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BRST Quantization of SU(2/1) Electro-Weak Theory in the Superconnection Approach – and the Higgs meson mass.

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Abstract

A superconnection, in which a scalar field enters as a zero form in the odd part of the superalgebra, is used in the geometrical BRST quantization of SU(2/1) electro-weak theory. The zero form field transforms like a matter field, in the context of BRST quantization. A quantum action is obtained from the principle of symmetric BRST/anti-BRST invariance. Evaluating the mass of the Higgs field, we exhibit the consistency between (a) the conventional derivation of the masses from spontaneous symmetry breakdown, just using the supergroup's gauge value for the coefficient of the scalar quartic potential in the superconnection approach, and (b) the direct treatment of superconnection components (gauge J=1 and scalar J=0) as a multiplet of the supergroup, used as a kinematical internal supersymmetry. This

result provides a foundation for the use of "internal" supergroups and supergauges at the gauge-mesons/Higgs-mesons level. Using renormalization group equations, we then evaluate the energy at which the SU(2/1) prediction for the other coupling it fixes, beyond the conventional SU(2)×U(1), namely $\sin^2\theta_W = .25$, materializes. Assuming SU(2/1) to be precise at this energy, we then apply the renormalization group equations to evaluate the corrections to the quartic coupling of the scalars and to the Higgs mass. Our result predicts Higgs mass to be $130 \pm 6\text{GeV}$ when we assume the top quark mass as 174GeV.

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1. Introduction

Some fifteen years have passed since the first suggestions of SU(2/1) electroweak theory[1, 2]. This is an *internal* supersymmetry or supergauge theory, with the gauged supergroup $\mathcal{G} = \text{SU}(2/1)$ containing both the Weinberg-Salam SU(2)×U(1) gauge group (as the even Lie subgroup $\mathcal{G}^+ \subset \mathcal{G}$) and its spontaneous symmetry breakdown (Goldstone-Higgs) fields $\phi, \bar{\phi}$, "gauging" the odd (we shall use the term "g-odd") part $\mathcal{G}^- \subset \mathcal{G}$. All couplings (including the quartic potential for the Higgs scalar) are thereby determined by the gauge's universality – up to possible quantum radiative corrections. This thus implies a much reduced ("superunifying") algebraic input and fewer parameters as arbitrary givens[3, 4]. The only feature of the symmetry breaking which was still put in "by hand" in ref.[1, 2] is the "trigger", the negative-squared-mass term in the Lagrangian. The physical fields and their interactions being the same as in SU(2)×U(1), their quantization and renormalizability would appear to be guaranteed, except for two new features, both deriving from the constraining superalgebraic embedding:

(a) The constraining supergroup (and the method of gauging it) might impose a ghost-structure different from that of the conventional SU(2)×U(1); the BRST transformations thus have to be adapted to the new ghost structure and proven in this new context.

(b) checking the renormalization of those couplings which are determined by the $\mathcal{G}$ supersymmetric embedding, beyond the usual gauge symmetry $\mathcal{G}^+ = [\text{SU}(2) \times \text{U}(1)] \subset \mathcal{G}$ – to ascertain the corrected values predicted by the supergroup, as modified by Relativistic Quantum Field Theory. These values should be compared with the experimentally deduced values, such as $\sin^2\theta_W$, the Higgs meson mass, etc..

Problem (a) has been studied at different stages, following the evolving different interpretations of the SU(2/1) supergauge multiplet:

(a1) In ref.[5], as in Ne'eman's original suggestion, the scalar field enters as a formal ghost, i.e. a one-form in dy , the "vertical" completion – within the principal fiber bundle's full dimensionality – of a one-form connection, which gauges (in dx) the "g-odd" part $\mathcal{G}^-$ of SU(2/1). The latter constituted an embarrassment for this first interpretation, since such vector-mesons are not observed. On the other hand, the presence of new physical or ghost fields makes it possible for the renormalization group equations to implement additional conservation laws, beyond those of $\mathcal{G}^+$.

(a2) In ref.[6, 7] and more successfully in ref.[8], the scalar field is a ghost-for-ghost in the g-odd part $\mathcal{G}^-$ of the supergroup (i.e. a 2-form in $dy \wedge d\bar{y}$) now gauged through a Kalb-Ramond-like two-form in dx , i.e. a Grassmann-even (we shall use the terminology "even w-grading") component of what has since become defined in mathematics by Quillen[9] as a "superconnection". Here too the model involves new ghost and field components.

(a3) Yet a third interpretation for the role of the scalar field was proposed by Ne'eman and Sternberg[10], using Quillen's generalization ansatz, beyond the original constructs of ref.[6]. The scalar field is assigned to the same "Grassmann-w-even/ group-g-odd" part of the SU(2/1) superconnection, but this time as a zero-form. A similar suggestion was made by Coquereaux et al.[11], who based their work on an application of Connes' non-commutative geometry[12, 13]. The latter foundation has the additional advantage of adding the notion of a matrix-derivative, for the discrete factor-space of the geometry, capable of supplying the negative-squared-mass "trigger" for the symmetry's spontaneous breakdown mechanism.

