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KEK Preprint 94-170
December 1994
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Submitted to Nucl. Instrum. Meth., A.

National Laboratory for High Energy Physics, 1994

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Characterization of the wide-pitch ohmic-side readout for a silicon micro-strip detector[★]

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ABSTRACT

A study of the ohmic-side structure for a strip-type semiconductor detector is presented in terms of the capacitance matrix configuration and the signal-partitioning characteristics, which give a comprehensive idea of both a narrow- and wide-pitch detectors with either a large or small width-to-pitch ratio of the strip configuration.

★ Submitted to Nucl. Instr. & Meth. Section A

1. Introduction

In the KEK B-factory experiment, i.e. the BELLE collaboration,^[1] we will employ a double sided silicon micro-strip detector as a vertex detector,^[2] whose junction side is assigned for a $r - \phi$ measurement, and the ohmic-side is for a z -coordinate measurement. The detectors are arranged in a barrel-type configuration around the beam-pipe. The particles traversing with a small azimuthal angle undergo multiple scattering from the beam pipe. In order to maintain compatibility with the detector's spatial resolution^[3] and the amount of multiple scattering, a wide-pitch design of the strip detector has been explored. Another benefit of the wide-pitch design is to save the signal's amplitude for a shallow particle track, to reduce the number of ganging strips, and eventually to reduce the effective detector capacitance. The research interest of this paper is to present a strategic study of the ohmic-side structure of strip-type semiconductor detectors which are frequently used for high energy physics experiments. The critical issues for the most recent silicon-strip detectors are their effective detector capacitance and the signal's generation properties. These issues have often been discussed separately. We try to present a comprehensive idea which includes these two issues together, where the effect of the decoupling capacitor can be taken into account. In order to evaluate the coupling capacitances between the electrodes of the silicon micro-strip detector we have employed PISCES-IIB,^{[4][5]} a Poisson-equation solver with an over-relaxation method^[6] and a conformal transformation with an elliptic-integral function.^{[7][8]} PISCES-IIB provides a very exact description of the depletion property of the detector as well as the electric fields around the readout electrodes, while the limitation on the available number of grid meshes is quite inadequate to access the broad geometry of the strip detector. Although the Poisson-equation solver can handle larger geometry than can PISCES-IIB, it still shares common trouble with PISCES-IIB. We choose to employ a conformal transformation approach with a partial cross check with the Poisson-equation solver. In chapter 2 we describe the general idea of a weighting field together with the equivalent capacitance matrix of a strip detector. In chapter 3 we apply the scenario for wide-pitch structures in

order to investigate their effective capacitances and signal-partitioning properties. Chapter 4 summarizes the discussion.

2. Theory of signal induction

2.1. WEIGHTING FIELD

Let the weighting fields^[9] corresponding to each readout electrode be

$$\{\Phi_1, \Phi_2, \dots, \Phi_n\}. \quad (2.1)$$

Readout electrodes 1 to n are hereafter called external electrodes, which are directly connected to a readout preamplifier. The junction-side node is assigned for the $(n+1)$ -th external nodes. The weighting functions generated from the implantation and/or the floating nodes, i.e. hereafter called internal nodes, are

$$\{\phi_1, \phi_2, \dots, \phi_{2n-1}\}, \quad (2.2)$$

where the odd-numbered nodes are connected to the external readout nodes via a decoupling capacitor (C_e), and the even-numbered nodes are either explicit or implicit floating nodes. When the k -th external electrode is stepped by v_0 volt, the l -th internal node is influence so as to raise its voltage by $v_l^{(k)} = f_l^{(k)} v_0$. Employing $f_l^{(k)}$, we can obtain the relations between $\{\Phi's\}$ and $\{\phi's\}$ as follows:

$$\Phi^{(k)} = \sum_{l=1}^{2n-1} f_l^{(k)} \phi_l, \quad (2.3)$$

where k runs over 1 to n . In practice, the voltage and charge relations between the readout and the implantation nodes are determined via an equation which includes the capacitance matrix of this system. Before moving on to detailed discussions,

we define three capacitance matrices (C_1 , C_2 , and C_3). Employing C_1 , C_2 , and C_3 , we can write the voltage-charge relation of the entire system as

$$\begin{pmatrix} \mathbf{q}_1 \\ \mathbf{q}_2 \end{pmatrix} = \begin{pmatrix} C_1 & C_2 \\ {}^tC_2 & C_3 \end{pmatrix} \begin{pmatrix} \mathbf{v}_1 \\ \mathbf{v}_2 \end{pmatrix}. \quad (2.4)$$

