Dissipation and memory capacity in the quantum brain model

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ABSTRACT

The quantum model of the brain proposed by Ricciardi and Umezawa is extended to dissipative dynamics in order to study the problem of memory capacity. It is shown that infinitely many vacua are accessible to memory printing in a way that in sequential information recording the storage of a new information does not destroy the previously stored ones, thus allowing a huge memory capacity. The mechanism of information printing is shown to induce breakdown of time-reversal symmetry. Thermal properties of the memory states as well as their relation with squeezed coherent states are finally discussed.
1. Introduction

The purpose of this paper is the study of the problem of memory capacity in the Ricciardi-Umezawa quantum model of brain[1] by resorting to recent results on dissipative systems in quantum field theory (QFT)[2].

Coupling coefficients and activity thresholds of artificial neuron units are central ingredients in neural network machines simulating the brain functions. Ricciardi and Umezawa[1] have observed that in the study of natural brain it is pure optimism to hope to determine the values of the coupling coefficients and the activity thresholds of all neurons by means of anatomical or physiological methods. On the other hand, specific activities of the natural brain persist in spite of changes in the number of alive neurons. In other words, the functioning of the whole brain appears not significantly affected by the functioning of the single neuron, neither physiological observations show the existence of special long-lived neurons or the existence of a large redundancy in specialized neuronal circuits. Besides the neurons, many other thousands of elements, as glia cells, play a rôle in the brain activity, which, again, is not critically dependent on the single cell functioning.

A characterizing feature of the brain activity is instead related with nonlocality, namely with the existence of simultaneous responses in several regions of the brain to some external stimuli. This suggests that the brain may be in states characterized by the existence of long range correlations among its elementary constituents; such long range correlations seem to play a more fundamental rôle than the functioning of the single cell in the brain activity.

Storing and recalling information appear as a diffuse activity of the brain not lost even after destructive action of local parts of the brain or after treatments with electric shock or with drugs. This suggests to model these memory activities as coding of the brain states whose stability is to be derived as a dynamical feature rather than as a property of specific neural nets which would be critically damaged by the above destructive actions.

Stable long range correlations and diffuse, nonlocal properties related with a code specifying the system state are dynamical features of quantum origin. Ricciardi and Umezawa[1] have thus proposed a quantum model where the el-
mentary constituents of the brain exhibit coherent behaviour and macroscopic observables are derived as dynamical output from their interaction.

Pioneering proposals relating advanced results in quantum optics, such as holography, with brain models were put forward by Pribram[3]. In more recent years, an analysis of non-algorithmic and non-computational character of brain functions has been made by Penrose[4], who has also proposed the quantum framework as the proper one to bridge microscopic dynamics with macroscopic functional activity of the brain.

For a general account of application of modern statistical mechanics and spin glass theory to brain system see refs. [5] and [6].

In the quantum model of Ricciardi and Umezawa the elementary constituents are not the neurons and the other cells and physiological units, which cannot be considered as quantum objects, but some dynamical variables, called corticons, able to describe stationary or quasi-stationary states of the brain.

A crucial assumption, based on the fact that the brain is an open system in interaction with the external world, is that information printing is achieved under the action of external stimuli producing breakdown of the continuous symmetry associated to corticons.

As well known, in spontaneously broken symmetry theories the Lagrangian is invariant under some group, say $G$, of continuous transformations; however, the minimum energy state, i.e. the ground state or vacuum, of the system is not invariant under the full group $G$, but under one of its subgroups. In this case, general theorems of QFT[7] show that the vacuum is an ordered state and collective modes (called Nambu-Goldstone bosons) propagating over the whole system are dynamically generated and are the carriers of the ordering information (long range correlations). In other words, order manifests itself as a global property dynamically generated and the quantum numbers characteristic of the collective mode acts as coding for the ground state: ordering and coding are thus achieved by the condensation of collective modes in the vacuum.

One important point, is that the collective mode is a gapless mode and therefore its condensation in the vacuum does not add energy to it. As a consequence, the stability of the ordering and of the coding is insured. Another consequence is that infinitely many vacua with different degrees of order may
exist, corresponding to different densities of the condensate. In the infinite volume limit these vacua are each other unitarily inequivalent and thus represent possible physical phases of the system, which thus appears as a complex system with many macroscopic configurations (phases). The actual phase is determined once one among the many degenerate vacua is selected as an effect of some external action.

Transitions among these vacua are in general not implementable (non-existence of unitary transformations relating different vacua) in the infinite volume limit; however, in the case of open systems these transitions may occur (phase transitions), for large but finite volume, due to coupling with external environment. The inclusion of dissipation leads thus to a picture of the system "living over many ground states" (continuously undergoing phase transitions)[8]. It is interesting to observe that even very weak (although above a certain threshold) perturbations may drive the system through its macroscopic configurations[8]. In this way, occasional (random) weak perturbations are recognized to play an important rôle in the complex behavior of living systems.

The observable specifying the ordered state is called order parameter and acts as a macroscopic variable since the collective modes present coherent dynamical behavior. The order parameter is specific of the kind of symmetry into play and may thus be considered as a code specifying the vacuum. The value of the order parameter is related with the density of condensed Goldstone bosons in the vacuum and specifies the phase of the system with relation to the considered symmetry. Since physical properties are different for different phases, also the value of the order parameter may be considered as a code number specifying the system state. In conclusion, code numbers specifying the phases may be organized in classes corresponding to different kinds of dynamical symmetry.

A typical example of spontaneous breakdown of symmetry is provided by the ferromagnet where the Lagrangian is invariant under the spin rotation group, but the ground state is invariant only under rotations around the direction of the magnetization. The collective modes are the spin-wave quanta or magnons and the system phases are indeed macroscopically characterized (coded) by the value of the magnetization, which is the order parameter. The magnetic order is thus a diffused, i.e. macroscopic, feature of the system.
The collective mode of the Ricciardi-Umezawa brain model has been called symmetron[9] and the information storage function is represented by the coding of the ground state through symmetron condensation.

