SEPTUM MAGNETS WITH 10 AND 20 KG FIELDS
FOR SLOW AND FAST EJECTION FROM THE CPS

by

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GENEVA
1965
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SEPTUM MAGNETS WITH 10 AND 20 KG FIELDS
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Summary

The following report summarizes the results of measurements and calculations on the new septum magnets (figs. 1 and 2) designed for both fast and slow ejection (resonant extraction system\(^1,2\)) from the CERN Proton Synchrotron (CPS). A first pair of magnets has been installed in the East Ejection Area in s.s. 58, and two further pairs (one for s.s. 62) are completed. One magnet reaches 10 kg with 0.6 \(^0\)\(^0\) stray field in front of the septum, the other 20 kg with -2.5 \(^0\)\(^0\) stray field.

1. Introduction

The ejection magnets can be considered as window frame magnets with C-shaped iron cores and a 20 mm air gap. The outer part of the coil consists of a current strip called septum which must be thin compared with the width of the window in order to obtain a high efficiency in the slow ejection process. On the other hand, it must carry a high current to obtain a high bending field which is maintained during the flat top of the CPS (which lasts up to 400 ms). The required bending strength in a short straight-section is 19 mrad at beam energies up to 28 GeV. A water-cooled septum has been designed which is 3 mm thick and carries a current of 16 kA at top field. At a repetition rate of 1 pulse every 2 secs this current can be maintained during 200 ms. The ejection magnet is split into two similar straight magnets which form an adjustable angle. The magnets are connected in series. The upstream magnet has 1 current turn, a 3 mm septum and a 10 kg nominal field. When the beam passes into the downstream magnet, it is already displaced. Therefore the downstream magnet is designed with a thicker septum (6.1 mm) which consists of 2 current turns. This magnet attains 20 kg.
Particular effort was taken to reduce the leakage field in front of the magnet to the order of $10^{-3}$ of the main field, because the beam circulates in front of the septum before it enters the air gap in order to be ejected. Since the thickness of the septum was restricted to 3 mm no magnetic screen could be used and all screening of the main field was done by the current sheet itself. The height of this current sheet is therefore carefully adjusted. Finally, when the leakage fields along the magnet had been reduced sufficiently, most of the stray fields were found near the ends of the magnets. A modification of the ends of the septum allowed to obtain positive and negative stray fields at the ends which to some extent cancel each other in their effect on the beam.

2. The principle

Fig. 3a shows a sectional view of a window-frame magnet, which is known to have a particularly uniform field since the field does not decrease near the coils. Cutting this magnet along the vertical plane of symmetry we obtain a C-type magnet, fig. 3b.

In order to maintain a uniform field in the gap the boundary conditions along the cut line must not be changed. By symmetry, there is no transverse field component on this line. Therefore one can satisfy
these boundary conditions placing current sheets with appropriate surface current densities on the line of cut. Only if the surface current density (dimension A/m) at any point of the current sheet equals the tangential field on its left surface, the right half space will be free of stray fields as required especially for the slow ejection process. Any departure from this ideal current distribution is considered as a source of perturbation. It can be calculated separately. Since the field in iron is $\mu$ times smaller than in air ($\mu = \text{rel. permeability}$) the total current flowing in part b is small compared with the current in part a (for $\mu = \infty$ it vanishes). Thus we would obtain a fair correction if we would replace these current sheets by simple correction windings in which the same current is concentrated. Unfortunately, the permeability changes a good deal when the fields are high and the correction currents should increase more than linearly with the main field. Therefore first we tried to reduce the necessary correction using very soft steel and a sufficiently large flux path. Then we paid particular attention to choose the correct height for the septum (see chapter 6).

3. Description of the iron core

Fig. 4 shows one block of the iron core. Its shape is identical for the 10 and the 20 kGauss magnet. The laminations for both magnets were cut with the same tool. However, two different magnetic materials have been employed. The flux path in iron is 5 cm large and larger than the flux path in the air gap. If the inner parts of the iron core tend to saturate, the flux can flow through the outer parts (no negative shims near the inner conductor). In view of a later application in which the magnet is to be moved rapidly into its position during the acceleration cycle, the weight of the core had to be small and the core was not enclosed in a supporting case but self-supporting and suspended on the ends. In order to allow for high flux density the steel of the 20 kG magnet is a 50% Co-alloy, "Vaccflux 50", which saturates at 24 kG and has $\mu_{\text{max}} = 8000$ at about 13 kG. The upstream
part of the ejection magnet which has 10 kG, has a much smaller stray field since this core is made of a 50% nickel alloy, "Permenorm 5000 H3", which saturates at 15.5 kG. Only prototypes were annealed and laminations assembled at CERN (from a slightly different material). On the final magnets annealing and block assembly were left to the steel manufacturer (Vacuumschmelze Hanau) who specifies a permeability of $\mu_{\text{max}} = 35000$ which is largely sufficient.

The laminations are 0.5 mm thick and insulated by oxyde layers to prevent distortion of the hysteresis curve by eddy currents during the pulse rise-time (30 msecs) and to prevent bypass of current through the magnet core which is not insulated from the septum. They are glued together with silicon resin (Rhodorsil) so that they form 5 cm blocks (fig. 4) which finally are baked in an oven. Since the saturation induction of 50% Co steel is not twice that of 50% nickel steel but only 24 kG : 16 kG, we could not pass twice the flux through the second magnet core. Therefore we increased the section of the inner conductor in the 20 kG magnet, thereby reducing the spacing between the inner conductor and the septum to 3 cm, while in the 10 kG magnet it is almost 4 cm.

4. Stray fields related to hysteresis loop

![Diagram](image)
In fig. 5 the induction in the conductors and the air gap are sketched. Hence the flux \(^1\) between the pole-faces is:

\[
\Phi = B \left( \frac{d}{2} + \frac{d_2}{2} \right) = (\mu_0 H) w_g
\]

(1)

where \( w_g \) is the mean width of flux path in the gap. Therefore the reluctance of the gap is

\[
R_g = \frac{1}{\mu_0} \cdot \frac{g}{w_g} \quad (g = \text{height of gap})
\]

(2)

and the magnetomotive force across the gap

\[
I_g = \Phi \cdot R_g \quad \text{(Hopkinson)}
\]

(3)

The actual values of our magnets are (20 mm gap):

<table>
<thead>
<tr>
<th>10 kG magnet :</th>
<th>( w_g = 4.6 ) mm</th>
<th>20 kG magnet :</th>
<th>( w_g = 41.5 ) mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R_g = \frac{0.435}{\mu_0} )</td>
<td>( (4a) )</td>
<td>( R_g = \frac{0.432}{\mu_0} )</td>
<td>( (4b) )</td>
</tr>
</tbody>
</table>

If we consider now the reluctance in the iron which is much smaller, we must notice that the equipotentials in iron do not coincide with the iron boundary and change with permeability. Fig. 6A and B show potential end flux lines in the case of \( \mu = \text{const.} \), which were determined by means of two analogue models on resistive paper. In one model \(^2\)

\(^{1}\) flux and reluctance in a 2-dimensional field refer to unit length of magnet.
the voltage corresponded to flux, in the other it corresponded to magnetic potential. In both cases the boundary conditions are: flux is entering and leaving the iron nowhere else but through a section of width \( w_g \) of the pole-faces where the normal component of flux density is constant. The magnetomotive force \( I_c \) between the two outer corners of the pole-faces divided by the total flux \( \Phi \) may be defined as core reluctance

\[
R_c = \frac{I_c}{\Phi} \quad (5)
\]