C_1 is an $(n+1)$ -by- $(n+1)$ diagonal matrix, C_2 is a $(2n-1)$ -by- $(n+1)$ matrix, and C_3 is a $(2n-1)$ -by- $(2n-1)$ symmetric matrix. Details concerning C_1 , C_2 , and C_3 are described in a later section. Vectors $\mathbf{v}_1$ and $\mathbf{v}_2$ represent the voltages on the external and internal nodes, respectively. Vectors $\mathbf{q}_1$ and $\mathbf{q}_2$ represent the charges on the external and internal nodes, respectively. In order to evaluate the weighting field, the k -th external electrode is set to v_0 , while the other external nodes are kept at 0 volt, i.e.

$${}^t\mathbf{v}_1 = (0 \quad \dots \quad 1^{(k)} \quad \dots \quad 0^{(n+1)}). \quad (2.5)$$

Since there is no low-impedance current flow path for the internal nodes, we can set the induced charge $\mathbf{q}_2$ to zero, which leads to

$${}^tC_2\mathbf{v}_1 + C_3\mathbf{v}_2 = 0. \quad (2.6)$$

By applying C_3^{-1} in Eq. (2.6) we obtain

$$\mathbf{v}_2 = -C_3^{-1}({}^tC_2\mathbf{v}_1). \quad (2.7)$$

At the very vicinity of the ohmic-side detector boundary we can set $f_l^{(k)} = [\mathbf{v}_2]_l/v_0$, with $\phi_l = 1$ at the surface of the l -th electrode, and $\phi_l = 0$ at the rest of the surface. As an effect of increasing the voltage of the k -th external electrode, the charge configurations for the external electrodes are modified as

$$\mathbf{q}_1 = C_1\mathbf{v}_1 + C_2\mathbf{v}_2. \quad (2.8)$$

Paying attention to the k -th element of $\mathbf{q}_1$ ($[\mathbf{q}_1]_k$) we obtain

$$[\mathbf{q}_1]_k = C_c v_0 + \sum_{l=1}^{2n-1} [\mathbf{C}_2]_{kl} [\mathbf{v}_2]_l, \quad (2.9)$$

from which we can evaluate the effective capacitance ($C_f^{(k)}$) for the k -th electrode as

$$C_f^{(k)} = C_c + \frac{1}{v_0} \sum_{l=1}^{2n-1} [\mathbf{C}_2]_{kl} [\mathbf{v}_2]_l, \quad (2.10)$$

where $[\mathbf{C}_2]_{kl}$, and $[\mathbf{v}_2]_l$ represent the (k, l) th matrix element of $\mathbf{C}_2$ and the l th vector element of $\mathbf{v}_2$, respectively. A useful relation for cross-checking is to make sure that

$$\sum_{k=1}^{n+1} [\mathbf{q}_1]_k = 0, \quad (2.11)$$

which results from static-charge conservation.

2.2. CAPACITANCE MATRIX

Figure 1 shows the capacitance elements appearing in this text, which should be referred to appropriately while considering discussions given in this section. The figure is simplified in order to eliminate some capacitance elements which may appear in the network repeatedly. Ext1, Ext2 and so on designate the external nodes. Int1, Int2 and so on designate the internal nodes. Matrix $\mathbf{C}_1$ is

$$\begin{pmatrix} C_c & 0 & 0 & 0 & 0 & 0 \\ 0 & C_c & 0 & 0 & 0 & 0 \\ 0 & 0 & C_c & 0 & 0 & 0 \\ 0 & 0 & 0 & C_c & 0 & 0 \\ 0 & 0 & 0 & 0 & C_c & 0 \\ 0 & 0 & 0 & 0 & 0 & C_G \end{pmatrix} \quad (2.12)$$

when we take $n=5$. In the actual study described in chapter 3 we employed a geometry of $n=9$; we show here the matrices with $n=5$ for simplicity of presentation.

