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Lectures by:
P.E. Beckmann
H. Rollnik
G. Höhler
U. Meyer-Berkhout
C. O'Ceallaigh

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PREFACE

The 1965 Easter School at Bad Kreuznach, Germany, was the fourth in a series that began in 1962. Whereas the first two Schools, held at St. Cergue (Switzerland) in 1962 and 1963, were devoted to introducing young emulsion physicists to the experimental possibilities offered at CERN, and to the modern developments of high-energy nuclear physics, the scope of the 1964 School which took place at Herceg Novi (Yugoslavia) was widened to include also the bubble chamber technique. For the 1965 School no restriction as to the experimental technique was applied: young research students working in elementary particle physics with any technique were eligible for admission, the only restrictions being due to the limited space available in the Kurhaus hotel and its lecture rooms.

The lectures of the 1965 Easter School at Bad Kreuznach started in the morning of 2 April. The School closed on 14 April with an after-dinner speech by Professor W. Jentschke. It was attended by about 90 students from 22 countries in Western Europe, Eastern Europe, Asia, and the United States.

The purpose of the School was to familiarize the students with the current theoretical and experimental situation in Elementary Particle Physics. Eighteen lecturers contributed to this end. Their respective lecture courses varied in length, some lasting two hours, and two (by Van Hove and by Veltman) consisting of six lessons of 90 minutes duration each.

In the interest of speedy publication of the Proceedings of the School we are reducing to a minimum any editorial changes in the manuscripts as they are kindly supplied by the lecturers. As a consequence, some topics will be treated several times by different authors. In some cases the notations used will not be consistent. It was felt, however, that any inconvenience derived from these facts is not serious, and is in any case amply out-weighted by the advantage that the Proceedings will be available sooner.

It should be noted that the lectures given by R.D. Tripp and M. Veltman will not appear in these Proceedings. Most of the material they presented may be found in CERN Report 65-7 (revised) by Dr. Tripp ("Baryon Resonances") and in Report 65-30 by Dr. Cabibbo and Dr. Veltman ("Weak Interaction").

Volume II contains the lectures by P.E. Beckman, H. Rollnik, G. Hühler, U. Neyer-Berkhout and C. O'Ceallaigh. We are grateful to the authors for their co-operation in preparing the manuscripts and checking the typed texts. Our thanks are due also to Miss S. Bloch and Miss H. Mutin for their careful typing; Mr. P. Theurillat and his colleagues of the Scientific Information Service, and Miss A. Lutke of the DSC Drawing Office for their preparation of certain figures; and to Mrs. K. Mackley for her careful checking of the final text.

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ELECTROMAGNETIC FORM FACTORS OF NUCLEONS

P.E. Beckmann,
Institut für theoretische Physik der Universität, Mainz

I. INTRODUCTION

In these lectures we shall discuss some aspects of the interaction between electrons and nucleons which go beyond quantum electrodynamics. They arise from the fact that the nucleon is not just a Dirac particle coupled to the electromagnetic field but a hadron and, as such, involved in strong interactions. The deviations of the nucleons behaviour in electromagnetic interactions from that of a Dirac particle are usually interpreted as being due to a structure of the nucleon originating in its coupling to pions and other hadrons. Although the effects of strong interactions are already reflected by the static properties of the nucleons, namely by their anomalous magnetic moments, they can be studied in more detail by electron-proton scattering, in particular in processes with large momentum change of the nucleon.

Since it is impossible in three lectures to give both an introduction and a comprehensive survey of the subject, we shall concentrate on those aspects which arise if one evaluates and interprets electron-proton scattering experiments at large momentum transfers 1).

II. GENERAL NOTATION

We consider the process of electron-proton scattering according to

\[ e^- + p \rightarrow e^- + p \]
\[ k + p = k' + p' \]  \hspace{1cm} (1)

\( k, p, k', \) and \( p' \) denote the particle four momenta \((k_0, \mathbf{k})\) such that

\[ k^2 = k_0^2 - \mathbf{k}^2 = m^2 \quad p^2 = p_0^2 - \mathbf{p}^2 = M^2 \]  \hspace{1cm} (2)

\( m \) and \( M \) are the masses of electron and proton, respectively. With the process we can associate a diagram as that of Fig. 1.

\[ \text{FIG. 1} \]
The scattering is described by a scattering amplitude $T$ such that the cross-section can be expressed as:

$$d\sigma = \frac{4}{|v_1 - v_2|} \frac{(2\pi)^2}{2p_0} \frac{dq'}{2q'} \frac{dq'}{2q'} \frac{\delta(p + k - p' - k')}{(2\pi)^6} |T|^2. \quad (3)$$

$|v_1 - v_2|$ is the relative velocity of the incoming particles. $T$ is connected with the $S$-matrix through:

$$<p'k' | \frac{S - 1}{2i} | pk> = \delta(p + k - p' - k') T. \quad (4)$$

It is more convenient to use variables $P$, $Q$, $s$, $t$ defined as:

$$P = (p + k) = (p' + k'); \quad P^2 = s$$
$$Q = (p - p') = (k' - k); \quad Q^2 = t. \quad (5)$$

Frequently $Q^2 = -t$ is also used in the literature.

For electron-proton scattering these variables have the following meaning:

- $P$: total energy-momentum four-vector
- $e-p$: four-momentum transfer
- $s$: square of c.m.s. energy
- $t$: invariant momentum transfer.

The process of electron-proton scattering through the substitution law is connected with proton-antiproton annihilation into an electron-positron pair. (Remember that an incoming particle with charge $e$ and momentum $p$ corresponds to an outgoing antiparticle with charge $-e$ and momentum $-p$.)

For $p\bar{p}$ annihilation $P$, $Q$, $s$, $t$ have a different meaning and assume different values:

- $e-p$ scattering
- $p\bar{p}$ annihilation

| $s$ : square of c.m.s. energy | inv. momentum transfer |
| $t$ : inv. momentum transfer | square of c.m.s. energy |

physical region: $t \leq 0$, $t > 4M^2$

The connection between $e-p$ scattering and $p\bar{p}$ annihilation has an immediate consequence: the squared matrix elements $|T|^2$, averaged over spin orientations, entering into the cross-section for unpolarized beam and target and without analysing polarization, is given by the same function, of course for different values of the variables $s$ and $t$. If $p\bar{p}$ annihilation proceeds through a finite number of angular momentum states with $l < L$, $|T|^2$ is of the form:

$$|T|^2 = A_0(t) + A_1(t) \cos \theta_t + A_2(t) \cos^2 \theta_t + \ldots + A_2^L \cos^2 \theta_t. \quad (6)$$

The dependence on $s$ is fully contained in $\cos \theta_t$, where $\theta_t$ is the angle between $p$ and $e^+$ in the c.m.s. of $p\bar{p}$ annihilation:

$$\cos \theta_t = \frac{2(s - M^2 - m^2) + t}{2(t/4 - M^2)^{1/2} (t/4 - m^2)^{1/2}}. \quad (7)$$
This implies for e–p scattering a particular dependence on $\text{ctg}^2 \frac{\theta}{2}$, where $\theta$ is the scattering angle of the electron in the laboratory system:

$$\cos \vartheta_L = \left(1 + \frac{1}{1 + \tau} \text{ctg}^2 \frac{\theta}{2}\right)^{1/2}; \quad \tau = \frac{-t}{4M^2} = \frac{q^2}{4M^2}$$

$$|T|^2 = A_0(t) + \left(1 + \frac{1}{1 + \tau} \text{ctg}^2 \frac{\theta}{2}\right)^{1/2} A_1(t) + \ldots + \left(1 + \frac{1}{1 + \tau} \text{ctg}^2 \frac{\theta}{2}\right)^2 A_{2L}(t).$$

An interaction proceeding via angular momentum 1 in p–p annihilation corresponds to the exchange of spin 1 in e–p scattering. Odd powers of $\cos \vartheta_L$ appear only if there is interference between contributions of different parity.

III. STRUCTURE OF THE SCATTERING AMPLITUDE

The electromagnetic interaction between electrons and protons can be described by the exchange of photons, i.e., of quanta of the electromagnetic field. We decompose the scattering amplitude into terms corresponding to different numbers of photons being exchanged (cf. Fig. 2).

![Diagram](image)

Such a decomposition arises if one uses perturbation theory for the electromagnetic interaction. We shall restrict our discussion mainly to the one-photon exchange contribution, which seems to describe most experimental data very well. This agreement might be correlated with the fact that the multiple-photon exchange term contains higher powers of the fine-structure constant $\alpha$, which is small ($\approx 1/137$). But we shall discuss explicitly methods to test the validity of the one-photon exchange approximation.

The one-photon contribution, according to the Feynman rules, turns out to be

$$T = \frac{1}{24} (-1)(2\pi)^4 \frac{\bar{U}(k') \gamma_\mu U(k)}{(2\pi)^2} \frac{-1}{Q^2} (-1)(2\pi)^4 \frac{<J_\mu>}{(2\pi)^2}$$

$$T = \frac{1}{2} \frac{e}{(2\pi)^2} \bar{U}(k') \gamma_\mu U(k) \frac{1}{t} <J_\mu>.$$

$<J_\mu>$ is the matrix element of the electromagnetic current between the states of the particle by which the electron is scattered:

$$<J_\mu> = (2\pi)^3 <\vec{p}' s'|J_\mu(0)|\vec{p} s>.$$
Here, \( s \) and \( s' \) are the spin quantum numbers of the proton. The electric charge \( e \) appearing as coupling constant in Eq. (10) is normalized such that \( e^2/4\pi = a \).

Usually one works with unpolarized electron beams and does not analyse the polarization of the outgoing electrons. The appropriate spin average of \(|T|^2\) can be written as:

\[
|T|^2 = \frac{1}{2} a \frac{1}{(2\pi)^3} \left\{ \left( k' + k \right) \mu \left( k' + k \right)_\nu \left( t_{\mu \nu} - q' \sigma \right) \right\} < J_\mu > < J_\nu >^*. \tag{12}
\]

If also the target is unpolarized and if the polarization of the outgoing target particles is not observed, the corresponding spin average entering into the cross-section can be written as:

\[
\frac{1}{2S+1} \sum_{\text{spins}} < J_\mu > < J_\nu >^* = e^2 \left[ a(t)(p' + p)_\mu (p' + p)_\nu + b(t)(t_{\mu \nu} - q' \sigma) \right]. \tag{13}
\]

This general form is independent of the magnitude \( S \) of the spin of the target particle. For a spin 0 and for a spin \( \frac{1}{2} \) particle, coupled only to the electromagnetic field, \( a(t) \) and \( b(t) \) according to the Feynman rules turn out to be:

\[
\begin{align*}
\text{spin 0} & : \quad < J_\mu > = e(p' + p)_\mu, \quad a(t) = 1, \quad b(t) = 0, \\
\text{spin } \frac{1}{2} & : \quad < J_\mu > = e \bar{U}(p') \gamma_\mu U(p) \rightarrow a(t) = 1, \quad b(t) = 1.
\end{align*} \tag{14}
\]

Here, of course, it has been assumed that the spin \( \frac{1}{2} \) particle has no anomalous magnetic moment. Particles with couplings to the electromagnetic field, as those indicated in Eq. (14), we shall call particles with point-like electric charges.

From Eq. (13) we obtain for \(|T|^2\):

\[
|T|^2 = \frac{\alpha^2}{4\pi} \frac{2}{t^2} \left[ 2[t(S - m^2) + (S - M^2 - m^2)] a(t) + t(t + 2m^2) b(t) \right]. \tag{15}
\]

This gives for the cross-section \(^3\):

\[
d\sigma = \langle d\sigma \rangle_{NS} \left( a(t) + \frac{t(t + 2m^2)}{t(S - m^2) + (S - M^2 - m^2)} b(t) \right). \tag{16}
\]

Here \( \langle d\sigma \rangle_{NS} \) is the cross-section for the scattering of electrons by a target with no spin and electric point charge. The differential cross-section in the laboratory system can be expressed as:

\[
\frac{d\sigma}{d\Omega} = \langle \frac{d\sigma}{d\Omega} \rangle_{NS} \left( a(t) + 2\tau \frac{\alpha^2}{2} b(t) \right), \tag{17}
\]

with

\[
\langle \frac{d\sigma}{d\Omega} \rangle_{NS} = \frac{d\sigma}{d\Omega}_{\text{Mott}} \frac{1}{N \sin^2 \frac{\Theta}{2}}, \quad \langle \frac{d\sigma}{d\Omega} \rangle_{\text{Mott}} = \frac{\alpha^2 \cos^2 \frac{\Theta}{2}}{4E^2 \sin^4 \frac{\Theta}{2}}. \tag{18}
\]


E is the laboratory energy of the incoming electron. \( E \gg m \) has been assumed. The functions \( a(t) \) and \( b(t) \) can be determined as follows: \( (d\omega/d\Omega)/[(d\omega/d\Omega)_{NS} \) for fixed \( t \) plotted versus \( \theta^2 \theta/2 \) gives a straight line. Slope and intercept give \( a(t) \) and \( b(t) \).

If in Eq. (15) we express the dependence on \( s \) in terms of \( \cos \theta_{\ell} \) we obtain

\[
|T|^2 = \frac{e^2}{\pi} \left[ 4\hat{N}^2 t (1+\tau) a(t) + t(t+2\hat{N}^2) \frac{b(t)}{2} + \left( \frac{t}{4} - \hat{N}^2 \right) \left( \frac{t}{4} - \hat{N}^2 \right) a(t) \cos^2 \theta_{\ell} \right].
\]

(19)

This is just the form expected for an interaction through angular momentum 1 with definite parity. This reflects that the photon has spin 1 and parity minus.

IV. STRUCTURE OF THE VERTEX FUNCTION, FORM FACTORS

We now turn to the discussion of \( \langle J_\mu \rangle \). The current \( J_\mu(x) \) plays the role of a source of the electromagnetic vector potential \( A_\mu(x) \) as can be seen from the familiar equation

\[
\Box A_\mu(x) = - J_\mu(x).
\]

If we compute the amplitude for the scattering of electrons by an external field, \( \langle J_\mu \rangle \) entering into the expression corresponding to Eq. (10) is the Fourier transform of the charge-current distribution producing the external field. If we have a static charge distribution only \( \langle J_\mu \rangle \neq 0 \).

For a point source \( \langle J_\mu \rangle = e \), while for a charge distribution

\[
\langle J_\mu \rangle = e \int \rho(x) \hat{\mathbf{q}} \cdot \hat{\mathbf{x}} \, d^3x = e f(t).
\]

(20)

The function \( f(t) \), i.e. the Fourier transform of the static charge distribution, is called "form factor". The scattering cross-section can be written as

\[
\sigma = (d\sigma)_p |f(t)|^2,
\]

(21)

where \( (d\sigma)_p \) is the cross-section for the scattering by a fixed point charge.

The transition to the scattering by a particle with finite mass will produce modifications due to recoil and associated acceleration. Even in classical physics we expect that:

a) the movement of the charge will in general make \( \langle \hat{J}_\mu \rangle \neq 0 \),

b) the acceleration will make the charge distribution vary during the scattering process. The effect a) can be eliminated by going to an appropriate co-ordinate system such that the contributions to \( \langle \hat{J}_\mu \rangle \) cancel for a particle without magnetic moment.

Consider a spin zero particle with point charge. We have \( \langle j_\mu \rangle = e(p' + p)_\mu \). We require \( \langle \hat{J}_\mu \rangle = 0 \). This corresponds to a frame of reference where \( p' = -p \). Such a system is well known in scattering theory. It is called the "Breit system" or "brickwall system". The momenta in the Breit system can be represented as in Fig. 3 (see next page).
Quantities in the Breit system we shall mark with a subscript $B$. We have
\begin{equation}
\langle j_0 \rangle_B = e(p_0' + p_0); \quad \langle j \rangle_B = 0
\end{equation}
for spinless point charge.

The general structure of $\langle j_\mu \rangle$ in the Breit system for a spinless particle follows from invariance considerations [which the limited space does not permit us to give in detail, c.f. reference 4]):
\begin{equation}
\langle j_0 \rangle_B = e(p_0' + p_0) G_E(t), \quad \langle j \rangle_B = 0 \quad \text{spin 0}.
\end{equation}
The function $G_E(t)$, which reflects the deviation from a point charge, is called the electric form factor of the particle.

Although the transition from Eq. (22) to Eq. (21) looks similar to the transition from a fixed point charge to a fixed extended charge, some caution is necessary when interpreting $G_E(t)$ as the Fourier transform of a charge distribution. The charge-current distribution $\rho_\mu(x)$ for a state $\varphi$ is the expectation value, i.e. the diagonal matrix element of $j_\mu(x)$:
\begin{equation}
\rho_\mu(x) = \langle \varphi | j_\mu(x) | \varphi \rangle,
\end{equation}
whereas into the scattering amplitude there enters the Fourier transform of
\begin{equation}
\langle j_\mu(x) \rangle = \langle \varphi' | j_\mu(x) | \varphi \rangle,
\end{equation}
where $\varphi'$ moves relative to $\varphi$. Therefore in general $\langle j_\mu(x) \rangle \neq \rho_\mu(x)$; only for $\varphi' = \varphi$, i.e. in forward scattering, we have $\langle j_\mu(x) \rangle = \rho_\mu(x)$. This quantum mechanical argument can be supplemented by a classical consideration: during scattering the charge distribution is accelerated and therefore - unless the charge distribution is rigid, which relativistically is impossible - $G_E(t)$ reflects the influence of a varying charge distribution. We therefore shall refrain from taking the Fourier transform of $G_E(t)$ literally to be "the" charge distribution of the particle.