In the present work, we reformulate the BRST quantization of the SU(2/1) theory, interpreting it according to (a3), namely the Ne'eman-Sternberg generalized superconnection approach. The version motivated by non-commutative-geometry, including the matrix derivative, will be treated elsewhere[14]. In what follows, symmetric BRST/anti-BRST transformation rules are worked out[5, 15] by apply-

ing the *horizontality* condition, deriving from Thierry-Mieg's original geometrical interpretation of the BRST constraints, as the Cartan-Maurer structural equations for a Principal Fiber Bundle[4, 16, 17, 18]. This interpretation does not involve new fields or ghosts.

A quantum Lagrangian is constructed, using the symmetric BRST/anti-BRST algebra[19]. Using renormalization group (RG) techniques, we now deal with two features due to SU(2/1), which are still allowed arbitrary values in SU(2)×U(1): the U(1) coupling g' (given the SU(2) coupling g) and the quartic coupling in the Higgs potential λ ; alternatively, we may deal with two related experimental quantities, $\sin^2\theta_W$ and the mass M(H) of the Higgs meson. In both cases we first rederive the SU(2/1) pre-renormalization predictions and discuss them in the light of the (a3) superconnection approach: for $\sin^2\theta_W$ this involves a transmutation with respect to odd and even gradings between SU(2/1) and the Grassmann manifold of the gauge n-forms. In the case of the Higgs mass, we learn more about the consistency of the superconnection approach with a kinematical (global) approach to the internal supersymmetry.

Having completed this "theoretical" discussion, we move on to the physical realization. The RG techniques are first used to evaluate the variation of the "running" gauge coupling parameter for the SU(2)×U(1) even subgroup. We use the result to fix the value of $q^2 = T$ at which SU(2/1) becomes precise. We can then use the same approach in the opposite direction for the "running" quartic coupling $\lambda(q^2)$ to evaluate the correction to the Higgs mass.

2. Superconnections and supercurvatures

We first write down the (overall anticommutative) superconnection as

$$\mathcal{J} = \begin{pmatrix} A & \phi \\ \bar{\phi} & B \end{pmatrix}. \quad (1)$$

If $\mathcal{J}$ is a $(n+m) \times (n+m)$ supermatrix, then A, B are $n \times n$ and $m \times m$ SU(2/1)

g-even submatrices, valued over the one-form gauge fields $A = A_\mu dx^\mu$, $B = B_\nu dx^\nu$; $\phi, \bar{\phi}$ are respectively $n \times m$ and $m \times n$ SU(2/1) g-odd submatrices, valued over the zero-form scalar fields $\phi, \bar{\phi}$; in the interest of simplicity in the physical treatment, we use the same symbol to denote both the forms – also multiplets of the $\mathcal{G}^+ = \text{SU}(2) \times \text{U}(1)$ even subgroup – and their realizations as SU(2/1) submatrices, at the risk of some confusion. $\mathcal{J}$ is thus itself an odd element of $\mathcal{U}$, a product supermanifold: $\mathcal{J} \in \mathcal{U}^- \subset \mathcal{U} = V(M) \otimes g$, i.e. it is an odd element within the direct product of $V(M)$, the Grassmann algebra of differential forms over a manifold M, by the abstract superalgebra $g = \text{su}(2/1)$ of $\mathcal{G}$. Returning to $V(M)$ – depending on the context, M might be, either just spacetime R^4 , or some larger manifold, such as the Principal Fibre Bundle $P(R, \mathcal{G})$, with structure group $\mathcal{G} = \text{SU}(2/1)$ (such extended M manifolds will be denoted by a tilde). $\mathcal{J}$ is "u-odd" because its A and B elements are w-odd but g-even, while its $\phi, \bar{\phi}$ elements are w-even but g-odd.

We now write down any two elements $u, u' \in \mathcal{U}$, with $u = (W \otimes h)$ and $u' = (W' \otimes h')$, where W, W' are differential forms of Grassmannian Z_2 "w-gradings" $|W|, |W'|$, and h, h' are su(2/1) supermatrices of fixed Lie supergroup Z_2 "g-grading" $|h|, |h'|$. For the $\mathcal{U}$ multiplication, we adopt the canonical convention[20]

$$(W \otimes h) \cdot (W' \otimes h') = (-1)^{|h||W'|} W \wedge W' \otimes (h \cdot h') \quad (2)$$

With the above convention and assuming $A, B, C, D \in V(M)$, we obtain the Ne'eman - Sternberg rule for a product of supermatrices (ref.[10])

$$\begin{pmatrix} A & C \\ D & B \end{pmatrix} \cdot \begin{pmatrix} A' & C' \\ D' & B' \end{pmatrix} = \begin{pmatrix} A \wedge A' + (-1)^{|D'|} C \wedge D' & A \wedge C' + (-1)^{|B'|} C \wedge B' \\ (-1)^{|A'|} D \wedge A' + B \wedge D' & (-1)^{|C'|} D \wedge C' + B \wedge B' \end{pmatrix}. \quad (3)$$