The corticon has been assumed[9] to be a two state system and the associated symmetry is a phase symmetry.

By following Fröhlich[10], Del Giudice et al.[11-18] have assumed that the symmetry to be spontaneously broken in living matter is the rotational symmetry for electrical dipoles. Such an assumption is phenomenologically based on the fact that living matter is made up by water and other biomolecules equipped with electric dipoles. The (electric) polarization density thus plays the rôle of order parameter and the associated Goldstone modes have been named dipole wave quanta ($dwq$). In the QFT approach to living matter the dynamical generation of collective modes thus shed some light on the problem of change of scale in biological systems, namely the problem of the transition from the microscopic scenario to the many macroscopic functional properties (many macroscopic configurations) of the living systems. Del Giudice et al. have shown the superradiant or “lasing” behaviour of water electrical dipoles[15] and the self-focusing propagation of the electrical field in ordered water[13], thus providing a conjecture for the formation of microtubules[13,18]. It has been shown[15] that the coherent interaction of water molecules with the quantized radiation field leads to a time scale for the coherent long range interaction much shorter ($10^{-14}\text{ sec}$) than the one of short range interactions. Water coherent domains are therefore protected from thermalization. Solitary wave propagation on biomolecular chains, as proposed by Davydov[19], has also been studied[12] and related with triggering of breakdown of symmetry.

Spontaneous breakdown of electric dipole rotational symmetry has revealed to be useful also in further developments of the quantum brain model (referred to as quantum brain dynamics (QBD)) worked out by Jibu and Yasue[20-24] who have identified the Ricciardi-Umezawa symmetron modes with $dwq$ and the corticon with the electric dipole field. They have obtained a first understanding of anesthesia and have elaborated a formalism for the superradiant propagation of electromagnetic field in cytoskeleton microtubules, also in relation to computational functions possibly associated to them[25].
Summing up, in the quantum brain model external stimuli aimed to information printing trigger the spontaneous breakdown of symmetry. The stability of the memory is insured by the fact that coding occurs in the lowest energy state and the memory nonlocal character is guaranteed by the coherence of the \( dwq \) (or symmetron) condensate.

The recall process is described as the excitation of \( dwq \) modes under external stimuli of a nature similar to the ones producing the memory printing process. When the \( dwq \) modes are excited the brain "consciously feels"[9] the pre-existing ordered pattern in the ground state.

Short-term memory is finally associated to metastable excited states of \( dwq \) condensate\[1\]. For a discussion on this point see also ref.[26].

The electrochemical activity observed by neurophysiology provides, according to Stuart et al.[9], a first response to external stimuli which, through some intermediate interaction, has to be coupled with the dipole field (or corticon) dynamics so to allow the coding of the ground state. One possibility, according to QFT approach to living matter[11-13], is that electrochemical activity may trigger, e.g. through ATP reaction, solitary dipolar waves on biomolecular chains. These solitary waves may in turn produce domains of nonzero polarization in the surrounding water molecules and the associated \( dwq \) condensation. In the original brain model it is conjectured that the formation of ordered local domains may play a relevant role in this intermediate coupling[9].

In the quantum brain model only one kind of symmetry is assumed (the dipole rotational symmetry). Thus there is only one class of code numbers. Suppose a vacuum of specific code number has been selected by the printing of a specific information. The brain then sets in that state and no other vacuum state is successively accessible for recording another information, unless a phase transition to the vacuum specified by the new code number is produced under the external stimulus carrying the new information. This will destroy the previously stored information (\textit{overprinting}): Vacua labelled by different code numbers are accessible only through a sequence of phase transitions from one to another one of them.

Such a problem of \textit{memory capacity} was already mentioned by Stuart et al.[9], who realized that the model was too simple to allow the recording of a
huge number of informations. Stuart et al. then proposed that the model could be extended in such a way to present a huge number of symmetries (a huge number of code classes) and "a realistic model would therefore require a vector space of extremely high dimensions" [9], which however would introduce serious difficulties and spoil its practical use.

The purpose of the present paper is to show that, by taking into account the fact that the brain is an open system with dissipative dynamics, one may reach a solution to the problem of memory capacity which does not require the introduction of a huge number of symmetries.

It will be shown that, even by limiting the analysis to one kind of symmetry, infinitely many vacua are accessible to memory printing in a way that in a sequential information recording the successive printing of information does not destroy the previous ones, thus allowing a huge memory capacity. Taking into account dissipation is crucial in reaching such a result.

Section 2 is devoted to the presentation of the dissipative quantum brain dynamics (DQBD). Its connection with thermal field theory and squeezed coherent states is discussed in sections 3 and 4, respectively.

2. Dissipative quantum dynamics of the brain

In this section the quantum model of the brain proposed by Ricciardi and Umezawa is extended to dissipative dynamics by resorting to some results on dissipative systems in QFT [2,27,28]. It will be shown that the problem of memory capacity may have a solution in the framework of dissipative quantum brain dynamics.

Let us start by the (trivial) observation that "only the past can be recalled". This means that memory printing breaks the time-reversal symmetry of the brain dynamics and is another way to express the (obvious) fact that brain is an open, dissipative system coupled with external world.

As a matter of fact, in the quantum brain model spontaneous breakdown of dipole rotational symmetry is triggered by the coupling of the brain with external stimuli. Here, however, our attention is focused on the fact that once the dipole rotational symmetry has been broken (and information has thus been recorded), then, as a consequence, time-reversal symmetry is also broken:
Before the information recording process, the brain can in principle be in anyone of the infinitely many (unitarily inequivalent) vacua. After information has been recorded, the brain state is completely determined and the brain cannot be brought to the state configuration in which it was before the information printing occurred (...NOW you know it!...).