Next, we turn to particles with spin $\frac{1}{2}$. We remember that $j_\mu$ corresponds to a source of photons, i.e. of particles with spin 1. For real photons ($Q^2 = 0$) only two spin orientations are possible with spin projections onto $\vec{Q}$: $S_{\text{ph}} = \pm 1$, corresponding to right- and left-handed circular polarization. Virtual photons ($Q^2 \neq 0$) also permit $S_{\text{ph}} = 0$. Now let $\vec{Q}$ be parallel to the 3-axis. Then circularly polarized photons carry a 3-component of angular momentum. Therefore a term corresponding to the interaction of circularly polarized photons will also exhibit a change of the spin projection of the particle (i.e. spin flip). The interaction with $S_{\text{ph}} = 0$ on the other hand will occur only without spin flip.
The four components \( \langle j_{\mu} \rangle \) are not independent as can be seen from current conservation:

\[
\partial_{\mu} \langle j_{\mu} (x) \rangle = 0 \rightarrow Q_{\mu} \langle j_{\mu} \rangle = 0 \rightarrow Q_0 \langle j_0 \rangle = - \vec{Q} \cdot \langle \vec{J} \rangle = 0 .
\] (26)

With our choice of \( z \)-axis this means that

\[
\langle J_3 \rangle_B = 0 .
\] (27)

Of the remaining terms \( \langle J_0 \rangle_B \) corresponds to the interaction of photons with \( S_{ph} = 0 . \) \( \langle J_{1,2} \rangle \) represent the source of transversely polarized photons. It is more convenient to work with circular polarization, i.e. with \( \langle J_{+} \rangle_B \):

\[
\langle J_{+} \rangle_B = \frac{1}{\sqrt{2}} \left[ \hat{z} \langle J_1 \rangle_B - i \langle J_2 \rangle_B \right] .
\] (28)

We now have

\[
\langle J_0 \rangle_B = e \delta_{s' s} \tilde{G}_E(t) \]
\[
\langle J_{+} \rangle_B = e \delta_{s' s} \hat{z} \tilde{G}_M(t) .
\] (29)

The term with \( \tilde{G}_M \) corresponds to the source of the field produced by the magnetic moment. We return to \( \langle J \rangle \) and express the result in terms of matrix elements between Pauli spinors \( \chi_{s'}, \chi_{s} \):

\[
\langle J_0 \rangle_B = e \chi_{s'}^{\dagger} \chi_{s} \tilde{G}_E(t) \]
\[
\langle J_{+} \rangle_B = \frac{ie}{2M} \chi_{s'}^{\dagger} \vec{Q} \times \vec{\sigma} \chi_{s} \tilde{G}_M(t) .
\] (30)

Now we normalize the functions \( \tilde{G}_E \) and \( \tilde{G}_M \) appearing in Eq. (30) such that for a particle without structure they become equal to one. We have for such a particle:

\[
\langle J_{-} \rangle = e \bar{U}(p', s') \gamma_{\mu} U(p, s) \rightarrow \langle J_0 \rangle_B = e \frac{2M}{2M} \chi_{s'}^{\dagger} \chi_{s}
\]
\[
\langle J_{+} \rangle_B = \frac{ie}{2M} 2M \chi_{s'}^{\dagger} \hat{z} \times \vec{\sigma} \chi_{s} .
\] (31)

Therefore our general result for spin \( \frac{1}{2} \) becomes:

\[
\langle J_0 \rangle_B = e \frac{2M}{2M} \chi_{s'}^{\dagger} \chi_{s} \tilde{G}_E(t) \]
\[
\langle J_{+} \rangle_B = \frac{ie}{2M} 2M \chi_{s'}^{\dagger} \hat{z} \times \vec{\sigma} \chi_{s} \tilde{G}_M(t) .
\] (32)

The normalization is such that \( \tilde{G}_E(0) = \) static charge in units of \( e, \tilde{G}_M(0) = \) static magnetic moment in units of \( e/2M, \) i.e. with anomalous magnetic moment \( \kappa: \tilde{G}_M(0) = 1 + \kappa .\)

The general expression for \( \langle j_{\mu} \rangle \) can now be written in terms of a covariant matrix between Dirac spinors:

\[
\langle j_{\mu} \rangle = e \bar{U}(p', s') \left\{ \frac{(p' + p) \cdot \gamma}{2M} \frac{\tilde{G}_E(t)}{1 - \tau} + \frac{1}{2M} \frac{r_{\mu}}{1 - \tau} \tilde{G}_M(t) \right\} U(p, s)
\]
\[
r_{\mu} = \frac{1}{2} \left[ \sigma \cdot \gamma \gamma_{\mu} (p' + p) \cdot \gamma - (p' + p) \cdot \gamma \gamma_{\mu} \sigma \cdot \gamma \right] .
\] (33)
If we use that
\[ \overline{U} r_\mu U = -i \frac{2M}{\mu} \overline{U} \gamma_\mu U - \overline{U} \sigma_{\mu\nu} Q_\nu U, \]
\[ (p' + p)_\mu \overline{U} U = 2M \overline{U} \gamma_\mu U + i \overline{U} \sigma_{\mu\nu} Q_\nu U, \]
the expression (33) can be rewritten as
\[ <J_\mu> = e \overline{U}(p', s') \left( \gamma_\mu F_1(t) - \frac{i}{2M} \sigma_{\mu\nu} Q_\nu F_2(t) \right) U(p, s) \]
\[ = e \overline{U}(p', s') \left( \gamma_\mu G_M(t) - \frac{(p' + p)_\mu}{2M} F_2(t) \right) U(p, s). \]

Here,
\[ F_1 = \frac{G_E + \tau G_M}{1 + \tau}, \quad F_2 = \frac{G_M - G_E}{1 + \tau}, \]
\[ G_E = F_1 - F_2, \quad G_M = F_1 + F_2; \]

\( F_1(t) \) and \( F_2(t) \) are called Dirac and Pauli form factors, respectively. The general expression for \( <J_\mu> \) now gives for the functions \( a(t) \) and \( b(t) \) appearing in the cross-section:
\[ a(t) = (1 + \tau) F_2^2 - 2F_2 G_M + G_M^2 = \frac{G_E^2 + \tau G_M^2}{1 + \tau} \]
\[ b(t) = G_M^2. \]

The cross-section may then be written as:
\[ d\sigma = (d\sigma)_{NS} \left( G_E^2 + \tau \left( 1 + 2 t s^2 \frac{G_M}{2} \right) G_M^2 \right) \cdot \frac{1}{1 + \tau}. \]

This formula is called the Rosenbluth formula. The fact that no interference terms between \( G_M \) and \( G_E \) appear is easily understood: \( G_E \) and \( G_M \) are the factors of the non-spin flip and spin-flip terms in the Breit system. When writing Eq. (37) use has been made of the fact that both \( G_E \) and \( G_M \) are real for electron-proton scattering. This property can be derived from time reversal invariance\(^{1}\).

V. TEST OF VALIDITY OF ONE-PHOTON APPROXIMATION

The one-photon approximation has two characteristic features:

a) a particular form of the dependence of \( |T|^2 \) on \( s \) and \( t \),
b) the form factors are real.

The particular dependence on \( s \) and \( t \) reflects the fact that spin 1 is exchanged. If the Rosenbluth formula is verified by experiment this means that the interaction goes via spin 1; but multiple photon exchange with exchanged total angular momentum equal to one and parity minus is not excluded. Furthermore, an additional interaction through \( J^P = 0^- \) would not modify the structure of the Rosenbluth formula since it would only give an additional
contribution to $A_0$ of Eq. (6) but no interference term. In general, a modification through exchange of states with $J^P$ other than $1^-$ and $0^-$ will produce additional terms in the cross-section formula with powers of

\[ \left( 1 + \frac{1}{1 + \tau} \cot^2 \frac{\theta}{2} \right)^\frac{1}{2} = \cos \theta_t. \]

Any deviations from the Rosenbluth formula can be expected to become more pronounced for small angles $\theta$. For instance, an additional $1^+$ interaction will make the cross-section become:

\[ \frac{d\sigma}{d\Omega} \left\{ a(t) + 2 \tau b(t) \frac{\theta}{2} + c(t) \frac{\theta^2}{2} \left( 1 + \frac{1}{1 + \tau} \cot^2 \frac{\theta}{2} \right) \right\} = \frac{d\sigma}{d\Omega}. \] (39)

The deviation of the $\theta$ dependence of the interference term from the form "constant + $\cot^2 \theta/2$ constant" increases for small $\theta$. Our result is that a test of the Rosenbluth formula basically is the test for the exchange of $J^P = 0^-, 1^-$. A more powerful test uses the measurement of the polarization of the outgoing nucleons and is based on the fact that the form factors are real. Since one works with an unpolarized electron beam and without analyzing the polarization of the outgoing electrons for simplicity we neglect the electron spin; its inclusion would not change the result. The scattering amplitude can be written as matrix between Pauli spinors

\[ T = \chi_{\sigma_f}^\dagger (f + i \sigma^\dagger \sigma^g) \chi_{\sigma}. \] (40)

The polarization can be written as:

\[ P = \frac{\sigma_+ - \sigma_-}{\sigma_+ + \sigma_-} = \frac{2 \text{Im}(f \sigma^g \cdot \hat{n})}{|f|^2 + |g|^2}. \] (41)

$\sigma_+$ and $\sigma_-$ are the cross-sections for the outgoing protons, spin being parallel or antiparallel to the unit vector $\hat{n}$. Now, in the one-photon exchange approximation both $f$ and $g$ are real. There should be no polarization if the approximation is valid. If there is an additional two-photon exchange we write

\[ f = e f_1 + e^2 f_2, \quad g = e g_1 + e^2 g_2. \] (42)

The factors $e$ indicate the different powers of the coupling constant for the electron. The indices 1 and 2 refer to one and two-photon exchange. Then

\[ P \sim (e f_1 + e^2 \text{Re} f_2) \text{Im} e^2 \sigma^g_2 \cdot \hat{n} + e^2 \text{Im} f_2 (e g_1 \cdot \hat{n} + e^2 \text{Re} g_2 \cdot \hat{n}) \left( 1 + \frac{1}{1 + \tau} \cot^2 \frac{\theta}{2} \right) \] (43)

Even small two-photon contributions can give rise to polarization since $P$ contains interference terms between one-photon and two-photon amplitudes. Basically polarization measurement is a test for the scattering amplitude being complex.

A third test of the one-photon approximation is the comparison of electron-proton scattering. Both electron and positron scattering are described by the same spin average $|T|^2$ of the square of the amplitude except that the electron coupling constant $e$ is replaced by $-e$. If we neglect spins we indicate the dependence on the coupling constant explicitly:

\[ T = e T_1 + e^2 T_2 \] (44)
where the indices 1 and 2 refer to single and double photon exchange, we have

$$|T|^2 = e^2 |T_1|^2 + e^4 |T_2|^2 + 2 e^3 T_1 \text{ Re } T_2 . \quad (45)$$

The interference term changes sign when going from electron to positron scattering:

$$\sigma_e^* - \sigma_{e^+} = - T_1 \text{ Re } T_2 . \quad (46)$$

VI. GENERAL STRUCTURE OF FORM FACTORS

We have introduced electric and magnetic form factors $G_E^p$ and $G_M^p$ (for the proton: $G_E^p, G_M^p$). A similar analysis can be done for the neutron giving $G_E^n$ and $G_M^n$. Since the static charge of the neutron is zero: $G_E^n(0) = 0$ and $G_M^n = \kappa_n$. But from the scattering of slow neutrons it is known that $G_E^n(t)/t = 0$. In practice the form factors of the neutron cannot be determined as easily as those for the proton. Since free neutrons are not available as target one has to use bound neutrons, e.g. deuterium, which causes difficulties due to the strong interaction between the neutron and the proton.

For treating the influence of strong interactions it is more convenient to introduce isoscalar and isovector form factors $G^S$ and $G^V$:

$$G^S = \frac{G^p + G^n}{2}, \quad G^V = \frac{G^p - G^n}{2} . \quad (47)$$

In nucleon-antinucleon annihilation $G^S$ corresponds to the isosinglet contribution ($I = 0$) and $G^V$ to the isotriplet contribution ($I = 1$) of the one-photon channel. We can write the inverse of Eq. (47) as matrix element between nucleon isospinors $\eta$:

$$G^p = \eta^+ \left( G^S + \tau^3 G^V \right) \eta^p . \quad (48)$$

The normalization is such that:

$$G_E^S(0) = \frac{1}{2}, \quad G_E^V(0) = \frac{1}{2}$$

$$G_M^S(0) = \frac{1}{2} + \frac{\kappa + \kappa_n}{2}, \quad G_M^V(0) = \frac{1}{2} - \frac{\kappa - \kappa_n}{2} . \quad (49)$$

The influence of strong interactions on the electromagnetic structure of the nucleons is most conveniently discussed within the frame of dispersion relations. It has been shown in perturbation theory that the form factors are functions analytic in the complex $t$-plane cut along the real axis from some value $t_0$ to $\infty$. The corresponding representation by a Cauchy integral gives the dispersion relation:

$$G(t) = G(0) + \frac{1}{\pi} \int_{t_0}^{\infty} \frac{\text{Im} G(t')}{(t' - t)^2} \text{ dt'} . \quad (50)$$
With this representation there is associated a decomposition into contributions of different intermediate states indicated in Fig. 4.

\[ \text{FIG. 4} \]

To each diagram corresponds a cut of the function \( G \) for values of \( t \) such that the particles in the intermediate state are on the mass shell, i.e. for \( t \geq (\sum m_i)^2 \), where \( m_i \) are the masses of the particles. The value of \( t_0 \) in Eq. (50) is given by the intermediate state of lowest mass. The intermediate states must satisfy the selection rules, i.e. they must have

- angular momentum \( J = 1 \) parity minus
- isospin \( I = 1 \) for isovector form factor
- \( I = 0 \) for isoscalar form factor.

Since the pions are bosons, two pions with \( J = 1 \) necessarily have \( I = 1 \). \( J = 1 \) and \( I = 0 \) can only be obtained with at least three pions; therefore

\[
t_0 = (2\mu)^2 \quad \text{for isovector form factors; } \mu = \text{pion mass},
\]

(51)

\[
t_0 = (3\mu)^2 \quad \text{for isoscalar}
\]

Each diagram of Fig. 4 is composed of diagrams corresponding to other processes, e.g. for the two-pion intermediate state corresponding to \( N^+ \bar{N} \to 2\pi \) and to the pion form factor. This is reflected in the rule for the computation of the weight function:

\[
\Im G = \sum_{\text{intermediate states } z} T^*_{\bar{N}N; z} \Gamma_{z1} G_1.
\]

(52)

Here \( T_{\bar{N}N; z} \) denotes the scattering amplitude for \( N^+ \bar{N} \to z \), \( \Gamma_{z1} \) denotes the electromagnetic vertex. Each of the functions entering into \( \Im G \) according to Eq. (52) itself is connected with amplitudes for other processes. For instance, the pion form factor entering into the two-pion contribution is related to the amplitude for pion-pion scattering as indicated in Fig. 5.

\[ \text{FIG. 5} \]
In practice it is impossible to take into account all relations between the functions appearing in Eq. (52) and the amplitudes for the related processes. But it is hoped that in the physical region for electron-nucleon scattering, i.e. \( t < 0 \), for small values of \( |t| \) the first portion of the cut will be more important than the far distant parts, for which the factor \( 1/(t'-t) \) appearing in the dispersion integral is smaller. Now, the first part of the cut is largely determined by the pion-pion interaction which is known to show strong resonances. Therefore, one frequently interprets the experimental results in terms of resonance models involving the \( \rho \), \( \omega \) and \( \varphi \) mesons.

Before turning to the approximations we shall mention some general considerations which impose constraints on parameters of the model. If a form factor \( G(t) \) for \( t \rightarrow -\infty \) vanishes sufficiently rapidly in the limit, the dispersion integral must cancel the constant term in Eq. (50). This gives

\[
G(0) = \frac{1}{\pi} \int_{t_0}^{\infty} \frac{\text{Im} \, G(t')}{t'} \, dt'.
\]  

(53)

If \( G(t) \) vanishes faster than \( \sim 1/|t| \), \( \text{Im} \, G(t) \) can not be \( \geq 0 \) throughout the interval of integration. In particular if \( \lim_{t \to -\infty} tG(t) = 0 \), we have

\[
\int_{t_0}^{\infty} \text{Im} \, G(t') \, dt' = 0.
\]  

(54)

Unfortunately no rigorous statement about the asymptotic behaviour can yet be given. We therefore shall not go into details\(^5\).

Another constraint is imposed by the behaviour at \( t = \Delta^2 \), i.e. the physical threshold for nucleon-antinucleon annihilation. The matrix element of the electromagnetic current in the c.m.s. for nucleon-antinucleon annihilation has the structure

\[
< J_\mu > \sim \chi_{-s'}^+ \left( \gamma^\mu \cdot \frac{p}{p^\mu} (G_M - G_E) + \sigma \cdot G_M \right) \chi_s.
\]  

(55)
The second term involving only $\tilde{\sigma}$ corresponds to a $3S$ contribution while the first term contains the $3D$ contribution. At threshold there should remain only the S-wave amplitude which requires: $G_N = G_E$ at $t = 4M^2$. Another argument for this relation to hold goes as follows: at threshold there exists no privileged direction in space. Therefore the angular distribution should be isotropic. Now, $|T|^2$ in general contains a term proportional to $\cos^2 \theta_t$ which should vanish at threshold:

$$\left( \frac{1}{4} - M^2 \right) \left( \frac{1}{4} - m^2 \right) a(t) \cos^2 \theta_t = - \left( \frac{1}{4} - m^2 \right) M^2 \left( G_E^2 - \frac{t}{4M^2} G_M^2 \right) \cos^2 \theta_t$$

$$\rightarrow 0 \text{ for } t \rightarrow 4M^2$$

(i.e.) $G_E^2 \rightarrow G_M^2$ for $t \rightarrow 4M^2$.

The reasoning for obtaining expressions to fit the form factors goes as follows: the pion-pion interaction at small energies is dominated by resonances (e.g. $\rho$, $\omega$, $\phi$). It is assumed that the contribution to the weight function in the dispersion relation for the nucleon form factors comes mainly from the vicinity of resonances. In the limit of an infinitely narrow resonance one has a $\delta$-function contribution to $\text{Im } G(t)$ leading to a form factor contribution:

$$\text{contribution of narrow resonance } \sim \frac{\text{constant}}{t - m_r^2}.$$  \hspace{1cm} (57)

$m_r$ is the mass of the particle associated with the resonance. The constant appearing in Eq. (57) is the product of the coupling constants $N - \bar{N}$ resonance and $\gamma$ resonance. This approximation corresponds to substituting one particle for a two or more pion intermediate state. For the two-pion contribution a $\rho$ meson is substituted according to Fig. 6.

Candidates for approximating the isoscalar form factors are the $\omega$ and $\phi$ mesons. It cannot be expected that all of the integral can be approximated by resonant contributions of the form (57). There will be a slowly varying background which sometimes is approximated by a constant. Thus one arrives at a formula of the structure:

$$G(t) \approx a + \sum \frac{b_i}{t - m_i^2}.$$  \hspace{1cm} (58)

A formula of this type with one pole term is known as "Clementel-Villu formula".

Our considerations leading to the expression (58) have only been a motivation but not a derivation. This should be borne in mind when applying it to the experiment.
We have added the constant term \( a \) in order to represent a slowly varying background. If the inclusion of such a term gives a good fit to the experiment it does not necessarily mean that the rigorous expression for \( G(t) \) contains a constant. An equally good fit can be obtained if instead of the constant \( a \) an additional pole term is used such that \( -\frac{b_1}{m_1^2} = a \) and \( m_1^2 \) sufficiently large. The constant term only simulates a contribution of a slowly varying \( \Im G(t) \) for large values of \( t' \). Since the Fourier transform of a constant is a \( \delta \)-function, the constant \( a \) sometimes is interpreted as being due to a hard core. But in the light of the fact that expression (58) is only an approximation this interpretation should not be taken too literally.

The number of pole terms necessary to give a good fit to the experiments is not known a priori. Of course, one expects the known resonances with appropriate quantum numbers to give contributions. There are the \( \rho \) meson for the isovector form factors and the \( \omega \) and \( \phi \) mesons for the isoscalar form factors. It turns out that a better fit is obtained if one pole more - a \( \rho' \) - is included for the isovector form factor. The apparent necessity to include such a pole does not necessarily mean that a corresponding resonance exists. It only says that the behaviour of the dispersion integral is thus better approximated.

The masses entering into (58) can not a priori be expected to be exactly those of the known resonances. The resonances have a finite width which in the case of the \( \rho \) is considerable. As pointed out by Ball and Wong, a more detailed calculation of the weight function entering into the isovector form factors shows that the maximum is shifted to lower energies and furthermore the region of small \( t' \) is weighted rather heavily. Therefore, in general, shift of the masses \( m_1 \) relative to the masses of the known resonances is expected.

The constants \( b_1 \) are restricted by the normalization at \( t = 0 \). Furthermore, sometimes the condition \( G_E^O = G_M^O \) at \( t = 4M^2 \) is used as a constraint. But it should be born in mind that this point lies outside the region where one can expect the approximation to hold.

At present the experiments indicate that there is no substantial deviation from the predictions of the one-photon approximation. The Rosenbluth formula with its particular dependence on \( \phi \), \( \theta/2 \) seems to hold, a measurement of polarization at \( t = 16 F^2 \) gives \( P = 0.031 \pm 0.025 \) which is no significant deviation from \( P = 0 \), and a comparison of \( e^+ \) and \( e^- \) scattering also does not indicate a substantial two-photon contribution.

If the scattering data are analysed in terms of form factors, the surprising result is that \( G_{E1}^O = G_{M1}^O + \kappa_p \), and \( G_0^O = G_0^M \) seem to be equal up to \( q^2 = 50 F^2 \). This equality in the region \( t < 0 \) is not expected to hold for all values of \( t \) since it contradicts \( G_E = G_M \) at \( t = 4M^2 \). If the experimental results are represented by a Clementel-Villii-type formula, the values for the parameters depend on the number of them left free. If one takes the masses of \( \rho, \omega \) and \( \phi \) and introduces a fictitious \( \rho' \) with \( m_{\rho'} = 940 \text{ MeV} \), one obtains \(^7\)
\[ G_E^s = \frac{1.24}{1 + q^2/15.8} - \frac{0.74}{1 + q^2/26.7} \]
\[ G_E^y = \frac{2.01}{1 + q^2/14.4} - \frac{1.51}{1 + q^2/25.0} \]
\[ G_M^s = \frac{1.12}{1 + q^2/15.8} - \frac{0.68}{1 + q^2/26.7} \]
\[ G_M^y = \frac{6.23}{1 + q^2/14.5} - \frac{3.87}{1 + q^2/25.0} \]

This fit does not include a constant term. But it is also possible to represent the data with \( \rho, \omega \) and \( \varphi \) only together with constant terms. This gives

\[ G_E^s = \frac{2.6}{1 + q^2/15.8} - \frac{3.1}{1 + q^2/26.7} + 1 \]
\[ G_E^y = \frac{0.9}{1 + q^2/14.5} - 0.4 \]
\[ G_M^s = \frac{3.6}{1 + q^2/15.8} - \frac{3.8}{1 + q^2/26.7} + 0.8 \]
\[ G_M^y = \frac{3.2}{1 + q^2/14.5} - 0.8 \]
REFERENCES

1) For more detailed information the reader is referred to:
and Boyd).
Nuclear Structure and Electromagnetic Interactions, N. MacDonald, ed., (Scottish
Universities Summer School in Physics 1964), to be published May 1965.
Further references to the original papers can be found in the above-mentioned articles.