It was determined from the resistance of the analogue model:\(^1\)

\[
R_c = \frac{3.42}{\mu \mu_0} \quad (5a)
\]

This magnetomotive force \( I_c \) is the cause of a leakage flux in front of the septum. From equs. (5) and (5) one obtains the ratio of magnetomotive forces on a path outside and inside the gap (or septum respectively):

\[
\frac{I_c}{I_g} = \frac{R_c}{R_g} \quad (6)
\]

Their sum equals the total number of current turns:

\[
nI = I_c + I_g \approx I_g \quad (7)
\]

Therefore the relative coercive force along the outer flux path, the "demagnetizing force" is

\[
\frac{I_c}{nI} \approx \frac{R_c}{R_g} \quad (8)
\]
Substituting from (5a) and (4a) or (4b) respectively, we find for our magnets:

\[
\begin{align*}
I_c &= \frac{7.3}{\mu} \quad (8a) \\
I_c &= \frac{6.6}{\mu} \\
\end{align*}
\]

\textbf{10 kg magnet} : \hspace{2cm} \textbf{20 kg magnet} :

\( I_c \) causes a negative field which is considered as a perturbation, because inside the gap it reduces the main field in a non-uniform way while outside it appears as a negative leakage field. Fig. 7 illustrates this leakage field by a computed field pattern.

The calculation is based on the assumption of \( \mu = \infty \) on the polefaces between which there is a given magnetomotoric force \( I_c \). The gap which is less interesting is supposed to extend far to the left \( (x \rightarrow -\infty) \) but the vertical extension of the iron boundary is limited to 6 cm., and also the effect of the "nose" at the corner has partly been considered. Although this nose has the effect of a shim (making the main field more uniform), its purpose is rather to keep the septum in place, when magnetic forces tend to push it outwards. By differentiation of the magnetic potential with respect to the coordinates the field \( \frac{dw}{dz} \) is obtained. The field on the midplane (x-axis) of the magnet is plotted in fig. 9. For details of calculation see Appendix 1. These results have also been checked on a third analogue model of resistive paper which yielded the field pattern in fig. 8 and the curve in fig. 10 taken from an internal report\(^4\). This analogue model, although suffering from anisotropy, includes more details of the actual magnet contour and considers varying magnetic potentials on the iron boundary. These potentials are known from the first analogue models (representing the iron core) and have been imposed on the boundary via voltage dividers of low resistance. The field curve which should have the value 1 in the gap allows some conclusion on the non-uniformity of field in the gap. Of course the curves in fig. 8 or fig. 10 have to be multiplied by the factor

\[
\frac{I_c}{nI} = \frac{7.3}{\mu} \quad \text{and} \quad \frac{I_c}{nI} = \frac{6.6}{\mu}
\]
\text{respectively.}
This factor can be regarded as the ratio between the unit perturbation field \( H_c \) and the main field \( H_g \) in the gap. If \( H_g \) is plotted versus \( H_c \) the curve resembles the hysteresis curve (axes interchanged) of the magnetic steel employed. If one assumes that at medium field the "average induction" \( B \) in iron is the same as in the gap, \( H_g \) corresponds to the B-axis of the hysteresis curve (fig. 11) while \( H_c \) divided by 7.3 and 6.6 respectively corresponds to the H-axis. One must however refer to the hysteresis loop ABCA which applies when \( B \) is never inverted. It starts in point A, where the negative abscissa is given approximately by the nominal coercivity or the remanent field \( H_o \) divided by our factor. Then, when the hysteresis curve intersects the B-axis in point B, the stray field vanishes and \( \mu \) appears to be infinite. At higher \( B \) the stray field becomes negative (i.e. opposed to the main field). Beyond the "knee" of the curve the coercive force as well as the leakage field increase progressively. The return branch CA of the hysteresis loop is of little interest, because the ejection will be terminated when the magnet is switched off. Obviously this leakage field is not a constant fraction of the main field.

5. **Description of the coil with septum**

The magnet coil which has 1 and 2 turns respectively consists of the inner conductor, the septum and the coil overhang or end connections between them. The pulsed septum has not only the hottest spots of the magnet but particularly in the upstream magnet it is also the part which is most exposed to radiation. Both effects would affect organic insulating materials and also increase outgassing when the magnet is pulsed inside the vacuum tank. Therefore the septum in the upstream magnet is not insulated from the iron core. A bypass of current through the magnet core is prevented by an insulating oxide layer between the laminations which is produced at the end of the heat treatment.

In the downstream magnet in which the septum consists of two current sheets only the inner current sheet is insulated.

There is a considerable force on the septum and it suggested
a design with a few turns only. The pressure is given by:

\[ p = \frac{B^2}{2 \mu_0} \]

With \( B = 2 \text{Tb/m}^2 \) we find 16 kg/cm\(^2\) on a surface of 2 x 52 cm\(^2\). The force is supported by the "noses".

The major problem was the high current density which amounts to \( j = 320 \text{ A/mm}^2 \) at nominal field and warms up the copper so that its average resistivity increases to about \( \rho = 2 \mu \Omega \text{ cm} \). Then the dissipated power per volume of copper is:

\[ P' = j^2 \rho = 2 \text{kW/cm}^3 \]

while the heat capacity of copper is only 0.815 cal/cm\(^3\) degree = 3.4 Ws/cm\(^3\) degree. So the temperature rises steeply from 20\(^\circ\)C to 70\(^\circ\)C - 90\(^\circ\)C during repetitive pulses. One had to provide for free heat expansion of the septum, for which the rabbets inside the noses represent a kind of slide bearings. Special attention was paid to assure smooth bearing surfaces to reduce wear by friction. In addition, the septum is protected by a chromium layer of 0.03 mm. At the ends the connections between septum and inner conductor are elastic: 16 parallel copper laminations are soldered into cooled blocks on the septum and inner conductor. Now the septum can expand independently. An additional looped stainless steel pipe (cf. fig. 12) of high resistance conducts the water coolant.

The septum of the magnets consists of one, and respectively, two similar thin copper conductors shown in fig. 13. They contain small rectangular holes of 1 mm x 1.23 mm in which water flows under pressure. The holes are distributed evenly, with a distance of 2.5 mm. There are however not 20 : 2,5 = 8 holes but only 7. The 8th hole is intersected by the polefaces which in principle act as magnetic mirrors.
so that the pattern seems to repeat to infinity. The holes in the conductor produce a similar perturbation as if there were a grid of wires with the same current density but opposite direction of current.

The calculation of this stray field (fig. 14) yields for both magnets 37 Gauss on the surface and an exponential decrease by a factor of 10 for every 0.92 mm of distance. We could not measure it*). It is more important that the septum fits correctly between the polefaces, if not, most of the stray field is due to this effect.

Since there must be some clearance to facilitate mounting, the dummy hole (number 8) was provided for compensation: the amount of lacking copper section was filled into this groove so that on an average the surface current density in the septum remains equal to the field. This compensation was done in particular on the inner current sheet of the magnet with 2 turns to provide space for a thin mylar insulation (fig. 13 b). The corners were rounded to reduce wear and improve breakdown strength (tests with up to 1500 V were made to assure safe operation with 500 V peaks). The copper section is 50 mm².

