the amplitude operator $A_n(p)$ is of n th order in the coupling constant λ and contains n factors $m(\tau)$ with different parameter values τ .

The evaluation of (18) is simplified by taking into account the explicit form (15) of P_- and P_+ . From the terms $P_- H_I P_+$ that appear in (18), only the following combinations do not vanish,

$$P_- d P_+ = d \quad \text{and} \quad P_+ d^\dagger P_- = d^\dagger. \quad (19)$$

This means that an operator $d^\dagger$ can only appear between two observations that constitute a transition from minus to plus (from the ground state of the detector to the excited state), and an operator d should correspond to the transition from plus to minus (from the excited state to the ground state). The product $d d^\dagger$ (or any power of such a product) may arise between two minuses (when the ground state is maintained) and the product $d^\dagger d$ may be located between two pluses. In these cases, the products correspond to intermediate excitation or de-excitation of the detector between two observations. The amplitudes for the corresponding processes contribute only to higher orders in perturbation theory, however. One can easily see from this that the amplitude $A_n(p)$ operator with the even (odd) number n is not zero only in case if the chain of pluses and minuses in the history p has even (odd) number of transitions from the series of pluses to the series of minuses and vice versa. Some simple examples illustrating this rule follow.

The simplest example is

$$p_0 = (- \dots -) \quad (20)$$

describing a history where the detector has been found in the ground state at each of the observation moments. Note that in higher order it could be excited temporarily between the moments of observation. Only amplitude operators $A_n(p_0)$ with even n are non-zero. The two operators of lowest order are $A_0(p_0)$ and $A_2(p_0)$ (we shall not actually use the second of these amplitude operators in the present paper). The first one is trivial:

$$A_0(p_0) = P(p_0) \mathbf{1}_\phi = |-\rangle\langle-| \mathbf{1}_\phi. \quad (21)$$

It corresponds to the case where nothing happens at all. Here $\mathbf{1}_\phi$ is an unity operator in the particle space. To obtain (21) we have used the fact that the projector product $P(p_0)$ is trivial in this case:

$$P(p_0) = P_- P_- \dots P_- = P_- = |-\rangle\langle-|. \quad (22)$$

The second order amplitude operator for the history p_0 is

$$\begin{aligned} A_2(p_0) &= -\frac{1}{2}|- \rangle \langle -| \sum_j \int_{\tau_{j-1}}^{\tau_j} d\tau'_1 \int_{\tau_{j-1}}^{\tau_j} d\tau'_2 \mathcal{P} H_I(\tau'_1) H_I(\tau'_2) \\ &= -\frac{\lambda^2 \mu^2}{2} |- \rangle \langle -| \sum_j \int_{\tau_{j-1}}^{\tau_j} d\tau'_1 \int_{\tau_{j-1}}^{\tau_j} d\tau'_2 e^{-iE|\tau'_1 - \tau'_2|} \mathcal{P} \Phi(x(\tau'_1)) \Phi(x(\tau'_2)), \end{aligned} \quad (23)$$

where an intermediate excitation between two observation instants τ_{j-1} and τ_j has taken place. In the calculation that led to (23), we have used the formulas (14) and (19).

Let us now consider a path of the following form

$$p_1 = (- \dots - + \dots +). \quad (24)$$

It describes a history of the detector which contains precisely one transition from the ground to the excited state. Let the last minus in the sequence (24) corresponds to the proper time τ equal to ζ_1 while the first plus corresponds to the time equal to ζ_2 . This means that at ζ_1 the detector was still in the ground state (as well as in all preceding moments of observation) while at ζ_2 it was already in the excited state (as well as in all succeeding observation moments). Note that the detector could be temporarily excited or temporarily transit into the ground state in these intervals. These processes contribute only in third and higher order of perturbation theory, however. The lowest non-zero amplitude operator $A_n(p_1)$ with the history p_1 of the form (24) is

$$\begin{aligned} A_1(p_1) &= -i\lambda \int_{\zeta_1}^{\zeta_2} d\tau' P_+ m(\tau') P_- \phi(x(\tau')) \\ &= -i\lambda \mu |+ \rangle \langle -| \int_{\zeta_1}^{\zeta_2} d\tau' \phi(x(\tau')) e^{iE\tau'}. \end{aligned} \quad (25)$$

One more example refers to the history

$$p_2 = (- \dots - + \dots + - \dots -) \quad (26)$$

with two transitions, the first from the ground state of the detector to the excited state (with the characteristic times ζ_1 and ζ_2) and the second backward (with the times ζ_3 and ζ_4). The corresponding amplitude operator is equal to

$$\begin{aligned} A_2(p_2) &= -\lambda^2 \int_{\zeta_3}^{\zeta_4} d\tau'_2 P_- m(\tau'_2) P_+ \phi(x(\tau'_2)) \int_{\zeta_1}^{\zeta_2} d\tau'_1 P_+ m(\tau'_1) P_- \phi(x(\tau'_1)) \\ &= -\lambda^2 \mu^2 |- \rangle \langle -| \int_{\zeta_3}^{\zeta_4} d\tau'_2 \int_{\zeta_1}^{\zeta_2} d\tau'_1 e^{iE(\tau'_1 - \tau'_2)} \phi(x(\tau'_2)) \phi(x(\tau'_1)). \end{aligned} \quad (27)$$

This amplitude corresponds to the situation when the detector is in the ground state at the end of the observation as well as in the beginning of it, however in the interval between an excitation is observed. In the following, we are interested in amplitudes up to first order perturbation theory. For a detector which is initially in the ground state, only the amplitude operators $A_0(p_0)$ and $A_1(p_1)$ need to be considered in this order.