2) We normalize our states such that for a spinless particle

$$< p' | p > = 2 p_0 \delta(p - p')$$

while for a particle with spin

$$< p's' | p' s > = 2 p_0 \delta(p - p') \delta_{ss'} .$$

Furthermore $\bar{U}(p, s') U(p, s) = 2M \delta_{ss'}$ is being used.

3) For a discussion of the structure of the cross-section see also

4) A more detailed discussion of the kinematic structure of the electromagnetic vertex
   can be found in
(Interscience).

5) Some aspects of the behaviour of form factors for high-momentum transfer are discussed
   by Sachs:
See also
A. Martin, Minimal interactions at very high transfers, CERN preprint.
T.T. Wu and C.N. Yang, Some speculations concerning high-energy large momentum transfer
   processes, Brookhaven preprint.


I. INTRODUCTION

The aim of these lectures is to explain what the investigation of photoproduction processes can teach us about the structure of strongly interacting particles. In the beginning of meson physics, experiments on the photoproduction of pions have led to a series of important discoveries: the existence of neutral pions was finally established\(^1\); the ratio of \(\pi^+\) and \(\pi^0\) production led to the invention of the first pion-nucleon resonant state\(^2\); also the existence and properties of the second \(N(1512)\) and third \(N(1688)\) \(\pi-N\) resonance was for the first time discussed using results of Cal-Tech and Cornell Electron synchrotron\(^3\).

Nowadays most of the structures in Elementary Particle Physics have been discovered by people working with hadrons - strongly interacting particles - only, but there are important properties of these resonances which can be studied only with photons.

Using photons for particle physics brings about experimental as well as theoretical complications. These lectures have to deal with the new theoretical problems which arise due to the peculiar properties of the photons. From a principal point of view there are only a few such peculiarities and these are of a more technical character:

- the spin 1 of the photon makes formulae clumsy;
- the vanishing mass of the photon introduces problems of gauge invariance;
- the photon breaks isospin invariance.

The last point does not introduce such great changes as one might expect because of the fortunate fact that the photon is coupled to other particles only "electromagnetically" by the small fine structure constant \(\alpha = 1/137\). Therefore it does not take part in the strong interconnection of the hadrons often referred to by the term "unitarity of the \(S\) matrix". More precisely, this fact allows the properties of the photons and the strongly interacting particles to be separated. In mathematical terms: any photoproduction amplitude \(T\) can be factorized according to

\[
T = A^\mu < \ldots | j_\mu | \ldots >
\]  

(1)

only if contributions of the order \(\sqrt{\alpha}\) are taken into account. Here \(A^\mu\) describes the photon wave function, and the Hilbert states in the matrix elements contain only hadrons. Equation (1) is valid for real as well as for virtual photons which are exchanged in electron processes. Table I gives examples for Eq. (1). All methods which have been invented for the hadron physics - analyticity methods, symmetries, etc. - can now be applied to the matrix elements of the current operator (with some minor change). Therefore a systematic theory exists for the matrix elements \(a^\alpha\) and \(b\) of Table I, while for the multiple processes \(c\) only special

\* Op. the preceding lectures by P. Beckmann.
models have been investigated. I shall not give a complete account of the systematic theory but I do intend to explain the general ideas and especially to point to the differences between photoproduction and hadronic two body-processes.

**Table I**

Examples for the matrix elements of the current operator

<table>
<thead>
<tr>
<th>matrix elements</th>
<th>investigated by</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) (&lt;N'</td>
<td>j_\mu</td>
</tr>
<tr>
<td>(&lt;\pi N'</td>
<td>j_\mu</td>
</tr>
<tr>
<td>(b) (&lt;\eta N'</td>
<td>j_\mu</td>
</tr>
<tr>
<td>(&lt;KY</td>
<td>j_\mu</td>
</tr>
<tr>
<td>(&lt;2\pi N'</td>
<td>j_\mu</td>
</tr>
<tr>
<td>(c) (&lt;\rho N'</td>
<td>j_\mu</td>
</tr>
<tr>
<td>(&lt;\omega N'</td>
<td>j_\mu</td>
</tr>
</tbody>
</table>

II. KINEMATICS AND INTERMEDIATE STATES FOR THE TWO-BODY PROCESSES

Let us start some brief remarks on the kinematics of the photoproduction of a meson \((m)\) and a baryon \((B)\):

\[
\gamma + N \rightarrow m \text{ (meson)} + B \text{ (baryon)}
\]

\[
K + P_1 = Q + P_2.
\]

The last equation contains four-momenta of the involved particles such that the different masses are given by

\[
K^2 = 0, \quad Q^2 = m^2; \quad P_1^2 = M_1^2; \quad P_2^2 = M_2^2.
\]
As usual we introduce the Lorentz invariant quantities:

\[ s = (K + P_t)^2 = W^2, \quad W = \text{total c.m.s. energy} \]
\[ t = (Q - K)^2 = m^2 - 2KQ = m^2 - 2K(\omega - q \cos \Theta) \quad (3) \]
\[ u = (P_x - K)^2 = M_x^2 - 2K(E_x + q \cos \Theta), \quad s + t + u = M_x^2 + M_y^2 + m^2 \]

and where \( K^2 = \frac{(s - M_x^2)^2}{4s} = (\text{c.m.s. momentum})^2 \) of the \( \gamma + N \) system \( (3a) \)

\[ q^2 = \frac{|s - (M_x + m)^2| |s - (M_x - m)^2|}{4s} = (\text{c.m.s. momentum})^2 \) of the \( m + B \) system \( (3b) \)

\[ \omega = \sqrt{q^2 + m^2} = \text{c.m.s. energy of the meson} \]

\[ E_x = \sqrt{q^2 + M_x^2} = \text{c.m.s. energy of the final baryon} \quad (3c) \]

\[ \Theta = \text{c.m.s. angle between meson and photon.} \]

From these relations the "physical domain" in the \( s-t \) plane can easily be calculated; i.e. these \( s, t \) points which can be realized by the photoproduction process (2). A necessary condition is:

\[ s \geq (M_x + m)^2; \quad t < 0. \]

Different from the elastic scattering case, the value \( t = 0 \) cannot be reached for finite \( s \); compare Fig. 1 which illustrates the situation for pion photoproduction. This fact has brought about some difficulties for the application of dispersion relations \(^4\) and the Regge pole hypothesis \(^5\,

All attempts for a detailed theoretical description of the dynamics of photoproduction start from analyticity properties \(^7\) of the photoproduction amplitude \( T \) which are assumed to be valid like in other two-body processes though they have never been proved. These assumptions maintain that \( T \) can be described by holomorphic functions in the physical \( s-t \) domain, the structure of which is determined by singularities with a simple physical meaning: their position and detailed properties can be traced back to the intermediate physical states which are allowed by the general conservation laws. To get a complete system of these singularities one must discuss the following three reactions at the same time \(^6\)

\[ \gamma + N \rightarrow m + B: \quad s\text{-channel} \quad (4a) \]
\[ \gamma + \bar{m} \rightarrow \bar{N} + B: \quad t\text{-channel} \quad (4b) \]
\[ \gamma + \bar{B} \rightarrow m + \bar{N}: \quad u\text{-channel} \quad (4c) \]

They are connected with each other by the crossing relation. Each process is named according to the variable which describes its total c.m.s. energy.

At the present time it appears hopeless to take account of all intermediate states. Even a more modest approach has not been carried through completely which consists in retaining only the resonant intermediate states. Tables II through IV give the presently known resonances which in principle can play a role in photoproduction:
Table II
Intermediate states in the s-channel
(if the incoming photon is replaced by a pion the situation remains the same)

<table>
<thead>
<tr>
<th>final state</th>
<th>intermediate (resonant) state</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi N$</td>
<td>$N(940)\ \Delta(1238)$</td>
</tr>
<tr>
<td></td>
<td>$N(1480)?$</td>
</tr>
<tr>
<td>or $K\Sigma$</td>
<td>$N(1512)$</td>
</tr>
<tr>
<td></td>
<td>$N(1688)\ \Delta(1920)$</td>
</tr>
<tr>
<td></td>
<td>$N(2190)\ \Delta(2360)$</td>
</tr>
<tr>
<td></td>
<td>$N(2700)$</td>
</tr>
<tr>
<td>$\eta N$</td>
<td>only $N^*(I=\frac{1}{2})$ intermediate states</td>
</tr>
<tr>
<td>$K\Lambda$</td>
<td></td>
</tr>
</tbody>
</table>

s-channel (Table II) - All known isobars = non-strange baryon resonances can occur for the $\pi N$ and $K\Sigma$ final state. For the production of $\eta N$ and $K\Lambda$ conservation of isospin in the upper bubble allows only the $I=\frac{1}{2}$ isobars.

Table III
Intermediate states in the u-channel

<table>
<thead>
<tr>
<th>final state</th>
<th>intermediate (resonant) states</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi N$</td>
<td>all isobars</td>
</tr>
<tr>
<td>$\eta N$</td>
<td>only $I=\frac{1}{2}$ - isobars</td>
</tr>
<tr>
<td>$K\Omega$</td>
<td>$\Lambda(1115)\ \Omega^<em>(1405); \ \Omega^</em>(1520)$; $\Omega^*(1815)$,</td>
</tr>
<tr>
<td>$K\Sigma$</td>
<td>$\Sigma(1189)\ \Sigma^<em>(1385); \ \Sigma^</em>(1660)$; $\Sigma^*(1765)$.</td>
</tr>
</tbody>
</table>

u-channel (Table III) - For $\pi N$ production all isobars can occur; in the $\eta N$ case only $I=\frac{1}{2}$ isobars are allowed, while for strange particle production all hyperon states with strangeness $-1$ can occur.

In both cases the photon can be replaced by a pion without changing any result. The deeper reason for this lies in the fact that the photon can be regarded as a mixture of an isoscalar and an isovector particle\(^1\). This can be inferred directly from the Gell-Mann-Nishijima relation:

$$Q = I_3 + Y/2$$  \hspace{1cm} (5)
For a local field theory which is finally the basis of all our considerations we
deduce from Eq. (5) because of $q = \int j_\mu(x) \, d^3x$ a decomposition of the current operator into an
isovector and an isoscalar component:
\begin{equation}
    j_\mu = j^{\nu}_\mu + j^{S}_\mu.
\end{equation}

The first term behaves with respect to isospin transformations like a neutral pion, which proves
our statement. Introducing Eq. (6) into the matrix element $<f|j_\mu|N>$ one finds by coupling
the isospin of $j^{\nu}_\mu$ to the isospin of the initial nucleon, the isospin decomposition for the
photoproduction amplitude
\begin{equation}
    T = T^{(0)} + T^{(1/2)} + T^{(3/2)},
\end{equation}
where $T^{(0)}$ originates in the isovector current and contains (in the s-channel) only the
(total) isospin $I = \frac{3}{2}$. The two other terms come from $j^{\nu}_\mu$ and belong to the isospin $I = \frac{1}{2}$
and $I = \frac{3}{2}$, respectively. For the production of the isoscalar $\eta$ particles and for $K + \Lambda$ only
$T^{(0)}$ and $T^{(1/2)}$ are different from zero while for pion and $K\Xi$ production all three terms
contribute.

### Table IV

Intermediate states in the t-channel for photoproduction and pion-nucleon scattering

<table>
<thead>
<tr>
<th>final state</th>
<th>intermediate (resonant) states</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi^+ N$</td>
<td>$\pi^+(140); \rho^+(763); \Lambda^+(1090); \eta^+(1215); \phi^+(1310)$</td>
</tr>
<tr>
<td>$\pi^0 N$</td>
<td>$\pi^0(763); \omega^0(783); \phi^0(1020); \rho^0(1215)$</td>
</tr>
<tr>
<td>$\eta^0 N$</td>
<td>$\rho^0(763); \omega^0(783); \phi^0(1020); \rho^0(1215)$</td>
</tr>
<tr>
<td>$K^+ Y$</td>
<td>$K^+(494); K^+(725); K^*(891); K_C^+(1215)$</td>
</tr>
<tr>
<td>$K^0 Y$</td>
<td>$K^0, K^{*0}, K_C^0$</td>
</tr>
<tr>
<td>$\pi N \to \pi N$</td>
<td>$\rho; \eta \pi(960); B; f^0(1250); E(1410)$</td>
</tr>
</tbody>
</table>

---

**t-channel (Table IV)** - Here conservation laws play an even more stringent role. Firstly we
have to distinguish between neutral and charged mesons to take charge
conservation into account. For the charged mode all charged meson resonances can enter as intermediate states. For the neutral mode, on the contrary, we have to pay attention to the charge conjugation invariance:
because the C parities of the photon resp. of the neutral pion and of
the $\eta$-particle are odd resp. even only meson resonances with odd C-
parity are possible intermediate states. Thus the following particles are
excluded: $\pi^0; \eta^0$; neutral components of $\Lambda$ and $\Lambda^0$, and finally the
$f^0$ particle.
This last fact distinguishes the photoproduction of pions clearly from pion-nucleon scattering that is indicated in the last line of Table IV. The allowed states for \( \pi^- N \) scattering are determined by their C parity; only even C parity states are allowed. The concept of C parity can also be applied to photoproduction if the decomposition (6) is used. The C operation

\[
G = Ce
\]

acts in the following way on the two components of \( j_\mu \):

\[
\begin{align*}
Gj_\mu^V G^{-1} &= j_\mu^V \\
Gj_\mu^S G^{-1} &= -j_\mu^S
\end{align*}
\]

(8a) (8b)

where the odd behaviour of the electromagnetic current under C has been taken into account.

In the diagram of Table IV the current \( j_\mu \) occurs only on the left hand bubble. Counting also the C parity of the pion resp. the \( \eta \) particle one finds the entries of Table V. Note, for example, that the exchange of a \( \rho \) particle contributes only to the isoscalar part in \( \pi \) production and only to the isovector part in \( \eta \) production.

**Table V**

<table>
<thead>
<tr>
<th>exchanged particle</th>
<th>G</th>
<th>contribution to</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \pi, \omega; \phi, A1, A2 )</td>
<td>-1</td>
<td>( T^{(1/2)} ) and ( T^{(3/2)} ) resp. ( T^{(0)} ) (isovector)</td>
</tr>
<tr>
<td>( \rho; B )</td>
<td>+1</td>
<td>( T^{(0)} ) (isoscalar)</td>
</tr>
</tbody>
</table>

For a theoretical description of the photoproduction process one must know the contribution of the different resonances. In the next section we start with qualitative discussion.

**III. RELATIVE IMPORTANCE OF THE DIFFERENT RESONANCES: QUALITATIVE DISCUSSION**

The first criteria to answer this question can be found by looking at the position of the resonances in the \( s-t \) plane: as a working principle we shall regard those resonances as most important which lie nearest to the discuss kinematical point. Therefore each resonance in the \( s \) channel will become important if the variable \( s \) passes through the mass value of the respective resonance. One encounters a usual resonance phenomenon. For the other channels the importance of the resonances can be easily discussed with the help of Fig. 2. Evidently the exchange of a pion should play an important role at least in the forward direction. Now pion exchange is by C invariance only possible for production of
charged pions. We expect a marked difference between the cross-sections for charged and neutral pions which indeed shows up in the experimental data. For the backward direction the nearest singularity is due to the exchange of a nucleon in the u-channel. It must be taken into account at any rate together with the pion exchange and the nucleon pole in the s-channel to preserve gauge invariance (cp. section VIII). The contributions of these three poles can be calculated completely by the formulae of (renormalized) perturbation theory once the pion-nucleon coupling constant \( f \) resp. the kaon-hyperon coupling constants and the magnetic moments of the baryons are known. Figure 3 gives the results for the best-known case: photo-production of pions\(^{10}\). Turning now to the role of the other mesonic resonances the situation is much more unclear. Especially the influence of the \( \rho \) meson is under vivid discussion in the present literature. In this section we only mention a reasoning using the Bronzan-Low quantum number \( A^{\prime \prime} \). This quantum number should be conserved in strong and electromagnetic interactions as long as virtual baryon states can be neglected. Table VI shows the values of \( A \) for the mesons in question and the result of the application of \( A \) conservation to the electromagnetic vertices of mesons: only \( \omega \) and the somewhat dubious \( A^0 \) particle can be coupled to photon and pion (resp. \( \eta^0 \)). If one could rely on this argument only the exchange of pions and very heavy subjects like \( A^1 \) is allowed for charged pion production. On the other hand, in the \( \pi^0 \) and \( \eta \) production, \( \omega \) exchange will presumably play a role. In connection with the questionable \( A \) parity of the \( A^2 \) resonance doubts have been raised against its applicability\(^{12}\).

### Table VI

Electromagnetic couplings of mesons allowed by \( A \) parity conservation

<table>
<thead>
<tr>
<th>( \gamma )</th>
<th>( \pi )</th>
<th>( \eta )</th>
<th>( K )</th>
<th>( \rho )</th>
<th>( \omega )</th>
<th>( \varphi )</th>
<th>( A^1 )</th>
<th>( A^2 )</th>
<th>( B )</th>
<th>( \kappa )</th>
<th>( K^* )</th>
<th>( K_C )</th>
</tr>
</thead>
<tbody>
<tr>
<td>A parity</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>?</td>
<td>+</td>
<td>?</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( \gamma \pi )</th>
<th>( \gamma \eta )</th>
<th>( \gamma K )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( V )</td>
<td>( V )</td>
<td>( V )</td>
</tr>
<tr>
<td>( ? )</td>
<td>( ? )</td>
<td>( ? )</td>
</tr>
</tbody>
</table>

We conclude this section with some remarks for very high energies (\( s \rightarrow \infty \)). s-channel resonances should no longer be observed; they overlap completely if present at all. On the other hand, the exchange of particles should become increasingly more important especially for particles with higher spin. We recall the well-known fact that the differential cross-section due to the exchange of a spin \( j \) particle in the t-channel behaves like\(^{13}\),
for small momentum transfers $t$.

Therefore vector particles like $\rho$, $\omega$ and $\phi$ are good candidates, even better, a spin 2 particle (perhaps $A_2$?). But it is generally believed that the $s$ dependence of Eq. (9) will be "damped" in some way in order to avoid conflicts with analyticity and unitarity\textsuperscript{14}).

This is achieved in the simplest way by replacing $j$ by a "Regge trajectory" $a(t)$ thus introducing a "moving spin" of the exchanged particle. Making the no: very convincing assumption that $a(t)$ is a linear function of $t$ with a slope given by proton-proton diffraction scattering one can draw some conclusions\textsuperscript{15): according to Fig. 4, $\rho$, $\omega$ and $A_2$ have the largest exponents $a(t)$ in the negative $t$ region. The energy variation of the photopion cross-section in the forward direction is therefore approximatively given by [because of $a(0) \approx 0.5$]

$$\frac{d\sigma}{dt} \approx \frac{1}{s^2} \text{ for } \gamma + p \rightarrow N^+\pi^-.$$

(10a)

Strange particle production is determined by an analogous argument by $K^*$ exchange leading to

$$\frac{d\sigma}{dt} \approx \frac{1}{s^2} \text{ for } \gamma + p \rightarrow N^+\eta.$$

(10b)

For the backward direction the exchange of excited baryons must be considered. The $\Delta(1235)$ trajectory seems to be the most important giving

$$\frac{d\sigma}{du} \approx \frac{1}{s^2} \text{ for pion and } \eta\text{-production},$$

(10c)

while for strange particle production the $Y^*_s(1585)$ exchange suggests

$$\frac{d\sigma}{du} \approx \frac{1}{s^2}.$$

(10d)

Of course all these formulae are guesses and describe only how the situation could be. For the formal details of the application of Regge poles to photoproduction we refer to the literature\textsuperscript{9,16,17,18}).

IV. MULTIPOLE ANALYSIS IN THE S-CHANNEL

In principle the $s$-channel resonances can be seen directly as maxima in the energy dependence of the production cross-sections. For the lower resonances this is indeed the case (see Fig. 5). The heavier isobars do not show up so clearly. Here the contributions of the different resonant states overlap each other and there are also non-resonant terms.