These conductors were electroformed: First, the 7 rectangular grooves were milled about 1 mm deep in a conductor of 2 mm thickness. The grooves were filled with a metal of low melting point, indium, and the surface smoothed. Then electrolytic copper was deposited and the conductor machined to its final dimension. Finally, the conductor

*) In a similar way one can find that the perturbation due to the holes in the inner conductor of the 10 kG magnet is 1% on the surface and 10/60 at 3,7 mm distance. In the 20 kG magnet it is much smaller.
was heated, the melting indium taken out and the holes rinsed with dilute hydrochloric acid. The inner conductor of both magnets consisted first of 2 square pipes of about 1 cm with 6 mm circular inner holes. In the magnet with 2 current turns the width of the inner conductor has been increased to 17 mm.

6. **Stray fields related to geometry of septum**

   If the 8th hole, i.e. the grooves, in the septum are not filled correctly, an excessive positive or negative current flows in the corner behind the noses and returns by the inner conductor as in a correction winding. While the field of a correct septum remains in the air gap, this stray field comes out. For this case the field pattern in Fig. 15 has been calculated for illustration (see Appendix 2). It is different from the one in Fig. 7 and attains high values near the corners. By differentiation of the magnetic potential one finds again the field. Its values on the midplane are plotted in Fig. 16. The curve falls off more steeply than that in Fig. 9. When the perturbation current is positive (grooves filled, septum too high) the field in the air gap is positive and can compensate for the negative field due to the core reluctance. In Fig. 17 the two types of stray fields are superimposed as in the case of an actual magnet. The curve with index 0 represents the negative stray field due to core reluctance only. The curve is normalized so that it attains the value -1 inside the air gap. One may fill the groove so that it produces a positive stray field which becomes +1 in the air gap. Superimposing these two fields one obtains curve 1. With an even larger positive stray field one obtains curves 2, 3 etc. which are more favourable because the two types of stray fields compensate at a certain distance, whereas the curves with negative index (grooves too deep, septum too small) are unfavourable. Unfortunately, the ratio of strength of the two types of stray fields is not fixed: the stray field of the first type changes according to the hysteresis loop while the stray field of the second type represents a constant fraction of the main bending field. Numerical example: if the grooves are not deep enough by 0,2 mm\(^2\) in a septum of 50 mm\(^2\) copper section this relative aberration of \(2 \times 0,2 \, \text{mm}^2/50 \, \text{mm}^2 \times 100 = 8 \%\) produces (according to Fig. 16) at 4 mm distance a stray field of 0,15 \%\(8 \% \times 1,2 = 1,2 \%\) of
the main field.- From fig. 15 one may conclude that the field variations inside the air gap which concern the main field are smaller than the stray field so that a reduction of the latter improves the former.

7. Stray fields from the ends of the magnet

Finally, when the stray field in front of the septum was of the order of $1^\circ/oo$ of the main field in the 20 kG magnet, and less in the 10 kG magnet, the measurements with an integrating long coil yielded still higher stray field values and it turned out that fringing fields at the ends had to be corrected. During the design of the prototypes we only minded not to make the coil overhang too large, because if the current in the end connection would flow in a wide loop, it would produce additional flux which would not return by the iron yoke but partly by the air path in front of the magnet. The situation near the end of the magnet (see fig. 20) can be explained in the following way:

![Diagram of the magnet core and septum current](image-url)
If one wants the field in front of the magnet to be zero, there
must be a current sheet (septum) similar to that in fig. 3b which satis-
ifies the boundary conditions. Then one maintains the same field as
found on the vertical plane of symmetry in fig. 3a, the only difference
being, that now we have to do not only with the uniform bending field
but also with the fringing field on the end which extends over some
distance beyond the end of the magnet core, and that the current density
must vary locally in the septum (fig. 20). Moreover, the ideal fringing
field just behind the current sheet should be two-dimensional and should
not traverse the current sheet. It is the same fringing field as on
the vertical plane of symmetry in the window-frame magnet of fig. 3a
with a window 2 x 8 cm and can be calculated approximately as a two-
dimensional field (Appendix 3). A plot of the end-field on the mid-
plane is shown in fig. 18.

If the current sheet would extend to infinity and would be
bounded in the same way as the field region is bounded by the pole-faces,
the pattern of current distribution would automatically be the same as
the two-dimensional field pattern, and no stray field would traverse the
current sheet. The end conductors which connect the current sheet
(septum) to the inner conductor are current sinks and the magnetic field
curls around them. However, the septum cannot extend far and is verti-
cally cut off. In the 20 kG magnet it consists actually of two over-
lapping current sheets one of which is connected with the upper, the other
with the lower end conductor, fig. 2. Calculations show that therefore
the current density is too high compared with the field on the midplane
(fig. 19). The difference corresponds to a perturbation current which
produces a negative stray field in front of the current sheet which
prevails over the smaller positive stray field that comes out beyond
the end of the current sheet. Therefore we increased the positive stray
field and reduced the negative stray field, cutting shorter the end of
the septum so that we obtained a compensation. An additional cause of
a negative stray field is the increased flux in the end blocks which
tends to saturate the iron, so that the field becomes too small compared
with the current density. In the 10 kG magnet where the septum consists
of a single current sheet the current density is better matched to the
field and there is less of a stray field. Actually our septum at the end is not a uniform current sheet because it contains water ducts and the current, when turning from the septum into the end conductors, does not immediately flow perpendicularly to the septum, as we would need a conductor material of high resistivity to refract the current lines. On the contrary, in our end conductor we have only 16 parallel copper laminations and holes for an intricate water circuit but qualitatively the explanation seems to be still valid.

8. Field measurements

Almost all field measurements were made with short current pulses of 100 ns including 20 ns rise-time and up to 16 kA from a laboratory power supply with 6-phase rectified current but no current-regulation. This limited the precision of measurements. In the ejection area the magnet current is 4-phase rectified and controlled to a precision of ± 2%. Fig. 21 shows a typical short test pulse. We used search coils of 0.3 m and 1.3 m length with an effective cross-section of about 4 cm x 4 cm. The signal was integrated and displayed on the vertical axis of an oscilloscope versus magnet current (shunt voltage) on the abscissa. 16 kA on the abscissa correspond to a main field of 10 kG and 20 kG respectively. So the "strayfield-hysteresis loop" could directly be observed (e.g. figs. 23, 24).

a) 20 kG magnet

Fig. 22 shows a plot of the main field in the magnet centre as measured with a Hall probe (Siemens FC 33) versus current. If the permeability would be infinite one would expect for two turns in an air gap of 2 cm:

\[
\frac{B}{I} = \frac{4\pi}{10} \cdot \frac{\text{kg}}{\text{kA}} = 1.256 \frac{\text{kg}}{\text{kA}}.
\]