4 Inertial detector and decoherence time

To be prepared for the discussion of a problem which will appear for the accelerated detector, we calculate in a first step for an inertial detector the first order transition amplitude $A_1(p_1)$ for a path of the form $p_1 = (- \dots - + \dots +)$. This means that the detector is initially in the ground state and is excited in the interval between ζ_1 and ζ_2 .

To evaluate the amplitude (25) for a detector on a trajectory $x(\tau) = 0$, $\tau = t$, we use the decomposition of the field operator

$$\phi = \int dk (a_k f_k + a_k^\dagger f_k^*) \quad (28)$$

with regard to the plane wave mode functions

$$f_k = \frac{1}{\sqrt{4\pi\omega}} e^{ikx - i\omega t}. \quad (29)$$

The annihilation operators a_k define the Minkowski vacuum via $a_k|0_M\rangle = 0$.

We obtain the following first-order amplitude operator:

$$A_1(p_1) = -i\lambda \mu |+ \rangle \langle -| \int \frac{dk}{\sqrt{4\pi\omega}} \left[a_k \int_{\zeta_1}^{\zeta_2} dt e^{i(E-\omega)t} + a_k^\dagger \int_{\zeta_1}^{\zeta_2} dt e^{i(E+\omega)t} \right]. \quad (30)$$

The evaluation of the integrals is trivial and we get

$$A_1(p_1) = -i\frac{\lambda\mu}{2\sqrt{\pi}} |+ \rangle \langle -| \int \frac{dk}{\sqrt{\omega}} \left[a_k S_k^-(\zeta_2, \zeta_1) \Delta t + a_k^\dagger S_k^+(\zeta_2, \zeta_1) \Delta t \right], \quad (31)$$

where we have introduced $\Delta t \equiv \zeta_2 - \zeta_1$ and

$$S_k^\pm(\zeta_2, \zeta_1) \equiv e^{\frac{i}{2}(E \pm \omega)(\zeta_1 + \zeta_2)} \frac{\sin\left(\frac{1}{2}(E \pm \omega)(\zeta_2 - \zeta_1)\right)}{\frac{1}{2}(E \pm \omega)(\zeta_2 - \zeta_1)}. \quad (32)$$

Note that because of the stationarity of the physical situation, the moduli of these amplitudes depend only on the interval Δt but not on ζ_1 and ζ_2 themselves:

$$R_k^\pm(\Delta t) := |S_k^\pm(\zeta_2, \zeta_1)|^2 = \left(\frac{\sin\left(\frac{1}{2}(E \pm \omega)\Delta t\right)}{\frac{1}{2}(E \pm \omega)\Delta t} \right)^2. \quad (33)$$

We see from (31) that two processes may be possible for an inertial detector which was initially in the ground state: It can be excited accompanied by either the absorption or the emission of a Minkowski particle. Resonant excitation ($E = \omega$) is possible only for the absorption process.

Let us concentrate on the situation where the quantum field is initially in the Minkowski vacuum state $|0_M\rangle$. In this case only excitation with emission is possible and the final state is, in first-order perturbation theory,

$$|\Psi\rangle = -i\frac{\lambda\mu}{2\sqrt{\pi}} \Delta t \int \frac{dk}{\sqrt{\omega}} S_k^+(\zeta_2, \zeta_1) a_k^\dagger |0_M, +\rangle. \quad (34)$$

Accordingly, the probability of the detector excitation during the interval Δt between ζ_1 and ζ_2 with emission of a particle with momentum k is

$$P_{\Delta t}(|0_M, -\rangle \rightarrow |1_k, +\rangle) = \frac{\lambda^2 \mu^2}{4\pi\omega} R_k^+(\Delta t) \Delta t^2. \quad (35)$$

How can the formula (35) be interpreted? In the absence of decoherence, and also in the limit $\Delta t \rightarrow \infty$ when the decoherence time is very large, the process under consideration is forbidden by energy conservation, since in this case Eq. (35) takes the form

$$R_k^+(\Delta t) \Delta t \rightarrow 2\pi\delta(E + \omega) \quad (36)$$

and both E and ω are greater than zero. It seems that this argument is not valid in the case with a 'free' evolution of finite duration Δt between the measurements of the detector state. Let us however consider the question in more detail.

