Relative spins and excitation energies of superdeformed bands in $^{190}\text{Hg}$: Further evidence for octupole vibration


Abstract

An experiment using the Eurogam Phase II gamma-ray spectrometer confirms the existence of an excited superdeformed (SD) band in $^{190}\text{Hg}$ and its very unusual decay into the lowest SD band over 3-4 transitions. The energies and dipole character of the transitions linking the two SD bands have been firmly established. Comparisons with RPA calculations indicate that the excited SD band can be interpreted as an octupole-vibrational structure.

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Recently, we have reported [1] the observation of an excited superdeformed (SD) band in $^{190}$Hg with the unusual property that it decays entirely to the lowest SD band [2,3]. The only other cases in even-even nuclei where transitions linking two SD bands (i.e. interband transitions) have been proposed are in $^{150}$Gd and $^{152}$Dy [4,5]. In all three nuclei, the presence of the interband transitions was inferred from the observed coincidence relationships. In the case of $^{190}$Hg, the excited SD band was populated very weakly: the gamma-ray intensities ranged from 0.02 to 0.06% with respect to population of the ground state. Hence, the possible interband transitions given in Ref. [1] were reported as tentative.

In general, the direct measurement of transitions between SD bands is of great interest because it can give critical information on the properties of these nuclei at extreme deformations. First, the fact that such transitions are so uncommon (excluding transitions between signature partner bands, e.g. Ref. [6]) suggests that atypical intrinsic SD structures are involved. Second, the accurate determination of relative energies, of relative spins and parities, and of branching ratios between in-band and out-of-band decay becomes possible. This allows for stringent new tests of mean-field calculations whose results have thus far mostly been compared with measured dynamic moments of inertia, $\mathcal{J}^{(2)}$.

The SD minima in both the A~150 and A~190 regions of superdeformation are calculated [7,8] to be soft with respect to octupole deformation. Thus, low-lying octupole-vibrational states should exist, and strong E1 transitions are expected to connect these vibrational levels to the lowest SD band. The nuclei in which these effects are likely to be observed are those in which the excitation energy of an octupole-vibrational band is low at the spins at which SD bands are fed. Calculations using the random phase approximation (RPA), described below, predict that at a rotational frequency $\hbar \omega=0.4$ MeV the lowest octupole-vibrational states in $^{190}$Hg, $^{192}$Hg and $^{194}$Hg occur at 0.37, 0.67 and 0.67 MeV above the lowest SD band, respectively. Thus, among the even-even Hg isotopes in which superdeformation has been observed, $^{190}$Hg is the best candidate for the observation of octupole vibrations. In Ref. [1] the excited band in $^{190}$Hg was interpreted as an octupole vibration. This interpretation is strengthened here by the firm identification of the interband
transitions and the experimental determination of their dipole character.

The results presented here are from a new measurement performed with the Eurogam Phase II gamma-ray spectrometer [9], which consists of 54 escape-suppressed Ge detectors. Of these, 30 are large-volume Ge detectors located in rings at 22, 46, 134 and 158 degrees with respect to the beam. The remaining 24 detectors, at 75 and 105 degrees, are so-called “clover” detectors, each consisting of four closely packed Ge crystals within a single cryostat inserted in an escape-suppression shield. The higher granularity of the “clover” detectors reduces Doppler broadening and allows measurements of linear polarization [10]. The experiment was performed using the reaction $^{156}$Gd($^{34}$S,4n) on a stack of two isotopically pure 0.5 mg/cm$^2$ Gd targets. The newly commissioned Vivitron accelerator was set to a beam energy of 153 MeV, but had not been calibrated, and the actual energy was estimated to be $\sim158$ MeV based on a comparison of the measured gamma-ray spectra with those from the earlier experiment performed at the 88-inch cyclotron at Lawrence Berkeley Laboratory [1]. An event was written to tape if four or more escape-suppressed Ge detectors recorded gamma rays in coincidence. A total of $4.4\times10^8$ events was acquired. When two crystals of a “clover” detector both registered signals, their energies were added in software, and the detector was treated as a single unit. The Doppler correction in such cases was calculated under the assumption that the primary gamma-ray entered the crystal that recorded the higher energy. When more than two crystals of a “clover” detector recorded signals, the information from the entire detector was disregarded for that event; cases such as these were found to arise mostly from pileup. Unfolding the coincidence events into all possible combinations of 3 and 4 energies yielded, respectively, $2.5\times10^9$ triples ($\gamma^3$) and $2.0\times10^9$ quadruples ($\gamma^4$) combinations. The data were subsequently analyzed by sorting all the $\gamma^3$ combinations into a three-dimensional histogram [11] and then extracting double-gated one-dimensional spectra with full background-subtraction and propagation of errors [12].