Thus, information printing introduces the arrow of time into brain dynamics. Due to memory printing process time evolution of the brain states is intrinsically irreversible.

Ricciardi and Umezawa[1] have studied the brain non-stationary or quasi-stationary states in the stationary approximation, thus avoiding damped oscillations. In the following discussion, on the contrary, we will consider non-stationary states without using the stationary approximation.

A central feature of the quantum dissipation formalism is the duplication of the field describing the dissipative system.

Let $a_\kappa$ and $\tilde{a}_\kappa$ denote the gapless dwq mode and the doubled mode required by canonical quantization of damped systems, respectively. $\kappa$ generically labels the field degrees of freedom, e.g. spatial momentum. The $\tilde{a}$ mode is the "time-reversed mirror image"[2] of the $a$ mode and represents the environment mode.

The canonical commutation relations (CCR) of the bosonic $a_\kappa$ and $\tilde{a}_\kappa$ operators are:

\[ [a_\kappa, a_\lambda^\dagger] = \delta_{\kappa,\lambda} = [\tilde{a}_\kappa, \tilde{a}_\lambda^\dagger] \ ; \ [a_\kappa, \tilde{a}_\lambda^\dagger] = 0 = [a_\kappa, \tilde{a}_\lambda] \ . \tag{1} \]

It is convenient[2] to work with the bosonic operators $A_\kappa$ and $\tilde{A}_\kappa$. These are related to $a_\kappa$ and $\tilde{a}_\kappa$ by the linear canonical transformations

\[ A_\kappa \equiv \frac{1}{\sqrt{2}}(a_\kappa + \tilde{a}_\kappa) \]

\[ \tilde{A}_\kappa \equiv \frac{1}{\sqrt{2}}(a_\kappa - \tilde{a}_\kappa) \]

which indeed preserve the CCR’s:

\[ [A_\kappa, A_\lambda^\dagger] = \delta_{\kappa,\lambda} = [\tilde{A}_\kappa, \tilde{A}_\lambda^\dagger] \ ; \ [A_\kappa, \tilde{A}_\lambda^\dagger] = 0 = [A_\kappa, \tilde{A}_\lambda] \ . \tag{2} \]

In the following, the $A$ modes will be called the "mirror modes".

Let $\{|\mathcal{N}_A, \mathcal{N}_{\tilde{A}} >\}$ be the set of simultaneous eigenvectors of $\hat{N}_A \equiv A_\kappa^\dagger A_\kappa$ and $\hat{N}_{\tilde{A}} \equiv A_{\tilde{\lambda}}^\dagger A_{\tilde{\lambda}}$, with $\mathcal{N}_A$ and $\mathcal{N}_{\tilde{A}}$ non-negative integers and let $|0 >_0 \equiv |\mathcal{N}_A = 0, \mathcal{N}_{\tilde{A}} = 0 >$ such that $A|0 >_0 = 0 = B|0 >_0$. 

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In ref. [2] it has been shown that the quantum dynamics of an (infinite) collection of damped harmonic oscillators $A_k$ is ruled by the Hamiltonian

$$H = H_0 + H_I \quad ,$$

(3a)

$$H_0 = \sum_k \hbar \Omega_k \left( A_k^\dagger A_k - \hat{A}_k^\dagger \hat{A}_k \right) \quad ,$$

(3b)

$$H_I = i \sum_k \hbar \Gamma_k \left( A_k^\dagger \hat{A}_k^\dagger - A_k \hat{A}_k \right) \quad ,$$

(3c)

where $\Omega_k$ is the frequency and $\Gamma_k$ is the coupling constant.

It is interesting to observe that in order to describe the dissipative system one does not need the details of the environment: its effective action on the system is globally represented by the action of the ”mirror image” of the system. We will comment more on this point in the following.

In order to take into account dissipativity we thus require that the memory state is a zero energy eigenstate of $H_0$ (vacuum) which therefore, at certain initial time, say $t_0 = 0$, is a condensate of equal number of modes $A_k$ and mirror modes $\hat{A}_k$ for any $k$. Clearly, we then have infinitely many memory states at $t_0 = 0$, each one corresponding to a different number $N_{A_k}$ of $A_k$ modes, for all $k$, provided $N_{A_k} - N_{\hat{A}_k} = 0$ for all $k$.

Let $|0 >_{N}$ denote the memory state with $N \equiv \{ N_{A_k} = N_{\hat{A}_k}, \forall k, at \ t_0 = 0 \}$ the set of integers defining the ”initial value” of the condensate, namely the code number (or simply the code) associated to the information recorded at time $t_0 = 0$.

At finite volume $V$, the memory state $|0 >_{N}$ can be then represented as a two-mode Glauber coherent state[29] (i.e. a generalized coherent state for $su(1,1)$):

$$|0 >_{N} = \exp \left( -i G(\theta) \right) |0 >_0 = \prod_k \frac{1}{\cosh \theta_k} \exp \left( - \tanh \theta_k J_+^{(k)} \right) |0 >_0 \quad ,$$

(4)

with $J_+^{(k)} \equiv A_k^\dagger \hat{A}_k^\dagger$ and

$$G(\theta) = -i \sum_k \theta_k \left( A_k^\dagger \hat{A}_k^\dagger - A_k \hat{A}_k \right)$$

(5)
In Eq. (4) the $\mathcal{N}$-set, $\mathcal{N}_0 = \{\mathcal{N}\}_{\kappa}, \forall \kappa, \text{ at } t_0 = 0\}, \text{ is related to }$

$$\mathcal{N}_\kappa = \mathcal{N}_0 < 0|A_\kappa^\dagger A_\kappa|0 >_{\mathcal{N}} = \sinh^2 \theta_\kappa,$$

and we will use the notation $\mathcal{N}_\kappa(\theta) = \mathcal{N}_\kappa$.