The process (35) goes back to the measurement process. It represents the back action of the measuring environment onto the measured system during the finite time interval Δt . If a measurement is performed in the time interval Δt , an uncontrollable transfer of energy from the environment to the measured system (or vice versa) is possible. The transferred amount of energy is of the order of

$$\Delta E = 1/\Delta t. \quad (37)$$

In the closed system including the environment as well as the measured system the total energy is of course conserved. However in the measured system (the detector and the field in our case)

conservation of energy may be violated by the amount ΔE during the measurement (decoherence) process. Comparing absolute values of the amplitude (31) for different energies and using Eq. (32) one can easily see that the scale of the (seeming) violation of energy conservation is indeed $1/\Delta t$. It means that the process of measurement of the duration Δt may lead to uncontrollable change of energy ΔE in the detector. If this change is more than the difference E between two levels of the detector, the detector cannot work properly. Therefore the condition $\Delta E \ll E$ has to be imposed for the detector working effectively. Consequently, we obtain a condition for Δt

$$E\Delta t \gg 1. \quad (38)$$

This is a condition on the decoherence time Δt_d of the system which must be fulfilled for a sensitive detector. It has to be included into our phenomenological approach.

Returning now to the formula (35), we see that Eq. (38) is just the condition that $\sin(E\Delta t)/(E\Delta t)$ may be approximately replaced by a delta-function or, in other words, Eq. (38) is the condition that the probability (35) may be neglected. Thus the processes under investigation are forbidden not only in an ideal situation when the decoherence is not taken into account, but also in the real situation if care is taken that the detector can work effectively due to an appropriate decoherence time. To conclude: for an inertial detector in the vacuum, no transitions take place if the condition (38) is respected.

5 Uniformly accelerated detector with continuous decoherence

To treat the case of a uniformly accelerated detector, it is convenient to introduce Rindler coordinates (η, ξ) , which are related to Minkowski coordinates (t, x) by means of the transformation (cf. e. g. [23])

$$t = \frac{1}{a} e^{a\xi} \sinh a\eta, \quad x = \frac{1}{a} e^{a\xi} \cosh a\eta. \quad (39)$$

They cover only the wedge labeled R in Fig. 1 in which the detector is moving. Lines $\xi = \text{const}$ represent world lines with uniform acceleration $ae^{-a\xi}$. We shall consider a detector moving on a world line with $\xi = 0$. The parameter η is the respective proper time.

The positive frequency plane wave mode solutions to the wave equation in Rindler coordinates are given by

$$p_k^R = \frac{1}{\sqrt{4\pi\omega}} e^{ik\xi - i\omega\eta} \quad (40)$$

with $\omega = |k|$. The quantization of the scalar field in wedge R can be performed according to the usual quantization procedure, but with Rindler modes instead of the Minkowski mode functions (29). This leads to the decomposition (which is valid in R)

$$\phi(x) = \sum_k \left(b_k^R p_k^R(x) + b_k^{R\dagger} p_k^{R*}(x) \right) \quad (41)$$

with creation and annihilation operators for Rindler particles. This quantization procedure is not equivalent to the usual one based on the Minkowski modes (29) [24]. Both procedures are related by a Bogoliubov transformation as discussed in the Appendix. We will use the Rindler quantization only as a tool for the calculations and will refer with physical statements to Minkowski particles, i. e. particles registered as usual by an inertial observer.

We want to calculate the first order amplitude $A_1(p_1)$ for the detector history $p_1 = (- \dots - + + \dots +)$, that corresponds to detector excitation from the ground state during the time interval between $\eta = \zeta_1$ and $\eta = \zeta_2$. Using the expression (41) for the field operator, the derivation of $A_1(p_1)$ proceeds formally analogous to the previous section. We obtain

$$A_1(p_1) = -i\lambda\mu|+\rangle\langle-|\int dk \frac{1}{\sqrt{4\pi\omega}} \left[b_k^R \int_{\zeta_1}^{\zeta_2} d\eta e^{i(E-\omega)\eta} + b_k^{R\dagger} \int_{\zeta_1}^{\zeta_2} d\eta e^{i(E+\omega)\eta} \right], \quad (42)$$

which can be evaluated to give

$$A_1(p_1) = -i\frac{\lambda\mu}{2\sqrt{\pi}}|+\rangle\langle-|\int \frac{dk}{\sqrt{\omega}} \left[b_k^R S_k^-(\zeta_2, \zeta_1) + b_k^{R\dagger} S_k^+(\zeta_2, \zeta_1) \right] \Delta\eta, \quad (43)$$

where $\Delta\eta = \zeta_2 - \zeta_1$ is the decoherence time and $S_k^\pm$ has been defined in (32).

To refer to Minkowski particles instead of Rindler particles, we use the Bogoliubov transformation (74) from the Appendix. To make a comparison easy, our physical particle interpretation is based on the Minkowski mode functions q_k^R and q_k^L introduced by Unruh [1] and discussed in the Appendix. The first-order amplitude for the history p_1 becomes

$$A_1(p_1) = -i\frac{\lambda\mu}{2\sqrt{\pi}}|+\rangle\langle-|\int \frac{dk}{\sqrt{2\omega \sinh(\pi\omega/a)}} \left[c_k^R e^{\pi\omega/2a} S_k^-(\zeta_2, \zeta_1) + c_{-k}^{L\dagger} e^{-\pi\omega/2a} S_k^-(\zeta_2, \zeta_1) + c_k^{R\dagger} e^{\pi\omega/2a} S_k^+(\zeta_2, \zeta_1) + c_{-k}^L e^{-\pi\omega/2a} S_k^+(\zeta_2, \zeta_1) \right] \Delta\eta. \quad (44)$$

We see that in general four types of processes may take place: The detector in R can be excited from the ground state with absorption of a R or a L Minkowski particle or with emission of a R or L Minkowski particle. Note that the mode functions for R (L) Minkowski particles are mainly concentrated in the right (left) wedge of Fig. 1.