A coincidence spectrum showing the excited SD band and its decay into the lowest SD band is presented in Fig. 1. The quality of the spectrum is greatly improved with respect to those shown in Ref. [1], both because of the factor of four increase in statistics and because of
the use of a lower beam energy, which avoided a number of interfering gamma-ray transitions from $^{188}$Hg. The sensitivity to dipole transitions was also enhanced by the presence of more detectors at angles close to 90°, where the angular distribution is peaked for this type of transition. As can be seen from Fig. 1, this experiment confirms the existence of the excited SD band, as well as its unusual pattern of decay to the lowest SD band. The transitions stand out clearly above the background, and the corresponding energies were determined more accurately (Fig. 2). The weak transitions at the top and bottom of the band are also seen in the present data with significantly better ratios of signal to noise.

More importantly, the interband transitions are now firmly established. The gamma-ray energies tentatively assigned in Ref. [1] to the interband transitions were found to be incorrect, and the actual transitions have energies of 812, 864 and 911 keV. The relative positions of the two SD bands is presented in Fig. 2. Compared with Ref. [1], the levels of the excited SD band have been shifted up by one transition, i.e. the excited band decays into the lowest SD band one level higher than previously thought. Based on the present data, the relative placement of the two bands is uniquely determined by the sums and differences of gamma-ray energies and by coincidence measurements. For example, the 911 keV transition is observed in coincidence with all members of the excited band down to the 511 keV transition, and with all members of the lowest SD band up to the 521 keV line, but not with the higher members. Weak evidence was also found for an additional linking transition, with an energy of 757 keV, lying above the three others (dashed arrow in Fig. 2), but its intensity was at the limit of the sensitivity of the experiment, leading only to an upper limit for this branch. The energies of the transitions in the lowest SD band were also measured more accurately than in Ref. [1]. These energies are: 316.9(4), 360 (unresolved doublet), 402.3(4), 442.98(6), 482.71(6), 521.30(6), 558.6(1), 594.9(1), 630.1(1), 664.1(1), 696.9(1), 728.5(4), 757.4(4), 783.5(6), and 801.8(8) (tentative).

In order to determine the relative spins of the two SD bands, DCO ratios (Directional Correlations from Oriented nuclei [13]) were extracted for the interband transitions. These were defined simply as the number of counts in the clover detectors (at side angles) divided by
the number of counts in the large-volume detectors (at forward and backward angles), after efficiency correction. These ratios were calibrated using transitions of known multipolarity, and were found to be approximately 1.3 for E1 transitions and 0.8 for E2 transitions. The ratios were extracted from single-gated and double-gated spectra under the assumption that the requirement of a gating transition or transitions did not significantly perturb the orientation of the nuclear angular momentum. This is a valid assumption because the gating transitions were measured at all angles by the nearly spherically symmetric array of detectors, and because the perturbation is negligible for $\lambda \leq J$, where $\lambda$ is the multipolarity of the gating transition [14]. The measured DCO ratios for the interband transitions are shown in Fig. 3, along with representative E1 and E2 transitions. Although the error bars are large, the clustering of these values around those of other dipole transitions indicates that they are of dipole character. Thus, it is highly probable that the two SD bands differ by one unit of angular momentum.