Actually, at top field we found 3% less in all three 20 kG magnets. The effective length is 0.535 m ± 0.3% (length of the iron core 0.520 m).
Fig. 23 and 24 show stray field displays on an oscilloscope picked up with the 0.3 m search coil the centre of which was 5, 10 and 20 mm distant from the outer surface of the septum. The resemblance to a hysteresis loop is obvious although the quantitative relation between this curve and the hysteresis loop is not exactly linear since the flux distribution in iron changes with permeability. Since coil measurements do not include the remanent field, the base line is uncertain. Therefore, additional measurements on this magnet were made with a Hall probe supplied with 3000 cycles A.C. current. The series of photos in fig. 25 were taken at various distances and show the absolute value of stray field during rise and flat top of a pulse (the returning trace is suppressed). One can see that up to 12 kA (corresponding to 15 kG) the stray field is small, then it increases progressively. It is plotted versus distance in fig. 26. Comparing this plot with calculated curves in fig. 17 one notices that the negative maximum of stray field is due to the two types of stray fields which compensate near the septum. One can also conclude that the magnetic steel really had the favourable properties given in fig. 11. Unfortunately, the second and third magnet cores (delivered later) were not so good and had about 8 Gauss remanent field. The slope of the stray field curve in fig. 24 is more negative than in fig. 23. These stray fields are very low compared with the 20000 Gauss main field in the gap. However, near the end the stray field attained a negative peak of -3.5% which was not yet compensated by the adjacent positive peak. (In s.s. 58 this magnet has been combined with an upstream magnet with positive stray field for compensation so that the integrated stray field along a trajectory at 4 mm distance became 24 Gauss m compared with 17900 Gauss m inside the gap (fig. 27)). In the following 20 kG magnets the end stray field has been corrected by separately cutting shorter the end of the septum by 9 mm. Fig. 26 and fig. 28 show end stray fields (measured with 0.3 m coil) before and after applying the correction to the second magnet. The initial slope of the stray field curve has become almost zero but at higher currents the negative stray field increases because the iron of the end blocks tends to saturate.
earlier than the rest of the magnet. One could improve future designs rounding the corners as well as the septum which must fit exactly between the iron boundaries. The integrating coil extended over 5 cm of the magnet core and 25 cm beyond its end. More details of this field at 4 mm distance from the septum are shown in fig. 30 where it is plotted along the magnet. One can see (curve 9.5 kG) that the stray field is negative near the end of the iron core and positive beyond the end of the septum where the main fringing field comes out. This situation was explained by the curves in fig. 19. Though positive and negative fields compensate at 9.5 kG they do not compensate at 20 kG when the iron partly saturates and the curve is shifted downwards.

Integrating measurements of stray field along the whole 20 kG magnet are shown in figs. 31 and 32. Dividing the integrated field by the effective length of the magnet one obtains the average stray field. One may compare with fig. 24 which concerns the same magnet. Obviously the end stray field contributes largely to the average. The average stray field is plotted versus distance in fig. 33. The values are still of the order of 2-4 0/60 of the main field. The remanent field did not decrease, i.e. the magnet was not demagnetized significantly even after several thousand pulses of lower level. Therefore the rising branch of the hysteresis curve was practically not affected by different cycling, as long as the field was not inverted. Therefore only few measurements were made with smaller pulses, but there seems to be a slight distortion of the loop by eddy currents during the steep rise of the magnetic field.

b) 10 kG magnet

The iron core of this magnet is 0.70 m long and the measured effective length 0.723 m. A plot of main field versus current is given in fig. 34. The advantage of the 20 kG ejection magnet lies in its high bending strength but the 10 kG magnet with 3 mm septum and larger window has less stray field because of the high permeability of Ni iron.
This can be seen in figs. 35 and 36 which show stray field loops measured (on the best magnet) with a short 0.3 m search coil at various distances from the septum. The result is much better than in figs. 23 and 24 for the 20 kG magnet. Additional measurements with an AC-supplied Hall probe show the absolute value of the stray field during rise-time and flat top of the 10 kG pulse (fig. 37). In fig. 38 the result is plotted versus distance so that one can draw a comparison with fig. 17 and fig. 26. At some distance from the magnet the curves might be distorted by the terrestrial field (up to 0.3 Gauss) but this is less important since the remanent field around the synchrotron magnets is even higher.

A simple criterion to distinguish between the share of the stray field related to permeability from that related to the geometry of the septum is that the latter becomes negligible at distances greater than 10-15 mm whilst at smaller distances it makes the initial slope of the stray field loop (figs. 35, 36) more positive. The average stray field in front of the whole magnet measured with a long coil is of course higher since it includes end effects. This can be seen on the photos in fig. 39 and the plot in fig. 40. We also displaced the coil vertically and measured a somewhat higher vertical stray field near the "noses". The end stray fields in the magnet with one turn are fortunately smaller than in the magnet with two turns. This has already been explained by the different shape of the current sheets. In one magnet the septum was shifted (by maximally 1 mm) towards one end. Therefore the integrated stray field at one end became more negative and at the other end more positive (fig. 41 and fig. 42).

Fig. 43 shows a typical plot of a stray field near one end of another septum which was better centred so that positive and negative stray fields are about equal.

A horizontal component of a stray field should not exist on the midplane if the magnets were perfectly symmetric to this plane. In the 20 kG magnet the horizontal component of the
stray field was actually quite small but some representative measurements with a long 13 m coil on the 10 kG magnet seem to be interesting and are shown in fig. 44: since the field lines in fig. 15 and fig. 17 are curved one can understand why the horizontal component of the stray fields above and below the midplane has an opposite sign. However, the curve on top is not simply the image and resembles the lower curve not very much because it turns out that during the flat top of the pulse (80 ms) all the curves are shifted or turned downwards. The effect is most evident on the midplane (central photo in fig. 44). This horizontal stray field has nothing to do with permeability and decreases rapidly with distance from the septum. It can be expected (and calculated neglecting the noses\(^3\)) if the septum moves upwards. The effect is proportional to the septum temperature, i.e. it increases linearly with the pulse length and almost quadratically with the current (fig. 45). The temperature change from 20\(^\circ\)C - 80\(^\circ\)C causes not only a longitudinal dilation but also a change in the height of the septum.

\[
\Delta h = a (t - t_o) \cdot h = 17 \cdot 10^{-6} (80^\circ - 20^\circ) \cdot 20\text{mm} = 0.02\text{mm}
\]

If the septum rests initially on the lower poleface and expands upwards its centre moves by 0.01 mm, i.e. 0.5\(^\circ\)/oo of the height of the septum and this results in about 0.2\(^\circ\)/oo = 2 Gauss change in the horizontal stray field. This effect has been observed only in the 10 kG magnet which had the smallest stray field. The stray field was also affected by mechanical tolerances.
9. **Measurements of temperature**

The total cross-section of the holes in one of the thin conductors is 8.6 mm$^2$, whilst its outer dimensions are 3 x 20 mm$^2$. When a pressure of 15 at/m is applied to a single conductor, the water flow is about 3.5 l/min, which means that the water in the holes has a speed of 6.8 m/sec. Therefore Reynolds' number is high and heat transfer quite efficient. The entire magnets need a pressure of 20 at.

Fig. 46 shows on top the increase and decrease of the temperature of water immediately after leaving the septum of the 20 kG magnet in the case of a current pulse on the bottom of 15 kA and 200 ms duration (including 20 ms rise-time). The time base is 0.5 sec/square, and the temperature scale is 12°C/square which correspond to 0.5 mV/square signal from a chromel-alumel-thermocouple. The initial temperature was 20°C. The septum temperature has also been measured with thermocouples. It is about the same as the water temperature shown on the photos. At the other end of the magnet, where the water enters, the temperature of the septum is close to 20°C.