The right-, bottom-most matrix element (C_G) comes from the common node of the junction side. C_G is the total capacitance for the junction-side electrode, plus the residual capacitances for very far strips which are not taken into account in the matrix element explicitly, whose actual presentation is defined later in Eq. (2.26). C_c is the capacitance between the implantation node and the readout node, whose unit capacitance is

$$\frac{\epsilon_{ox} \epsilon_0}{t_{ox}} \rightarrow 0.173 \text{ fF}/\mu\text{m}^2 \quad (2.13)$$

for a 0.2nm-thick silicon dioxide insulator film. Table 1 shows the typical parameters employed here to evaluate the capacitance. Matrix $\mathbf{C}_2$ is

$$\begin{pmatrix} -C_c & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -C_c & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -C_c & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -C_c & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -C_c \\ -C_g^{(1)} & -C_g^{(2)} & -C_g^{(3)} & -C_g^{(4)} & -C_g^{(5)} & -C_g^{(4)} & -C_g^{(3)} & -C_g^{(2)} & -C_g^{(1)} \end{pmatrix} \quad (2.14)$$

where

$$C_g^{(1)} = C_b^{(1)} + C_i^{(1)} - (C_i^{(4)} + C_i^{(3)} + C_i^{(2)} + C_i^{(1)}), \quad (2.15)$$

$$C_g^{(2)} = C_b^{(2)} + C_i^{(2)} - (2C_i^{(4)} + C_j^{(3)} + C_i^{(2)} + C_j^{(1)}), \quad (2.16)$$

$$C_g^{(3)} = C_b^{(1)} + C_i^{(1)} - \{2(C_i^{(4)} + C_i^{(3)}) + C_i^{(2)} + C_i^{(1)}\}, \quad (2.17)$$

$$C_g^{(4)} = C_b^{(2)} + C_i^{(2)} - \{2(C_i^{(4)} + C_j^{(3)} + C_i^{(2)}) + C_j^{(1)}\}, \quad (2.18)$$

and

$$C_g^{(5)} = C_b^{(1)} + C_i^{(1)} - \{2(C_i^{(4)} + C_i^{(3)} + C_i^{(2)} + C_i^{(1)})\}. \quad (2.19)$$

$C_i^{(k)}$'s and $C_j^{(k)}$'s are the coupling capacitance measured from the odd-numbered internal nodes (implantation nodes) and from the even-numbered internal nodes

(floating nodes), respectively. $C_i^{(1)}$ is the total capacitance of the internal implantation node. $C_i^{(2)}$ is the total capacitance of the internal floating node. The evaluation procedure of $C_i^{(k)}$, $C_j^{(k)}$, $C_i^{(1)}$, and $C_i^{(2)}$ is described in chapter 3 with the aid of the conformal transformation of the elliptic integral function. $C_b^{(1)}$ and $C_b^{(2)}$ are the capacitances of the n^+ implantation node and the floating node to the junction-side. The junction capacitance of the p-i-n diode is described as

$$\frac{\epsilon_{si}\epsilon_0}{l_d} \rightarrow 0.351aF/\mu m^2, \quad (2.20)$$

where ϵ_0 is the permittivity of the vacuum, ϵ is the dielectric constant of silicon, and l_d is the thickness of the depletion region. Taking the width of the n^+ implantation as w and the pitch of the periodic structure as p , we obtain