A measurement of the polarizations of the interband transitions was also attempted using the clover detectors, based on the asymmetry between Compton scattering parallel and perpendicular to the reaction plane. Although the technique was successful for transitions with strengths as low as a few percent relative to the $^{190}$Hg channel, the statistics available in this experiment were not sufficient to allow such a measurement for the very weak interband transitions of interest.

Assuming that the excited band has a transition quadrupole moment of $18(3)$ eb, equal to that of the lowest SD band [2], it is possible to extract the partial half-lives of the interband transitions. These are shown in Table I, along with the transition strengths in units of Weisskopf units (W.u.) under the assumptions of E1 and M1 multipolarity. As argued in Ref. [1], it is unlikely that M1 transitions between different quasiparticle configurations would occur with such short partial half-lives; the B(M1) values in Table I are much larger than those typically observed in deformed nuclei [15]. Therefore, these transitions are very likely of E1 character. The B(E1) values in Table I are about two orders of magnitude stronger than typical E1 transitions in heavy nuclei [15], but are similar to those observed
among actinide nuclei exhibiting strong octupole collectivity in the normally deformed well [16].

These findings reinforce the arguments put forward in Ref. [1], i.e. there is strong evidence that these states are of octupole-vibrational character (for the range of rotational frequencies over which E1 transitions are observed), rather than members of a negative-parity two-quasiparticle band. We have therefore carried out RPA calculations based on the cranked Nilsson model for the octupole-vibrational modes. The equilibrium quadrupole deformation, $\delta_{eq} = 0.454$, and pairing gaps, $(\Delta_n, \Delta_p) = (0.8, 0.6)$ MeV, were determined by means of the conventional shell correction method with smoothed pairing gaps [17]. The form of the Hamiltonian is the same as in Ref. [18]. A more detailed discussion of the calculations will be given in a future paper [19]. All calculated quantities given here for the excited SD band, including Routhians and aligned angular momenta, are differences between the relevant value for the excited band itself and that of the lowest SD band.

A comparison between the observed and calculated $I(2)$ moments of inertia for the excited band is shown in Fig. 4a. The excited SD band has an average $I(2)$ value of $123 \hbar^2$MeV$^{-1}$, which is unusually large for an SD band in an even-even $\sim$190 nucleus. It is essentially constant over the entire range of frequencies where the band is observed. The calculations not only accurately reproduce the measured $I(2)$, but also match well with the observed excitation energy of the vibrational band relative to the lowest band (Fig. 4b). (It should be noted that although generator coordinate calculations have not been carried out for $^{190}$Hg, such calculations [8] predict phonon energies for nearby nuclei which are nearly twice as large as those resulting from the RPA calculations.) Finally, preliminary calculations of the E1 transition rates yield values of the order of $10^{-3}$ to $10^{-4}$ Weisskopf units, consistent with the observed rates.

The agreement found above between theory and experiment makes it interesting to examine the calculated wave-functions for insight into the physical structure of the observed states in the excited SD band. The structure of the lowest excited RPA state evolves smoothly as a function of rotational frequency, but for purposes of discussion it is convenient to define
three regions of frequency: $\hbar \omega = 0 - 0.2$ MeV, $0.2 - 0.4$ MeV, and above 0.4 MeV. In the first region, the RPA states make up an octupole-vibrational multiplet. At zero frequency, the K=2, 0, 3 and 1 modes occur at 0.99, 1.28, 1.35 and 1.41 MeV, respectively. These modes become increasingly mixed by the Coriolis force as the rotational frequency increases. Thus, in the second region of frequency, which corresponds roughly to that at which the SD band is observed experimentally, the lowest vibrational state becomes rotationally aligned. At $\hbar \omega = 0.35$ MeV, its aligned angular momentum (relative to that of the lowest SD band) reaches a maximum value of $\sim 3\hbar$, which corresponds to a completely rotationally aligned octupole phonon. The similarity between the slopes of the theoretical and experimental Routhians in Fig. 4b indicates that the calculations reproduce the observed alignment of the excited band. In the highest region of frequency, the lowest RPA state gradually loses its collective vibrational character, eventually becoming a two-quasineutron state. The low excitation energy of the octupole-vibrational band at high spins in $^{190}$Hg compared to $^{192}$Hg and $^{194}$Hg is attributable to the much lower phonon energies of the low-K members of the octupole-vibrational multiplet in this nucleus. More precisely, the close spacing in energy of the different modes leads to stronger Coriolis mixing and a smaller excitation energy for the lowest rotation-aligned state in $^{190}$Hg than in any other even-even Hg isotope where superdeformation has been observed.