The magnet can also be de-operated. Tests were made with 5300 A d.c. corresponding to 6.6 kGauss. Fig. 47 shows on top: when the current is switched on the temperature rises with essentially the same time constant as it falls during cooling in fig. 46. On the bottom of fig. 47, the dc current is shown for reference (2500 A/cm). It declined because the current in our laboratory power supply was not regulated and the total load resistance increased with temperature after switching on. Further examples in fig. 48 in which the time base is 0.1 sec/square show also the time delay in temperature measurements (and a perturbation on the 1 mV signal at the end of the magnet pulse). If a maximum temperature is exceeded these thermocouples via an amplifier and relay switch off the current of the magnet. From measurements in fig. 46-48 the limits on current pulses of various lengths and repetition rates from 1 to 5 sec have been derived (fig. 49). In de-operation the inner conductor is warmer than the septum. With respect to outgassing of the insulating material the limit was set to 5.3 kA.
10. Magnet parameters

The following table summarizes the main parameters of the magnets:

<table>
<thead>
<tr>
<th></th>
<th>upstream magnet</th>
<th>downstream magnet</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nominal field</td>
<td>10 kG</td>
<td>20 kG</td>
</tr>
<tr>
<td>Nom. bending strength</td>
<td>0.723 Wb/m</td>
<td>2x0.535 = 1.07 Wb/m</td>
</tr>
<tr>
<td>Number of current turns</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Current for nom. field</td>
<td>16.1 kA</td>
<td>16.4 kA</td>
</tr>
<tr>
<td>Air gap</td>
<td>(20 - 0.04) mm</td>
<td></td>
</tr>
<tr>
<td>Length of iron core</td>
<td>700 mm</td>
<td>520 mm</td>
</tr>
<tr>
<td>Useful window</td>
<td>19 mm x 38 mm</td>
<td>19 mm x 29 mm</td>
</tr>
<tr>
<td>Thickness of septum</td>
<td>3 mm</td>
<td>6.1 mm</td>
</tr>
<tr>
<td>Water flow (demineralized)</td>
<td>4 - 5 lit/min</td>
<td>7 - 8 lit/min</td>
</tr>
</tbody>
</table>

ACKNOWLEDGEMENTS

Many people were involved in this work who cannot all be mentioned. Within the MPS/RF Group the author thanks first of all Mr. H. Fischer and Mr. U. Jacob for their support and many helpful discussions; further Mr. R. Stierlin, and Mr. W. Wünsche who made all the drawings of the magnets right from the beginning, Mr. R. Godet and his collaborators who succeeded in electroforming the septum (we first tried out another septum without internal water ducts), and finally many people mostly from the Atelier West among whom Mr. G. Vuffray and Mr. E. Nicolle carried out the final assembly of the magnets and the vacuum tank. Mr. G. Prinadel aided me during measurements. My thanks are also due to Mr. W. Remmer for making the last computer programme.

H.-H. Unstüttler

Distribution: (open)

MPS Scientific and Technical Staff
RF Group

PS/5016
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Appendix 1 (Field calculation)

Although similar calculations were presented before starting the design of the magnet, the following calculation is more complete since it includes partly the effect of the "noses" on the perturbation field and considers the fact that the vertical iron boundary does not extend to infinity. Since the magnet is long compared with the air gap, the field is essentially two-dimensional and can be calculated by the method of conformal mapping. The contour ABCDD'A' of the half gap (BC corresponds to the "nose") is mapped

![Diagram](image)

Fig. 50

from the z-plane into the upper half t-plane. The Schwarz-Christoffel differential equation of this conformal transformation is

$$\frac{dz}{dt} = f (t + t_1) t^{-1/2} (t - 1)^{-1}$$  \hspace{1cm} (1)

The factor $f$ will be determined afterwards to normalize the height of the gap. We integrate
\[ z = f \int_0^t \frac{t + t_1}{t^{1/2}(t-1)} \, dt = f \int \frac{\frac{dt}{t^{1/2}}}{2t^{1/2}} + f(1 + t_1) \int \frac{dt}{t^{1/2}(t-1)} \]  

(2)

In the second term we substitute

\[ t^{1/2} = r \]  

(3)

\[ \frac{dt}{2 \cdot t^{1/2}} = dr \]  

(4)

and obtain

\[ f(1 + t_1) \int \frac{2 \frac{dr}{r^2 - 1}} {r - 1} = -2f(1 + t_1) \tanh^{-1} r = -2f(1 + t_1) \tanh^{-1} (t^{1/2}) \]  

(5)

and

\[ z = 2f(1 + t_1) \left[ \frac{t^{1/2}}{1 + t_1} - \tanh^{-1} t^{1/2} \right] \]  

(6)

Real values of \( t < 1 \) correspond to the pole-face and \( t > 1 \) to the midplane of the magnet. Integrating \( dz/dt \) along a small half-circle around \( t = 1 \) in the upper half-plane we find that in the expression for \( z \) (equ. 6) only \( \tanh^{-1} t^{1/2} \) is varying by \( + \pi i/2 \). This variation corresponds to the gap height. Therefore we chose the factor \( f \) to be such that the height of the gap becomes 1:

\[ z = \frac{2}{\pi} \left[ \frac{t^{1/2}}{1 + t_1} - \tanh^{-1} t^{1/2} \right] \]  

(8)

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Then we have

\[ f = \frac{1}{\pi \left(1 + t_1\right)} \]  \hspace{1cm} (7)

\[ \frac{dz}{dt} = \frac{1}{\pi (1 + t_1)} \cdot \frac{t + t_1}{t^{1/2} (t-1)} \]  \hspace{1cm} (9)

The parameter \( t_1 \) determines the "nose". For \( t = -t_1 \) we obtain

\[ z_1 = \frac{2i}{\pi} \left[ \frac{(-t_1)^{1/2}}{1 + t_1} - \text{tanh} \left(-t_1\right)^{1/2} \right] \]

\[ = iy_1 = \frac{2i}{\pi} \left[ \frac{t_1^{1/2}}{1 + t_1} - \arctan \left(t_1\right)^{1/2} \right] \]

\((i^2 = -1)\)

From this formula one can calculate \( t_1 \) by iteration if the size of the "nose" is given. Now we assume that we have the following potential distribution on the boundary: the potential is zero on the midplane \( D'A' \) and \( 1 \) on the iron boundary along \( DCBA_1 \). Beyond \( A_1 \) we assume that the field lines go vertically into free space along the line \( A_1A \) on which the potential drops steadily to zero. Therefore the potential in the t-plane is

\[ v = 0 \quad \text{for real} \quad t > 1 \]

\[ v = 1 \quad \text{for real} \quad -t_2 < t < 1 \]

\[ 0 < v < 1 \quad \text{for real} \quad t < -t_2 \quad \text{and} \quad \frac{dv}{dt} \text{ real} \]

In order to satisfy these boundary conditions we can use a further Schwarz-transformation to map a semi-infinite strip with uniform potential distribution in the \( v \)-plane into the \( t \)-plane in such a way that we obtain the required potential distribution (\( \nu = \text{complex potential function} \), see fig. 51).
The Schwarz differential equation is now:

$$\frac{dw}{dt} = -f \cdot (t + t_2)^{-1/2} \cdot (t-1)^{-1} \quad (10)$$

The factor $f$ has to be determined afterwards. We integrate:

$$w = -f \cdot \int_0^t \frac{dt}{(t+t_2)^{1/2} (t-1)} = -f \cdot \int \frac{dt}{(t+t_2)^{1/2} [(t+t_2) - (1+t_2)]} \quad (11)$$