$$C_b^{(1)} = 0.351 * 10^{-18} * w \text{ [F}/\mu m] \quad (2.21)$$

and

$$C_b^{(2)} = 0.351 * 10^{-18} * (p - w) \text{ [F}/\mu m]. \quad (2.22)$$

Matrix C_3 is

$$\begin{pmatrix} C_T^{(1)} & -C_i^{(4)} & -C_i^{(3)} & -C_i^{(2)} & -C_i^{(1)} & 0 & 0 & 0 & 0 \\ -C_i^{(4)} & C_T^{(2)} & -C_i^{(4)} & -C_j^{(3)} & -C_i^{(2)} & -C_j^{(1)} & 0 & 0 & 0 \\ -C_i^{(3)} & -C_i^{(4)} & C_T^{(1)} & -C_i^{(4)} & -C_i^{(3)} & -C_i^{(2)} & -C_i^{(1)} & 0 & 0 \\ -C_i^{(2)} & -C_j^{(3)} & -C_i^{(4)} & C_T^{(2)} & -C_i^{(4)} & -C_j^{(3)} & -C_i^{(2)} & -C_j^{(1)} & 0 \\ -C_i^{(1)} & -C_i^{(2)} & -C_i^{(3)} & -C_i^{(4)} & C_T^{(1)} & -C_i^{(4)} & -C_i^{(3)} & -C_i^{(2)} & -C_i^{(1)} \\ 0 & -C_j^{(1)} & -C_i^{(2)} & -C_j^{(3)} & -C_i^{(4)} & C_T^{(2)} & -C_i^{(4)} & -C_j^{(3)} & -C_i^{(2)} \\ 0 & 0 & -C_i^{(1)} & -C_i^{(2)} & -C_i^{(3)} & -C_i^{(4)} & -C_T^{(1)} & -C_i^{(4)} & -C_i^{(3)} \\ 0 & 0 & 0 & -C_j^{(1)} & -C_i^{(2)} & -C_j^{(3)} & -C_i^{(4)} & C_T^{(2)} & -C_i^{(4)} \\ 0 & 0 & 0 & 0 & -C_i^{(1)} & -C_i^{(2)} & -C_i^{(3)} & -C_i^{(4)} & C_T^{(1)} \end{pmatrix} \quad (2.23)$$

where

$$C_T^{(1)} = C_i^{(1)} + C_c + C_b^{(1)} \quad (2.24)$$

and

$$C_T^{(2)} = C_i^{(2)} + C_b^{(2)}. \quad (2.25)$$

There is a supplementary equation for C_G , which has already appeared in matrix C_1 ,

$$C_G = C_g^{(5)} + 2(C_g^{(4)} + C_g^{(3)} + C_g^{(2)} + C_g^{(1)}). \quad (2.26)$$

2.3. CHARGE SHARING WITH THE CAPACITANCE MATRIX

The induced signal for the k -th electrode is described in terms of the weighting field $\Phi^{(k)[10]}$ as

$$i^{(k)}(t) = q_p \nabla \Phi^{(k)} * \mathbf{u}(t), \quad (2.27)$$

where $\mathbf{u}(t)$ is the velocity vector of the drifting carrier and q_p is the amount of charge drifting together. The velocity of the carrier ($\mathbf{u}(t)$) is controlled by the electric field ($\mathbf{E}$) and the magnetic field ($\mathbf{B}$),

$$\mathbf{u}(t) = \mu_{(e,h)} \mathbf{E} + \mu_{(e,h)} \mu_{H(e,h)} \mathbf{E} \times \mathbf{B}, \quad (2.28)$$

where $\mu_{(e,h)}$ is the carrier mobility for electrons or holes, and $\mu_{H(e,h)}$ is the Hall mobility. We do not go into detail here concerning the effect of the B field for simplicity of discussion. For electrons, we assign a negative mobility for convention, while for holes we assign a positive mobility. In semiconductor devices, a stimulus of the energy deposit generates the same amount of electrons and holes simultaneously. The electrons and holes drift in opposite directions from each

other, eventually arriving at (x_1, y_1) and (x_2, y_2) , respectively. The total charge appearing on the k -th electrode is