The $\theta$-set is conditioned by the requirement that $A$ and $A$ modes satisfy the Bose distribution at time $t_0 = 0$:

$$\mathcal{N}_\kappa(\theta) = \sinh^2 \theta_\kappa = \frac{1}{e^{\beta E_\kappa} - 1},$$

where $\beta = \frac{1}{k_B T}$ denotes the inverse temperature at time $t_0 = 0$ ($k_B$ is the Boltzmann constant). We thus recognize $\{0 >_{\mathcal{N}}\}$ as a representation of the $CCR’s$ at finite temperature, equivalent with the Thermofield Dynamics representation $\{0(\theta(\beta)) >\} [30,31]$.

We note that $|0 >_{\mathcal{N}}\}$ is normalized to 1 for all $\mathcal{N}$ and that in the infinite volume limit $\{0 >_{\mathcal{N}}\}$ and $\{0 >_{\mathcal{N}’}\}$ are representations of the $CCR’s$ each other unitarily inequivalent for different codes $\mathcal{N} \neq \mathcal{N}’$. We have thus at $t_0 = 0$ the splitting, or foliation, of the space of states into infinitely many unitarily inequivalent representations of the $CCR’s$. The freedom thus introduced by the degeneracy among the vacua $|0 >_{\mathcal{N}}$, for all $\mathcal{N}$, plays a crucial role in solving the problem of memory capacity. A huge number of sequentially recorded informations may coexist without destructive interference since infinitely many vacua $|0 >_{\mathcal{N}}$ are independently accessible. Recording information of code $\mathcal{N}’$ does not necessarily produce destruction of previously printed information of code $\mathcal{N} \neq \mathcal{N}’$, contrarily to the nondissipative case, where differently coded vacua are accessible only through a sequence of phase transitions from one to another one of them. In the present dissipative case the ”brain (ground) state” may be represented as the collection (or the superposition) of the full set of memory states $|0 >_{\mathcal{N}}$, for all $\mathcal{N}$. Alternatively, one may also think of the brain as a complex system with a huge number of macroscopic states (the memory states).

In order to better clarify this point it is useful to consider the dynamical group structure associated with our system. For each $\kappa$, the underlying group is $SU(1,1)[2]$. Let us neglect for the moment the suffix $\kappa$ for simplicity. The
two-mode realization of the algebra $su(1,1)$ is generated by

$$J_+ = A^\dagger \tilde{A}^\dagger \ , \quad J_- = J_+^\dagger = \tilde{A}A \ , \quad J_3 = \frac{1}{2}(A^\dagger A + \tilde{A}^\dagger \tilde{A} + 1) \ , \quad (8)$$

$$[J_+, J_-] = -2J_3 \ , \quad [J_3, J_\pm] = \pm J_\pm \ . \quad (9)$$

The Casimir operator $C$ is given by $C^2 \equiv \frac{1}{4} + J_3^2 - \frac{1}{2}(J_+J_- + J_-J_+) = \frac{1}{4}(A^\dagger A - \tilde{A}^\dagger \tilde{A})^2$. We thus see that the eigenstates of $H_0$ can be expressed in terms of the basis of simultaneous eigenstates of $C$ and of $(J_3 - \frac{1}{2})$ in the representation labelled by the value $j \in \mathbb{Z}_\frac{1}{4}$ of $C$, \{[j, m >; m \geq |j|]\}:

$$C|j, m > = j|j, m > \ , \quad j = \frac{1}{2}(N_A - \bar{N}_A) \ ; \quad \left(J_3 - \frac{1}{2}\right)|j, m > = m|j, m > \ , \quad m = \frac{1}{2}(N_A + \bar{N}_A) \ . \quad (10)$$

The memory state corresponds to the choice $j = 0$ (for all $\kappa$) and we see that, at certain time $t$ there are (for each $\kappa$) $m$ coexisting, independent eigenstates of $C$ (of course, by reintroducing the $\kappa$ suffix, we have $m_\kappa = N_{A_\kappa} = \bar{N}_{\bar{A}_\kappa}$ and the $m$-set, $m \equiv \{m_\kappa\}$, corresponds to the $N$-set).

As a result, the $SU(1,1)$ structure of the dissipative dynamics introduces $m$-coded "replicas" of the system (foliation of the state space) and information printing can be performed in each replica without destructive interference with recorded informations in the other replicas. In the nondissipative case the "$m$-freedom" is missing and consecutive information printing produces overprinting.

The non-existence in the infinite volume limit of unitary transformation which may map one representation of code $N$ to another one of code $N'$ guarantees that the corresponding printed informations are indeed different or distinguishable informations ( $N$ is a good code) and that each information printing is also protected against interference from other information printing (absence of confusion among informations). The effect of finite (realistic) size of the system may however spoil unitary inequivalence and may lead to "association" of memories. We will comment more on this point in section 4.

We have $[H_0, H_I] = 0$. The commutativity of $H_0$ with $H_I$ ensures that the number $(N_{A_\kappa} - \bar{N}_{\bar{A}_\kappa})$ is a constant of motion for any $\kappa$.

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We therefore realize that in the dissipative dynamics ruled by the Hamiltonian \((3a)\), although \(\mathcal{N}_{A_k}\) and \(\mathcal{N}_{-A_k}\) are allowed to separately change in time, their difference is kept constantly zero during time evolution.