We now define the supercurvature as

$$\mathcal{F} = d\mathcal{J} + \mathcal{J} \cdot \mathcal{J}, \quad (4)$$

where $\mathbf{d} = \begin{pmatrix} d & 0 \\ 0 & d \end{pmatrix}$ and $d = dx^\mu \frac{\partial}{\partial x^\mu} \otimes \mathbf{1}$ (where $\mathbf{1}$ is the identity for an $\text{su}(2/1)$ submatrix). Here one can see that $\mathbf{d}$ acts as a derivation and has $w = 1, u = 1$. For instance, if $\mathbf{d}$ acts on a g-odd supermatrix $\mathcal{P}^- = \begin{pmatrix} 0 & C \\ D & 0 \end{pmatrix}$ multiplied by an arbitrary supermatrix $\mathcal{Q}$, then

$$\mathbf{d} (\mathcal{P}^- \cdot \mathcal{Q}) = (\mathbf{d} \mathcal{P}^-) \cdot \mathcal{Q} + (-1)^{\|\mathcal{P}^-\|} \mathcal{P}^- \cdot \mathbf{d} \mathcal{Q} \quad (5)$$

where $\|\mathcal{P}^-\|$ is the total u-grading of $\mathcal{P}^-$, i.e. the sum (mod 2) of the $\text{SU}(2/1)$ "g-degree", $g=1$ in this case, and the form "w-degree" of the entries of $\mathcal{P}^-$, namely of C and D in equation (3). Thus $\|\mathcal{P}^-\| = 1$ if C, D are valued over w-even forms, and $\|\mathcal{P}^-\| = 0$ if C, D be valued over w-odd forms (this formulation embodies the fact that the overall u-grading of $\mathbf{d}$ is $1+0 = 1$). From (1) and (4) with the help of the rule (3), we write the supercurvature $\mathcal{F}$ in its component form

$$\mathcal{F} = \begin{pmatrix} dA + A \wedge A + \phi \bar{\phi} & d\phi + A\phi - \phi B \\ d\bar{\phi} - \bar{\phi}A + B\bar{\phi} & dB + B \wedge B + \bar{\phi}\phi \end{pmatrix}. \quad (6)$$

3. Horizontality condition, BRST/anti-BRST symmetry

To find the BRST/anti-BRST transformation rules, we use the horizontality condition[4, 16, 17], the essence of the Maurer-Cartan equation, after extending M , in the Grassmannian $V(M)$, to the entire Principal Bundle P , as in ref.[4]. To reproduce a Curci-Ferrari[15] symmetric algebra, we use a doubled structure-group $\mathcal{G} \otimes \mathcal{G}$ as fiber, $M = P(R, \mathcal{G} \otimes \mathcal{G})$, as in ref.[5, 6, 8, 19]. We generalize the supercurvature as

$$\tilde{\mathcal{F}} = \tilde{\mathbf{d}} \tilde{\mathcal{J}} + \tilde{\mathcal{J}} \cdot \tilde{\mathcal{J}} \quad (7)$$

where $\tilde{\mathbf{d}}, \tilde{\mathcal{J}} \in \tilde{\mathcal{U}}$ are extensions of $\mathbf{d}$ and $\mathcal{J}$, respectively, over the above complete bundle manifold, now including the Grassmann forms over the gauge-fiber coordinate y and its dual $\bar{y}$; we thus project over a section in the bundle as

follows,

$$\begin{aligned} \tilde{\mathbf{d}} &= \mathbf{d} + \mathbf{s} + \bar{\mathbf{s}}, \\ \tilde{\mathcal{J}} &= \mathcal{J} + \mathcal{C} + \bar{\mathcal{C}}. \end{aligned} \quad (8)$$

$\mathcal{C}$ and $\bar{\mathcal{C}}$ are obtained from $\mathcal{J}^+$ by replacing dx^μ by dy^N , and by $d\bar{y}^M$, respectively, i.e. the "vertical" components of the $\mathcal{J}^+$ (g-even) gauge connection, contracted over the vertical differentials,

$$\begin{aligned} \mathcal{C} &= \begin{pmatrix} A_N dy^N & 0 \\ 0 & B_N dy^N \end{pmatrix} \equiv \begin{pmatrix} c & 0 \\ 0 & t \end{pmatrix}, \\ \bar{\mathcal{C}} &= \begin{pmatrix} A_M d\bar{y}^M & 0 \\ 0 & B_M d\bar{y}^M \end{pmatrix} \equiv \begin{pmatrix} \bar{c} & 0 \\ 0 & \bar{t} \end{pmatrix}. \end{aligned} \quad (9)$$

c and t are thus the g-even w-odd 1-form anticommuting scalar ghost fields for $\text{SU}(2)$ and for $\text{U}(1)$ respectively. Note that the M -vertical supermatrices of (9) have no g-odd submatrices because the corresponding ones in $\mathcal{J}$ of (1) are zero-forms $\phi, \bar{\phi}$. This feature represents an important advantage of the generalized superconnection[9, 10] treatment, i.e. there are no new ghost fields, a difficulty encountered in ref.[6], for instance.

BRST/antiBRST operators $\mathbf{s}$ and $\bar{\mathbf{s}}$ are vertical derivations on the fiber and on its dual, respectively,

$$\mathbf{s} = \begin{pmatrix} s & 0 \\ 0 & s \end{pmatrix}, \quad \bar{\mathbf{s}} = \begin{pmatrix} \bar{s} & 0 \\ 0 & \bar{s} \end{pmatrix} \quad (10)$$

where $s = dy^N \frac{\partial}{\partial y^N} \otimes \mathbf{1}$, $\bar{s} = d\bar{y}^M \frac{\partial}{\partial \bar{y}^M} \otimes \mathbf{1}$.