If the initial state of the particle field is the Minkowski vacuum $|0_M\rangle$, only emission processes can take place and the final state in first order perturbation theory becomes

$$|\Psi(\zeta_2, \zeta_1)\rangle = -i\frac{\lambda\mu\Delta\eta}{2\sqrt{\pi}} \int \frac{dk}{\sqrt{2\omega \sinh(\pi\omega/a)}} \left[e^{-\pi\omega/2a} S_k^-(\zeta_2, \zeta_1) c_k^{L\dagger} + e^{\pi\omega/2a} S_k^+(\zeta_2, \zeta_1) c_k^{R\dagger} \right] |0_M, +\rangle. \quad (45)$$

The probability for detector excitation between ζ_1 and ζ_2 with the emission of an R Minkowski particle with quantum number k , which will be called *process R*, is

$$P_{\Delta\eta}(|0_M, -\rangle \rightarrow |1_k^R, +\rangle) = \frac{\lambda^2\mu^2}{4\pi\omega} \left(1 + \frac{1}{e^{2\pi\omega/a} - 1} \right) R_k^+(\Delta\eta)\Delta\eta^2. \quad (46)$$

The corresponding probability for detector excitation in the interval $\Delta\eta$ accompanied by the emission of an L particle, called *process L*, is

$$P_{\Delta\eta}(|0_M, -\rangle \rightarrow |1_k^L, +\rangle) = \frac{\lambda^2\mu^2}{4\pi\omega} \frac{1}{e^{2\pi\omega/a} - 1} R_k^-(\Delta\eta)\Delta\eta^2. \quad (47)$$

As discussed in the previous section, one has to impose the condition $E\Delta\eta \gg 1$ in order to keep the disturbance introduced by the environment to the detector sufficiently small. Under this condition, the process R with a probability proportional to $R_k^+(\Delta\eta)$ will not take place and we are left with process L as the real physical process.

We considered so far the situation of a selective measurement when it is known what output the measurement gave. In the present case this means that it is known that the detector has been excited between the specific moments ζ_1 and ζ_2 . Having an application to realistic situations in mind (cf. Sec. 1), we should also consider the situation in which this knowledge is not available. We therefore have to use Eq. (12) which refers to non-selective measurements.

If we know nothing about the output of the detector (but we know that the detector worked and therefore influenced the field), the summation in this formula should be performed over all possible paths p . Among these paths are the following: the path p_0 consisting of minuses, the paths p_1 with all possible locations of the transition, all possible paths of the form p_2 etc. However amplitudes corresponding to paths p_2 are not lower than of the second order in the coupling constant. Therefore, in a calculation up to the first order, one may keep in the sum (12) only the path p_0 and all paths of the form p_1 . The corresponding amplitudes have already been calculated. Substituting them into Eq. (12) and taking the Minkowski vacuum as an initial state, we obtain for the density matrix of the final state up to order λ^2

$$\rho' = |0_M, -\rangle\langle 0_M, -| + \sum_j |\Psi(\eta_j, \eta_{j-1})\rangle\langle\Psi(\eta_j, \eta_{j-1})|. \quad (48)$$

Here, the summation should be performed over all values of j corresponding to the total time interval $T = \eta_N - \eta_0$.

The density matrix (48) describes the mixed state of the composite system detector and field after the time interval T during which continuous decoherence took place. Probabilities for specific outcomes can be derived from the density matrix in the usual way. For example the probability to find the system in the given state ρ after the time T passed is equal to

$$\text{Prob}_T(\rho) = \text{Tr}(\rho' \rho).$$

With the help of this formula one can easily calculate the probability to find the detector in the excited states and the field in an one-particle state containing an L Minkowski particle (process L):

$$\text{Prob}_T(|0_M, -\rangle \rightarrow |1_k^L, +\rangle) = \frac{\lambda^2 \mu^2}{4\pi\omega} \frac{1}{e^{2\pi\omega/a} - 1} R_k^-(\Delta\eta) \Delta\eta T. \quad (49)$$

This is our final result. Again we have to take for $\Delta\eta$ the decoherence time Δt_d which is characteristic for the particular environment realized in the experimental setup.

Note that the probability (49) turns out to be $N = T/\Delta\eta$ times larger than the corresponding probability (47) calculated under the assumption that time of the detector excitation is known. This is a direct consequence of fact that alternative times of the detector excitation are classical alternatives, so that probabilities instead of amplitudes should be summed up.