Formally, at finite volume \(V\), the time evolution of the memory state \(|0 >_{\mathcal{N}}\) is given by

\[
|0(t) >_{\mathcal{N}} = \exp \left( -i t \frac{H}{\hbar} \right) |0 >_{\mathcal{N}} = \exp \left( -i t \frac{H_I}{\hbar} \right) |0 >_{\mathcal{N}} = \prod_k \frac{1}{\cosh (\Gamma_k t - \theta_k)} \exp \left( \tanh (\Gamma_k t - \theta_k) J_+(^{(k)}) \right) |0 >_{\mathcal{N}},
\]

which is again a generalized coherent state for \(su(1,1)\). In obtaining Eq.(11) we used the commutativity between \(H_I\) and \(G(\theta)\).

Let us observe that the vacuum \(|0(t) >_{\mathcal{N}}\) is specified by the initial value \(\mathcal{N}\), at \(t_0 = 0\), of the condensate.

We note that \(\mathcal{N} < 0(t)|0(t) >_{\mathcal{N}} = 1, \ \forall t\), and that, provided \(\sum_k \Gamma_k > 0\),

\[
\lim_{t \to \infty} \mathcal{N} < 0(t)|0 >_{\mathcal{N}} \propto \lim_{t \to \infty} \exp \left( -t \sum_k \Gamma_k \right) = 0 .
\]

Using the customary continuous limit relation \(\sum_k \mapsto \frac{V}{(2\pi)^3} \int d^3 \kappa\), in the infinite-volume limit we have (for \(\int d^3 \kappa \Gamma_k\) finite and positive)

\[
\mathcal{N} < 0(t)|0 >_{\mathcal{N}} \xrightarrow{V \to \infty} 0 \ \forall t ,
\]

\[
\mathcal{N} < 0(t)|0(t') >_{\mathcal{N}} \xrightarrow{V \to \infty} 0 \ \forall t,t' , \ t \neq t' .
\]

In the infinite volume limit, time evolution of \(|0 >_{\mathcal{N}}\) would be rigorously frozen according to Eq. (13) (the states \(|0(t) >_{\mathcal{N}}\) and the associated Hilbert spaces are each other unitarily inequivalent for different time values \(t \neq t'\) in the infinite volume limit); however, in realistic situations, a finite life-time may be possible due to effects of the system boundaries (cf. Eq.(12)).

Time evolution of the memory state \(|0 >_{\mathcal{N}}\) is thus represented as the trajectory of "initial condition" specified by the \(\mathcal{N}\)-set in the space of the representations \(\{|0(t) >_{\mathcal{N}}\}\) of the \(CCR\)'s. The non-unitary character of time-evolution implied by damping is consistently recovered in the unitary inequivalence among representations at different times in the infinite-volume limit.
We also have
\[
\mathcal{N} < 0(t) |0 > = \exp \left( - \sum_k \ln \cosh (\Gamma_k t - \theta_k) \right),
\] (14)
which shows that at time \( t = \tau \), with \( \tau \) the largest of the values \( \tau_k = \frac{\theta_k(N)}{\Gamma_k} \), the memory state \( |0 >_{\mathcal{N}} \) is reduced (decayed) to the "empty" vacuum \( |0 >_0 \): the information has been forgotten. At the time \( t = \tau \) the state \( |0 >_0 \) is available for recording a new information.

It is interesting to observe that in order to not completely forget certain information, one needs to "restore" the \( \mathcal{N} \) code, which corresponds to "refresh" the memory by brushing up the subject (external stimuli maintained memory).

We observe that the number of modes of type \( A_k \) is given, at each instant \( t \), by
\[
\mathcal{N}_{A_k}(\theta, t) \equiv \mathcal{N} < 0(t)|A_k^\dagger A_k|0(t) >_{\mathcal{N}} = \sinh^2(\Gamma_k t - \theta_k)
\] (15)
and similarly for the modes of type \( \tilde{A}_k \). Eq. (15) shows that the assigned initial condition is satisfied (cf. Eq.(6)) and that, as already observed above, the information code is washed out after a time \( t = \tau \). We will comment more on this point in the following section.

Finally, we note that at each \( t \)
\[
\begin{align*}
\frac{1}{\cosh (\Gamma_k t - \theta_k)} A_k^\dagger |0(t) >_{\mathcal{N}} &= \frac{1}{\sinh (\Gamma_k t - \theta_k)} \tilde{A}_k |0(t) >_{\mathcal{N}}, \\
\frac{1}{\cosh (\Gamma_k t - \theta_k)} \tilde{A}_k^\dagger |0(t) >_{\mathcal{N}} &= \frac{1}{\sinh (\Gamma_k t - \theta_k)} A_k |0(t) >_{\mathcal{N}},
\end{align*}
\] (16)
which show that the creation of a mode \( A_k \) is equivalent to the destruction of a mode \( \tilde{A}_k \) and vice-versa. This leads us to interpreting the \( \tilde{A}_k \) modes as the holes for the modes \( A_k \) [2] (see also [30]).

In the following sections we discuss the connection of the DQBD with thermal field theory and squeezed coherent states.

3. Thermal field theory

The state \( |0(t) >_{\mathcal{N}} \) may be written as[2,30]:
\[
|0(t) >_{\mathcal{N}} = \exp \left( -\frac{1}{2} S_A \right) |\mathcal{I} > = \exp \left( -\frac{1}{2} S_{\tilde{A}} \right) |\mathcal{I} >,
\] (17)
where \(|\mathcal{I}| \equiv \exp \left( \sum_k \mathcal{A}_k^\dagger \mathcal{A}_k^\dagger \right) |0 \rangle > 0\) and

\[
S_A \equiv - \sum_k \left\{ \mathcal{A}_k^\dagger \mathcal{A}_k \ln \sinh^2 (\Gamma_k t - \Theta_k) - \mathcal{A}_k \mathcal{A}_k^\dagger \ln \cosh^2 (\Gamma_k t - \Theta_k) \right\} .
\] (18)

\(S_A\) is given by an expression similar to (18) with \(\mathcal{A}_k\) and \(\mathcal{A}_k^\dagger\) replacing \(\mathcal{A}_k\) and \(\mathcal{A}_k^\dagger\), respectively. Since \(\mathcal{A}_k\) and \(\tilde{\mathcal{A}}_k\) commute (see (2)), we shall simply write \(S\) for either \(S_A\) or \(S_{\tilde{A}}\). It is known\(^{[2,30]}\) that \(S\) can be interpreted as the entropy operator for the dissipative system.