From the horizontality condition

$$\tilde{\mathcal{F}} = \mathcal{F}, \quad (11)$$

we obtain the BRST/anti-BRST transformation rules:

$$(dy)^1 : \mathbf{s} \mathcal{J} + \mathbf{d} \mathcal{C} + \mathcal{C} \cdot \mathcal{J} + \mathcal{J} \cdot \mathcal{C} = 0, \quad (12)$$

$$(d\bar{y})^1 : \bar{\mathbf{s}} \mathcal{J} + \mathbf{d} \bar{\mathcal{C}} + \bar{\mathcal{C}} \cdot \mathcal{J} + \mathcal{J} \cdot \bar{\mathcal{C}} = 0. \quad (13)$$

$$(dy)^2 : \mathbf{s} \mathcal{C} + \mathcal{C} \cdot \mathcal{C} = 0, \quad (14)$$

$$(d\bar{y})^2 : \bar{\mathbf{s}} \bar{\mathcal{C}} + \bar{\mathcal{C}} \cdot \bar{\mathcal{C}} = 0, \quad (15)$$

$$(dyd\bar{y})^1 : \bar{\mathbf{s}} \mathcal{C} + \mathbf{s} \bar{\mathcal{C}} + \mathcal{C} \cdot \bar{\mathcal{C}} + \bar{\mathcal{C}} \cdot \mathcal{C} = 0. \quad (16)$$

We now proceed to decompose the horizontal parts of the 2-form (7) according to the SU(2/1) g-degree, i.e. $\mathcal{J}$ into $\mathcal{J}^+ + \mathcal{J}^-$, where $\mathcal{J}^+ = \begin{pmatrix} A & 0 \\ 0 & B \end{pmatrix}$, $\mathcal{J}^- = \begin{pmatrix} 0 & \phi \\ \bar{\phi} & 0 \end{pmatrix}$, also noting that $\mathbf{d}$, $\mathbf{s}$ and $\bar{\mathbf{s}}$ are g-even matrices (the identity), whose entries are one-form differential operators, i.e. with w-degree = 1. We list the even and odd parts separately.

$$\text{g-even part} : \mathbf{s} \mathcal{J}^+ + \mathbf{d} \mathcal{C} + \mathcal{C} \cdot \mathcal{J}^+ + \mathcal{J}^+ \cdot \mathcal{C} = 0, \quad (17)$$

$$\bar{\mathbf{s}} \mathcal{J}^+ + \mathbf{d} \bar{\mathcal{C}} + \bar{\mathcal{C}} \cdot \mathcal{J}^+ + \mathcal{J}^+ \cdot \bar{\mathcal{C}} = 0, \quad (18)$$

$$\mathbf{s} \mathcal{C} + \mathcal{C} \cdot \mathcal{C} = 0, \quad (19)$$

$$\bar{\mathbf{s}} \bar{\mathcal{C}} + \bar{\mathcal{C}} \cdot \bar{\mathcal{C}} = 0, \quad (20)$$

$$\bar{\mathbf{s}} \mathcal{C} + \mathbf{s} \bar{\mathcal{C}} + \mathcal{C} \cdot \bar{\mathcal{C}} + \bar{\mathcal{C}} \cdot \mathcal{C} = 0. \quad (21)$$

$$\text{g-odd part} : \mathbf{s} \mathcal{J}^- + \mathcal{C} \cdot \mathcal{J}^- + \mathcal{J}^- \cdot \mathcal{C} = 0, \quad (22)$$

$$\bar{\mathbf{s}} \mathcal{J}^- + \bar{\mathcal{C}} \cdot \mathcal{J}^- + \mathcal{J}^- \cdot \bar{\mathcal{C}} = 0. \quad (23)$$

By introducing an auxiliary g-even field supermatrix $\mathcal{E}$, such that

$$\mathbf{s} \bar{\mathcal{C}} \equiv \mathcal{E}, \text{ i.e., } \begin{pmatrix} s\bar{c} & 0 \\ 0 & s\bar{t} \end{pmatrix} \equiv \begin{pmatrix} l & 0 \\ 0 & m \end{pmatrix}, \quad (24)$$

we can fix the remaining BRST/anti-BRST transformation rules,

$$\bar{\mathbf{s}} \mathcal{C} = -\mathcal{E} - \mathcal{C} \cdot \bar{\mathcal{C}} - \bar{\mathcal{C}} \cdot \mathcal{C}, \quad (25)$$

$$\mathbf{s} \mathcal{E} = 0, \quad (26)$$

$$\bar{\mathbf{s}} \mathcal{E} = -\bar{\mathcal{C}} \cdot \mathcal{E} + \mathcal{E} \cdot \bar{\mathcal{C}}. \quad (27)$$

We note that (17) and (18) are the usual BRST/anti-BRST transformation rules for the one-form gauge field[19], and (22) and (23) are those of the Higgs scalar;

in the latter case, the geometrical construction reproduces the conventional transformation rules for matter fields.

One can easily check that the nilpotency of BRST/anti-BRST transformations is satisfied, $\mathbf{s}^2 = \bar{\mathbf{s}}^2 = 0$.