In conclusion, the existence of an excited SD band in $^{190}$Hg and its anomalous pattern of decay to the lowest SD band have been confirmed. The energies of the transitions connecting the two bands have been measured. From the measured DCO ratios, the relative spins of the two SD bands have been determined. These data provide further support for the hypothesis that this excited SD band corresponds to rotations built on an octupole-vibrational configuration. RPA calculations reproduce the experimental data very well, and suggest that the octupole phonon is rotationally aligned in the observed range of spins.

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REFERENCES


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FIGURES

FIG. 1. Sum of combinations of double coincidence gates yielding events in which the excited SD band was populated. The pairs of coincidence gates consisted of combinations of transitions in the excited SD band with other transitions in the excited band, plus transitions in the excited band with transitions in the lowest SD band.

FIG. 2. Proposed level-scheme of SD bands in $^{199}$Hg.

FIG. 3. DCO ratios for selected transitions in $^{199}$Hg, as defined in the text. Circles, squares and triangles are used, respectively, for E1 transitions between normally deformed states, E2 transitions between states in the lowest SD band, and transitions between the two SD bands.

FIG. 4. Comparison of observed and calculated properties of SD bands in $^{199}$Hg versus rotational frequency, $\hbar\omega$: (a) dynamic moment of inertia $\mathcal{I}^{(2)}$; (b) difference, $E'-E'(\text{lowest SD})$, between Routhians of excited bands and that of the lowest SD band, where $E'\equiv E-\omega_\pi I_\pi$. Both vibrational and nonvibrational states are calculated in the RPA, but only the lowest four excited states of both signatures are shown here, and these are the vibrational states at low frequencies (see text for details). The theoretical $\mathcal{I}^{(2)}$ values were obtained by calculating the difference in $\mathcal{I}^{(2)}$ between the excited and lowest SD bands and adding a Harris polynomial as a reference. The coefficients of the Harris polynomial were $\mathcal{I}_0=82.6$ MeV$^{-1}$ and $\mathcal{I}_1=113.0$ MeV$^{-3}$, fitted to the levels of the lowest SD band up to the 728 keV transition.
<table>
<thead>
<tr>
<th>energy of transition (keV)</th>
<th>branching ratio of inter-band transition</th>
<th>partial half-life (fs)</th>
<th>B(E1) (W.u.×10⁻³)</th>
<th>B(M1) (W.u.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>911</td>
<td>&gt;0.5</td>
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<td>&gt;1.4</td>
<td>&gt;0.15</td>
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<tr>
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<td>0.35(4)</td>
<td>260(60)</td>
<td>1.2(3)</td>
<td>0.13(3)</td>
</tr>
<tr>
<td>812</td>
<td>0.29(4)</td>
<td>260(70)</td>
<td>1.5(4)</td>
<td>0.16(4)</td>
</tr>
<tr>
<td>757</td>
<td>&lt;0.3</td>
<td>&gt;200</td>
<td>&lt;2.4</td>
<td>&lt;0.26</td>
</tr>
</tbody>
</table>

**TABLE I.** Information obtained for the transitions linking the excited and lowest SD bands in this experiment. The branching ratio shown in the second column is the ratio between the out-of-band intensity and the total intensity deexciting a particular level, i.e. $I_\gamma(I+1 \rightarrow I) / [I_\gamma(I+1 \rightarrow I-1) + I_\gamma(I+1 \rightarrow I)]$. The partial half-lives and transition strengths are inferred from the data as explained in the text.