$$[q_1]_k = \int_0^\infty \{i_e^{(k)}(t) + i_h^{(k)}(t)\} dt \rightarrow q_p \int_{(x_1, y_1)}^{(x_2, y_2)} \nabla \Phi^{(k)} ds, \quad (2.29)$$

where $\mathbf{s}$ represents a curved coordinate along the drifting carrier, and ds is tangential to the trajectory. Eq. (2.29) is virtually independent of the original location where the electron-hole pair is created. If the drift velocity is always directed along the y -coordinate, i.e. $x_1 = x_2$ and $d\mathbf{s} = dy$, Eq. (2.29) becomes simply

$$[q_1]_k = q_p \int_{(x_1, y_1)}^{(x_1, y_2)} \frac{d\Phi^{(k)}}{dx} dx \rightarrow q_p (\Phi^{(k)}(x_1, y_2) - \Phi^{(k)}(x_1, y_1)). \quad (2.30)$$

Driving field The electric field which drives the electron and hole carriers to drift in the depletion area could be approximated as

$$E_y = \frac{2V_0}{l_d^2} y \quad (2.31)$$

and

$$\phi = -\frac{V_0}{l_d^2} y^2, \quad (2.32)$$

where the ohmic side electrodes are on the $y = 0$ plane, and the junction-side electrodes are on the $y = l_d$ plane. We did not take into account the effect of the horizontal electric field (E_x), which might be important in discussing the time-dependent nature of signal capture.

Smearing of the weighting field On the junction side, due to the high attractive field to the p^+ implantation, the drifting charge, i.e. hole, eventually arrives at the very vicinity of the p^+ electrode. This is the reason why the measured particle

track appears to be digitized. For the ohmic side, the electric field which drives the carriers is sufficiently weak to compete with the thermal diffusion. In order to derive a realistic response, we should smear the weighting field at the side of the ohmic contact in terms of x_1 in Eq. (2.30) by

$$\sigma = \sqrt{Dt}, \quad (2.33)$$

where D is the diffusion constant, defined as

$$D = \frac{\mu kT}{q}, \quad (2.34)$$

and t is the drift time before being captured by the readout electrode,

$$t = \int_{y_1}^{y_2} \frac{dy}{\mu E_y}. \quad (2.35)$$

By employing Eqs.(2.35) and (2.31), we obtain

$$t = \frac{l_d^2}{2V_0\mu} \ln(y_2/y_1). \quad (2.36)$$

In Eq. (2.34) k is the Boltzmann constant, T the absolute temperature, and q the electronic charge. Eventually, Eq. (2.33) is reorganized as

$$\sigma = l_d \sqrt{kT \ln(y_2/y_1) / (2qV_0)}. \quad (2.37)$$

Putting $kT/q = 26mV$, $y_2/y_1 = 100$ for convenience, $l_d = 300\mu m$, and $V_0 = 80V$, we have $\sigma = 8.2 \mu m$. We try to take three values in the later analysis, i.e. $\sigma = 5.0, 10.0$, and $15.0 \mu m$, to smear the weighting field. There exist other smearing sources for an actual measurement: the distribution of the charge on a line segment, the Landau fluctuation for primary ionization, and electronic noise, which are not taken into account explicitly.

3. Simulation study

3.1. COUPLING CAPACITANCE

Elliptic integral of the first kind The elliptic integral

$$w = \int_0^z \frac{du}{\sqrt{(1-u^2)(1-\kappa^2 u^2)}} \quad (3.1)$$

maps the upper- z plane to the inside of the rectangle of the w -plane. The corners $(K, 0)$, (K, K') , $(-K, K')$, and $(-K, 0)$ appearing in the w -plane correspond to the coordinates on the real-axis on the z -plane $(1, 0)$, $(1/\kappa, 0)$, $(-1/\kappa, 0)$, and $(-1, 0)$, respectively). K and K' are described as