4. Quantum action and the $\sin^2\theta_W = .25$ prediction

We follow Baulieu and Thierry-Mieg[19] in the construction of a BRST/anti-BRST invariant quantum Lagrangian, adapting the method to the superconnection version (a3), i.e. we add a BRST/anti-BRST closed form to the Yang-Mills classical Lagrangian.

$$\mathcal{L}_{qu} = \frac{1}{4} \text{Tr} \left\{ \mathcal{F} \cdot \mathcal{F}^* - \mathbf{s} \bar{\mathbf{s}} (\mathcal{J} \cdot \mathcal{J}^*) + \alpha \mathbf{s} (\bar{\mathcal{C}} \cdot \mathcal{E}^*) \right\} \quad (28)$$

where, α is a parameter, and $\star$ denotes taking duals over the base manifold (the x^μ variable) for each differential form entry W , W' in the supermatrices (2) and taking hermitian conjugates for the supergroup supermatrices h , h' themselves.

Note the use of the Trace (Tr) rather than the Supertrace (str), due to the fact that e.g. the overall u-superparity of the $\mathcal{J}$ gauge connection is odd - g=0, w=1, u=w+g=1 for A and B , or g=1, w=0, u=1 for ϕ and $\bar{\phi}$. As a result, *the overall connection multiplet $\mathcal{J}$ represents operators of a single statistical mode (fermionic) and thus relates to a symmetry rather than to a supersymmetry*. Similarly, the curvature (field-strengths) will represent an overall bosonic multiplet, the A,B parts with g=0, w=2, u=0 mod 2; the g-odd parts, namely the kinetic energies of the ϕ , $\bar{\phi}$, on the other hand, will have g=1, w=1, u=0 mod 2. The total grading is thus the same for all components here too.

This is why we have to use U(3) traces rather than U(2/1) supertraces. This metamorphosis is caused by the Superconnection formalism's transmutation of g-oddness into w-oddness, preserving the u-grading sum. Had SU(2/1) been used

abstractly as a gauge group (i.e. a structure-group in a Principal Fiber Bundle, with spacetime as base manifold $P(M, SU(2/1))$), the connection should have been a 1-form in its entirety, both in A, B and in the odd part; the latter would then have been commutative, from being both g-odd and w-odd. This is the formalism (a1) of ref.[1]; in ref.[6] the g-odd feature is transmuted into w-oddness by valuing the odd parameters of $SU(2/1)$ themselves over the Grassmann manifold, using w-odd forms in the vertical parameter dy. This turned the g-odd w-odd part of the connection into a g-even w-even vertical 2-form (a2 above). In the present generalized superconnection formalism, that vertical 2-form is replaced by a horizontal 0-form, preserving the w-parity, without changing the g-even result of (a2). The emergence of traces rather than supertraces is thus a direct result of the inter-compensation between g and w parities; it is the overall u-parity which matters and which has not changed in (a1)-(a3).

After a somewhat tedious, but straightforward calculation, one can check that

$$\text{Tr} \{ \mathbf{s} \bar{\mathbf{s}} (\mathcal{J}^- \cdot \mathcal{J}^-^*) \} = 0, \quad (29)$$

$$\text{Tr} \{ \mathbf{s} \bar{\mathbf{s}} (\mathcal{J}^+ \cdot \mathcal{J}^+^*) \} = 2 \text{Tr} \{ \mathcal{J}^+ \cdot (\mathbf{d} \mathcal{E})^* + \mathbf{d} \bar{\mathcal{C}} \cdot (\mathcal{D} \mathcal{C})^* \} \quad (30)$$

where $\mathcal{D} \mathcal{C} = \mathbf{d} \mathcal{C} + \mathcal{J}^+ \cdot \mathcal{C} + \mathcal{C} \cdot \mathcal{J}^+$, and

$$\text{Tr} \{ \alpha \mathbf{s} (\bar{\mathcal{C}} \cdot \mathcal{E}^*) \} = \text{Tr} \{ \alpha \mathcal{E} \cdot \mathcal{E}^* \}. \quad (31)$$

Thus $\mathcal{L}_{qu}$ becomes

$$\mathcal{L}_{qu} = \frac{1}{4} \text{Tr} \{ \mathcal{F} \cdot \mathcal{F}^* - 2 \mathcal{J}^+ \cdot (\mathbf{d} \mathcal{E})^* - 2 \mathbf{d} \bar{\mathcal{C}} \cdot (\mathcal{D} \mathcal{C})^* + \alpha \mathcal{E} \cdot \mathcal{E}^* \}. \quad (32)$$

This quantum Lagrangian is fully BRST/anti-BRST invariant.

We now apply this result to the $SU(2/1)$ supergauge. The superconnection $\mathcal{J}$ is Lie superalgebra-valued, $\mathcal{J} = i \mu_I \mathcal{J}^I$, $I = 1, 2, \dots, 8$, where μ_I 's ([1]) are the same as $SU(3)$ λ -matrices, except for

$$\mu_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -2 \end{pmatrix}.$$

Our above discussion relating to the result that traces, rather than supertraces, supply the invariants – is also relevant here and explains the original (otherwise hard to justify) normalization of the μ_I according to $\text{tr} (\mu_I)^2 = 2$, which yields the prediction $\sin^2 \theta_W = .25$ as a result of the ratio of coefficients in μ_3 and μ_8 ([1]). As noted in our discussion of the trace, $\mathcal{J}^I = A^I$ for $I=1, 2, 3$; $\mathcal{J}^8 = B$, all one-form (gauge) fields; $\mathcal{J}^J = \phi^J$ for $J=4, 5, 6, 7$; these are zero-forms: $\sqrt{2} \phi^\pm = \mathcal{J}^4 \mp i \mathcal{J}^5$, $\sqrt{2} \phi^0 = \mathcal{J}^6 - i \mathcal{J}^7$, $\sqrt{2} \bar{\phi}^0 = \mathcal{J}^6 + i \mathcal{J}^7$. We thus write