Note that \(|0(t) >_{\mathcal{N}}\) depends on time only through the exponential of \(\frac{1}{2} S_A\) (or respectively, \(\frac{1}{2} S_{\tilde{A}}\)) whose operatorial part depends uniquely on the \(A\) (\(\tilde{A}\)) variables: thus Eq. (17) may be regarded as the projection on the (sub)system \(A\) (\(\tilde{A}\)) with the elimination of the \(\tilde{A}\) (\(A\)) variables.

The time variation of \(|0(t) >_{\mathcal{N}}\) at finite volume \(V\) is given by\(^{[2,28]}\),

\[
\frac{\partial}{\partial t} |0(t) >_{\mathcal{N}} = - \left( \frac{1}{2} \frac{\partial S}{\partial t} \right) |0(t) >_{\mathcal{N}} .
\] (19)

which shows that \(i \left( \frac{1}{2} \hbar \frac{\partial S}{\partial t} \right)\) is the generator of time-translations, namely time-evolution is controlled by the entropy variations. It is an interesting feature of the present treatment of dissipation that the same operator \(S\) that controls time evolution also defines the dynamical variable whose expectation value is formally the entropy: this feature indeed reflects the irreversibility of time evolution (breakdown of time-reversal symmetry) characteristic of dissipative systems, namely the choice of a privileged direction in time evolution (arrow of time).

In order to study the stability condition to be satisfied at each time \(t\) by the state \(|0(t) >_{\mathcal{N}}\) let us introduce the free energy functional\(^{[2,30]}\)

\[
\mathcal{F}_A \equiv \mathcal{N} < 0(t) | \left( H_A - \frac{1}{\beta} S_A \right) |0(t) >_{\mathcal{N}} .
\] (20)

\(\beta\) is a strictly positive function of time representing the inverse temperature:

\(\beta(t) = \frac{1}{k_B T(t)}\); \(H_A\) is the part of \(H_\theta\) relative to the \(A\)-modes only, namely \(H_A = \sum_k \hbar \Omega_k \mathcal{A}_k^\dagger \mathcal{A}_k\). Let \(\Theta_k \equiv \Gamma_k t - \Theta_k\) and \(E_k \equiv \hbar \Omega_k\). The stationarity condition

\[
\frac{\partial \mathcal{F}_A}{\partial \Theta_k} = 0 , \quad \forall \kappa ,
\] (21)
gives $\beta(t)E_\kappa = -\ln \tanh^2(\Theta_\kappa)$. This finally leads to
\begin{equation}
\mathcal{N}_\kappa(\theta, t) = \sinh^2(\Gamma_\kappa t - \theta_\kappa) = \frac{1}{e^{\beta(t)E_\kappa} - 1},
\end{equation}
which is the Bose distribution for $A_\kappa$ at time $t$.

Again, this allows us to recognize $\{|0(t)\rangle_{\mathcal{N}}\}$ as a representation of the $CCR$'s at finite temperature, equivalent with the Thermofield Dynamics representation[30,31].

One can see (cf. Eq. (18)) that the entropy $S(t) = \langle 0(t)|S|0(t)\rangle_{\mathcal{N}}$ is a decreasing function of time in the interval $(t_0 = 0, \tau)$ meaning that the memory state, although not conserved in time, is however "protected" from "going back" to the "unrecorded" or "blank" vacuum state (memory cancellation). Of course, here it is crucial the energy exchange with the environment and we are also assuming finite volume effects. In the infinite volume limit, as already noticed, time evolution would be frozen and stability rigorously ensured. One can also see that the entropy, for both $A$ and $\bar{A}$ system, grows monotonically with $t$ from value 0 at $t = \tau$ to infinity at $t = \infty$. However, the difference $(S_A - S_{\bar{A}})$ is constant in time: $[S_A - S_{\bar{A}}, H] = 0$. Since the $\bar{A}$-particles are the holes for the $A$-particles, $S_A - S_{\bar{A}}$ is, in fact, the (conserved) entropy for the complete system.

Also, it can be shown[2,31] that, as time evolves, the change in the energy $E_A \equiv \sum_\kappa E_\kappa \mathcal{N}_\kappa$ and in the entropy is given by
\begin{equation}
dE_A = \sum_\kappa E_\kappa \mathcal{N}_\kappa dt = \frac{1}{\beta}dS_A,
\end{equation}
i.e.
\begin{equation}
dE_A - \frac{1}{\beta}dS_A = 0.
\end{equation}
When $\frac{\partial \beta}{\partial \tau} = -\frac{1}{k_A^2 T^2} \frac{\partial T}{\partial \tau} \approx 0$, namely changes in inverse temperature are slow, eq. (24) can directly be obtained by minimizing the free energy (20): $d\mathcal{F}_A = dE_A - \frac{1}{\beta}dS_A = 0$. $E_A$ is thus recognized as the internal energy of the system. Eq. (24) also expresses the first principle of thermodynamics for a system coupled with environment at constant temperature and in absence of mechanical work. One may define as usual heat as $dQ = \frac{1}{\beta}dS$. Thus the change in time of condensate (Eq. (23)) turns out into heat dissipation $dQ$. 

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In conclusion, time evolution of the $\mathcal{N}$-coded memory state is represented as a trajectory of initial condition $\mathcal{N} = \{ \mathcal{N}_\lambda \}$ running over the space of the representations $\{|0(t) >, \mathcal{N}\}$, each one minimizing the free energy functional.