$$K(\kappa) = \int_0^{\pi/2} \frac{d\theta}{\sqrt{1-\kappa^2 \sin^2 \theta}} \quad (3.2)$$

and

$$K'(\kappa) = K(\sqrt{1-\kappa^2}), \quad (3.3)$$

respectively. When the metal strip is located $(-1, 0)$ to $(1, 0)$ between the semi-infinite planes whose edges are located at $(-1/\kappa, 0)$ and $(1/\kappa, 0)$ the capacitance for the strip to the rest of the world is

$$C = \frac{4\epsilon K(\kappa)}{K'(\kappa)}, \quad (3.4)$$

where ϵ is the effective permittivity of the environmental medium. We take it as an average over silicon and silicon-dioxide, i.e. 7.9. In order to take into account the effect from the lower-half plane we include a factor 2 in the above equation. We also mention that the capacitance of the slit geometry depends only on the ratio of the geometry. We take the coordinate origin at the center of the strip. When the

strip covers from $(-a, 0)$ to $(a, 0)$, i.e. $w = 2a$, and the edge of the semi-infinite plane is located at $(-b, 0)$, and $(b, 0)$, i.e. $s = b - a$, we have $\kappa = a/b = w/(w + 2s)$. We take s as being as small as 3 to 5 μm . This space is required due to a mathematical reason to prevent a singularity in the procedure of integrating Eq. (3.2) as well as a practical reason of the silicon process. The sensitivity of s on the effective strip capacitance is discussed in the next section.

Coupling to the far segments In order to estimate the coupling to the adjacent strips we need to estimate the coupling capacitance for an arbitrary segment. Assuming that a segment is located from $(x_1, 0)$ to $(x_2, 0)$, where x_1 , and x_2 are larger than b , we have a mirror image of these coordinates at $(ab/x_1, 0)$, and $(ab/x_2, 0)$. The image charge appearing in this region is opposite and equal to the charge appearing in the x_1 - x_2 segment. Integration of the charge located in ab/x_1 to ab/x_2 gives the capacitance for x_1 - x_2 segment. Employing Eq. (3.2) within the integration region of $\theta_1 = \sin^{-1}(b/x_1)$ to $\theta_2 = \sin^{-1}(b/x_2)$ we obtain a partial capacitance,

$$C'(x_1, x_2, \kappa) = \frac{2\epsilon}{K'(\kappa)} \int_{\theta_2}^{\theta_1} \frac{D(b/\sin\theta)d\theta}{\sqrt{1-\kappa^2 \sin^2 \theta}}, \quad (3.5)$$

where $D(x) = \exp(-(x - \sqrt{ab})/l_c)$ is used to incorporate the shielding effect from the junction side electrode. $\sqrt{ab}$ in $D(x)$ is chosen because $(\sqrt{ab}, 0)$ is the pivot of the mirror image between the strip and the semi-infinite plane. We recognize that the damping factor $(D(x))$ should exist based on physical considerations, and that the exponential function adopted here keeps consistency with the result from the Poisson-equation solver. Figure 2 (a), (b) and (c) show the consistency with the Poisson-equation solver and the conformal transformation for electrode widths of 20, 40 and 80 μm . The vertical axis is the coupling from the central strip of width w to a strip segment for each 5 μm . Each figure is accompanied by three analytically calculated lines with $l_c = 160, 200$ and 240 . The agreement between the two approaches appears to be quite good when we take an appropriate cut-off length (l_c) . A systematic disagreement between the Poisson-equation solver and

the conformal transformation appears at the right-most area, which is due to the boundary of the grid-mesh of the Poisson-equation solver. The effect of l_c on the effective strip capacitance is discussed in the next section. In terms of $C(x_1, x_2, \kappa)$, we can write $C_i^{(k)}$'s as