$$\mathcal{J} = i \begin{pmatrix} \tau_a A^a - \frac{1}{\sqrt{3}} B \cdot \mathbf{1} & \sqrt{2} \Phi \\ \sqrt{2} \Phi^\dagger & -\frac{2}{\sqrt{3}} B \end{pmatrix} \quad (33)$$

where $\tau_a = \mu_a$, $a = 1, 2, 3$ are the Pauli matrices, $\mathbf{1}$ is a 2 by 2 unit matrix, and $\Phi \equiv \frac{1}{\sqrt{2}} \begin{pmatrix} \mathcal{J}^4 - i \mathcal{J}^5 \\ \mathcal{J}^6 - i \mathcal{J}^7 \end{pmatrix} = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$. From (1), (6) and (33), the supercurvature $\mathcal{F}$ is given by

$$\mathcal{F} = i \begin{pmatrix} F_A - \frac{1}{\sqrt{3}} F_B + 2i \Phi \Phi^\dagger & \sqrt{2} (d\Phi + i A \Phi + i \frac{1}{\sqrt{3}} B \Phi) \\ \sqrt{2} (d\Phi^\dagger - i \Phi^\dagger A - i \frac{1}{\sqrt{3}} B \Phi^\dagger) & -\frac{2}{\sqrt{3}} F_B + 2i \Phi^\dagger \Phi \end{pmatrix} \quad (34)$$

where

$$F_A = (dA_a + i(A \wedge A)_a) \tau_a \quad \text{with} \quad A = A_a \tau_a, \quad a = 1, 2, 3,$$

and $F_B = dB \cdot \mathbf{1}$.

Introducing coupling parameters goes as usual.

We assign a dimensionless parameter g for the superconnection (i.e., $\mathcal{J} \rightarrow g \mathcal{J}$).

Then we scale the curvature squared term in the Lagrangian by $\frac{1}{g^2}$:

$$\begin{aligned} \frac{1}{4g^2} \text{Tr} \mathcal{F} \cdot \mathcal{F}^* = \\ \frac{1}{2} (F_A)_a \wedge (F_A)_a^* + \frac{1}{2} F_B \wedge F_B^* - (D\Phi)^\dagger \wedge (D\Phi)^* + \lambda (\Phi^\dagger \Phi) \wedge (\Phi^\dagger \Phi)^*, \end{aligned}$$

where $*$ denotes taking a dual of a differential form, and $\lambda \equiv 2g^2$, and

$$D\Phi = d\Phi + ig A \Phi + i \frac{1}{\sqrt{3}} g B \Phi.$$

If we also write

$$\begin{aligned}\mathcal{C} &= i\mu_j c_j, \quad j = 1, 2, 3, 8, \\ \bar{\mathcal{C}} &= i\mu_j \bar{c}_j, \\ \mathcal{E} &= i\mu_j b_j,\end{aligned}$$

then

$$-\frac{1}{2}\mathcal{A}_{ev} \cdot (\mathbf{d}\mathcal{E})^* - \frac{1}{2}\mathbf{d}\bar{\mathcal{C}} \cdot (\mathcal{D}\mathcal{C})^* + \frac{1}{4}\alpha\mathcal{E} \cdot \mathcal{E}^* = -A_j \wedge (db_j)^* - d\bar{c}_j \wedge (Dc_j)^* + \frac{1}{2}\alpha b_j \wedge b_j^*,$$

where

$$Dc_j = dc_j + ig[\tilde{A}, c]_j \quad \text{with} \quad \tilde{A} = A_j t_j, \quad j = 1, 2, 3, 8.$$

Thus the quantum action $\mathcal{S}_{qu} = \int_{M_4} \mathcal{L}_{qu}$ is given by

$$\begin{aligned}\mathcal{S}_{qu} &= \int d^4x \left\{ -\frac{1}{4}F_{\mu\nu}F_j^{\mu\nu} + A_{\mu j}\partial^\mu b_j + (\partial_\mu \bar{c}_j)(D^\mu c)_j \right. \\ &\quad \left. - \frac{\alpha}{2}b_j^2 + (D_\mu \Phi)^\dagger (D^\mu \Phi) - \lambda(\Phi^\dagger \Phi)^2 \right\},\end{aligned}\quad (35)$$

where $j = 1, 2, 3, 8$ and μ, ν are Lorentz indices with metric $g_{\mu\nu} = \text{diag}(1, -1, -1, -1)$.