To formulate a mathematical apparatus for such considerations we have to generalize the well-known partial wave expansion

$$\sum_{l} (2l+1) e^{i\delta l} \sin \delta \frac{d\sigma}{d\Omega} \cos \theta$$

(11)

for the scattering of spinless particles for our case. Complications arise because of the spins of photons and baryons. In the older treatments\textsuperscript{19}) also the zero mass of the photon.
introduces troubles. But the brilliant "helicity-formalism" invented by Jacob and Wick gives a simple solution for our problem. In this lecture we shall apply this formalism directly to the photoproduction, referring to the original papers for its deeper foundation.

We recall the basic definition: each spinning particle is described by its momentum \( \vec{p} \) and its spin component in the direction of \( \vec{p} = \text{helicity} \lambda \):

\[
| \vec{p}, \lambda > .
\] (12)

In general \( \lambda \) can take the \( 2j+1 \) values

\[
\lambda = -j, \ldots, +j
\] (12a)

but for massless particles only

\[
\lambda = \pm j
\] (12b)

is allowed. The helicity does not change under rotations, but is reversed under space reflection. Moreover, it is not Lorentz invariant. Therefore we shall use throughout the c.m.s. system. The photoproduction process (2) for spin-zero mesons and spin \( \frac{1}{2} \) baryons can be described by the following states (see Fig. 6):

a) **Initial state (in the c.m.s. frame):**

\[
| k, \Theta_i; \lambda, \nu_i >
\]

\( k = | \vec{k}_\text{c.m.} | = \text{c.m.s. momentum of the incoming particles [cp. Eq. (3a)]} \)

\( \Theta_i = \text{initial angle (will be put equal to zero later on)} \)

\( \lambda = \text{helicity of the photon; } \lambda = \pm 1 \)

\( \nu_i = \text{helicity of the incoming baryon; } \nu_i = \pm \frac{1}{2} \).

Note that the photon has no \( \lambda = 0 \) (longitudinal) component.

b) **Final state:**

\[
| q, \Theta_f; \nu_2 >
\]

\( q = | \vec{q}_\text{c.m.} | = \text{c.m.s. momentum of the final particles [cp. Eq. (3b)]} \)

\( \Theta_f = \text{final angle} \)

\( \nu_2 = \text{helicity of the final baryon; } \nu_2 = \pm \frac{1}{2} \).

The production amplitude

\[
| q, \Theta, \nu_2 | T | k, 0; \lambda, \nu_i >
\] (15)
contains 8 functions of \( W \) and \( \Theta \) resp. \( s \) and \( t \) (count all possibilities \( \lambda = \pm 1, \nu_t = \pm \frac{1}{2}, \nu_s = \pm \frac{1}{2} \)!) but only 4 are independent. Parity conservation ensures that the process with

\[ \lambda, \nu_1; \nu_2 \quad \text{and} \quad -\lambda, -\nu_1; -\nu_2 \]

are directly connected. One has only to apply a reflection on the production plane. Therefore we can restrict to

\[ \nu_2 = \pm \frac{1}{2} \quad (16) \]

and must consider the following 4 amplitudes \(^{21}\)

\[ H^+(s,t) = \langle q, \Theta; \nu_2 = \pm \frac{1}{2} | T | k, 0; \lambda = \pm 1, \nu_t = \pm \frac{1}{2} \rangle \quad (17a) \]

no helicity-flip amplitudes

\[ \phi^+(s,t) = \langle q, \Theta; \nu_2 = \pm \frac{1}{2} | T | k, 0; \lambda = \pm 1, \nu_t = -\frac{1}{2} \rangle \quad (17b) \]

helicity-flip amplitudes.

By the way, these definitions can easily be generalized to virtual photons which occur in inelastic electron scattering. One has only the additional possibility of a longitudinal polarized photon: \( \lambda = 0 \) and therefore two longitudinal amplitudes

\[ H^0(s,t) \quad \text{and} \quad \phi^0(s,t), \quad (17c) \]

which can be obtained from Eq. (15) by putting \( \lambda = 0 \).

In order to find the wanted partial wave decomposition one has to develop the states (13) and (15) in terms of eigenstates of the total angular momentum. This can be easily done because the helicity is a rotational invariant quantity. One gets

\[ |q; \Theta_f; \nu_2 > = \sum_{J,M} |W; JM; \nu_2 > \frac{2J+1}{4\pi} d^J_M - \nu_2 (\Theta_f) , \quad (18a) \]

and

\[ |k; \Theta_1; \lambda, \nu_1 > = \sum_{J,M} |W; JM; \lambda \nu_1 > \frac{2J+1}{4\pi} d^J_M \lambda - \nu_1 (\Theta_1) \quad (18b) \]

where the known functions

\[ < JM | e^{-i\Theta J_z} | J, M' > = d^J_{MM'} (\Theta) \]

have been introduced and the energy dependence of the states has been indicated by the total c.m.s. energy \( W \).

Contrary to the usual situation the angular momentum states occurring in equations (18a) and (18b) are not eigenstates of the parity operator. Instead one has the parity property:

\[ F |W; JM; \nu_2 > = (-1)^J |W; JM; -\nu_2 > \quad \text{for the } \pi \text{ B state} \quad (19a) \]

\[ F |W; JM; \lambda, \nu_1 > = (-1)^J |W; JM; -\lambda, -\nu_1 > \quad \text{for the } \gamma \text{ B state} \quad , \quad (19b) \]

where we have used the proper phase normalizations of the state vectors and have assumed:
negative parity for the meson and positive parity for the baryons\textsuperscript{*}). (19c)

Now parity eigenstates can be constructed in a simple way. Let us start with pion-baryon states:

\[
|W; J, M, \pi > = \frac{1}{\sqrt{2}} (|W; J, M, \nu_2 > \pm |W; J, M, -\nu_2 >) = |W; J, M, t >
\]

where the parity \(\pi\) is given by

\[
\pi = \pm (-1)^{J-\frac{1}{2}} \overset{\text{def}}{=} (-1)^t .
\]

Here we have in addition introduced the orbital angular momentum \(l\) of the \(\pi N\) system which is formally defined by the second equation of (20a).

These state vectors (20) are the proper quantities to describe the isobars which by definition have a well-defined spin and parity.

Repeating this procedure for the photon-baryon state one encounters a somewhat more complicated situation. The appropriate eigenstates of \(J^z\) and \(P\) are

\[
|W; J, M; \pi; \lambda > = \frac{1}{\sqrt{2}} (|W; J, M; \lambda, \nu_1 = +\frac{1}{2} > \pm |W; J, M; -\lambda, \nu_1 = -\frac{1}{2} >)
\]

with

\[
\pi = \pm (-1)^{J-\frac{1}{2}} = \pm (-1)^t .
\]

The parameter \(\lambda\) in the l.h.s. of Eq. (21) no longer denotes the helicity of the states but distinguishes between two different eigenstates belonging to the same total angular momentum \(J\) and the same parity \(\pi\). Therefore: for given spin \(J\) and parity there are two possible states of the \(\gamma B\) system. Correspondingly we have two photoproduction amplitudes for each isobar:

\[
A_{\lambda}^{J, \pi}(W) = \langle JM; \pi | T(W) | JM; \pi; \lambda > \quad \text{with} \quad \lambda = \pm 1 .
\]

The matrix elements on the r.h.s. are independent on \(W\) by rotational invariance\textsuperscript{**}).

These two amplitudes are usually characterized by the terms "electric and magnetic multipoles". But these quantities are not directly given by Eq. (22). They are therefore introduced by a consideration starting from the photon states only

\[
|\vec{k}, \lambda > ,
\]

which describe a single photon with momentum \(\vec{k}\) and helicity \(\lambda\). Repeating the step leading to Eq. (18) we arrive at eigenstates of the total angular momentum of the photon which we characterize by the quantum numbers \(L, m\textsuperscript{\dagger}\):
\[ |L, m; \lambda \rangle \quad \text{with} \quad \lambda = \pm 1. \quad (24) \]

Now the electric resp. magnetic multipole states of order \( L \) are defined by (20)

\[ |EL, m \rangle = \frac{1}{\sqrt{2}} (|L, m; \lambda = +1 \rangle + |L, m; -1 \rangle) \quad (25a) \]

\[ |ML, m \rangle = \frac{1}{\sqrt{2}} (|L, m; +1 \rangle - |L, m; -1 \rangle). \quad (25b) \]

They belong to the parities (cp. footnote of p. 27, \((-1)^L \) for the electric multipoles \((26a)\)

\[ -(-1)^L \) for the magnetic multipoles. \quad (26b) \]

To get angular momentum states for the photon-baryon system we couple to Eqs. (25a) and (25b) spin \( \frac{1}{2} \) states \( u_s \) according to the well-known recipe

\[ |JM; EL \rangle = \sum_{m, s} (L, m; \frac{1}{2}, s |JM \rangle |EL, m \rangle u_s \quad \text{with} \quad J = L \pm \frac{1}{2}. \quad (27a) \]

\[ |JM; ML \rangle = \sum_{m, s} (L, m; \frac{1}{2}, s |JM \rangle |EL, m \rangle u_s \quad (27b) \]

These states are the basis for the usual definition of the electric and magnetic multipole amplitudes. Using the \( \pi N \) states (20) the electric multipole amplitudes are given by

\[ <JM; 1 | T(\gamma) | JM; EL \rangle. \]

Because of parity conservation we find, using Eq. (20a) and Eq. (26a),

\[ L = t \pm 1, \]

therefore by \( J = L \pm \frac{1}{2} \) only the matrix elements

\[ <J = t + \frac{1}{2}, M; t | T(\gamma) | t + \frac{1}{2}, M; E(t+1) \rangle = \sqrt{(t+1)(t+2)} E_{t+1}(W) \quad (28a) \]

(electric multipole of order \( L = t+1 \))

and

\[ <J = t - \frac{1}{2}, M; t | T(\gamma) | t - \frac{1}{2}, M; E(t-1) \rangle = \sqrt{t(t-1)} E_{t-1}(W) \]

(electric multipole of order \( L = t-1 \))

are different from zero.

*) Because of \( \lambda > 0 \) the value \( L = 0 \) is excluded by the analogue of Eq. (18).
On the r.h.s. we have introduced conventional factors and a notation due to Feld(3). Observe that the index in \( E_{\pm} \) does not coincide with the multipole order \( L \). Analogously we have for the magnetic multipole amplitudes by parity conservation

\[
L = t
\]

and the non-vanishing matrix elements are given by

\[
< J = t \pm \frac{1}{2}, M_t; t \mid T(M) \mid t \pm \frac{1}{2}, M_t^\pm > = \sqrt{t(t+1)} M_t^{\pm} (y)
\]

(magnetic multipole of order \( L = t \)).

In this case the index \( t \) coincides with the multipole order.

It remains to establish the connection with earlier introduced amplitudes \( A^J_{\lambda, \pi} \) [Eq. (22)]. We shall not go into the details of the calculation but merely quote the results(4):

\[
A^J_{\lambda} = t \pm \frac{1}{2}, \pi = \frac{(t+1)(2t+3)}{2} (t+1, \lambda; \frac{1}{2}, -\frac{1}{2}) | t+\frac{1}{2}, \lambda - \frac{1}{2} \rangle E_{\pm}
\]

\[
+ \frac{t(2t+1)}{2} (t, \lambda; \frac{1}{2}, -\frac{1}{2}) | t+\frac{1}{2}, \lambda - \frac{1}{2} \rangle M_{\pm}
\]

and

\[
A^J_{\lambda} = t - \frac{1}{2}, \pi = - \frac{(t-1)(2t-1)}{2} (t-1, \lambda; \frac{1}{2}, -\frac{1}{2}) | t-\frac{1}{2}, \lambda - \frac{1}{2} \rangle E_{\pm}
\]

\[
+ \frac{(t+1)(2t+1)}{2} (t, \lambda; \frac{1}{2}, -\frac{1}{2}) | t-\frac{1}{2}, \lambda - \frac{1}{2} \rangle M_{\pm}
\]

[In both equations the parity \( \pi \) is given by Eq. (20a).]

Now we have written down all necessary definitions and formulae to find the wanted expansion of the helicity amplitudes \( H^\pm, \phi^\pm \) by a straightforward calculation. We give the result in Table VII in an explicit form where all Clebsch-Gordan coefficients have already been worked out and the \( d^J_{MM'} \) functions expressed in terms of Legendre polynomials \( P_\ell (\cos \theta)^{2\pi} \).

A great advantage of the amplitudes \( H^\pm \) and \( \phi^\pm \) lies in the fact that differential cross-sections and polarizations can be expressed in a simple way. We give three examples:

a) The differential cross section for photons with circular polarization but unpolarized baryons is given by

\[
\frac{d\sigma^\pm}{d\Omega} = \frac{g}{\kappa} \left( |H^\pm|^2 + |\phi^\pm|^2 \right),
\]

while the cross-section for unpolarized \( \gamma \) rays follows from

\[
\frac{d\sigma}{d\Omega} = \frac{1}{2} \left( \frac{d\sigma^+}{d\Omega} + \frac{d\sigma^-}{d\Omega} \right).
\]
Table VII

Multipole expansion of the helicity amplitude

\[
\frac{d\sigma}{d\Omega} = \frac{\alpha}{k} \left( |H^*|^2 + |H^-|^2 \right); \quad P(\theta) \frac{d\sigma}{d\Omega} = \frac{\alpha}{k} \text{Im}(\tilde{H}^* H^* + \tilde{H}^- H^-)
\]

\[
H^+ = -\frac{1}{\sqrt{2}} \sin \frac{\theta}{2} \sum_{l} \left( p_t^l + p_t^l \right) \left( (l+2) [E_{l+} + M(l+1)_-] + l [M_{l+} - E(l+1)_-] \right)
\]

\[
H^- = -\frac{1}{\sqrt{2}} \sin \frac{\theta}{2} \sum_{l} \left( p_t^l - p_t^l \right) \left( E_{l+} - M(l+1)_- - M_{l+} - E(l+1)_- \right)
\]

\[
\Phi^+ = -\frac{1}{\sqrt{2}} \cos \frac{\theta}{2} \sum_{l} \left( p_t^l - p_t^l \right) \left( (l+2) \left( E_{l+} - M(l+1)_- \right) + l \left( M_{l+} + E(l+1)_- \right) \right)
\]

\[
\Phi^- = -\frac{1}{\sqrt{2}} \cos \frac{\theta}{2} \sum_{l} \left( p_t^l + p_t^l \right) \left( E_{l+} - M(l+1)_- \right) - l \left( M_{l+} + E(l+1)_- \right)
\]

\[
H^0 = -\frac{1}{\sqrt{2}} \cos \frac{\theta}{2} \sum_{l} (l+1) \left( p_t^l - p_t^l \right) \left[ L_{l+} - L(l+1)_- \right] \frac{\sqrt{k^2}}{k_0}
\]

\[
\Phi^0 = -\sin \frac{\theta}{2} \sum_{l} (l+1) \left( p_t^l + p_t^l \right) \left[ L_{l+} + L(l+1)_- \right] \frac{\sqrt{k^2}}{k_0}
\]

[Note: a factor \( \frac{1}{2} \) in Eq. (31a) has been cancelled because we use only four amplitudes instead of eight !]

b) The polarization \( P(\theta) \) of the final baryon which is perpendicular to the production plane can be calculated from

\[
P(\theta) = \frac{\alpha}{k} \text{Im}(\tilde{H}^* H^* + \tilde{H}^- H^-);
\]

(32)

c) For linearly polarized photons linear combinations of \( H^\pm \) resp. \( \Phi^\pm \) occur. We describe these photons by a polarization vector \( \hat{e} \) which is perpendicular to the photon momentum \( \hat{K} \) and has an angle \( \varphi \) with respect to \( - (\hat{K} \times \hat{q}) \)

\[
\cos \varphi = \frac{\hat{e} \cdot (\hat{K} \times \hat{q})}{|\hat{K} \times \hat{q}|}
\]

Expressing \( \hat{e} \) in terms of \( \hat{e}^\pm \) which corresponds to the helicity states, one arrives by simple algebra at the following amplitudes

\[
H(\hat{e}) = \frac{1}{\sqrt{2}} \left( H^+ e^+ - H^- e^- - i\varphi \right);
\]

(33a)

\[
\Phi(\hat{e}) = \frac{1}{\sqrt{2}} \left( \Phi^+ e^+ - \Phi^- e^- - i\varphi \right).
\]
The differential cross-section for linearly polarized photons is given by

\[
\frac{d\sigma}{d\Omega} = \frac{\alpha}{k} \left( |\mathcal{M}(\mathbb{C})|^2 + |\mathcal{\Phi}(\mathbb{C})|^2 \right).
\]

(33b)

For further use we collect in Table VIII the differential cross-sections for pure multipoles, and in Table IX the cross section and polarization if only the values \( t = 0 \) and \( 1 \) of the angular momentum of the pion are important.

Table VIII

Angular distribution of photoproduction for different pure multipoles
(This distribution depends only on \( J \) and \( L \) but not on the parity of the \( \pi N \) state.)

<table>
<thead>
<tr>
<th>( \pi N ) system</th>
<th>multipole</th>
<th>matrix element</th>
<th>( \frac{d\sigma}{d\Omega} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{1}{2}^+ ), ( P_{1/2} )</td>
<td>M1</td>
<td>( M_{1-} )</td>
<td>(</td>
</tr>
<tr>
<td>( \frac{1}{2}^- ), ( S_{1/2} )</td>
<td>E1</td>
<td>( E_{0+} )</td>
<td>(</td>
</tr>
<tr>
<td>( \frac{3}{2}^+ ), ( P_{3/2} )</td>
<td>M1</td>
<td>( M_{1+} )</td>
<td>(</td>
</tr>
<tr>
<td></td>
<td>E2</td>
<td>( E_{1+} )</td>
<td>(</td>
</tr>
<tr>
<td>( \frac{5}{2}^- ), ( D_{3/2} )</td>
<td>E1</td>
<td>( E_{3-} )</td>
<td>(</td>
</tr>
<tr>
<td></td>
<td>M2</td>
<td>( M_{2-} )</td>
<td>(</td>
</tr>
<tr>
<td>( \frac{5}{2}^- ), ( D_{5/2} )</td>
<td>N2</td>
<td>( M_{5+} )</td>
<td>(</td>
</tr>
<tr>
<td></td>
<td>E3</td>
<td>( E_{5+} )</td>
<td>(</td>
</tr>
<tr>
<td>( \frac{5}{2}^+ ), ( F_{5/2} )</td>
<td>E2</td>
<td>( E_{5-} )</td>
<td>(</td>
</tr>
<tr>
<td></td>
<td>M3</td>
<td>( M_{5-} )</td>
<td>(</td>
</tr>
</tbody>
</table>
Table IX

Differential cross-section and the polarization of the second baryon including all multipoles with \( \ell = 0,1 \)

\[
\frac{d\sigma}{d\Omega} = A + B \cos \theta + C \cos^2 \theta
\]

\[
A = \frac{q}{k} \left[ \frac{7}{2} |M_{1+}|^2 + |E_{0+}|^2 + |M_{1-}|^2 + \frac{7}{2} |E_{1+}|^2 + 2 \text{Re} \left\{ M_{1+}^* (M_{1-} - 3 E_{1+}) + 3 M_{1-}^* E_{1+} \right\} \right]
\]

\[
B = \frac{q}{k} 2 \text{Re} \left\{ E_{0+}^* (M_{1+} - M_{1-} + 3 E_{1+}) \right\}
\]

\[
C = \frac{q}{k} \left[ -\frac{7}{2} |M_{1+}|^2 + \frac{7}{2} |E_{1+}|^2 - 3 \text{Re} \left\{ M_{1-}^* (M_{1-} - 3 E_{1+}) - 3 M_{1+}^* E_{1+} \right\} \right]
\]

\[
a = - \frac{q}{k} \text{Im} \left\{ E_{0+}^* (M_{1+} + 2 M_{1-} + 3 E_{1+}) \right\}
\]

\[
b = 3 \frac{q}{k} \text{Im} \left\{ M_{1-}^* (M_{1-} + 3 E_{1+}) \right\}
\]

\[
\frac{d\sigma}{d\Omega} \propto \sin \theta \left[ a + b \cos \theta \right];
\]

For linearly polarized photons one finds from Eqs. (33a) and (33b) for this case

\[
\frac{d\sigma}{d\Omega} \propto \frac{d\sigma}{d\Omega}|_{\text{unpol.}} + a \sin^2 \theta \cos^2 \varphi
\]

with

\[
a = - \frac{7}{2} |M_{1+}|^2 + \frac{7}{2} |E_{1+}|^2 - 3 \text{Re} \left\{ M_{1-}^* (M_{1-} - 3 E_{1+}) - 3 M_{1+}^* E_{1+} \right\}.
\]

We stress the important rules which are contained in these results:

1) the angular distribution depends only on the spin \( J \) and the multipole order \( L \) but not on the parity;

2) polarizations occur only if different multipoles interfere which leads to an over-all factor \( \sin \theta \);

3) the formulae are invariant under the simultaneous replacement of

\[
\theta \rightarrow \pi - \theta, \text{ i.e. } \cos \theta \rightarrow - \cos \theta; \sin \theta \rightarrow \sin \theta
\]

and of

\[
E_{1\pm} \rightarrow -(-1)^L E_{1\pm}
\]

resp.

\[
M_{1\pm} \rightarrow -(-1)^L M_{1\pm}
\]

(By this rule the asymmetry coefficient \( B \) is due to an interference of different "parity" multipoles.) This invariance based on parity conservation helps very much when discussing the multipole expansion qualitatively.
V. PHENOMENOLOGICAL DISCUSSION OF THE s-CHANNEL RESONANCES

According to the results of the last section each s-channel resonance with a given spin and parity can lead to two different multipole amplitudes and we have to ask for each isobar: which multipole resonates? In this section we discuss the empirical evidence for an answer to this question. In order to avoid the complications due to the near singularity in the t-channel introduced by one-pion exchange we restrict ourselves to the production of neutral pions. Figure 7 shows the coefficients of the angular distribution

$$\frac{d\sigma}{d\Omega} = A + B \cos \theta + C \cos^2 \theta + D \cos^3 \theta + E \cos^4 \theta$$  \hspace{1cm} (34a)

for energies below 1 GeV.\(^6\) One clearly recognizes the maxima corresponding to the first and second isobar, while the third πN resonance (with \(E_\gamma = 1050\) MeV) lies just outside the region.