Substitution:

$$\left(\frac{t + t_2}{1 + t_2}\right)^{1/2} = \alpha; \quad d\alpha = \frac{dt}{(1+t_2)^{1/2} \cdot 2(t+t_2)^{1/2}} \quad (12)$$

$$w = -f \frac{2 \alpha}{(1+t_2)^{1/2}} \frac{d\alpha}{\alpha^2 - 1} = \frac{2f}{(1+t_2)^{1/2}} \ -1 \ \tanh \alpha \quad (13)$$

$$w = u + iv = \frac{2f}{(1+t_2)^{1/2}} \ -1 \ \tanh \left(\frac{t + t_2}{1 + t_2}\right)^{1/2} \quad (14)$$
We choose the factor \( f = (1+t_2)^{1/2}/\pi \) so that the potential \( v \) varies between 0 and 1 while \( u \) measures the flux:

\[
w = \frac{2}{\pi} \tanh^{-1} \left( \frac{t+t_2}{1+t_2} \right)^{1/2}
\]  

\( (15) \)

\[
\frac{dw}{dt} = \frac{-(1+t_2)^{1/2}}{\pi(t+t_2)^{1/2}(t-1)}
\]

\( (16) \)

The inverse function is:

\[
t = (1 + t_2) \tan^2 \frac{\pi}{2} w - t_2
\]

\( (17) \)

which can be substituted into eqn. ( ) so that we have explicitly \( z(u+iv) \). This result was programmed directly in complex arithmetic for the IBM 7090 to obtain the illustration in fig. 7. Differentiating the potential \( w \) with respect to \( z \) we obtain the field from eqn.(9) and eqn. (16) we have

\[
\frac{dw}{dz} = \frac{dw}{dt} \frac{dt}{dz} = -\frac{(1+t_1)}{(t+t_1)} \left( \frac{(1+t_2)t}{(t+t_2)} \right)^{1/2}
\]

\( (18) \)

For real \( t > 1 \) one obtains points \( z = x(t) - i \) on the midplane of the magnet in which the field is vertical and has real values \( dw/dz \) (fig. 9).
Appendix 2

For a second time we use eqn. (8) to transform the boundary into the upper \( t \) half-plane and assume that the parameter \( t_1 \) which determines the "rose" is known. For simplicity we let the vertical iron boundary extend to infinity so that we do not need to know \( t_2 \). In addition, we now have to determine a third point \( t = -t_3 \) which is the image point of the groove in the septum (point \( z_3 \) in the \( z \)-plane).

If the copper section in point \( z_3 \) is too great, there is a positive perturbation current around which the magnetic field curls. This current returns by the inner conductor which is supposed to be at the extreme left in \( D, D' \). So we also have to set up a similar curl in \( t = -t_3 \) and \( t = 1 \) with opposite signs in the \( t \)-plane (logarithmic potential):

\[
w(t) = \frac{1}{\pi} \log(t-1) - \frac{1}{\pi} \log(t+t_3)
\]

(19)

\[
w = u + iv = \frac{1}{\pi} \log \left( \frac{t-1}{t+t_3} \right) = \frac{1}{\pi} \log \left( 1 \frac{1+t_3}{t+t_3} \right)
\]

(20)
Again \( u = \text{const.} \) corresponds to field lines and \( v = \text{const.} \) to potential lines. The factor \( 1/\pi \) has been added in order to limit the variation of \( v \) to \( 0 \leq v < 1 \). The inverse function \( t(u) \) is

\[
t = \frac{l + t^2}{1 - \exp(\pi \cdot w)} - t_3
\]

(21)

and substituting it into \( z(t) \) one obtains again \( z(w) = z(u + iv) \).

This formula yielded the field lines \( u = \text{const.} \) drawn in fig. 15. The derivative of \( v \) with respect to \( t \) is (cf. equ. (19))

\[
\frac{dv}{dt} = \frac{1}{\pi} \left( \frac{1}{t-1} - \frac{1}{t+t_3} \right) = \frac{1+t_3}{\pi (t+t_3)(t-1)}
\]

(22)

So the field in a point \( z(t) \) is given by (cf. equ. (9)):

\[
\frac{dw}{dz} = \frac{dv}{dt} \cdot \frac{dt}{dz} = \frac{(1+t_3)^{\frac{1}{2}}}{(t+t_3)^{\frac{1}{2}}} \frac{1+t_3}{(t+t_3)}
\]

(23)

Here again the real part of \( (dw/dz) \) gives the vertical component of the field and the negative imaginary part the horizontal component of the field. The curve in fig. 16 is calculated with the formula given above for a septum of 3 mm thickness in which the groove (dummy 8th hole) in the septum is located under the poleface 1.5 mm behind the "nose". Fig. 15b illustrates a similar case in which the septum is thin and the groove is placed in the corner. Although in this calculation the vertical iron boundary extends to infinity this simplification has little effect because of the smaller range of this type of stray field.
Appendix 3  (Field and current density at the end of septum)

First we start with the calculation of the current density in the terminal of a septum consisting of 1 current sheet, or of two overlapping ones. Afterwards an ideal, two-dimensional fringing field is calculated which has no components perpendicular to the septum. Our surface current density which has the same dimension as the field \([A/m]\) should be matched to this ideal field. The difference between our actual current density distribution and the ideal one is regarded as a perturbation current density which produces the stray field near the ends of the septum. The septum of the first magnet (fig. 1) consists of a single current sheet which is symmetric to the horizontal median line (median plane of magnet) so that we need to consider only the upper half current sheet. The contour of such a current sheet can be simplified in the following way in the \(z\)-plane:

![Diagram](image)

**Fig. 53**

It can be mapped into the upper half plane by a Schwarz-Christoffel transformation \(^5\) \(^7\) which has the differential equation

\[
\frac{dz}{dt} = \left( \frac{l+b^2}{\pi} \right) \cdot (t+b^2)^{-1} \cdot t^{1/2} \cdot (t-1)^{-1}
\]

\[\text{norm. factor}\]
from which the transformation \( z(t) \) itself is obtained by integration:

\[
z = \left(\frac{1+b^2}{\pi}\right)^t \int_0^{t/2} \frac{1/t^2}{(t+b^2)(t-1)} \, dt
\]

(25)

Substitute:

\[
t^{1/2} = r
\]

\[
t = r^2
\]

\[
dt = 2r \, dr
\]

\[
z = \frac{2}{\pi} \int_0^r \frac{(1+b^2) \, r^2 \, dr}{(r^2+b^2)(r^2-1)}
\]

\[
= \frac{2}{\pi} \int_0^r \left(\frac{1}{r^2-1} + \frac{b^2}{r^2-b^2}\right) \, dr
\]

\[
= \frac{2}{\pi} \left( b \, \arctan \frac{r}{b} - \tanh^{-1} r \right) = \frac{2}{\pi} \left( ib \, \tanh^{-1} \frac{r}{ib} - \tanh^{-1} r \right)
\]

(26)

(27)

The origin of the coordinate system in the \( z \)-plane lies in the corner, point \( C \), and the function \( \tanh^{-1} \) has to be defined suitably to obtain a single-valued representation \( z(t) \). The inverse function \( t(z) \) can be calculated by iteration (e.g. Newton's interpolation formula for complex numbers).
As a next step a complex potential function \( w \) has to be established which describes voltage and current in the current sheet. We know that the current comes from a source at the far left (D) and disappears in a sink when it enters the end conductor. For the moment this sink may be located in point \( z' \) and may have its image in \( t = t' \) while the source \( D \) at infinity to the left is mapped into \( t = 1 \).