5. Kinematical and dynamical evaluations of the Higgs mass

We note that the SU(2/1) curvature contains, in its main diagonal, both the SU(2)×U(1) field-strength and a quadratic term in Φ , whose square (in the Lagrangian) will reproduce the quartic term of the Higgs field potential $L(\Phi)$, with the SU(2/1) fixing the value of the coupling λ relative to the SU(2) gauge coupling g ,¹

$$\lambda = 2g^2, \quad L(\Phi) = -\kappa^2 \Phi^\dagger \cdot \Phi + \lambda(\Phi^\dagger \cdot \Phi)^2. \quad (36)$$

The mass of the Higgs field, as related to that of the W bosons gauging SU(2), is given by[21]

$$(M(\Phi))^2 = \frac{2\lambda}{g^2}(M_W)^2 = 4(M_W)^2, \quad M(\Phi) = 2M_W \quad (37)$$

¹The quadratic term in Φ in the Lagrangian can be generated either by hand or through the Coleman-Weinberg mechanism. Note that the mass squared ratio of M_W and M_Φ does not depend on κ^2 .

where we have applied the value of λ as given by (36). The derivation of this result applies the conventional spontaneous symmetry breakdown dynamical calculation, as given for instance in [21], with the symmetry only providing in addition the value of the quartic coupling, relative to the SU(2) gauge coupling. On the other hand, the above result coincides with the one obtained by one of us[22], in a kinematical calculation, which assumed SU(2/1) to act as a global, low-energy symmetry. In that derivation, the mass operator is taken – by the Wigner-Eckhardt theorem – to be proportional in its matrix elements to those of the generator μ_6 , since we have for the vacuum-expectation value $v = \langle 0|\phi^6|0 \rangle$. For Dirac fermions, this operator connects the two chiral states $\langle \Psi_L|\mu_6|\Psi_R \rangle \neq 0$, provided both exist. For bosons, as it is not diagonal, we have to evaluate M^2 ,

$$\langle \chi_K|M^2|\chi_K \rangle = \sum_N \langle \chi_K|\mu_6|\chi_N \rangle \langle \chi_N|\mu_6|\chi_K \rangle = n^2 \sum_N |c_{K6N}|^2 \quad (38)$$

For the gauge and Higgs boson masses, the K,N indices relate to the adjoint representation of the superalgebra; so does the index "6" of the operator. For the W,Z and photon masses, $K = 1, 2, 3, 8$ and $N = 4, 5, 6, 7$; this result then coincides with that of the conventional calculation, in which the squared masses are generated by squaring the isospin-hypercharge covariant derivative of ϕ^6 in $\mathcal{J}^-$; the c_{K6N} are thus the isospin-hypercharge transitions of $|\chi_6 \rangle$, coinciding with the generator algebra for the adjoint representation.

As to the mass of the Higgs meson, it arises in the squared curvature of the supergroup through the symmetric c_{6NK} coefficients, i.e. the d_{6NK} of SU(3), yielding the above (36) result for λ . The same coefficients will appear in the Wigner-Eckhardt formula of (38).

6. Renormalization of the supergroup couplings and the Higgs meson mass.

Within the supergauge multiplet there are two couplings which are arbitrary in the conventional Weinberg-Salam theory but are now given by the supergroup: g' and λ . Following the experimental practice, we may replace g' (given g) by the angle θ_W , through $\tan\theta_W = g'/g = 1/\sqrt{3}$, i.e. $\theta_W = \pi/6$ in $SU(2/1)$ in this superconnection (a3) version, as we discussed. However, we do not know of a conservation law, cancelling radiative corrections, for couplings given solely by the internal supergroup. Moreover, this (a3) version does not involve (a3) fields and ghosts beyond those appearing in the conventional gauge plus Higgs mechanism. The implication is that those couplings whose value is given only by the supergroup, beyond those supplied by the even subgroup, are indeed affected by radiative corrections and thus do undergo renormalization. Using the Renormalization Group equations, we should be able to estimate the corrections.

In one case – the angle θ_W – we know the group value and may compare it to the experimentally observed one. This should enable us to evaluate the energy $q^2 = E_s$ at which the symmetry becomes precise. We may then invert the procedure for λ : assume the supergroup value to hold at the energy E_s and evaluate the correction for $E \sim 100\text{GeV}$. The corrected value for λ can then be used to reevaluate the Higgs mass, i.e. its mass after inclusion of renormalization effects. This is then our strategy in what follows.

Note that our quantum action from $SU(2/1)$ has the same fields content as in the usual $SU(2) \times U(1)$.²

The renormalization group equations depend only on the fields content, thus we can use the result from the standard case: For the gauge couplings[23],

$$\frac{1}{[g_i(M)]^2} = \frac{1}{[g_i(M_0)]^2} + 2b_i \ln \frac{M}{M_0}, \quad i = 1, 2, 3 \quad (39)$$

²Here we did not include matter fields in our action, but the way in which they introduced, i.e., Yukawa couplings, etc., is the same as in the usual $SU(2) \times U(1)$ case.

where

$$\begin{aligned} b_1 &= \frac{1}{12\pi^2} \left(-\frac{5}{3}N_g - \frac{1}{8} \right), \\ b_2 &= \frac{1}{12\pi^2} \left(-N_g - \frac{1}{8} + \frac{11}{2} \right), \\ b_3 &= \frac{1}{16\pi^2} \left(-\frac{4}{3}N_g + 11 \right), \end{aligned}$$

N_g is the number of generations, and g_1, g_2, g_3 denote the gauge couplings of $U(1)$, $SU(2)$, $SU(3)$, respectively.