It is interesting to compare this formula with the corresponding result for the coherent in-out evolution of the closed detector-field system (primary calculation of the Unruh effect). It can be obtained either by direct application of standard first-order perturbation theory or, which amounts to the same, by taking in Eq. (47) the limit of a very large time between two measurements ($\Delta\eta = T \rightarrow \infty$). Because of

$$\lim_{\Delta\eta \rightarrow \infty} R_k^\pm(\Delta\eta) \Delta\eta = 2\pi\delta(E \pm \omega) \quad (50)$$

we get in this limit the standard formula for the Unruh effect without continuous decoherence [23]

$$P_T(|0_M, -\rangle \rightarrow |1_k^L, +\rangle) = \frac{\lambda^2 \mu^2}{2\omega} \frac{1}{e^{2\pi\omega/a} - 1} \delta(E - \omega) T. \quad (51)$$

Although this result refers strictly speaking to a coherent process of infinite duration, it has been taken as a basis for practical applications in writing it as a probability per unit time

$$\frac{1}{T} P_T(|0_M, -\rangle \rightarrow |1_k^L, +\rangle) = \frac{\lambda^2 \mu^2}{2\omega} \frac{1}{e^{2\pi\omega/a} - 1} \delta(E - \omega). \quad (52)$$

On the other hand, from our result (49) which reflects continuous decoherence, we obtain

$$\frac{1}{T} \text{Prob}_T(|0_M, -\rangle \rightarrow |1_k^L, +\rangle) = \frac{\lambda^2 \mu^2}{4\pi\omega} \frac{1}{e^{2\pi\omega/a} - 1} R_k^-(\Delta t_d) \Delta t_d. \quad (53)$$

We know already that $\Delta t_d \gtrsim 1/E$ is necessary for an efficiently working detector. If we are concerned only with the behaviour of the detector, we should integrate (52) and (53) over all particle momenta. For $\Delta t_d \gg 1/E$, which seems to be realistic, the limit (50) may be applied and our result agrees with the standard formula (52). For $\Delta t_d \approx 1/E$, this is also the case in a first approximation because of

$$\int d\omega R_k^-(\Delta t_d) \Delta t_d = 1 \quad (54)$$

Differences between the two cases arise only if we use a narrow-band detector (with a bandwidth smaller than $1/\Delta t_d$) and measure the energy of the emitted particle. For the detector with coherent evolution, one will find in this case a sharp spectrum at $\omega = E$ (cf. (51)), whereas for a detector under the influence of the environment one will see a spectrum with a finite width (cf. (53) with $R_k^-(\Delta t_d)$ of (33)).

We therefore obtain the result that the total detector excitation rates (integrated over the photon energy) almost agree, independent of whether the effect of the environment is taken into account or not. This seems surprising at first. The reason for it may be seen in the fact that P_T of (51) and, because of (50), essentially also $P_{\Delta\eta}$ of (47) are linear in time. $P_{\Delta\eta}$ is smaller than P_T by a factor $1/N$. But the necessary transition to Prob_T of (49) amounts to a multiplication by N .

6 Localization of the detector excitation and the emitted photons

We now turn to our second problem. We want to localize the trajectory of an emitted particle as well as the event of excitation of the accelerated detector. To do this we have to treat one single history of the form $p_1 = (- \dots - + + \dots +)$ where the excitation of the detector is located in the time interval between the two observations at ζ_1 and ζ_2 . Our task is to find out in which way the trajectories of the emitted photons are correlated with this event.

To obtain localized statements, we will refer to the detector history p_1 of (24). To localize the particles emitted or absorbed in the transition, we introduce as in [4, 7] *Minkowski wave packets* f_{km} and *Rindler wave packets* p_{km}^R

$$f_{km}(t, x) = \frac{1}{\sqrt{\epsilon}} \int_k^{k+\epsilon} dke^{-ikm} f_k(t, x), \quad p_{km}^R(\eta, \xi) = \frac{1}{\sqrt{\epsilon}} \int_k^{k+\epsilon} dke^{-ikm} p_k^R(\eta, \xi) \quad (55)$$

They are localized functions that are obtained by superposition of the plane waves (29) and (40) in a momentum interval ϵ . A particular Minkowski wave packet is labeled by the two *packet parameters* (k, m) , where k/ϵ and $m\epsilon/2\pi$ have to be integers. The momentum parameter k gives the energy $\omega = |k|$ of the wave packet and its sign specifies its direction of propagation. The trajectory parameter m fixes the null trajectory of the wave packet's maximum according to

$$\begin{aligned} (t - x)_{\max} = \text{const} &= -m & \text{for } k > 0 \\ (t + x)_{\max} = \text{const} &= +m & \text{for } k < 0 \end{aligned} \quad (56)$$

This can be seen from a stationary phase analysis of the wave packet mode functions (55). For details see [4]. For small ϵ , when $\omega^{-1/2}$ is approximately constant over the range of integration, the form of the wave packet can be given explicitly: in this case, it is proportional to $\sin[(\epsilon/2)(x \mp t - m)]/(x \mp t - m)$. Note that a wave packet that passes the left (right) wedge of Fig. 1 has always $m < 0$ ($m > 0$). The momentum width of a wave packet is ϵ and its spatial width is $2\pi/\epsilon$. Rindler wave packets can be analyzed in a completely analogous way. One obtains, for example, for the trajectory of their maximum

$$\begin{aligned} (\eta - \xi)_{\max} = \text{const} &= -m & \text{for } k > 0 \\ (\eta + \xi)_{\max} = \text{const} &= +m & \text{for } k < 0 \end{aligned} \quad (57)$$

instead of (56).