1. \(\Delta(1235) = N^*_{1/2}\) with \(3/2^+\) \((E_\gamma = 350\) MeV\)

Here our question has been answered uniquely. The angular distribution can be well represented by

$$5 - 3 \cos^2 \theta$$  \hspace{1cm} (34b)

with a small asymmetry coefficient B which goes through zero at resonance. Assuming the spin-parity assignment \(3/2^+\) we immediately deduce with the help of Table VIII:

the magnetic dipole amplitude \(M_{1+}\) resonates at the \(\Delta(1235)\) resonance . (35)

Moreover one finds from experiment the behaviour

$$|M_{1+}|^2 \sim q^3$$

for small energies. This agrees with the expected threshold law\(^27\)

$$M_{1+} \sim q^{2l+1}$$

for the expected value \(l = 1\).

2. \(N(1512) = N^*_{1/2}\) with \(1/2^-\) \((E_\gamma = 750\) MeV\)

In this case the situation is not quite as clear. From the angular distribution which is given again by Eq. (34) we already conclude that the spin must be \(1/2\). If, in addition, we again rely on the result of the detailed analysis of \(\pi N\) scattering\(^28\) and accept the negative parity we are led to an

electric dipole \(E_{1-}\) resonant amplitude. (36)

This assignment fits very well with the polarization measurements of the recoiling proton. These have been done for several energies below 900 MeV for the c.m.s. angle \(\theta = 90^\circ\). In the region of the first resonance the polarization is quite small\(^29\),

$$\text{e.g., } P \left(\frac{E}{2}\right) = (14 \pm 6)\% \text{ for } E_\gamma = 520 \text{ MeV},$$  \hspace{1cm} (37)
but approaching the second resonance it increases up to 60-80% and stays so in the measured energy range (≤ 850 MeV) [cp. Fig. 830]. The small polarization can be understood from the formula of Table IX: it is due to the interference of the small non-resonant amplitude $E_{0^+}$ with $M_{1^+}$. The large polarization between the first and the second resonance will be just expected by the assignment (36); it is due to an interference between two resonant amplitudes $E_{2^+}$ and $M_{1^+}$

$$P\left(\frac{\pi}{2}\right) \frac{d\sigma}{d\Omega} = -4 \frac{d}{k} \text{Im}(E_{2^+}^* M_{1^+})$$

On the other hand, it has been argued that Eq. (36) is in contradiction with polarization measurements in the process

$$\gamma + n \rightarrow p + \pi^-,$$

where one observes again a negative polarization31)

$$P\left(\frac{\pi}{2}\right) = -0.26 \pm 0.06 \text{ for } E_\gamma = 715 \text{ MeV.}$$

These authors31) expect a change of sign in the isospin $I = \frac{1}{2}$ amplitude $E_{2^+}$ if one goes over from $\gamma + p \rightarrow \pi^0 + p$ to Eq. (39) and arrives at a contradiction to the experimental result. This is indeed the case if the second resonance has an isovector character (cp. Table X). In addition even in $\pi N$ scattering the situation around the second resonance is controversial32,33).

* cp. J.J. Sakurai32).

---

**Table X**

| Isospin decomposition for $\pi^{+,-}$ and $\eta$ production on nucleons |
|--------------------------|-------------------|-------------------|-------------------|
| $\gamma + p \rightarrow \pi^0 + p$ | $T^{(0)} + \frac{1}{2} T^{(1/2)} + \frac{3}{2} T^{(3/2)}$ | $T^{(0)} + T^{(+)}$ |
| $\gamma + p \rightarrow \pi^+ + n$ | $\sqrt{2} (T^{(0)} + \frac{1}{2} T^{(1/2)} - \frac{3}{2} T^{(3/2)})$ | $\sqrt{2} (T^{(0)} + T^{(-)})$ |
| $\gamma + n \rightarrow \pi^0 + n$ | $- T^{(0)} + \frac{1}{2} T^{(1/2)} + \frac{3}{2} T^{(3/2)}$ | $- T^{(0)} + T^{(+)}$ |
| $\gamma + n \rightarrow \pi^- + p$ | $\sqrt{2} (T^{(0)} - \frac{1}{2} T^{(1/2)} + \frac{3}{2} T^{(3/2)})$ | $\sqrt{2} (T^{(0)} - T^{(-)})$ |
| $\gamma + p \rightarrow \eta + p$ | $T^{(0)} + T^{(1/2)}$ |
| $\gamma + n \rightarrow \eta + n$ | $T^{(0)} - T^{(1/2)}$ |
3. $N(1688) = \frac{\sin^2}{\cos^2} \left( \frac{1}{2} \right)$ with $\frac{3}{2}^+ (E = 1050 \text{ MeV})$

This is the first isobar where presumably both possible multipoles, the electric quadrupole $E_{3/2}^-$ and the magnetic octupole $M_{3/2}^-$ contribute appreciably. From the angular distribution (Fig. 7) one finds $E < 0$ which, according to Table VIII, indicates $J = \frac{5}{2}^-$. On the other hand, the cross-section in forward direction (see Fig. 9) has been found to be quite small$^{35}$. This can be understood$^{34, 37}$ if the ratio of $E_{3/2}^-$ and $M_{3/2}^-$ is:

$$R = \frac{E_{3/2}^-}{M_{3/2}^-} = 2.$$  \(41\)

In this case the differential cross-section contains a factor $\sin^2 \theta$.

The argument for the positive parity is rather weak$^{38}$). Again the large observed polarization is in favour of this assignment: it makes possible a large contribution to $P(\pi/2)$ because of an interference between the third and second resonance.

4. Higher Isobars

The CEA results$^{39}$ on the pion photoproduction between 1 and 4 GeV show some structures (Fig. 10) but it seems premature to conclude anything about the existence and properties of isobars.

Concluding this section we remark that in the photoproduction of strange particles $\gamma + p = K^+ + \Lambda, K^0 + \Sigma^+$ only the third and higher isobars can be seen directly. The experimental results$^{40}$ are too meagre to identify any resonance though the $N(1688)$ lies in the accessible region. But theoretical studies indicate that $s$-channel resonances below the strange particle threshold and exchanges of resonances in the $t$- and $u$-channel play an important role$^{41}$.

VI. DETAILED THEORETICAL DISCUSSION OF THE $s$-CHANNEL RESONANCES

The empirical evidence summarized in the last section shows maxima in the multipole amplitudes of energies where also the $\pi N$ scattering has a resonance. This is true at least for the first three isobars. Qualitatively this can be understood with the "compound nucleus" picture taken over from nuclear physics$^{42(a)}$. If the isobar can be understood as such a compound system according to Bohr's independence assumption$^{42}$) its properties should be the same whether it is produced in $\pi N$ or in $\gamma N$ collisions.

For a more detailed theoretical development of these ideas the Watson Theorem has played an important role$^{43}$). For energies below the two-meson threshold it gives an exact relation between the multipole amplitudes of a given order $E_{l \pm}^J, M_{l \pm}^J$ and the corresponding meson-nucleon scattering amplitudes:

$$f_{l \pm}^J(s) = \frac{1}{q(s)} e^{i \delta l \pm} \sin \delta l \pm \text{ refers to } J = \frac{l \pm}{2},$$  \(42\)

(I denotes the isospin).
This connection follows from time reversal invariance and the unitarity of the S matrix under the assumption that the only energetically possible states are the π N and γ N systems and if higher orders in the electric charge e are neglected.

For a given value of spin J, parity π resp. l and isospin I, the S matrix can be written in the form:

\[
\begin{array}{c|ccc}
\pi N & S_{t \pm}^I, & i E_{t \pm}^I, & i M_{t \pm}^I \\
\hline
\gamma N & i E_{l \pm}^I, & 1 + \ldots & \ldots \\
& i M_{l \pm}^I, & \ldots & 1 + \ldots \\
\end{array}
\]  

for \( n \leq (M_0 + 2m)^2 \). \hspace{1cm} (43)

Use has already been made of time reversal invariance from which the symmetry of the S matrix follows \(^{44}\). \( S_{t \pm}^I \) denotes the S-matrix element for \( \pi N \) scattering with given \( t \) and \( J = t \pm \frac{1}{2} \). (By the helicity consideration of Section V one can prove that only one independent scattering amplitude exists for each \( J, l \).) The dots indicate Compton scattering amplitudes which are of second order in e. The factors "i" have been introduced in accordance with the definition \( S = 1 + iT \).

The unitarity condition gives for the first line of Eq. (43):

\[
|S_{t \pm}^I|^2 = 1 - \eta_{t \pm}^I = e^{2i\delta_{t \pm}^I} \text{ with real } \delta_{t \pm}^I
\]

neglecting second order terms in e. The orthogonality between the first and the second resp. the first and the third line gives (retaining only the linear terms in e):

\[
S_{t \pm}^I E_{l \pm}^I = E_{l \pm}^I, \quad S_{l \pm}^I M_{t \pm}^I = M_{l \pm}^I.
\]

Introducing Eq. (42) one gets:

\[
\text{Im } E_{t \pm}^I = q \cdot r_{t \pm}^I E_{l \pm}^I; \quad \text{Im } M_{t \pm}^I = q \cdot r_{l \pm}^I M_{l \pm}^I. \hspace{1cm} (45)
\]

By observing that the l.h.s. of these relations must be real one arrives at the equivalent result:

\[
E_{t \pm}^I = \pm |E_{t \pm}^I| e^{i\delta_{t \pm}^I}; \quad M_{t \pm}^I = \pm |M_{l \pm}^I| e^{i\delta_{t \pm}^I}. \hspace{1cm} (46)
\]

The phases of the photoproduction amplitudes are equal to the corresponding scattering phases up to a multiple of \( \pi \).

Returning now to the description of resonances we find that all dynamical theories maintain: for resonances, not only the phase of the multipole amplitude is given by the scattering amplitude but the multipole is proportional to \( f_{t \pm}^I \):

\[
E_{t \pm}^I(s) \text{ or } M_{t \pm}^I(s) = C \frac{f_{t \pm}^I(s)}{s} \text{ for resonances}, \hspace{1cm} (47)
\]
where the factor \( C \) depends only weakly on the energy.

The most elaborate theory exists, of course, for the \( \frac{3}{2}^+ \) isobar. Here the proportionality

\[
\frac{3}{2}(s) = \frac{k}{q} \frac{\mu_v}{\rho_{1/2}} f_{1/2}^2(s)
\]

has been first proven in the static Chew-Low theory\(^ {45} \).

Here we have introduced the isovector magnetic moment of the nucleons

\[
\mu_v = \frac{1}{2}(\mu_p - \mu_n) = 2.35 \frac{e}{2M}
\]

(49a)

and the (renormalized) \( \pi N \) coupling constant

\[
\frac{f^2}{4\pi} = 0.08
\]

(49b)

All other \((1+)\) multipoles vanish, especially

\[
F_{1/2}^2(s) \approx 0
\]

(50)

The physical reason for these results (46) and (47) can be found in large magnetic coupling to the isovector magnetic moment which already gives the largest contribution to \( \pi^0 \) production in the Born approximation (cp. Fig. 3). After the advent of relativistic dispersion theory\(^ {46} \) it was found that Eq. (46) is also consistent with the relevant dispersion relations if all terms of order \( \omega/M \) are neglected.

General mathematical conditions which can lead to Eq. (45) have been discussed by P. Stichel\(^ {47} \). He introduces the frequently used "irreducible" amplitudes\(^ {48} \)

\[
f_{\text{irr}}(s) = \frac{f(s)}{1 + i\alpha f(s)} \quad \text{and} \quad M_{\text{irr}}(s) = \frac{M(s)}{1 + i\alpha f(s)}
\]

(51)

where we have dropped for a moment all indices. As long as Eq. (44) and Eq. (46) are valid one has

\[
\text{Im} f_{\text{irr}}(s) = \text{Im} M_{\text{irr}}(s) = 0
\]

i.e. these functions are real up to threshold for two-particle production. But they can have pole singularities if the denominator in Eq. (51) vanishes. Indeed, if we assume

\[
f_{\text{irr}}(s) = \frac{\gamma(s)}{s - s_0} \quad [\gamma(s): \text{slowly varying near } s_0]
\]

(52a)

we find a Breit-Wigner behaviour for the scattering amplitude:

\[
f(s) = \frac{\gamma(s)}{s - s_0 - i\alpha\gamma(s)}
\]

(52b)

*) To prove the first result one conveniently uses \( \text{Im} f(s) = q[f(s)]^2 \) which follows from the reality of \( \delta(s) \) in Eq. (42).
Now the trivial relation
\[
\frac{M(s)}{f(s)} = \frac{\gamma_{\text{irr}}(s)}{\gamma_{\text{irr}}(s)}
\]
leads to
\[
M(s) = \frac{(s-s_0) \gamma_{\text{irr}}(s)}{\gamma(s)} f(s) .
\]

This formula provides us with a necessary and sufficient condition for the validity of Eq. (47): \( M_{\text{irr}}(s) \) must have a pole at the same position \( s_0 \):
\[
M_{\text{irr}}(s) = \frac{\Gamma(s)}{s-s_0} , \quad \Gamma(s): \text{slowly varying around } s_0 .
\]

\( M^{1/2} \) evidently exhibits such a behaviour while \( E^{1/2} \) does not.

This last statement has to be modified somewhat for the semi-phenomenological "isobar model" proposed by Gourdin and Salin \(^{49} \). The authors treat the unstable resonant states \( N \) with the same Feynman rules as stable particles only replacing in the propagators
\[
\frac{1}{M^2-s} \text{ by } \frac{1}{M^2-s-i\Gamma M}
\]
with \( \Gamma = \text{width} \) of the resonance.

In addition, one has to introduce a variety of coupling constants: for the \( \gamma NN^* \) and \( NN^* \pi \) vertices. Counting helicity states \(^*) one finds for the \( \gamma \)-coupling two parameters and for the pion-coupling one parameter independent of the spin of \( N^* \) (assuming, of course, spin \( 1/2 \) for \( N^0 \)). For the spin \( 1/2 \) resonances the Rarita-Schwinger formalism \(^{51} \) has been used.

Thereby introducing the following interaction Hamiltonians
\[
H_{\pi NN^*} = \frac{\lambda_1}{m_\pi} \bar{\psi} \gamma^\mu \psi A^\mu + h.c.
\]
\[
H_{NN^*} = e \frac{C_1}{m_\pi} \bar{\psi} \gamma_5 \gamma_5 \psi A^\mu + e \frac{C_2}{m_\pi} \bar{\psi} \gamma_\mu \gamma_5 \psi \partial^\nu A^\nu + h.c. ,
\]
where
\[
\psi = \text{nucleon spin or operator,}
\phi = \pi\text{-meson operator,}
A^\mu = \text{electromagnetic potential.}
\]

The \( \frac{3}{2}^- \) particle has been described by four Dirac spinors \( \psi_\mu (\mu = 0, \ldots, 3) \) obeying the auxiliary conditions
\[
\gamma^\mu \psi_\mu = \partial^\mu \psi_\mu = 0 ,
\]
thus reducing the arbitrary components to four. The authors \(^{51} \) obtain:

\(^{*) \text{This can be done as in Section V using the Breit system.}\)
\[ \frac{f_{1+}^{3/2}}{q^2} = \frac{1}{12\pi} \left( \frac{m_\pi}{1} \right)^2 \left( E_1 + M \right) \frac{1}{M^{3/2} - s - i\Gamma_\gamma M} \]  

(57)

\[ \frac{M_{1+}^{3/2}}{q_k} = \frac{\mu_{1+}^{3/2}}{M^{3/2} - s - i\Gamma_\gamma M} \]  

(58)

with

\[ \mu_1 = \frac{-1}{24\pi} \frac{\sigma_{\lambda_1}}{m_\pi} \sqrt{\frac{E_2 + M}{E_1 + M}} \left[ C_1 + \frac{E_4 + M}{m_\pi} C_2 \right] \]  

(58a)

\[ \epsilon_{1+}^* = \frac{-1}{24\pi} \frac{\sigma_{\lambda_1}}{m_\pi} \sqrt{\frac{E_2 + M}{E_1 + M}} \left[ -C_1 + \frac{E_4 + M}{m_\pi} C_2 \right]. \]  

(58b)

From a theoretical point of view the great number of parameters is a bad feature of the model, especially if one varies also the width \( \Gamma_\gamma \) as the authors do. On the other hand, the simple formulae of the model are convenient for practical calculation and can be easily generalized for several isobars.

Taking the first three isobars into account, in addition to the Born approximation, Salin found an over-all fit to the existing data (Fig. 11a and b). The best fit parameters lead to

\[ \frac{E_{1+}^{3/2}}{M_{1+}^{3/2}} = -0.045 \]

in a certain contradiction to Eq. (50).

As has been stressed by Höhler\(^{32}\) the model contains an over-simplification which we like to mention because of its general significance. In the formulae used, the isobar makes contributions only to the resonant amplitudes \( M_{1+} \), resp. \( E_{1+} \). But we must expect an influence on non-resonant multipole amplitudes for two reasons:

a) The isobar in the \( s \)-channel is in general a virtual off-shell particle and such particles with spin \( J > \frac{1}{2} \) are known to contain also lower spin values. This is due to the failure of the subsidiary condition \((56a)\) for virtual particles\(^{33}\). Relativity also allows changes in parity which is brought about by the small components of the Dirac spinors;

b) The isobars can also occur in the \( u \)-channel.

Both effects are not small as shown by Höhler and co-workers\(^{31}\) using the dispersion relation approach. For an illustration we show in Fig. 12 the influence of the first isobar on the \( E_{1+} \) electric dipole matrix element: \( (E_{1+})_{23} \). In fact, also Gourdin and Salin need such an \( E_{1+} \) matrix element which they introduce by a so-called subtraction term whereby another parameter enters\(^{31}\). In their discussion of the \( \pi^+ \) photoproduction these authors have of course also taken account of the influence of one-pion exchange. This process will be discussed in the next section.

\(^{*)}\) For a more complete discussion, cp. the seminar given by Professor Höhler.
VII. THE ONE-PION EXCHANGE, GAUGE INVARIANCE AND THE COMPLETE BORN APPROXIMATION

Turning now to the t-channel we have at first to deal with the one-pion intermediate state. The connected one-pion exchange processes have been studied extensively for inelastic hadronic processes. For photoreactions a new problem arises in this connection which is due to the gauge invariance of the photoproduction amplitude.