We close this section by observing that the time evolution above discussed is different from the quantum decay process of memory involving the virtual dynamics of instantons which is discussed in ref. [23]. Dissipation has been treated above as a realistic physical feature of the brain considered as an open system. The memory states thus obey a truly dissipative dynamics not considered in [23]. The virtual dynamics of instantons with quantum fluctuations due to tunnel effect can also be considered in the present dissipative framework in the same fashion as it has been studied by Jibu ad Yasue in ref. [23]. However, it deals with QFT features which are different from the ones presented in this paper.

4. Squeezing and concluding remarks

We now briefly discuss the relation between the memory states and the squeezed coherent states which emerges in the dissipative dynamics above presented.

It is easy to show that the operator $\exp \left( -iG(\theta) \right)$ (see Eq. (5)) is rewritten in terms of the operators $a$ and $\tilde{a}$ as

$$
\exp \left( -iG(\theta) \right) = \prod_k \exp \left( -\frac{\theta_k}{2} \left( a_k^2 - a_{\tilde{k}} \right) \right) \exp \left( \frac{\theta_k}{2} \left( \tilde{a}_k^2 - \tilde{a}_{\tilde{k}} \right) \right)
$$

$$
= \prod_k \hat{S}_a(\theta_k) \hat{S}_{\tilde{a}}(-\theta_k), \quad (25)
$$

with $\hat{S}_a(\theta_k) \equiv \exp \left( -\frac{\theta_k}{2} (a_k^2 - a_{\tilde{k}}) \right)$ and similar expression for $\hat{S}_{\tilde{a}}(-\theta_k)$ with $\tilde{a}$ and $\tilde{a}^\dagger$ replacing $a$ and $a^\dagger$, respectively.

The operators $\hat{S}_a(\theta_k)$ and $\hat{S}_{\tilde{a}}(-\theta_k)$ are the squeezing operators for the $a_k$ and the $\tilde{a}_k$ modes, respectively, as well known in quantum optics[32]. The set $\theta \equiv \{ \theta_k \}$ as well as each $\theta_k$ for all $k$ is called the squeezing parameter.

An expression similar to Eq. (25), but with $\theta_k$ replaced by $-\Gamma_k t$, is obtained for the time evolution operator $\hat{U} \equiv \exp \left( -it \frac{H_F}{\hbar} \right)$ (cf. Eq. (3c)).
From Eqs.(4) and (11) we thus conclude that the memory state and its time evoluted state are squeezed coherent states.

To illustrate the effect of the squeezing, let us focus our attention only on the \( a_\kappa \) modes for sake of definiteness. For the \( \tilde{a} \) modes we can proceed in a similar way.

As usual, for given \( \kappa \) we express the \( a \) mode in terms of conjugate variables of the corresponding oscillator. By using dimensionless quantities we thus write \( a = X + iY \), with \([X, Y] = \frac{i}{2}\). The uncertainty relation is \( \Delta X \Delta Y = \frac{1}{4} \), with \( \Delta X^2 = \Delta Y^2 = \frac{1}{4} \) for (minimum uncertainty) coherent states. The squeezing occurs when \( \Delta X^2 < \frac{1}{4} \) and \( \Delta Y^2 > \frac{1}{4} \) (or \( \Delta X^2 > \frac{1}{4} \) and \( \Delta Y^2 < \frac{1}{4} \)) in such a way that the uncertainty relation remains unchanged. Under the action of \( \exp (-iG(\theta)) \) the variances \( \Delta X \) and \( \Delta Y \) are indeed squeezed as

\[
\Delta X^2(\theta) = \Delta X^2 \exp(2\theta) \quad \Delta Y^2(\theta) = \Delta Y^2 \exp(-2\theta) .
\]

For the tilde-mode similar relations are obtained for the corresponding variances:

\[
\Delta \tilde{X}^2(\theta) = \Delta \tilde{X}^2 \exp(-2\theta) \quad \Delta \tilde{Y}^2(\theta) = \Delta \tilde{Y}^2 \exp(2\theta) .
\]

For positive \( \theta \), squeezing then reduces the variances of the \( Y \) and \( \tilde{X} \) variables, while the variances of the \( X \) and \( \tilde{Y} \) variables grow by the same amount so to keep the uncertainty relations unchanged. This reflects, in terms of the \( A \) and \( \tilde{A} \) modes, the constancy of the difference \( \mathcal{N}_{A_\kappa} - \mathcal{N}_{\tilde{A}_\kappa} \) against separate, but equal, changes of \( \mathcal{N}_{A_\kappa} \) and \( \mathcal{N}_{\tilde{A}_\kappa} \) (degeneracy of the memory states labelled by different codes).

In conclusion, the memory code of a specific information, namely the \( \theta \)-set \( \{ \theta_k(\mathcal{N}_\kappa) \} \) (cf. Eq.(6)), is nothing but the squeezing parameter classifying the squeezed coherent states in the hyperplane \((X, \tilde{X}; Y, \tilde{Y})\). Note that to different squeezed states (different \( \theta \)-sets) are associated unitarily inequivalent representations of the \( CCR \)'s in the infinite volume limit: in dissipative quantum brain dynamics the huge (infinite) number of squeezed states, labelled by the squeezing parameter \( \theta \equiv \{ \theta_k \} \), constitute the memory capacity.
As observed above, time evolution also contributes to squeezing. We find

\[
\Delta X^2(\theta, t) = \frac{1}{4} \exp (-2(\Gamma t - \theta)) , \\
\Delta Y^2(\theta, t) = \frac{1}{4} \exp (2(\Gamma t - \theta)) .
\]

(27)

Similar relations hold for the tilde-mode (with the exponential factors exchanged). In Eq. (27) we used the values for the variances for the vacuum \(|0 \equiv |0, \emptyset >\).