It will turn out to be useful to introduce the concepts of equivalent and mirror modes. For a given Rindler wave packet mode (k, m) in the right wedge, we introduce its *equivalent Minkowski mode* as the wave packet (k_α, m_α) the maximum of which moves along the same trajectory (see Fig. 1) with an energy ω_α that is obtained from the Rindler energy ω according to the Doppler shift formula. This leads to [4]:

$$(k, m) \xrightarrow{\text{equiv.}} (k_\alpha, m_\alpha) := (e^{-am} k, a^{-1} e^{am}). \quad (58)$$

The *mirror mode* of this Minkowski wave packet (k_α, m_α) is defined as the Minkowski wave packet (k_β, m_β) with the same energy, but with a trajectory which is obtained by 'reflection' at one of the horizons $t \pm x = 0$ (Fig. 1):

$$(k_\alpha, m_\alpha) \xrightarrow{\text{mirror}} (k_\beta, m_\beta) := (k_\alpha, -m_\alpha). \quad (59)$$

The equivalent mode of a right wedge Rindler wave packet always passes the right wedge (because always $m_\alpha > 0$), while its mirror mode crosses the left wedge ($m_\beta < 0$). Consequently, the index α denotes in the following always Minkowski particles in R, while an index β refers to Minkowski particles in L.

We can quantize the field with respect to the wave packet modes (55). The decomposition of the field operators in terms of Minkowski and Rindler wave packets reads

$$\begin{aligned}\phi(x) &= \sum_{km} (a_{km} f_{km} + a_{km}^\dagger f_{km}^*) \\ &= \sum_{km} (b_{km}^R p_{km}^R + b_{km}^{R\dagger} p_{km}^{R*}),\end{aligned}\quad (60)$$

leading to wave packet creation and annihilation operators. We only need the field decomposition in the right wedge because of the local interaction between detector and field.

Rindler and Minkowski wave packet creation and annihilation operators are connected by a Bogoliubov transformation. As has been shown in [4], it can be approximated by the following expression:

$$b_{km}^R = \alpha_k^* a_{k_\alpha m_\alpha} - \beta_k^* a_{k_\beta m_\beta}^\dagger. \quad (61)$$

According to this, a Rindler wave packet (k, m) is connected with its equivalent Minkowski mode (k_α, m_α) passing the right wedge and the mirror mode (k_β, m_β) passing L. The Bogoliubov parameters α_k and β_k appearing in (61) are given by

$$\left. \begin{matrix} \alpha_k \\ \beta_k \end{matrix} \right\} = \mp i \left(\frac{a}{2\pi\omega} \right)^{\frac{1}{2}} e^{\pm i\omega\pi/2a} \left| \Gamma \left(1 + i \frac{k}{a} \right) \right|. \quad (62)$$

For their squares we find

$$|\alpha_k|^2 = 1 + \frac{1}{e^{2\pi\omega/a} - 1}, \quad |\beta_k|^2 = \frac{1}{e^{2\pi\omega/a} - 1}. \quad (63)$$

After having introduced wave packets as the basis of our calculations, we are now able to obtain localized statements not only about the time interval of detector excitation, but also about the particles involved in this process. The localization is thereby obtained in referring to one particular detector history p_1 of (24). The corresponding first-order transition amplitude for the path p_1 can be evaluated from the general formula (25). Inserting the decomposition (60), we obtain

$$A_1(p_1) = -i \frac{\lambda\mu}{\sqrt{4\pi\epsilon}} |+\rangle\langle-| \sum_{km} [b_{km}^R M_{km}^-(\zeta_2, \zeta_1) + b_{km}^{R\dagger} M_{km}^+(\zeta_2, \zeta_1)] \quad (64)$$

where

$$M_{km}^\pm(\zeta_2, \zeta_1) = \int_k^{k+\epsilon} \frac{dk}{\sqrt{\omega}} e^{\pm i k m} S_k^\pm(\zeta_2, \zeta_1) \quad (65)$$

and $S_k^\pm$ has been defined in (32).

Using the Bogoliubov transformation (61) between Rindler and Minkowski wave packets, we can express this first-order amplitude $A_1(p_1)$ with reference to Minkowski packet states:

$$\begin{aligned}A_1(p_1) &= -i \frac{\lambda\mu}{\sqrt{4\pi\epsilon}} |+\rangle\langle-| \sum_{k,m} [a_{k_\alpha m_\alpha} \alpha_k^* M_{km}^-(\zeta_2, \zeta_1) - a_{k_\beta m_\beta}^\dagger \beta_k^* M_{km}^-(\zeta_2, \zeta_1) \\ &\quad + a_{k_\alpha m_\alpha}^\dagger \alpha_k M_{km}^+(\zeta_2, \zeta_1) - a_{k_\beta m_\beta} \beta_k M_{km}^+(\zeta_2, \zeta_1)]\end{aligned}\quad (66)$$

Here, (k_α, m_α) and (k_β, m_β) denote the equivalent and mirror Minkowski modes of the Rindler mode (k, m) (cf. (58) and (59)).