Since the complex potential around a single unit source in \( t = 1 \) in the t-plane is given by

\[
w = u + iv = -\frac{1}{2\pi} \log (t-1)
\]

we add a term for the potential of the sink in \( t = t' \) and obtain

\[
w = \frac{1}{2\pi} \log (t-t') - \frac{1}{2\pi} \log (t-1) = \frac{1}{2\pi} \log \frac{t-t'}{t-1}.
\]

Moreover the potential must satisfy the boundary condition that the current does not traverse the boundary of the septum i.e. that it remains in the upper half of the t-plane. If we place a second source and sink pair at complex conjugate points in the lower half plane, this condition will be satisfied by symmetry. So finally the complex potential is given by:

\[
w = \frac{1}{2\pi} \log \frac{t-t'}{t-1} + \frac{1}{2\pi} \log \frac{t-t'}{t-1} \quad (28)
\]

where \( \bar{t}' \) is the complex conjugate of \( t' \). We differentiate \( w \) with respect to \( z \) to obtain the current density:

\[
\frac{dw}{dz} = \frac{dw}{dt} \cdot \frac{dt}{dz}
\]

\[
= \left( \frac{1}{2\pi} \frac{(t-1)}{(t-t')(t-1)} + \frac{1}{2\pi} \frac{(\bar{t}'-1)}{(t-\bar{t}')(t-1)} \right) \frac{dt}{dz} \quad (29)
\]
and from equation (24):

\[
\frac{\text{d}w}{\text{d}z} = \left( \frac{1}{2\pi} \frac{(t'-1)}{(t-t')(t-l)} + \frac{(\xi'-1)}{2\pi (t-\xi') (t-l)} \right) \cdot \left( \frac{\pi}{1+b^2} \right) \frac{(t+b^2)(t-l)}{t^{1/2}}
\]

\[
\frac{\text{d}w}{\text{d}z} = \frac{t+b^2}{2 \cdot t^{1/2}(1+b^2)} \left( \frac{t'-1}{t-t'} + \frac{\xi'-1}{t-\xi'} \right)
\]

(30)

The actual end conductor is not a thin wire located in z', but consists of n parallel laminations, so that we deal with a distributed current sink. For simplicity reasons let us assume that the current in these laminations flows perpendicularly to the current sheet \(^*)
.

If we consider one lamination extending straight from \(z_1\) to \(z_2\), our sink is distributed on a straight line of length \(|z_2 - z_1|\) every element of which contributes only by

\[
\frac{|\text{d}z'|}{|z_2 - z_1|} = \frac{\text{d}z'}{z_2 - z_1}
\]

(31)

Therefore we have to integrate over these contributions from \(z_1\) to \(z_2\) in order to obtain the new distribution of current density. \(\text{d}z'\) maps into \(\text{d}t'\) in the t-plane:

\[
\text{d}z' = \left( \frac{1+b^2}{\pi} \right) \frac{t'^{1/2}}{(t'-1)(t'+b^2)} \text{d}t'
\]

(32)

\(^*)\) This is necessary to obtain a two-dimensional end field and this could be better achieved if the laminations were slitted still further, or if the end conductor would consist of a material of higher resistivity so that the current lines are refracted when they enter it.
Moreover we have to add also the complex conjugate sink to satisfy the boundary condition but we carry out the integration only for the upper half plane and add the complex conjugate part afterwards.

\[ \left( \frac{dw}{dz} \right)_{z_2} = \int_{z_1}^{z_2} \frac{(t+b^2)(t'-1)}{2t^{1/2}(1+b^2)(t-t')} \cdot \frac{dz'}{z_2 - z_1} \]

\[ = \frac{1}{z_2 - z_1} \int_{t_1}^{t_2} \frac{(t+b^2)(t'-1)(1+b^2)t'^{1/2}}{2t^{1/2}(1+b^2)(t-t') \pi(t'-1)(t'+b^2)} \; dt' \]

\[ = \frac{(1+b^2/t)}{2\pi(z_2 - z_1)} \int \frac{(t'/t)^{1/2}}{(t'/t-1)(t'/t+b^2/t)} \; d(t'/t) \]

This integral is similar to that in equation (25). Therefore by analogy to equation (27) we easily find the result

\[ w' = \left( \frac{dw}{dz} \right)_{z_2} = \frac{1}{\pi(z_2 - z_1)} \left[ \left( \frac{1}{h/t^2} \right) \tanh^{-1} \left( \frac{t'/t}{ih/t^2} \right) - \tanh^{-1} \left( \frac{t'/t}{h/t^2} \right) \right]^{t_2}_{t'=t_1} \]

\[ + \frac{1}{\pi(z_2 - z_1)} \left[ \cdots \right]^{t_2}_{t'=t_1} \]

(33)
where we have added the second bracket containing contributions of
the source and sink in the lower half plane. Formulae (27) and (33)
have been programmed for the computer for the case of a conductor
which consists of sixteen copper laminations. For a given point \( z \)
first the image point in the \( t \)-plane is calculated and then \( t \) is put
into equation (33) in order to evaluate the field. For points on the
median line of the magnet \( t \) is real \( t > 1 \) and therefore \( w' \) is also
real which means that the current density has no vertical component on
the median line (symmetry). Equations (27) and (33) can also be used
to calculate the fringing field at the end of a magnet, if this fringing
field is two-dimensional. For such a case we turn back to fig. 3b in
which the window is approximately \( 2 \times 4 \) cm and to fig. 3a where we
have a window of \( 2 \times 8 \) cm. In particular on the vertical plane of
symmetry at the end of this magnet the field can be calculated as a two-
dimensional fringing field. In this case our region ABCDA in the \( z \)-plane
(fig. 53) corresponds to a longitudinal section through the magnet and
its end part. It represents the upper half gap. BCD corresponds to
the iron boundary, DA corresponds to the mid-plane and AB would corre-
spend to a neutral pole but since we do not have a neutral pole we must
use formulae (27) and (33) in the limit case \( b \to \infty \). In this case
equation (27) becomes

\[
z = \frac{2}{\pi} \left( t^{1/2} - \tanh^{-1} t^{1/2} \right)
\]  

(34)

and (33) becomes

\[
w' = \frac{1}{\pi(z_2 - z_1)} \left[ \left( t'/t \right)^{1/2} - \tanh \left( t'/t \right)^{1/2} \right] t_2^{t_2} + \frac{1}{\pi(z_2 - z_1)} \left[ \ldots \right] t_1^{t_1}
\]

(35)
The line \( z_1 z_2 \) had been considered as a sectional view of one of the laminated conductors and represented a distributed current sink. Now it represents the (distributed) centres of curls of the magnetic field. Formulae (34) and (35) have also been programmed for the computer and fig. 18 is a plot of the ideal field on the median plane at the end of the magnet in fig. 3a. This is also the field which should be obtained near the current sheet of the septum magnet in fig. 3b, if the field would not traverse the current sheet. The other curve in fig. 18 shows the current density which decreases more steeply than the field and becomes zero at point A, where the septum is cut off. Therefore near point A the fringing field partly penetrates the septum and a positive stray field can be observed (fig. 43). On the other hand, our septum is longer than the magnet core, so that the corner C of the magnet current sheet is shifted by 1 mm to the right beyond the corner C of the iron core. This has the effect that the field curve in fig. 18 starts first to decrease and lies under the curve of the current density until they intersect. Therefore we have a negative stray field near the corner (this negative field is even more striking when the corner starts to saturate as in the case of the 20 kg magnet). We had the possibility of cutting off the septum (thereby reducing b) in such a way that the integrated stray field at the end became approximately zero for low and medium fields as long as the corner did not yet saturate. However, it should be remembered that this calculation cannot be expected to give true quantitative results because it is based on too many simplifying assumptions and the actual 3-dimensional field must decrease a bit more rapidly than calculated.