Eqs. (27) (and the corresponding ones for the tilde-mode) show the time behaviour of the squeezing and we can recover the analysis of time evolution made in the previous sections. Again, we note the rôle of the characteristic time \(\tau \equiv \frac{\theta}{\Gamma}\). We also see that in the limit \(t \to \infty\) the variances of the variables \(Y\) and \(\bar{X}\) become infinity making them completely spread out.

Let us now summarize the main points of the discussion presented in this paper.

In dissipative quantum brain dynamics infinitely many vacua coexist and a huge number of informations may be sequentially recorded without destructive interference.

The problem of memory capacity in the quantum brain model, arising from the fact that vacua labelled by different code numbers belonging to the same class are accessible only by a sequence of phase transitions, finds a solution in the intrinsic dissipative character of brain dynamics.

As we have indeed stressed, the process of information printing by itself produces the breakdown of time-reversal symmetry and thus introduces the arrow of time into brain dynamics. The key point is that the resulting dissipative dynamics cannot be worked out without the introduction of the time-reversed image (the tilde-system) of the original system. As a consequence, energy degeneracy is introduced and the brain ground state may be represented as a collection (or superposition) of infinitely many degenerate vacua or memory states, each of them labelled by a different code number and each of them independently accessible to information printing (without reciprocal interference). Many information storage levels may then coexist thus allowing a huge memory capacity.
Differently stated, the brain system may be viewed as a complex system with (infinitely) many macroscopic configurations (the memory states). Dissipation, which is intrinsic to the brain dynamics, is recognized to be the root of such a complexity, namely of the huge memory capacity.

Time evolution of the $\mathcal{N}$-coded memory state is represented as a trajectory of initial condition $\mathcal{N} = \{\mathcal{N}_A\}$ running over the states $|0(t)\rangle_\mathcal{N}$, each one minimizing the free energy functional.

Memory states have also been shown to be squeezed coherent states.

Let us close the paper with few more comments.

The QFT approach to living matter does not require the introduction of other symmetries than the dipole rotational symmetry (and the electromagnetic gauge symmetry, see ref. [13]). In the case other symmetries could be required in future developments of such an approach, to each broken symmetry will be associated a code class. Then the memory state will carry as many labels (codes) as many dynamical symmetries are broken. In such a case, the Goldstone modes associated to a specific label may interfere with the Goldstone modes associated to some other label of the same state. This may produce fluctuations in their condensation and thus originates the mechanism of ”association” of memories, by which some information is recorded with some ”confusion” due to the presence of elements belonging to a different information; or also, the recalling of some information may trigger the recalling of some other information.

As already observed, association of memories may also occur when, as in the present paper, only one kind of symmetry is considered. In such a case, ”interferences” are due to the realistic (finite) size of the system (boundaries effects) making the memory states not exactly orthogonal (unitary non-equivalence is spoiled).

We remark that the memory state is not invariant under $H_I$ (see Eq.(11)), while the Hamiltonian $H$ commutes with $H_I$. Therefore, in addition to breakdown of time-reversal (discrete) symmetry, already mentioned in the previous sections, we also have spontaneous breakdown of time translation (continuous) symmetry. Dissipation (i.e. energy non-conservation) has been thus described in this paper (see also ref. [2]) as an effect of breakdown of time translation and time-reversal symmetry.

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Finally, according to the original quantum brain model, the recall process is described as the excitation of $dwq$ modes under an external stimulus which is "essentially a replication signal" [9] of the one responsible for memory printing. When $dwq$ are excited the brain "consciously feels" [9] the presence of the condensate pattern in the corresponding coded vacuum. The replication signal thus acts as a probe by which the brain "reads" the printed information.

In this connection we observe that the $dwq$ may acquire an effective nonzero mass due to the effects of the system finite size [12]. Such an effective mass will then introduce a threshold in the excitation energy of $dwq$ so that, in order to trigger the recall process an energy supply equal or greater than such a threshold is required. Non sufficient energy supply may be experienced as a "difficulty in recalling". At the same time, however, the threshold may positively act as a "protection" against unwanted perturbations (including thermalization) and cooperate to the memory state stability. In the opposite case of zero threshold any replication signal could excite the recalling and the brain would fall in a state of "continuous flow of memories".

As for the "replication signal", it is interesting to observe that in DQBD the $\tilde{A}$ system is indeed a "replication" of the $A$ system and plays in fact a central rôle in the recalling process: Eqs.(16) show that the creation (excitation) of the $A$ mode is equivalent, up to a factor, to the destruction (from the memory state) of the $\tilde{A}$ mode. In this sense the coupling term of the $\tilde{A}$ mode with the $A$ mode in the Hamiltonian can be seen as a self-interaction term of the $A$ system, thus confirming the rôle of $\tilde{A}$ system in "self-recognition" processes.

Remarkably, the tilde-system also represent the environment effects and cannot be neglected since the brain is an open system. Therefore the tilde-modes can never be eliminated from the brain dynamics: the tilde-modes thus might play a rôle as well in the unconscious brain activity. This may provide an answer to the question "as whether symetron modes would be required to account for unconscious brain activity" [9].

Moreover, we have seen that the $\tilde{A}$ system is the time-reversed image of the $A$ system. Thus the $A$ system is the "mirror in time" system. This fact, together with the rôle of the $\tilde{A}$ modes in the self-recognition processes, leads us to conjecture (also accepting the literary image of consciousness as a "mirror")
that tilde-system is actually responsible for consciousness mechanisms: Consciousness emerges as a manifestation of the dissipative quantum dynamics of the brain.

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References


A.S. Davydov, Physica Scripta 20, 387 (1979)