If now again the initial state of the field is the Minkowski vacuum $|0_M\rangle$, we obtain from the amplitude (66) for the final state of the system

$$|\Psi\rangle = -i \frac{\lambda\mu}{\sqrt{4\pi\epsilon}} \sum_{km} [-a_{k_\beta m_\beta}^\dagger \beta_k^* M_{km}^-(\zeta_2, \zeta_1) + a_{k_\alpha m_\alpha}^\dagger \alpha_k M_{km}^+(\zeta_2, \zeta_1)] |0_M, +\rangle \quad (67)$$

Two processes may happen: Excitation of the detector in the time interval between ζ_1 and ζ_2 (transition from $-$ to $+$ in p_1) with emission of a particle in the equivalent mode of a given Rindler wave packet, or emission into its mirror mode.

As in the preceding sections, we must impose the condition $\Delta\eta \gg 1/E$ in order to deal with a properly working detector. This excludes the expression containing $M_{km}^\pm$ in (64) and (67), as can be seen from the definitions (65) and (32). Accordingly, only excitation with emission into the Minkowski mirror mode (k_β, m_β) is possible.

To obtain a correlation between the event of detector excitation and the trajectories of the emitted particles, let us now investigate for which trajectory parameters m the functions $M_{km}^\pm(\zeta_2, \zeta_1)$ defined in (65) assume their maximal values. This can be done by using the method of stationary phase. We obtain

$$-\sigma m = \frac{1}{2}(\zeta_2 + \zeta_1) \equiv \bar{\zeta}, \quad (68)$$

where $\sigma = \text{sign}(k)$. Approximately, the sum over m in Eqs. (64), (66), (67) reduces to one term given by (68). Note that according to (57), the trajectory parameter $\pm m$ gives directly the time η_{max} at which the Rindler wave packet crosses the trajectory $\xi = 0$. Using this, Eq. (68) can be written

$$\eta_{max} = \frac{1}{2}(\zeta_2 + \zeta_1) = \bar{\zeta} \quad (69)$$

We see that the Rindler wave packets are most likely to be absorbed during the time interval where the detector makes the transition i.e. when its proper time η lies between ζ_1 and ζ_2 . The respective dashed null world lines go to the left or to the right in Fig. 2. This reflects completely what one would expect intuitively.

More interesting is that the condition (68) fixes also the trajectories of the emitted Minkowski wave packets in (67). Only emission of Minkowski particles into wave packet modes (k_β, m_β) with m_β obtained from (68) and (59) occurs. These are the mirror modes of the Rindler modes just specified (see Fig. 2). There can be emission of left-moving ($k_\beta < 0$) or right-moving ($k_\beta > 0$) particles. This leads to one particular pair of wave packet trajectories where emission of Minkowski particles is possible, as shown in Fig. 2. The point P where these two trajectories intersect can be identified with the point where the respective particles are produced. We see that this space-time point is symmetrical (with regard to the origin of the Minkowski coordinate system) to the point where the detector has been excited. Note that in the momentary rest frame of the accelerated detector, the creation of the particles and the excitation of the detector happen at the same time. The role which the detector history played in the derivation above stresses again the importance of the influence of the localized decoherence process during measurement for the localization of the creation of the Minkowski particles.

Without going into the details of the calculation, we want to mention that it is also possible to perform a similar calculation for an accelerated detector that is initially in the excited state $|+\rangle$ and makes a transition to the ground state in the interval between ζ_1 and ζ_2 . This process, which corresponds to a history of the form

$$\hat{p}_1 = (++++\dots - - - - \dots)$$

is nothing but the generalization of the familiar phenomenon of spontaneous emission. To obtain a localization of the detector transition as well as the emitted radiation, one has to calculate the amplitude $A_1(\hat{p}_1)$ using a wave packet basis. Imposing the condition (38) one finds in contrast to above that only emission into the equivalent mode (k_α, m_α) is possible. Again, a stationary phase analysis of the amplitude correlates the de-excitation event with the trajectories of the emitted particles. As expected in this case, we find that the trajectories of the left- and right-moving wave packets, which are most likely to be emitted, intersect on the trajectory of the detector at the time $\bar{\zeta}$ of (68) where the detector makes the transition.

7 Conclusion

We considered in this paper the Unruh effect, (i. e. the excitation of an accelerated particle detector in the vacuum), taking into account the effects of continuous decoherence. This physical phenomenon is a consequence of the fact that information about the detector state is recorded by the interaction with an environment. As compared with a coherent in-out process in which

detector and field are treated as a closed system, certain interference effects disappear due to the decoherence, and therefore probabilities of some processes may become different.

The continuous decoherence was described above phenomenologically in an approximate way by continuous measurement on the detector, represented by von Neumann projections that are separated by periods of free evolution. This leads to a description in terms of detector histories.