$$C_i^{(4)} = C(w/2 + s, p - w/2, \kappa), \quad (3.6)$$

$$C_i^{(3)} = C(p - w/2, p + w/2, \kappa), \quad (3.7)$$

$$C_i^{(2)} = C(p + w/2, 2p - w/2, \kappa), \quad (3.8)$$

$$C_i^{(1)} = C(2p - w/2, 2p + w/2, \kappa) \quad (3.9)$$

and

$$C_i^{(1)} = 2 * C(w/2 + s, \infty, \kappa), \quad (3.10)$$

where w is the width of the readout strip, p the pitch, and s the narrow gap between the readout strip and the intermediate floating region. Taking a composite pattern of a given geometry, we obtain the following coupling capacitances for $C_j^{(k)}$'s:

$$C_j^{(3)} = C(p - w'/s, p + w'/2, \kappa'), \quad (3.11)$$

$$C_j^{(1)} = C(2p - w'/2, 2p + w'/2, \kappa') \quad (3.12)$$

and

$$C_i^{(2)} = 2 * C(w'/2 + s, \infty, \kappa'), \quad (3.13)$$

where $w' = p - w$, and $\kappa' = w'/(w' + 2s)$.

3.2. EFFECTIVE STRIP CAPACITANCE

Employing the general formula Eq. (2.10) with the capacitance matrices defined in the previous section, we show the effective strip capacitances for various configurations in Figure 3 (a), (b) and (c), where the bulk capacitance over the strip pitch (p) was removed from the effective capacitance before drawing the figure. The figures are accompanied by solid and dotted lines, which are from the experimental formula and the Ilusson's approximation with an elliptic integral, which appeared in ref 7. We checked these figures for three issues while taking into account the consistency with the existing and/or known results. The first thing is the sensitivity on the cut-off parameter (l_c). As discussed in the previous section, we found that the Poisson-equation solver and the segment capacitance from Eq. (3.5) provided consistent result for $l_c = 200 \text{ } \mu m$ and its vicinity. The effective capacitance again supports this results. The second concerns the sensitivity on the gap length (s). When we take the gap length as about $4 \text{ } \mu m$, we can obtain a consistent results with the known data. The third is the scalability on the w/p ratio. We show the effective capacitances for three pitches ($p=50, 100$, and $200 \text{ } \mu m$) with $l_c = 200 \text{ } \mu m$ and $s = 4 \text{ } \mu m$. We can find a virtually perfect scalability over a w/p ratio of 0.2 to 0.8. We eventually find that the effective capacitance derived here is quite close to the experimentally known result for wide a parameter range of w/p , p , l_c and s .

3.3. SIGNAL PARTITIONING

Employing Eq. (2.7) to estimate the weighting field, we can evaluate the signal's response for a given periodic geometry. Figure 4 (a), (b), (c), and (d) show the effective signal responses in terms of distance from the center of the readout strip, where the decoupling capacitance (C_c) is assumed to cover the full width of the readout electrode, i.e. $C_c = 0.173 * 10^{-15} * w [F/\mu m]$. When the strip pitch is $50 \text{ } \mu m$ and the smearing parameter (σ) is larger than $10 \text{ } \mu m$, we obtain a very smooth response curve as a function of the distance, which is comfortable for

interpolating coordinates between the adjacent readout strips. While the figure is for a geometry with $w/p = 0.4$, we could still obtain a good interpolation capability even with $w/p = 0.2$, owing to subsidiary reasons of smearing. When the strip pitch is $100\ \mu m$, the response curve is far from a linear response, even for the case of $w/p=0.6$. It appears to be useless to increase the strip width further, since the linear-response region becomes narrower in any case. Choosing a larger w/p , we obtain a penalty of the larger strip capacitance. For a strip pitch of $200\ \mu m$, the signal-partitioning property is more or less digital. In order to maintain the benefit of the charge-sharing capability we can subdivide the floating-electrode region, which substantially recovers smooth charge sharing characteristics. In practice, we modify the capacitance matrix as to be compatible with an every-two strip readout with a $p = 100\ \mu m$ structure. Comparing two figures for a geometry of $w = 60\ \mu m$ with $p = 200\ \mu m$ and for a geometry of $w = 60\ \mu m$ with the every-two strip readout of $p = 100\ \mu m$, we can convince ourselves that the every-two readout scheme is more superior than the simple $p = 200\ \mu m$ structure.