For the (top-quark) Yukawa and Higgs couplings[24],

$$\frac{dg_t}{dM} = \frac{1}{16\pi^2 M} \left\{ \frac{9}{2}g_t^3 - \left(\frac{17}{12}g_1^2 + \frac{9}{4}g_2^2 + 8g_3^2 \right)g_t \right\}, \quad (40)$$

$$\begin{aligned} \frac{d\lambda}{dM} &= \frac{1}{16\pi^2 M} \left\{ 24\lambda^2 + 12\lambda g_t^2 - 6g_t^4 - 3(g_1^2 + 3g_2^2)\lambda \right. \\ &\quad \left. + \frac{3}{8} [(g_1^2 + g_2^2)^2 + 2g_2^4] \right\}, \quad (41) \end{aligned}$$

where g_t, λ denote the top-quark Yukawa and Higgs couplings, respectively.

We used MATHEMATICA to solve these equations numerically: First we obtained the unbroken scale E_s of $SU(2/1)$ using (39), by finding the scale where $g_2 = \sqrt{3}g_1$. Then we solved the eqs.(40,41) with the boundary conditions such that $\lambda = 2g_2^2$ at E_s and $M_t = 174\text{GeV}$ in the low energy range ($E \sim 100 - 200\text{GeV}$).

We assumed three generations ($N_g = 3$), and used $\sin^2\theta_W(M_W) = 0.229 \pm 0.005$ [23], with $\alpha_Q^{-1} = 128.80 \pm 0.05$, $\alpha_2^{-1} = 29.5 \pm 0.6$, $\alpha_3^{-1} = 8.332$ where $\alpha_i^{-1} = \frac{4\pi}{g_i^2}$ and $\frac{1}{g_Q^2} = \frac{1}{g_1^2} + \frac{1}{g_2^2}$.

In solving the equations, we used the relation $g_t(M) = \frac{\sqrt{2}}{v}M_t = \frac{\sqrt{2}}{246}M_t$ where v is the vacuum expectation value of Φ , under the assumption that $SU(2/1)$ theory would fit these low energy experimental values.

The result we obtained is a Higgs meson mass of $130 \pm 6\text{GeV}$.

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References

- [1] Y. Ne'eman, Phys. Lett. B81 (1979) 190.
- [2] D. B. Fairlie, Phys. Lett. B82 (1979) 97.
- [3] Y. Ne'eman and J. Thierry-Mieg, in "Differential Geometrical Methods in Mathematical Physics" (Proceedings of the Aix-en-Provence and Salamanca 1979 International Conferences), P.L. Garcia, A. Perez-Rendon and J.M. Souriau eds., Lecture Notes in Mathematics No.836, Springer-Verlag, Berlin/Heidelberg/NewYork (1980), pp. 318-348.
- [4] Y. Ne'eman and J. Thierry-Mieg, Proc. Natl. Acad. Sci. USA 77 (1980) 720.
- [5] J. Thierry-Mieg and Y. Ne'eman, Nuovo Cimento 71 A (1982) 104.
- [6] J. Thierry-Mieg and Y. Ne'eman, Proc. Natl. Acad. Sci. USA 79 (1982) 7068.
- [7] Y. Ne'eman and J. Thierry-Mieg, Phys. Lett. B108 (1982) 399.
- [8] C.Y. Lee and Y. Ne'eman, Phys. Lett. B264 (1991) 389; *ibid.* B269 (1991) 477.
- [9] D. Quillen, Topology 24 (1985) 89.
- [10] Y. Ne'eman and S. Sternberg, Proc. Natl. Acad. Sci. USA 87 (1990) 7875; *ibid.* in "Symplectic Geometry and Mathematical Physics", (Proc. Intern. Conf. Diff. Geom. Meth. in Phys., Marseilles 1990), P. Donato et al. eds., Birkhauser, Boston (1991) pp. 326-354.
- [11] R. Coquereaux, G. Esposito-Farese and G. Vaillant, Nucl. Phys. B353 (1991) 689; R. Coquereaux, Phys. Lett. B 261 (1991) 449.
- [12] A. Connes, in "The interface of mathematics and particle physics", D. Quillen, G. Segal and S. Tsou eds. (Oxford Univ. Press, Oxford, 1990).

- [13] A. Connes and J. Lott, Nucl. Phys. B (Proc. Suppl.) 18B (1990) 29.
- [14] D. S. Hwang, C. Y. Lee and Y. Ne'eman, in preparation.
- [15] G. Curci and R. Ferrari, Nuovo Cimento 30 A (1975) 155.
- [16] J. Thierry-Mieg, J. Math. Phys. 21 (1980) 2834.
- [17] J. Thierry-Mieg, Nuovo Cimento 56 A (1980) 396.
- [18] M. Quiros, F. J. De Urries, J. Hoyos, M. L. Mazou, and E. Rodriguez, *ibid.* 22 (1981) 1767; L. Bonora and M. Tonin, Phys. Lett. B98 (1981) 48.
- [19] L. Baulieu and J. Thierry-Mieg, Nucl. Phys. B197 (1982) 477; *ibid.* B228 (1983) 259.
- [20] L. Corwin, Y. Ne'eman and S. Sternberg, Rev. Mod. Phys. 47 (1975) 573.
- [21] E.S. Abers and B.W. Lee, Physics Reports 9C (1973) 1.
- [22] Y. Ne'eman, Phys. Lett. B181 (1986) 308.
- [23] L. Maiani, in Proceedings of the NATO Advanced Study Institute on Z^0 Physics, Cargese 1990, ed. M. Levy *et al.* (Plenum Press, New York, 1991).
- [24] A. Sirlin and R. Zucchini, Nucl. Phys. B266 (1986) 389; H.E. Haber, Lectures given at TASI-90, June, 1990.