A similar correction has also been applied to the magnet with 2 current turns in which the current density is different since the septum consists of 2 overlapping current sheets (cf. fig. 2). These current sheets are no longer symmetric to the midplane. Therefore the current does no longer flow parallel to the midplane but since the two current sheets are overlapping the effects of the vertical components of the current density on the midplane cancel and only the horizontal components count. The calculation has been carried out with the same formulae, (27) and (33), but for other points and with other constants.
Fig. 19 shows the result for a septum which has been cut off first at 32 mm and then at 23 mm, the latter curve fitting better. Since the current density curve decreases even more steeply in this type of septum, and deviates more from the curve of the ideal field, the peaks of the negative and positive stray field are more pronounced in this magnet than in the 10 kG magnet (cf. fig. 30 and 43).
FIG. 10. Champ de fuite d'o à la réductance des lèges
Airmant à septum
HYSTERESIS LOOPS
(SKETCHED FROM DATA SHEETS)

FIG. 11
CALCULATED FIELD AND CURRENT DENSITY NEAR END OF SEPTUM MAGNET WITH 2 CURRENT Turner

FIG 19

CURRENT DENSITY IN SEPTUM

CURRENT DENSITY IN SHORT SEPTUM (AFTER CORRECTION)

IDEAL FIELD

END OF IRON CORE END OF SEPTUM END OF SEPTUM

DISTANCE
MAIN FIELD VERSUS CURRENT

FIG. 22

EFFECTIVE LENGTH 0.525 m
MAGNET CORE NO 3B, 26-6-65
Fig. 21
Test pulse
sweep: 20 ms/square
vert.: 2500 Amp/square

Fig. 23
Stray field versus current
in front of first 20kG-magnet
vert.: 7.4 Gauss/square
horiz.: 5 kA/square

Fig. 24
Stray field
in front of second 20kG-magnet
vert.: 7.4 Gauss/square
horiz.: 5 kA/square
Fig. 25

Stray field of 20kG-magnet measured with Hall probe
vertical scale: ± 5 Gauss/square
horiz. scale: 2.5 kAmp/square

Distance from septum
4 mm
6 mm
8 mm
10 mm
15 mm
20 mm
FIG. 26
STRAY FIELD VERSUS DISTANCE
Fig. 27
Integrated stray field
along a particle trajectory
at 4mm distance from upstream
magnet and 8mm from the end of
the downstream magnet. The
magnets formed an angle of
7.5 mrad.
Vert.: 5.4 Gauss • m/square
Horiz.: 5 kA/square

Fig. 28
Stray field
on right end before correction
vert.: probably 4.5 Gauss • m/square

Fig. 29
Stray field
on right end after correction
vert.: 9 Gauss • m/square

horizontal: 5 kA/square
distance from septum: 5mm, 10mm, 20mm
FIG 30.
LEFT END OF SEPTUM MAGNET 20 KGAUSS
FIELD MEASURED AT 4 mm DISTANCE

COORDINATE ALONG MAGNET

$B_x = 9.5 \text{ kGauss}$

$B_y = 2.0 \text{ kGauss}$

1 cm
Integrated stray field of 20kG-magnet

Fig. 31
vert.: 20 Gauss/square
horiz.: 5 kA/square
(effective length 0.535m)

Fig. 32
vert.: 10 Gauss/square
horiz.: 5 kA/square
VERTICAL STRAY FIELD ON MIDPLANE
MEASURED WITH LONG INTEGRATING COIL
MAGNET 20 KGauss

FIG. 33

DISTANCE FROM SEPTUM
FIG. 34

EFFECTIVE LENGTH 0.723 m

MAGNET CORE NO. 2A 27.9.65
Stray field of 10kG-magnet
in front of the septum, excluding the ends

Fig. 35
vert.: 0.86 Gauss/square
horiz.: 5 kA/square

30mm

20mm

15mm

Fig. 36
vert.: 1.71 Gauss/square
horiz.: 5 kA/square

10mm

8mm

5mm
Fig. 37

Stray field of 10kG-magnet measured with Hall probe versus magnet current.

vert.: ± 2.5 Gauss/square
horiz.: 2.5 kA/square
FIG. 38
STRAY FIELD MEASURED WITH HALL PROBE
10 KGauss Magnet
Fig. 39
Average stray field in front of 10 kG septum magnet
vert.: 4 Gauss/square
horiz.: 5 kA/square
(efl. length 0.723 m)
AVERAGE VERTICAL STRAY FIELD ON MIDPLANE OF 10 K GAUSS SEPTUM MAGNET

FIG. 40
Fig. 41
Stray field
at right end
of 10kG-magnet
integrated by means of
0.3m search coil as in
figs. 28, 29
vert.: 2.2 Gauss \cdot m/square
horiz.: 5 kA/square

Fig. 42
Stray field
at left end
(same scale as fig. 40)
FIG. 43

VERTICAL STRAY FIELD AT END OF SEPTUM
LEFT END OF SEPTUM MAGNET
(10 K GAUSS MAIN FIELD)

END OF IRON CORE
Horizontal stray field of 10kG-magnet at 5mm distance from septum

Fig. 44
Horizontal stray field above, below and on midplane
vertical scale: 2.5 Gauss/square
horizontal scale: 5 kAmp/square

Fig. 45
Horizontal stray field on midplane with pulse flat top lasting 80ms and 180ms.
Same scale as in fig. 44
Temperature measurements

Fig. 46
Top: 12 degrees/square
Current: 5 kA/square
Time base: 0.5 sec/square

Fig. 47
Temperature: 12 degrees/square
Current: 2500 A/square
Time base: 0.5 sec/square

Pulses:

16kA x 0.18sec

Fig. 48
Temperature: 24 deg/square
Time base: 0.1 sec/square

12kA x 0.18sec

12kA x 0.38sec
FIG. 4.9

CURRENT LIMITS (I) IN SEPTUM MAGNETS FOR DIFFERENT PULSELENGTHS (T) AND REPETITION TIME (T)

WATER FLOW = 2.8 LIT./MIN IN 2 MWG. MAGNET
= 3.4 LIT./MIN IN 4 MWG.

SEPTUM TEMP: 670 OC ABOVE WATER INPUT TEMP.

PULSELENGTH + [s]