Because of the continuity equation for the electric current $\partial^\mu j_\mu = 0$ the matrix element of Eq. (1) must obey the condition

$$K^\mu < \ldots | j_\mu | \ldots \rangle = 0 \ .$$  \hspace{1cm} (59)

For a real photon we can replace $A_\mu$ in Eq. (1) by the four-vector $\epsilon_\mu$ of the photon polarization so that each photoproduction amplitude can be written in the form

$$T = \epsilon^\mu < \ldots | j_\mu | \ldots \rangle \ .$$  \hspace{1cm} (60)

Incidentally this is true exactly to each order in $\alpha$. Because of Eq. (59) we have the rule: by replacing $\epsilon^\mu$ in the photoproduction amplitude by the photon four-momentum $K^\mu$ one must get zero.

Let us now go back to the one-pion exchange diagram. The application of Feynman rules gives the following result (we have omitted the nucleon spinors for simplicity)

![One-pion exchange diagram](image)

$$J^{(-)}_\alpha = 2 \epsilon^\mu Q_\mu \frac{1}{E - t} g \cdot \gamma_5 \ ,$$  \hspace{1cm} (61)

where $g^2/4\pi = 15$ is the usual pseudoscalar coupling constant and the isospin factor

$$J^{(-)}_\alpha = (-1) \epsilon_{\alpha\beta\gamma} \tau_\beta = \frac{1}{2} \{ \tau_\alpha, \gamma_5 \}$$  \hspace{1cm} (62a)

describes the fact that only charged pions can be exchanged. Replacing $\epsilon^\mu$ in Eq. (61) by $K^\mu$ one gets

$$J^{(-)}_\alpha = g\gamma_5 \ .$$  \hspace{1cm} (61')

Thus gauge invariance is violated in the one-pion exchange approximation. This result seems plausible if one recalls that in this process a proton emits a $\pi^+$ changing into a neutron. We have to take account of the current of the proton also to get a result which obeys charge conservation. Therefore we write down the relevant contributions of the nucleon pole diagram:
The electric $\gamma N$ coupling is usually described by $e\gamma_\mu$ which contains an orbital current contribution $e F_\mu / M$ and a current due to the normal magnetic moment $^\ast$. For our purposes we need only the first so that we are led to

$$
\gamma a \gamma S \frac{1}{M^2 - s} e \frac{1 + \tau_3}{2} 2 e^\mu P_{1 \mu} + e \frac{1 + \tau_3}{2} 2 e^\mu P_{2 \mu} \frac{1}{M^2 - u} \gamma a \gamma S =
$$

$$
= e \gamma S \left( \left( J_a^{(o)} + J_a^{(+)} \right) \left[ \frac{e^\mu P_{1 \mu}}{M^2 - s} + \frac{e^\mu P_{2 \mu}}{M^2 - u} \right] + J_a^{(-)} \left[ \frac{e^\mu P_{1 \mu}}{M^2 - s} - \frac{e^\mu P_{2 \mu}}{M^2 - u} \right] \right),
$$

(63)

where in addition to Eq. (62a) the isospin quantities

$$
J_a^{(o)} = \tau_a, \quad J_a^{(+)} = \frac{1}{2}(\tau_a \tau_3 + \tau_3 \tau_a) = S_a
$$

(62b)

have been introduced which are different from zero also for the neutral pions. In accordance with this fact the factor of $J_a^{(o)}$ in Eq. (63) vanishes if $e^\mu$ is replaced by $k^\mu$, while the factor of $J_a^{(-)}$ just leads to a result which compensates Eq. (61'). [Looking for the origin of the isospin factors in Eq. (63) we observe that $J_a^{(o)}$ is due to first term in the electric charge $e 1 + \tau_3/2$ of the proton and thus describes the isoscalar part while $J_a^{(\pm)}$ due to $\tau_3$ has an isovector character.] These gauge properties can be expressed most conveniently with the help of the "invariant"

$$
M_2 = 2 i \gamma S F^{\mu \nu} P_\mu Q_\nu = 2 i \gamma S \left( P \cdot e Q \cdot K \cdot P \cdot K Q \cdot e \right),
$$

(64)

where

$$
P = \frac{1}{2}(P_+ + P_-) \quad \text{and} \quad F^{\mu \nu} = e^{\mu} k^\nu - e^{\nu} k^\mu
$$

denote the antisymmetric tensor of the electric and magnetic field strength for a plane wave photon. Therefore $M_2$ is an evident gauge invariant quantity. By an elementary calculation (61) and (63) can be found to be proportional to $M_2$:

$$
eg \left[ \left( J_a^{(o)} + J_a^{(+)} \right) \frac{1}{(M^2 - s)(M^2 - u)} + J_a^{(-)} \frac{u - s}{(M^2 - s)(M^2 - u)} \right] M_2.
$$

(63')

This result is contained in Table XII where the complete result of an evaluation of the pion and nucleon pole diagrams can be found. In Fig. 3 we already have given the cross-sections following from this result. I would like now to interrupt the formal theory and discuss a very practical application of these considerations. Several years ago S. Drell proposed the diagram of Fig. 13 as a source of a high intense pion beam produced parallel to the incoming photon $^\ast$. Evidently this one-pion exchange diagram again violates gauge invariance.
Table XII

Pole-terms = Born approximation for pion photoproduction
\( q_{\mu \nu} = e^\mu k^\nu - e^\nu k^\mu; \ P = \frac{1}{2}(P_1 + P_2); \ k_s = 0.065; \ k_v = 1.845 \)

<table>
<thead>
<tr>
<th>Normal magnetic moment contribution</th>
<th>Electric orbital current</th>
<th>Anomalous magnetic moment contributions</th>
</tr>
</thead>
<tbody>
<tr>
<td>( M_1 = i \gamma_5 \hat{g} k = )</td>
<td>( M_2 = 2 i \gamma_5 (P \cdot \epsilon \cdot q \cdot k - \bar{k} \cdot P \cdot q \cdot \epsilon) = )</td>
<td>( M_3 = \gamma_5 \hat{k} \bar{P} \cdot \epsilon )</td>
</tr>
<tr>
<td>( = \frac{1}{2} \gamma_5 \hat{g} \mu \nu \gamma_{\mu} \gamma_{\nu} )</td>
<td>( = 2 i \gamma_5 \hat{g} \mu \nu \mu \nu q_{\mu} )</td>
<td>( = 2 \gamma_5 \hat{g} \mu \nu \gamma_{\mu} q_{\nu} )</td>
</tr>
<tr>
<td>( M_4 = \gamma_5 \hat{k} \bar{P} \cdot \epsilon - i M_S \hat{k} )</td>
<td>( = 2 \gamma_5 \hat{g} \mu \nu \gamma_{\mu} \mu \nu P - 2 N M )</td>
<td></td>
</tr>
</tbody>
</table>

\( J_a^{(+)} = \gamma_a \) | \( \frac{-e g}{2} \left[ \frac{1}{M^2 - s} + \frac{1}{M^2 - u} \right] \) | \( \frac{\kappa_s e g}{2M} \left[ \frac{1}{M^2 - s} - \frac{1}{M^2 - u} \right] \)

\( J_a^{(-)} = \bar{\gamma}_a \) | \( \frac{-e g}{2} \left[ \frac{1}{M^2 - s} - \frac{1}{M^2 - u} \right] \) | \( \frac{\kappa_v e g}{2M} \left[ \frac{1}{M^2 - s} - \frac{1}{M^2 - u} \right] \)

\( J_a^{(-)} = \frac{1}{2} [\tau_a \tau_3] \) | \( \frac{-e g}{2} \left[ \frac{1}{M^2 - s} - \frac{1}{M^2 - u} \right] \) | \( \frac{\kappa_v e g}{2M} \left[ \frac{1}{M^2 - s} - \frac{1}{M^2 - u} \right] \)
Stichel and Scholz have investigated a model where this lack can be remedied—quite analogously to the way described above. They restricted themselves to a production of a $\Delta(1235)$ isobar on the nucleon bubble (see Fig. 14) which they again describe by the Harita-Schwinger formalism [cp. Eqs. (55) and (56)]. Due to the derivative coupling in Eq. (55) they have to calculate four diagrams (see Fig. 15) in order to arrive at gauge invariance. The coupling of the new "catastrophic" diagram (III) follows from Eq. (55) by replacing $\phi^\mu$ by $ieA^\mu$. The evaluation for high energies ($s \to \infty$) gives a remarkably large change relative to the simple Drell diagram (Fig. 16).

VIII. DETAILED FORMULATION OF THE ANALYTICITY PROPERTIES

The decomposition for the nucleon and pion pole terms of Table XII which just give the renormalized Born approximation is very convenient also in the general case. It can therefore be proven that a photoproduction amplitude can in general be written as

$$T = \sum_{i=1}^{N} \left[ A_i^{(s)}(s,t) J^{(s)}_\alpha + A_i^{(+)}(s,t) J^{(+)}_\alpha + A_i^{(-)}(s,t) J^{(-)}_\alpha \right] \phi_{\alpha i}.$$  \hspace{1cm} (65)

Accordingly we have for a specified charge mode four independent amplitudes in agreement with the existence of four helicity amplitudes. The connection of $H^\mu, \phi^\mu$ with the $A_1$ - s is somewhat involved. One normally uses in an intermediate step the functions $F_i$ and $F_i^\mu$ defined in Table XIII. With their help the helicity amplitudes can be calculated according to Table XIV.

People trained in analyticity properties will suspect that the functions $A_i(s,t,u)$ obey a Mandelstam representation in the variables $s$, $t$ and $u$. Unfortunately, gauge invariance again lead to complications. Of course, we can only guess the validity of this representation. But perturbation theory gives a good tool to guess the correct answer. Looking at Table XII one observes that $A_1^{(s,t,u)}$, $A_2^{(s,t,u)}$, $A_2^{(t,u)}$ have the expected simple pole behaviour, but $A_2^{(s,t)}$ looks different. Indeed it was in the $A_2$ amplitudes that gauge troubles occur. But a more detailed investigation by Ball has shown that the usual Mandelstam relation

$$A_i(s,t,u) = \text{pole terms} + \frac{1}{\pi} \int ds' \frac{\rho_s(s')}{s'-s} + \frac{1}{\pi} \int du' \frac{\rho_u(u')}{u'-u} + \frac{1}{\pi} \int dt' \frac{\rho_t(t')}{t'-t} +$$

$$+ \frac{1}{\pi^2} \int ds' dt' \frac{\rho_{1s}(s',t')}{(s'-s)(t'-t)} + \frac{1}{\pi^2} \int ds' du' \frac{\rho_{1u}(s',u')}{(s'-s)(u'-u)} + \frac{1}{\pi^2} \int dt' du' \frac{\rho_{1t}(t',u')}{(t'-t)(u'-u)} ,$$

should be conjectured for $i = 1, 3, 4$ with pole terms given by Table XII. Each term in Eq. (66) also carries the isospin indices $(z,0)$. On the other hand $A_2^{(s,t,u)}$ should obey a similar relation with pole terms having only the more complicated form of Table XII and the single dispersion integrals should be dropped. As usual one has to add crossing relations the validity of which can be read off from Table XII for the pole terms:
\[ F_1 = A_1 + (W - M) A_4 - \frac{t - 1}{2(W - M)} (A_3 - A_4) = \frac{8 \pi W}{W - M} \left( \frac{E_4 + M}{E_4 + M} \right)^{1/2} \left( \frac{F_1}{q} \right) \]

\[ F_2 = -A_1 + (W + M) A_4 - \frac{t - 1}{2(W + M)} (A_3 - A_4) = \frac{8 \pi W}{W + M} \left( \frac{E_4 + M}{E_4 + M} \right)^{1/2} \left( \frac{F_2}{q} \right) \]

\[ F_3 = (W - M) A_3 + A_3 - A_4 = \frac{8 \pi W}{W - M} \left( \frac{E_4 + M}{E_4 + M} \right)^{1/2} \left( \frac{F_3}{q} \right) \]

\[ F_4 = -(W + M) A_3 + A_3 - A_4 = \frac{8 \pi W}{W + M} \left( \frac{E_4 + M}{E_4 + M} \right)^{1/2} \left( \frac{F_4}{q} \right) \]

Table XIII

The intermediate functions \( F_1 \) resp. \( F_1 \) in terms of \( A \):

(the isospin indices \((0, \pm)\) must be added to each symbol)

\[ H^- = -\frac{1}{\sqrt{2}} \sin \theta \cos \frac{\omega}{2} (F_3 + F_4) \]

\[ H^+ = -\sqrt{2} \sin \frac{\omega}{2} (F_3 + F_4) + H^- \]

\[ \Phi^+ = \frac{1}{\sqrt{2}} \sin \theta \sin \frac{\omega}{2} (F_3 - F_4) \]

\[ \Phi^- = -\sqrt{2} \cos \frac{\omega}{2} (F_3 - F_4) + \Phi^+ \]

Table XIV

The connection between the \( F_1 \) functions and the helicity amplitudes

These formulae differ somewhat from those given in Ref. 15 because of a slight difference in the polarization vectors \( \vec{e}^+ \) and \( \vec{e}^- \). We use \( \vec{e}^+ = -\lambda/\sqrt{2} (\vec{e}_1^+ + \lambda \vec{e}_2) \)

\( (\lambda = \pm 1) \) where \( \vec{e}_2 \) lies in the plane defined by \( \vec{k} \) and \( \vec{q} \).

\[ A_1^{(\pm, \sigma)}(s, t, u) = \pm A_1^{(\pm, \sigma)}(u, t, s) \quad (67) \]

where the upper sign holds for: \(0, + \) and \( i = 1, 3, 4 \) resp. - and \( i = 2 \) and the lower sign holds for: \(0, + \) and \( i = 2 \) resp. - and \( i = 1, 3, 4 \). A general proof of Eq. (67) can be given with the help of the general crossing relations and the odd behaviour of \( j_\mu \) under \( C \) conjugation. By well-known methods one deduces from Eq. (66) one-dimensional dispersion relations. For example, one has for fixed \( t \):
\[ A^{(\pi^0)}_1(s,t) = \text{pole terms} + \frac{1}{\pi} \int_0^\infty ds' \text{Im} A^{(\pi^0)}_1(s',t) \left[ \frac{1}{s' - s} + \frac{1}{s' - u} \right], \quad (68) \]

where \( u = 2 M^2 + m^2 - s - t \).

A recent evaluation of these relations has been performed by W. Schmidt\(^{29}\) for energy region around the first isobar. Here one normally (following Chew et al.\(^{46}\)) approximates the imaginary part in Eq. (68) by the contribution of the magnetic dipole matrix element \( M^{\frac{3}{2}}_{\pi^+} \). This leads with help of Tables VII, XIII and XIV to

\[ \text{Im} A^{+}_1 = C^+(s) \text{Im} \frac{M^{\frac{3}{2}}_{\pi^+}}{s} f_4(s,t); \quad \text{Im} A^{-}_1 = 0 \quad (69) \]

with

\[ f_1 = \frac{3}{2} \text{Re}(W) - \frac{1}{W + M}; \quad f_2 = -3 \]

\[ f_3 = \frac{3}{2} (t - 1) \frac{1}{W + M} + \frac{1}{W - M}; \quad f_4 = \frac{3}{2} (t - 1) \frac{1}{W + M} + \frac{1}{W - M} \]

\[ C^+ = 4s \left( \frac{t}{t - 1/\nu} \right)^{\frac{1}{2}} \frac{1}{\nu} \frac{1}{[(W + M)^2 - 1]^{\nu/2}} \quad (69b) \]

For \( M^{\frac{3}{2}}_{\pi^+} \) occurring in Eq. (69) the approximation \( (48) \) was used and experimental phase shifts were taken to calculate \( f^{\frac{3}{2}}_{\pi^+}(s) \). The agreement with recent experimental results\(^{59}\) on \( \pi^+ \) production is fairly good especially for large angles (see Fig. 17a and b). On the other hand, the Goudin-Salin model\(^{49}\) gives a somewhat better fit to the same data\(^{60}\). But one must keep in mind that the formulae \( (68) \) and \( (69) \) are different from the isobar model and do not contain any free parameter.

We should remark finally that improvement of the relation \( (48) \) has been discussed with dispersion relations\(^{64}\) but the gained numerical results have been criticized\(^{52}\). In this treatment the possible influence of the exchange of a \( \rho \) meson (the \( \omega \) does not contribute in \( \pi^+ \) production) has been neglected. This is in accord with \( A \)-parity arguments of Section IV. We discuss this problem in some more detail in the next section.

**IX. THE ELECTROMAGNETIC COUPLING OF VECTOR MESONS**

At the last High-Energy Conference at Dubna, Professor Baldin called the determination of the coupling strength between the photon, the pion, and the vector mesons (\( \rho, \omega \)) "a problem of the day"\(^{62}\). Yet the results so far reported are rather conflicting. This section reviews the evidence for the magnitude of the coupling constants \( g_{\gamma \rho \rho} \) and \( g_{\gamma \omega \pi} \). We start with precise definition: the coupling between the photon, the pseudoscalar pion, and a vector particle can be written in a unique and gauge invariant way\(^{63}\):

*) In preparing this section the author has made use of the material presented by H. Joos at DESY in December 1964.
where $k^\rho, \delta^\sigma$ are the four momenta of the photon and the vector meson, respectively. The vector particle carries the index "$\mu$". In terms of this constant the decay width of $\rho(\omega)$ into $\pi + \gamma$ is found to be

$$\Gamma_{\rho(\omega) \rightarrow \pi + \gamma} = \frac{1}{24} \frac{g_{\rho \pi \pi}^2 (\omega)}{4\pi} m_\rho \left(1 - \frac{m_\rho}{m_\pi}\right)^3.$$  \hspace{1cm} (71)

Because of differences in conventional factors the best way to report values of the wanted quantity is by this width.

To calculate the influence of the exchange of a vector particle on the single pion production we need in addition the nucleon-$\rho(\omega)$ coupling which contains a Dirac and a Pauli-like term:

$$\left( g_{\rho \pi} \gamma_{\mu} + g_{\rho \pi} \frac{1}{2M} \sigma_{\mu \nu} \delta^\nu \right).$$  \hspace{1cm} (72)

Using the well-known propagator of a vector particle

$$\frac{1}{m_\rho^2 - t} \left( g_{\mu \nu} - \frac{\delta_{\mu \nu}}{m_\rho^2} \right),$$

one finds for the $\rho$-exchange diagram

$$\frac{g_{\rho \pi \pi}}{m_\pi} \frac{1}{m_\rho^2 - t} \left[ g_{\rho \pi \pi} m_\pi - \frac{g_{\rho \pi \pi}^2}{2M} (tM_\pi + M_\pi) \right] J^{(o)}_\alpha.$$  \hspace{1cm} (73)

Here we have used the invariants of Table XII. The occurrence of the isospin factor $J^{(o)}_\alpha$ expresses the fact that the $\rho$ exchange only contributes to the isoscalar current (cp. Section III). For an $\omega$ exchange one has merely to change the index $\rho$ into $\omega$ and to replace $J^{(o)}_\alpha$ by $J^{(+)}_\alpha = \delta_{\alpha 3}$. (Because of $I_\omega = 0$ no dependence on $\tau$ matrices arises.) In this treatment we have regarded the vector particles as stable particles. This can be done better with the aid of the Mandelstam representation. Professor Hühler will discuss this subject in his seminar. The major change which comes about consists of an extra constant term to $A^{(o)}_\alpha$ [the factor of $M_\pi$ in Eq. (73)]. Moreover a connection between the bracket in Eq. (73) and the nucleon form factor is established. The physical basis of this connection can also be expressed in a simple model. By introducing a direct $\gamma - \rho$ coupling
one gets a contribution to the isovector electromagnetic form factors through the diagram

\[ \frac{e}{2 \gamma p} \frac{m_{\rho}^2}{g_{p,1}} \epsilon_{\mu} \epsilon_{\nu}(p) \cdot \epsilon_{\rho} \]  

\[ \text{(74)} \]

where we have dropped the nucleon spinors. Originally one was inclined to identify Eq. (75) directly with the isovector form factors thus being led to

\[ \frac{g_{p,2}}{g_{p,1}} = (K' - K') = 3.7 \]  

\[ \text{(76a)} \]

If, on the other hand—one makes use of a two-pole fit to the form factors and identifies Eq. (75) only with the \(\rho\) pole, recent results give\(^{65}\)

\[ \frac{g_{p,2}}{g_{p,1}} \approx 3 \]  

\[ \text{(76b)} \]

These values agree with the result from an analysis of nucleon-nucleon scattering\(^{66}\)

\[ g_{p,1} = 3.26; \quad g_{p,2} = 12.1; \quad \frac{g_{p,2}}{g_{p,1}} = 3.7 \]  

\[ \text{(77)} \]

Relying on this ratio, the contribution of \(\rho\) exchange to photoproduction contains only one open parameter

\[ \Lambda = \frac{g_{\pi\rho} g_{\rho\pi}}{8 \pi m_{\pi}} \]  

which could be extracted from the experimental results on \(\pi^+\) production if all other contributions are known. Unfortunately this supposition is not fulfilled. But the different authors\(^{67,68,69}\) agree that \(\Lambda\) is small. To give an order of magnitude we quote the result by A.I. Lebedev\(^{67}\)*

\[ \Gamma_{\rho \to \pi Y} < 0.1 \text{ MeV} \]  

\[ \text{(78)} \]

*) Knowing also the absolute value of the \(\rho\) N coupling [see Eq. (77)] one can calculate the radiative width of the \(\rho\) from \(\Lambda\).
This upper bound contradicts strongly with a recent result from the investigation of the process
\[ \gamma + p \rightarrow \rho^0 + p \text{ at } 2 \text{ GeV}. \]

Using the measured cross-section (13.2 \( \mu \)barn) one finds with the one-pion exchange model where in addition to \( \Gamma_{\rho \rightarrow \pi \gamma} \) only the well-known \( \pi N \) coupling enters
\[ \Gamma_{\rho \rightarrow \pi \gamma} = 1.65 \text{ MeV}. \] (79)

This value is somewhat larger than
\[ \Gamma_{\rho \rightarrow \pi \gamma} = 0.5 \text{ MeV}, \]
deducted from MacLeod et al.\(^{69}\) from the two-pion production at lower energies (1 GeV).