4. Summary

Based on a general approach of the capacitance matrix and the weighting field, we built a quick evaluation code for the ohmic-side structure. Instead of solving the Poisson equation numerically, we employed an analytic formula with a conformal transformation. In order to take into account the shielding effect of the junction-side electrode, we artificially introduced a cut-off parameter (l_c). The effective strip capacitance calculated here well reproduced the existing models and experimental formula. We could find that the scaling property on w/p was well maintained up to $p = 200\ \mu m$ or more. Since we concentrated on the total charge for the readout instead of the current waveform, the discussions were simplified. The weighting "potential" which we required was well approximated by a weighting "potential" on the internal nodes. In order to estimate the signal's response, we smeared the weighting "potential" with an appropriate smearing parameter which

was derived from the thermal diffusion. While the signal's response was well shared between the adjacent strips for narrower pitch detectors, say $p = 50\ \mu m$, for wider pitch detectors the signal's response became more or less digital unless we employed a special readout scheme, such as an every-two strip readout. An experimental study with a real strip detector is in progress for both a field-plate separation detector and a p^+ -stop separation detector of strip pitches of 100 to 250 μm .

ACKNOWLEDGEMENTS

This work is a part of the developmental effort for the silicon micro-strip vertex detector of the BELLE collaboration. The directors of the physics department of KEK, S. Iwata, F. Takasaki, and M. Kobayashi, are acknowledged for their encouragement during this work.

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TABLE CAPTIONS

- 1: Parameter for capacitance evaluation

Table 1. Parameters for capacitance evaluation

Parameters	unit	
Permittivity of vacuum (ϵ_0)	F/cm	8.854e-14
Dielectric constant of silicon (ϵ_{si})	-	11.9
Dielectric constant of silicon dioxide (ϵ_{ox})	-	3.9
Detector thickness (l_d)	μm	300
Thickness of gate oxide (t_{ox})	μm	0.2

FIGURE CAPTIONS

- 1) Capacitance network
- 2) Consistency with the Poisson equation solver
 - a) $w = 20 \mu m$
 - b) $w = 40 \mu m$
 - c) $w = 80 \mu m$
- 3) Capacitance in terms of w/p

The horizontal axis is the w/p ratio. The vertical axis is the effective strip capacitance minus the bulk capacitance.

 - (a) Sensitivity on l_c for $p = 100 \mu m$, and $s = 4 \mu m$. The three curves are for $l_c = 160, 200$, and $240 \mu m$
 - (b) Sensitivity on s for $p = 100$, and $l_c = 200$. The three curves are for $s = 3, 4$, and $5 \mu m$.
 - (c) Scaling for p for $l_c = 200 \mu m$, and $s = 4 \mu m$. The three curves are for $p = 50, 100$, and $200 \mu m$
- 4) Signal's response at the ohmic side
 - a) $p = 50$, $w/p = 0.4$ with smearing of 5, 10, and 15 μm .
 - b) $p = 100$, $w/p = 0.2, 0.4$, and 0.6 with smearing of 10 μm .
 - c) $p = 200$, $w/p = 0.2, 0.4$, and 0.6 with smearing of 10 μm .
 - d) $p = 100$, $w/p = 0.2, 0.4$ and 0.6 with every-two readout and smearing of 10 μm

FIG2. (A)

Figure 1

FIG. 2 (c)

FIG. 2 (B)

CAPACITNACE IN pF/cm

FIG. 3 (B)

CAPACITANCE IN pF/cm

FIG. 3 (A)

CHARGE COLLECTION EFFICIENCY

FIG. 4 (A)

CAPACITANCE IN pF/cm

FIG. 3 (C)

CHARGE COLLECTION EFFICIENCY

FIG. 4 (C)

CHARGE COLLECTION EFFICIENCY

FIG. 4 (B)

FIG. 4 (D)