We studied to lowest order in a perturbation scheme two physically different situations: First we treated the case of non-selective measurements where the detector history is not known. The probability for detector excitation with emission of a particle was calculated and compared with the results obtained in the usual approach in which decoherence is neglected. It turned out that the total detector excitation rates almost agree independent of whether the effect of the environment is taken into account or not.

The second main point dealt with the case of selective measurements, where a definite detector history is considered. This allows the localization of the detector excitation in space and time. It was shown that the space-time region of particle creation can also be localized and correlated with the space-time event of the detector excitation.

Finally we mention that only because the continuous decoherence was taken into account, the localization of the detector excitation (and of the accompanying photon creation) could be considered correctly. It was shown that the precision with which these events may be localized depends on the decoherence time.

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Appendix: Rindler, Minkowski, and Unruh modes

The quantization of a real massless scalar field in the right Rindler wedge has been carried out in Sec. 5. It will be useful to perform the same procedure for the left wedge L. To this end, the left wedge can be covered with Rindler coordinates, which are obtained from (39) by replacing (t, x) by $(-t, -x)$. We can also introduce the left wedge mode functions

$$p_k^L = \frac{1}{\sqrt{4\pi\omega}} e^{ik\xi + i\omega\eta} \quad (70)$$

where again $\omega = |k|$. The quantization with respect to both left and right wedge mode functions leads to

$$\phi(x) = \sum_k \left(b_k^R p_k^R(x) + b_k^{R*} p_k^{R*}(x) + b_k^L p_k^L(x) + b_k^{L*} p_k^{L*}(x) \right) \quad (71)$$

instead of (41), with the left wedge annihilation and creation operators b_k^L, b_k^{L*} .

It is well known [1, 23] that it is possible to form linear combinations of left and right wedge Rindler mode functions that are positive frequency with respect to the Minkowski time coordinate t . They are given by the Unruh modes

$$\begin{aligned} q_k^R &= N_k^R (p_k^R + e^{-\frac{\pi\omega}{a}} p_{-k}^{L*}) \\ q_k^L &= N_k^L (p_{-k}^{R*} + e^{\frac{\pi\omega}{a}} p_k^L) \end{aligned} \quad (72)$$

Note that the R (L) mode functions are located mostly in the right (left) wedge. The normalization factors are $N_k^R = (1 - e^{-2\pi\omega/a})^{-1/2}$ and $N_k^L = e^{-\pi\omega/a} (1 - e^{-2\pi\omega/a})^{-1/2}$.

We can use as well the Unruh modes as a basis for the quantization of the field. The decomposition of the field operator reads then

$$\phi(x) = \int dk \left(q_k^R c_k^R + q_k^L c_k^L + q_k^{R*} c_k^{R\dagger} + q_k^{L*} c_k^{L\dagger} \right), \quad (73)$$

leading to creation and annihilation operators for the L and R modes. They obey the commutation relations

$$[c_{k_1}^L, c_{k_2}^{L\dagger}] = [c_{k_1}^R, c_{k_2}^{R\dagger}] = \delta(k_1 - k_2).$$

All other commutators vanish. As the Unruh modes are positive frequency with respect to Minkowski time, the annihilation operators annihilate the Minkowski vacuum: $c_k^L |0_M\rangle = c_k^R |0_M\rangle = 0$.

The Bogoliubov transformation between Rindler and Unruh creation and annihilation operators can be obtained by comparison of (71) and (73). It is given by

$$\begin{aligned} b_k^R &= \frac{1}{\sqrt{2 \sinh(\pi\omega/a)}} \left(e^{\frac{\pi\omega}{2a}} c_k^R + e^{-\frac{\pi\omega}{2a}} c_{-k}^{L\dagger} \right) \\ b_k^L &= \frac{1}{\sqrt{2 \sinh(\pi\omega/a)}} \left(e^{\frac{\pi\omega}{2a}} c_k^L + e^{-\frac{\pi\omega}{2a}} c_{-k}^{R\dagger} \right). \end{aligned} \quad (74)$$

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Fig. 1: Trajectories of equivalent and mirror wave packet modes. Solid lines denote Minkowski modes, the dashed line a Rindler mode. The Minkowski wave packet (k_α, m_α) is the equivalent mode to the Rindler wave packet (k, m) . (k_α, m_α) and (k_β, m_β) are Minkowski mirror modes. $(k_\alpha, m_\alpha > 0)$ always crosses the right wedge R, while $(k_\beta, m_\beta < 0)$ moves through the left wedge L.

Fig. 2: Localization of the detector excitation and of the particles involved. If the detector is excited in the time interval between $\eta = \zeta_1$ and $\eta = \zeta_2$, those Rindler wave packets (k, m) have the maximal probability to be absorbed, whose maximum crosses the detector trajectory during this time interval. The Minkowski wave packets emitted in the process are most likely to be found in the mirror modes (k_β, m_β) . The trajectory of right-moving $(k_\alpha > 0, m_\alpha)$ and left-moving $(k_\beta < 0, m_\beta)$ wave packets intersect in the point P, which lies symmetrically with respect to the origin of the Minkowski coordinates to the point of detector excitation.