Turning now to the \( \omega \) coupling we expect from the A-parity argument given in Section III a larger value of \( \xi_{\pi \omega} \). In fact for high energies\(^{39}\) neutral pions are produced much more strongly than charged ones:
\[ d\sigma^\pi \approx (5-10) d\sigma^\omega \text{ for about } 2 \text{ GeV}. \]

Assuming that vector-meson exchange is responsible for this difference one has indeed
\[ \xi_{\gamma \rho} \ll \xi_{\pi \omega}. \]

It is clear from this short survey that much more information is wanted. For high energies one needs more detailed experimental results. For low energies the theoretical description of the "other" contribution should be improved\(^{70}\).
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Fig. 1

Physical Region for $y + N \rightarrow \pi + N$

Kinematics for photoproduction of pions in the $s$-$t$ plane

Fig. 2
a) $\gamma + p \rightarrow \pi^0 + p$

$$
\frac{d\sigma}{dn} = \left(ef \frac{M}{W}\right)^2 \frac{1}{E_2 + q \cos \Theta} \frac{q}{kW} \left\{(1 + g_p')^2 (\omega - q \cos \Theta)^2 + \frac{q^2}{2} \sin^2 \Theta \left[g_p'^2 \left(\frac{W}{M}\right)^2 - \frac{W}{k^2 (E_2 + q \cos \Theta)}\right]\right\}.
$$

b) $\gamma + p \rightarrow \pi^+ + n$

$$
\frac{d\sigma}{dn} = 2 \left(ef \frac{M}{W}\right)^2 \frac{q}{k} \left\{1 - \frac{q^2}{2k^2} \frac{\sin^2 \Theta}{(\omega - q \cos \Theta)^2} - (g_p + g_n) \frac{\omega - q \cos \Theta}{W} + \frac{1}{4W(E_2 + q \cos \Theta)} \left[(g_p' + g_n)^2 (\omega - q \cos \Theta)^2 + (g_p'^2 + g_n'^2) \frac{q^2 M^2}{W^2} \sin^2 \Theta\right]\right\}.
$$

The renormalized Born approximation for the pion photoproduction

$$
g_p' = g_p - 1 = 1.79; \quad g_n = -1.91 \quad (h = c = m_p = 1)
$$

Fig. 3
Fig. 7a

Fig. 7b
\[ \theta_{CM} = 90^\circ \]

Fig. 8

\[ \gamma + p \rightarrow \omega + p \]

Fig. 9
Photoproduction of $\pi^0$ up to 4 GeV

Fig. 10
Fig. 11
Fig. 12

Fig. 13

Fig. 14
Fig. 15

Fig. 16
Fig. 17a

Fig. 17b
SPECIAL MODELS AND PREDICTIONS FOR PION
PHOTOPRODUCTION (LOW ENERGIES)

G. Höhler,
Institut für Theoretische Kernphysik der
Technischen Hochschule, Karlsruhe.

I. INTRODUCTION

The theoretical investigations on pion photoproduction can be classified into two
groups: the phenomenological analysis and the attempts to treat the dynamics of the
\( \pi N \) system.

In the phenomenological analysis\(^1\) only the general theoretical principles are
used, namely

a) Lorentz and gauge invariance (including space and time reflection);

b) the principle of minimal electromagnetic interaction, which states that the electro-
    magnetic interaction has to be introduced by\(^{\text{\( \Delta \)}}\)
    \( p_{\mu} \rightarrow p_{\mu} - eA_{\mu} \). It leads to a relation
    between the photoproduction processes in different charge states;

c) the unitarity condition. Together with the time reversal invariance it allows one to
deduce the 'final state theorem', according to which the phase of a multipole amplitude
is equal to the scattering phase shift in the final state, if the energy is below the
inelastic threshold

\[
M_{f^\pm} = \pm |M_{f^\pm}| e^{i\delta_{f^\pm}}.
\]  

In (b) and (c) the electromagnetic field is treated only to lowest order.

The aim of the phenomenological analysis (or multipole analysis) is analogous to
that of the phase-shift analysis in \( \pi N \) scattering. One tries to determine the multipole
amplitudes from the experimental data, since these amplitudes are much better suited for a
comparison with the predictions of a dynamical theory than the cross-sections.

A dynamical theory which allows one to calculate the photoproduction amplitudes from
first principles does not yet exist. In recent years all attempts to predict the amplitude
were based on the dispersion relation approach\(^2\), which has a more modest aim. One tries
to calculate the photoproduction amplitudes from some consequences of the axioms of field
theory\(^3\) together with the experimental information from other reactions, as for instance
\( \pi N \) and ep scattering.

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Editor's Note: Professor Höhler presented this paper both at the CERN School at Bad Kreuznach
and at the International Symposium on Electron and Photon Interactions held at Hamburg in
June 1965. When the words "this Conference" occur in the text of the paper it is the
latter Meeting to which reference is made. Thanks are due to the Springer-Verlag for
permission to reprint this article.
At present the dispersion relation approach is not a systematic theory. In order to obtain a prediction one has to make drastic approximations, which were found using the static model of Chew and Low as a guide. There is no reliable way to estimate the errors.

Omitting all indices the dispersion relation at fixed $t$ for the production amplitude reads

$$\text{Re} \ A(s,t) = A_{\text{pole}}(s,t) + \frac{1}{\pi} \int_{\tilde{s}}^{s} \frac{ds'}{s'-s} \text{Im} \ A(s',t) \left[ \frac{1}{s'-s} + \frac{1}{s'-\tilde{s}} \right]$$

where $s = \tilde{W}^{2}$, $\tilde{W}$ = total energy in the c.m. system, $t$ = invariant momentum transfer squared, $\tilde{s} = -s - t + 2M^{2} + 1$, $m_{\pi} = 1$, $M$ = mass of the nucleon.

The pole term follows from the one-nucleon intermediate states. It is also called the 'Born term', since it happens to agree with the amplitude calculated from the Feynman graphs of Fig. 1, if the pseudoscalar $\pi N$ coupling is used ($f^{2} = e^{2}/4\pi^{2} = 0.081$) and the electromagnetic coupling of the anomalous moment $\mu'$ of the nucleon is taken into account explicitly.

The most important contribution to $\text{Im} \ A(s',t)$ in the dispersion integral is expected to belong to the isobar intermediate state

$$\pi + N \rightarrow \Delta \rightarrow \pi + N,$$

where $\Delta$ denotes the $\Delta$-resonance at the total c.m. energy $\tilde{W} = 1236$ MeV. This transition can proceed via the magnetic dipole amplitude $M_{33}$ and the electric quadrupole amplitude $E_{33}$ only. If all the other contributions to $\text{Im} \ A$ are neglected and $\text{Re} \ A$ is calculated from Eq. (2), the result will be called in the following the 'isobar approximation'.

II. THE ISOBAR APPROXIMATION

1. The resonant multipoles

The first problem in all investigations using the isobar approximation is to find an expression for the energy dependence of the resonant multipoles $M_{33}$ and $E_{33}$.

CGLM derived approximate 'dispersion relations' for the resonant partial waves by a projection of the fixed- $t$ dispersion relations, neglecting the non-33 contributions to the dispersion integral and considering the static limit and some recoil corrections. They found that the 'dispersion relations' for the resonant $\pi N$ partial wave $f_{33}$ and the resonant multipole $M_{33}$ agree, if the formula

$$M_{33} = \frac{u_{\gamma}}{T} \frac{k}{q} f_{33}; \quad f_{33} = e^{i\alpha_{33}} \sin \theta_{33}$$

is assumed to hold and only a certain part of the Born term of $M_{33}$ is taken into account,
namely the static limit of the anomalous magnetic part plus the recoil correction of order \(1/M\)
to the electric part. In Eq. (3) \(\mu_V = (\mu_p - \mu_n)/2\) denotes the isovector part of the total magnetic moment of the nucleon, \(k\) the photon momentum, and \(q\) the pion momentum in the c.m. system\(^*\).

CGLN also gave an estimation of the remaining part of \(M_{33}\) and of \(E_{33}\), assuming that these multipole amplitudes have a similar behaviour as in the static model. The contribution of these terms is small in comparison with Eq. (3).

Several authors have tried to improve the result of CGLN. Recently Finkler\(^7\) used the Omnès method, assuming that \(f_{33}\), \(M_{33}\), and \(E_{33}\) have the same real phase at all energies.

His further assumption that all non-33 contributions to the dispersion integrals can be neglected has to be discussed critically, since Donnachie and Hamilton\(^8\) had found an appreciable \(T = 0\) \(\pi\pi\) contribution in their investigation of the resonant \(f_{33}\) amplitude.

Finkler corrected Eq. (3) by a factor on the right-hand side, which is about 0.9 at the resonance and decreases at higher energies, similar to Mckinley's result\(^9\). Furthermore, he obtained a large correction to the CGLN estimate of the \(E_{33}/M_{33}\) ratio, which is of special importance for the \(\pi^0\) production with polarized \(\gamma\) rays\(^10\). A detailed discussion has not yet been made, but it seems that the discrepancy found in earlier discussions will be reduced considerably by Finkler's treatment.

2. The work of Ball and Schmidt

Ball\(^11\) approximated \(\text{Im} A\) by the contribution of \(\text{Im} m_{33}\) alone, taking \(M_{33}\) from Eq. (3). Then he evaluated the dispersion integrals and calculated the cross-sections without further approximations. Unfortunately his numerical results are not reliable, because there is an error in his Eq. (8.29), and also in the \(D\) coefficient for \(\pi^0\) production.

Ball's work was continued by Schmidt\(^12-14\), who calculated predictions for all measured quantities up to 500 MeV, except for the polarization of the recoil proton in \(\pi^0\) production, which was treated by Müllensiefen\(^15\).

Since \(f^2\) and \(c_{33}\) were taken from the scattering data, the prediction is an absolute one, it contains no adjustable parameters.

A comparison with the experimental data shows that the Ball-Schmidt calculation leads to a reasonable zero order approximation for the photoproduction cross-sections, including the measurements with polarized \(\gamma\) rays and the \(\pi^-/\pi^+\) ratios. However, one should notice that the agreement is mainly due to the fact that the cross-sections are dominated by the pole terms and the resonance effects. From the experience with the isobar approximation \(\text{in } \Pi N\) scattering\(^**\)\(^16\) one would expect that the Ball-Schmidt result could be quite wrong for some of the small multipoles, especially for those leading to \(T = \frac{1}{2}\) final states.

Figure 2 shows a contour diagram of the difference between Schmidt's prediction for \(\pi^+\) production and the experimental data, interpolated by a Moravskik fit. The deviation amounts to 15% at its maximum near 280 MeV. In the region of small angles the extrapolation of the experimental data is only a crude estimate, and the magnitudes of the deviations will be much better known after the completion of the new measurements at Orsay and Bonn.

\(^*\) Other derivations of Eq. (3) are discussed in part 9.3 of Ref. 1, and part 2.2,3 of Ref. 6.

\(^**\) See Fig. 6 of Ref. 16.
Baldin\textsuperscript{17}) has pointed out that it is interesting to consider the cross-section at a fixed momentum transfer $t$ which is equal to its value at threshold, since in this case the integrand of the dispersion integral (2) is used in the physical region of $(s',t)$ only. In all other cases there is an additional uncertainty following from the extrapolation of $\text{Im} A(s',t)$ into the unphysical region. According to Fig. 3 the agreement between the Ball-Schmidt prediction and the experimental data is good up to 260 MeV. It is interesting to notice that in this region the data can be as well described by the cross-section calculated from the pole term alone (= second order Born approximation). At higher energies there is an increasing and rather large deviation (cf. the curve $t = -0.87$ in Fig. 2) which must be due to an error of $\text{Im} A$ in the physical region.

The error caused by the extrapolation into the unphysical region should be especially large for the excitation curve at $180^\circ$; however, Fig. 2 shows that in this case the Ball-Schmidt prediction agrees very well with the experiments.

Although the recent $\pi^+$ production data at $90^\circ$ near threshold are very accurate, they do not allow one to test the dispersion integral contribution, since the Born term is so much larger (Fig. 4). Unfortunately, one cannot use these data for an accurate determination of the coupling constant $f^2$, since one expects several other slowly energy-dependent contributions from the dispersion integral, which cannot be estimated in a reliable way. The same difficulty is present, if one wants to test the well-known relation between the Panofsky ratio, the difference of the $\pi N$ S-wave scattering lengths, and the photoproduction data\textsuperscript{18}).

The experimental data on $\pi^+$ production with polarized $\gamma$ rays are compared with the prediction in Fig. 5.

The experimental information on $\pi^0$ production is not as good as for $\pi^+$ production. Figures 6, 7, and 8 show that the Ball-Schmidt prediction again is a zero order approximation. However, it is easier to find large deviations, since the Born term is smaller than for $\pi^+$ production and partly compensated by an indirect effect of the resonance. Therefore, the cross-sections are more sensitive to the 'small' multipoles, for which the isobar approximation is not reliable. For instance, there is a large discrepancy\textsuperscript{13}) in the energy dependence of $B$ and $C$ ($\sigma = A + B \cos \theta + C \cos^2 \theta$) below 300 MeV and in the ratio $\sigma/C$, which follows from the experiments with polarized $\gamma$ rays\textsuperscript{10}).

It is astonishing that the simple Ball-Schmidt calculation describes so many features of pion photoproduction, although it does not contain adjustable parameters. If one discusses the comparison with the experimental data, one should keep in mind that appreciable corrections are expected from several other contributions to the amplitude, but at present they cannot be calculated in a reliable way. The most interesting experiments are those which show deviations from the prediction far outside the errors, since they help to identify those parts of the theory which are in need of improvement.

The work of Schmidt is only the simplest version of the isobar approximation. It should be improved by taking into account the unitarity condition for the $33$-multipoles in a better way. First steps in this direction have already been made by CGLN\textsuperscript{2}), Ball\textsuperscript{11}), and by McKinley\textsuperscript{9}). It will be interesting to see to what extent Finkel's careful treatment\textsuperscript{7}) of the unitarity condition for the resonant multipoles diminishes the discrepancies found by Schmidt.
3. Feynman graphs

Amati and Fubini have pointed out that in the limit of a narrow resonance the isobar approximation of the dispersion integral leads to the same result as the evaluation of the Feynman graphs of Fig. 9. The first term in the integrand of Eq. (2) corresponds to the graph I, and the second term to II.

It is interesting to notice that the $1/(s'-s)$ term in the dispersion integral gives a large contribution to Re $E_{2+}^{(s)}$ and thereby to a $J = \frac{1}{2}$ final state. For the corresponding graph I in Fig. 9 this is somewhat unexpected, since the intermediate isobar state has the spin $\frac{1}{2}$. However, if one starts from the usual interaction Lagrangian one finds an additional term in the interaction Hamiltonian which leads to the $J = \frac{1}{2}$ final state.*

Gourdin and Salin** treated the isobar intermediate states in such a way that the graph I does not contribute to $J = \frac{1}{2}$ final states. It is not clear to me how these calculations can be justified from the general theoretical principles.

The comparison between Schmidt's calculation and the Feynman graph formulae gives the values of the $\gamma NN^*$ coupling constant, the $NN^*$ coupling constant following from a similar treatment of $NN$ scattering. Furthermore, it shows that the description of the finite width by a constant imaginary part of the isobar mass ($NN^* + i\Gamma$) is not sufficient for quantitative purposes. If one wants to use a Breit-Wigner type formula one has to assume a strongly energy-dependent width.

III. CORRECTIONS TO THE ISOBAR APPROXIMATION

1. Final-state corrections to the non-33 multipoles

In the isobar approximation the unitarity condition is not fulfilled for the non-33 multipoles. An estimation of the final-state corrections was given by CGLN, it was improved by taking into account relativistic kinematics in the paper of McKinley, but the theoretical derivation is still more or less doubtful. Also the addition of these corrections to the isobar approximation amplitude does not lead to a better agreement with the experiments.

In my opinion one does not gain much information if the experiments are compared with a prediction which contains many uncertainties. It is better to determine the multipoles by a phenomenological analysis (part 4) and to compare each multipole with the theoretical expression as given, for instance, in the paper of McKinley.

*) I am much indebted to Professor H. Umezawa, Professor A. Visconti, and Dr. G. von Gehlen for discussions on this question. Also, I would like to thank Dr. F. Hadjiconnou for sending me a preprint which treats a closely related aspect of this problem.

**) Compare the interesting discussion of the isobaric model in the review article of Gell-Mann and Watson.
2. The $\rho$-exchange contribution

In the isobar approximation the dispersion integral does not contribute to the isoscalar part $A^0_1$ of the amplitude. But if one considers the fixed-s dispersion relation\textsuperscript{11} or the Cini-Fubini approximation to the Mandelstam representation\textsuperscript{24}, one is led to expect a contribution from the $\rho$-meson exchange in the $t$ channel (Fig. 10a). Ball\textsuperscript{11} succeeded in expressing the result by the isovector part of the electromagnetic nucleon form factor which is assumed to be dominated by the $\rho$-exchange effect (Fig. 10b).

If an empirical fit to the experimental form factors and the new data for the $\rho$ resonance are inserted into Ball's result, one finds for the $\rho$-exchange contribution to the amplitude\textsuperscript{18} (we give $A^0_1$ only)

$$A^0_{1\rho}(s,t) = 4.1 \frac{\Delta}{e} \mu'_V \left[ 0.50 + \frac{0.60 t}{18 - t} \right].$$

For energies in the region of the first resonance or below, $t$ is so small that the first term in the bracket is dominating and a multipole decomposition shows that the main contributions belong to $E_{0+}$ and $M_{1-}$. It will be very difficult to distinguish the $\rho$-exchange parts from other corrections to these multipoles (for instance, from final-state interactions or high-energy contributions to the dispersion integrals) as long as one considers $\pi^+$ or $\pi^0$ production directly. The situation is more favourable if the data are combined in such a way that the isoscalar part is isolated or enhanced ($\pi^-/\pi^+$ ratio). At present there is no convincing evidence for the $\rho$-exchange effect\textsuperscript{18}. Ball's coupling constant $\Delta$ is smaller than 0.5 e, unless the $\rho$ exchange is masked by another correction to the isobar approximation.

Gourdin et al.\textsuperscript{24} have also noticed the relation to the nucleon form factor, but instead of evaluating the first term in the bracket of Eq. (4) they suggested treating it as an adjustable parameter. The same suggestion was made by de Tollis et al.\textsuperscript{25}. Since McKinley\textsuperscript{26} neglected this term which is the dominating one in Ball's result, without giving a reason, his treatment of the $\rho$-exchange effect is questionable.

The values of $\Delta$ given in the experimental papers should not be compared with each other without a critical examination of the underlying theoretical analysis. In several cases the results for $\Delta$ differ not only because of the experimental data but also because of different assumptions and definitions.
IV. PHENOMENOLOGICAL ANALYSIS

1. Summary of the results of the phase shift analysis

Because of the close connection between pion photoproduction and \( \pi N \) scattering it is useful to summarize first our knowledge of the \( \pi N \) system as obtained from the scattering data, which have considerably improved during the last year\(^*\)). The quantum numbers of the 2nd, 3rd, and 4th resonances are now well established and it is clear that other strong effects occur mainly in the states \( P_{11}, S_{11}, S_{13} \) (indices: \( 2T, 2J \))\(^**\)). The following table gives some of the properties which are relevant for photoproduction. All energies are \( \gamma \)-laboratory energies, \( E_\gamma = T_\pi + 150 \) MeV.

| \( T \)  | \( J^P \) | multipoles   | \( \delta = 90^\circ \) \( E_\gamma \) (MeV) | \( \eta_{\text{min}} \) | \( |\text{Re} f| = \text{max} \) at \( E_\gamma \) (MeV) |
|---------|-----------|-------------|---------------------------------|----------------|---------------------------------|
| \( \Delta \) (1236) 1st res. | \( \frac{1}{2} \) | \( \frac{3}{2}^+ P_{33} \) | \( M_{\frac{1}{2}^+}, E_{\frac{1}{2}^+} \) | 345 | 1.0 | 290, 480 |
| \( N \) (1525) 2nd res. | \( \frac{1}{2} \) | \( \frac{3}{2}^- D_{13} \) | \( M_{\frac{3}{2}^-}, E_{\frac{3}{2}^-} \) | 770 | 0.25 | 680 |
| \( N \) (1680) 3rd res. | \( \frac{1}{2} \) | \( \frac{3}{2}^+ F_{15} \) | \( M_{\frac{3}{2}^+}, E_{\frac{3}{2}^+} \) | 1040 | 0.6 | 960, 1120 |
| \( \Delta \) (1920) 4th res. | \( \frac{1}{2} \) | \( \frac{3}{2}^+ F_{37} \) | \( M_{3^+}, E_{3^+} \) | 1495 | 0.2 | 1290, 1680 |
| \( N \) (1400) ? | \( \frac{1}{2} \) | \( \frac{3}{2}^- P_{11} \) | \( M_{1^-} \) | \( \approx 750 \) (\( \delta \approx 80^\circ \)) | 0.2 | 490 |

\( \delta \) is the real part of the phase shift, \( \eta \) the absorption parameter. The last column gives the energy at which the real part of the resonant scattering amplitude has its maximum. It is an estimate of the position of a peak or a dip caused by the interference between the real part of the resonant multipole and a slowly varying real background amplitude.

In the discussion of the question whether the \( P_{11} \) phenomenon is a 'resonance' one should keep in mind that the notion of a resonance is not sharply defined. There is a continuous transition to several other phenomena and therefore to a certain extent it is a matter of convention and of convenience how to define a resonance. For instance, the

\(^*\)) Compare the papers presented at the Royal Society Meeting in London (11 February 1965), to be published in the Proceedings of the Royal Society.

\(^**\)) Note added in proof: recently several authors have found evidence for a \( T = \frac{1}{2} \frac{3}{2}^- \) \((D_{13})\) resonance near \( N(1680) \).
properties of a resonance are considerably changed if a threshold is nearby or if there is a large background in the same partial wave. Also, in many models a resonance occurs together with strong variations in other partial waves, and it might be unsuitable to consider it separately.

It will be very interesting to see if the $P_{11}$ phenomenon occurs in photoproduction as inconspicuously as in $\pi N$ scattering, or if it is enhanced for some reason as in the final state of pp scattering at small momentum transfer and high energies $^{26}$. Presumably the 'shoulder' in the total $\pi^+ p$ cross-sections near $T_{\pi} = 750$ MeV is not caused by a resonance, but it seems $^{27}$ that an important contribution comes from a strong variation in $S_{33}$.

2. The work of Gourdin and Salin

In their well-known work on the 'isobaric model' Gourdin and Salin $^{21}$ have described the $\pi^+$ and $\pi^0$ production data up to $E = 800$ MeV by an ansatz which uses Breit-Wigner type formulae for the resonances, treating the coupling constants, the widths, and several background terms as adjustable parameters. This investigation was performed three years ago. In the meantime our knowledge of the $\pi N$ system has considerably improved, and the present status leaves little hope that the simple ansatz of Gourdin and Salin, or Rashid and Moravcsik $^{22}$ is adequate for a quantitative description of photoproduction.

Of course, the ansatz could be extended by admitting additional parameters for the background multipoles and introducing the important energy dependence $^{23}$ of the resonance widths $\Gamma$ ($\Gamma_{33}$ for $\pi N$ scattering changes by a factor of two between $\alpha_{33} = 45^\circ$ and $135^\circ$). But there remains the question of uniqueness which has lead to so many difficulties in the simpler case of $\pi N$ scattering, and has not yet been discussed in photoproduction.

3. The work of Schmidt, Schwidersky and Wunder

As mentioned in II.2, the general features of pion photoproduction below 500 MeV are well described by the results of Schmidt's evaluation of the isobar approximation. Therefore, it seems reasonable to consider the possibility that the exact multipole amplitudes differ only by small corrections from the multipoles of the Ball-Schmidt approximation. In order to find these corrections, Schmidt $^{28}$ has calculated the variation of his prediction for the cross-section $\sigma_{BS}$ if the real and imaginary part of one of the $s, p, \text{ or } d$-wave multipoles is changed by a small amount. The result is plotted at fixed energies as a function of angle. It allows easy discussion of the different possibilities for corrections of $\sigma_{BS}$ which lead to a better agreement with the experimental data.

This method corresponds to a multipole analysis which is limited to sets of multipoles in the neighbourhood of the Ball-Schmidt amplitudes.
The $\pi^0$ and $\pi^+$ production data near the 2nd resonance were analysed by Schmidt, Schwidersky and Wunder$^{29}$, assuming that the cross-sections can be described by the Ball-Schmidt amplitude and additions (which are not necessarily small) to $E_2^-, M_2^-$, and a few other multipoles. The isobar approximation to the dispersion relation approach cannot be applied in this region because the polynomial expansion in $\cos \Theta$ of $\text{Im} A(s', t)$ does not converge any more. However, this is no objection against using the empirical fact that the extension of Schmidt's calculations to these energies reproduces the general features of the non-resonant background.

It turns out that a good fit can be obtained (Fig. 11) for a resonance-like behaviour of $E_2^-$ and $M_2^-$. However, the present data admit several solutions and there could be others which were not found because of the restricted assumptions.

The discussion of the polarization of the recoil proton in $\pi^0$ production$^{15,20}$ has shown that a broad peak above 500 MeV is expected from the background effects alone. It would be very interesting to look for a superimposed structure at the energy of the 2nd resonance, taking into account that according to the recent results of the phase shift analysis its width is much narrower than formerly supposed$^{27}$).

Finally, we compare in Fig. 12 the position of the resonances defined by $\delta = 90^\circ$ with the sum of the total $\pi^0$ and $\pi^+$ cross-sections. This quantity was chosen in order to eliminate all interference terms which could cause a shift of the peaks. It is seen that in all three cases there is a shift to the low-energy side, which presumably has to be explained for the higher resonances in the same way as for the well-known first resonance.

V. CONCLUDING REMARKS

In earlier summaries on the status of photoproduction several simple models played an important role which were not treated above, for instance, the model of Peierls$^{30}$ which has lead to the correct predictions for the quantum numbers of the 2nd and 3rd resonances.

Unfortunately, there are many indications that photoproduction is more complicated than assumed in these models and cannot be described quantitatively by a small number of simple terms.

We have seen that in the low-energy region there is a remarkable success of the isobar approximation to the dispersion relation approach. Although it does not contain adjustable parameters, the predictions for the absolute values of the cross-sections are in most cases in reasonable agreement with the experimental data. So it seems that this approximation is a good starting point for further improvements.

At present there is no convincing evidence for the contributions of $\omega$ and $\rho$ exchange processes to single-pion photoproduction. This question deserves further study since it would be very interesting to determine the coupling constants. Possibly one will encounter similar difficulties as those mentioned by Professor van Hove$^{31}$) for $\pi N$ charge exchange scattering. In this case the data seem to be compatible with the assumption of the exchange of a $\rho$ Regge pole$^{32}$. 
Presumably further progress in the theory of pion photoproduction will be made in a similar way as in pion-nucleon scattering during the last years. The phenomenological analysis of the forthcoming data will give information on the energy dependence of the multipole amplitudes which can be compared with the results of a more detailed study of their dispersion relations. This does not necessarily mean that the theory will become more and more complicated since one could hope that someone will find a simple physical picture for the dominating parts of the production amplitude.
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31) L. van Hove, invited paper at this conference.

Figure captions

Fig. 1 Feynman graphs of the Born terms.

Fig. 2 Contour diagram of the difference between the experimental differential cross-sections and Schmidt's prediction in mb/st. The curve denoted by \( t = 0.87 \) gives the relation between energy and angle if the four-momentum transfer \( t \) is equal to its value at threshold.

Fig. 3 \( \pi^+ \) cross-section for a momentum transfer equal to its threshold value (cf. Fig. 2).

Fig. 4 \( \pi^+ \) cross-section at 90° c.m. near threshold. \( \sigma_p \) = Born cross-section.

Fig. 5 \( \pi^+ \) excitation curve for plane-polarized \( \gamma \) rays.

Fig. 6 Excitation curve for \( \pi^0 \) production at 90° c.m.s.

Fig. 7 Excitation curve for \( \pi^0 \) production at 0°. Dashed line: estimation of the correction from \( \text{Im } M_{33} \cdot \text{Im } E_{0+} \).

Fig. 8 \( \pi^0 \) angular distributions at 360 and 450 MeV. Dashed line as in Fig. 7.

Fig. 9 Feynman graphs for isobar intermediate states.

Fig. 10 Graphs for \( \rho \)-exchange processes.

Fig. 11 \( \pi^0 \) production in the region of the 2nd resonance. Experimental points: \( \Delta \) gives the difference between the data and Schmidt's calculation which is assumed to describe the main background effects. Solid line: best fit obtained by additional contributions in \( E_{2\alpha}, E_{2\alpha}, E_{0+} \).

Fig. 12 Sum of total \( \pi^0 \) and \( \pi^+ \) cross-sections. \( \delta \) = real part of the resonant phase shift.
Fig. 1

\[ \gamma + p \rightarrow n + \pi^+ \]

\(0 < \Delta < 0.5\)

Fig. 2
\( \gamma + p \rightarrow n + \pi^+ \)

- \( \Theta = 45^\circ \)
- \( \Theta = 90^\circ \)
- \( \Theta = 135^\circ \)

\( R(\theta) = \frac{d^2 \sigma}{dE d\Omega} \sin^2 \theta_{cm} \)

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\( k \frac{d^2\sigma}{d\Omega dE} = A \) [\text{mb/sr]}]

- Schmidt
- Polterm

\( \gamma + p \rightarrow p + \pi^0 \)

\( \Theta = 90^\circ \)

---

Fig. 5

Fig. 6
Fig. 7

\[ \gamma + p \rightarrow p + \pi^* \quad \theta = 0^\circ \]

Fig. 8

\[ E_T = 360 \text{ MeV} \quad E_T = 450 \text{ MeV} \]
THE ELECTROMAGNETIC STRUCTURE OF THE NUCLEON

U. Meyer-Berkhout,
Deutsches Elektronen-Synchrotron, Hamburg

During the past ten years electron-nucleon scattering experiments have provided convincing evidence that the nucleon has structure. In other words, the observed scattering of high-energy electrons from nucleons is quite different from the scattering that would be expected if the proton and neutron were point particles with their actual charges and magnetic moments. The term "structure" is somewhat misleading in that it may convey the idea of some kind of measurable spatial distribution of charge and current inside the nucleon. As was discussed in Dr. Beckmann's lectures, it is better to avoid such spatial concepts, since the principles of relativity and quantum mechanics imply that it is difficult to attach a meaning to them. The results of electron scattering experiments are scattering amplitudes as function of the momentum of the incident and scattered electron. These scattering amplitudes can be expressed in terms of the electromagnetic form factors. Limiting ourselves to the electromagnetic electron-nucleon interaction, the electromagnetic structure of the nucleon in its ground state is fully described by a total number of four form factors which are functions only of $q^2$, the Lorentz-invariant squared energy-momentum transfer. In the early times of electron scattering work, one used the Dirac and Pauli form factors $F_1(q^2)$ and $F_2(q^2)$ for proton and neutron. At present, the electric and magnetic form factors of proton and neutron $G_E(q^2)$ and $G_M(q^2)$ are preferred. These are linear combinations of the Dirac and Pauli form factors

$$G_E(q^2) = F_1(q^2) - \frac{q^2}{4M^2} \kappa F_2(q^2) \quad G_M(q^2) = F_1(q^2) + \kappa F_2(q^2).$$

The four form factors - two for the proton and two for the neutron - are sufficient to account for elastic electron-nucleon scattering, and there is absolutely no reason to introduce the concept of spatial structure. However, implicitly one frequently falls back into the habit of using spatial concepts. For example, when we ask whether there exists a central core in the charge or magnetic moment structure of the proton, we are using a spatial concept, but we really mean the behaviour of the proton form factors at large momentum transfers.

In this talk it will be assumed that you are familiar with the definition of the form factors. Furthermore, some knowledge of the Rosenbluth equation gives the differential cross-section for elastic electron-nucleon scattering. The basic underlying assumptions upon which the derivation of the Rosenbluth equation is based are:

1. Pure electromagnetic interaction between the electron (or muon, which is also used as a probe) and the nucleon. In other words, it is assumed that all other electron-nucleon interactions (e.g. weak interaction) can be neglected. The assumption is justified in the presently accessible energy range.
2. Validity of quantum electrodynamics. In principle it is possible that part of the observed structure, that is part of the variation of the form factors with $q^2$, arises from a break-down of quantum electrodynamics. However, this is not very likely since quantum electrodynamics has been tested thoroughly in experiments involving essentially only muons and electrons. One good but incomplete check is the comparison of the scattering of muons and electrons by protons or heavier nuclei at the same four momentum transfer. This will measure the ratio of the muon to electron electromagnetic form factors because all other parts of the Feynman diagram are the same. Such experiments have been done at CERN and at Brookhaven National Laboratory. The results indicate that in the thus far investigated range of four momentum transfers which extends up to about $1 \text{ (GeV/c)}^2$ no measurable differences exist between the electron and muon vertex functions. Although quite a few additional experiments have been carried out to test the validity of quantum electrodynamics, one still does not know whether quantum electrodynamics will eventually break down at very high momentum transfers. From our point of view, such a break-down could result either in a change of the photon propagator $1/q^2$ or a deviation of the electron (or muon) form factor from unity at high momentum transfers. In fact, unexpected results were obtained in a recent experiment performed by Blumenthal et al.\textsuperscript{1) at the Cambridge electron accelerator on wide-angle production of symmetrical electron pairs produced by photon in a carbon target. In this experiment, deviations from the predictions based on the Bethe-Heitler equation are observed\textsuperscript{1). If confirmed, it may mean a break-down of quantum electrodynamics. But it is too early to make such a drastic statement since a great many corrections enter into the experiment and the results are still of a preliminary nature. One may also hope to learn more about the validity of quantum electrodynamics from colliding beam electron-electron scattering experiments now being performed with the Stanford storage rings. For the time being we shall be naive and assume that electrons and muons have no structure, whereas the strongly interacting nucleon has structure which is associated with the cloud of strongly interacting particles surrounding the nucleon. Our simple-minded approach implies admitting that we do not have the slightest idea of what is the origin of the mass difference between electron and muon.

3. The third basic assumption upon which the derivation of the Rosenbluth equation is based is the one-photon exchange approximation. It is assumed that the elastic electron-nucleon interaction is mediated by the exchange of a single virtual photon. Two-photon exchange processes which in principle can contribute to the elastic $e^{-N}$ cross-section are neglected. At first sight it may seem reasonable to assume that contributions due to two-photon exchange graphs are small, since they are reduced by one power of $\alpha = 1/137$ relative to the one-photon exchange contribution. However, we shall see that the situation is not quite that simple. We shall have to come back later to the problem of justifying the one-photon exchange approximation.

In the one-photon exchange diagram the circle around the proton vertex is meant to indicate that no attempt is made to specify in detail how the interaction occurs. The blob simply symbolizes all effects of any number of strongly interacting virtual particles which contribute to nucleon structure.
The four momentum of the virtual photon is given by

\[ q = \left[ q^\mu, q^\rho \right], \]

where \( q \) is the transferred momentum and \( q^\rho = i\Delta E \) the transferred energy. Units are used in which \( h = c = 1 \). The Lorentz-invariant square of the four-momentum transfer is then given by

\[ q^2 = (q^\rho)^2 - \Delta E^2. \]

In electron nucleon scattering experiments one is dealing with spacelike four-momentum transfers. If using this metric which was also used by Dr. Beckmann in his lectures one has positive \( q^2 \) values for electron-nucleon scattering. For the crossed-reaction - nucleon-antinucleon annihilation to electron-positron pairs - the four momentum of the intermediate photon is timelike, i.e. \( q^2 \) is negative and smaller than \(-4M^2\). This is immediately obvious in the centre-of-mass system in which the four-momentum transfer has only space components for elastic electron-nucleon scattering since \( \Delta E = 0 \). In other words, as long as we have elastic scattering the virtual photon carries no energy but only momentum in the centre-of-mass system. In the crossed channel the intermediate photon carries no momentum but only energy in the centre-of-mass system. Since the square of the transferred energy must be at least \( 4M^2 \) one has \( q^2 \geq 0 \) and \( q^2 \leq -4M^2 \).

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Dr. Zichichi will discuss what is known about the behaviour of form factors in the region of time-like momentum transfers. Therefore, we can limit ourselves here to a discussion of the form factors in the region of space-like four-momentum transfers.

The Rosenbluth equation expresses the differential cross-section \( d\sigma/d\Omega \) for elastic scattering through an angle \( \theta \) in the laboratory system

\[
\frac{d\sigma}{d\Omega} = \left( \frac{d\sigma}{d\Omega} \right)_{NS} \left[ \frac{G_E^2 + \tau G_M^2}{1 + \tau} + 2\tau G_M^2 \tan^2 \frac{\theta}{2} \right].
\]

The Rosenbluth equation contains only two adjustable parameters which are the two form factors \( G_E(q^2) \) and \( G_M(q^2) \) describing the photon-proton interaction. In case of electron-neutron scattering these form factors have to be replaced by the corresponding neutron form factors. For space-like momentum transfers, information on the behaviour of the four nucleon form factors is available up to momentum transfers of \( 6.81 \) (GeV/c)^2. When expressed in units of an inverse squared length this corresponds to a \( q^2 \) value of \( 175 \) \( 1/\text{fm}^2 \) \( 1 \) (GeV/c)^2 \( \approx 25.7 \) \( 1/\text{fm}^2 \). The symbol \( \tau \) is used to denote the invariant squared four-momentum transfer \( q^2 \) divided by \( 4M^2 \) where \( M \) is the mass of the nucleon:

\[ \tau = \frac{q^2}{4M^2}. \]
If \( q^2 \) is expressed in units of \((\text{fermi})^{-2}\) then the dimensionless parameter \( r \) is given by \( r \equiv q^2/90 \). The squared four-momentum transfer \( q^2 \) can be expressed through the energies \( E \) and \( E' \) of the incident and scattered electron and the scattering angle \( \theta \) in the laboratory system in the following way

\[
q^2 = 4EE' \sin^2 \frac{\theta}{2}
\]

The cross-section

\[
\left( \frac{\mathrm{d}^2 \sigma}{\mathrm{d}^2 \mathbf{q}} \right)_{\text{NS}} = \frac{q^2}{q^4} \left( \frac{E'}{E} \right)^2 \frac{1}{tq^2} \sin^2 \frac{\theta}{2}
\]

occurring as a factor in the Rosenbluth equation, is the relativistic generalization of the Rutherford cross-section and gives the cross-section for the scattering of a relativistic electron in the pure Coulomb field of a hypothetical spinless particle of mass \( M \).

Dr. Beckmann has discussed in his lectures the reasons why one prefers to use the electric and magnetic form factors \( G_E(q^2) \) and \( G_M(q^2) \) instead of the Dirac and Pauli form factors \( F_1(q^2) \) and \( F_2(q^2) \). There were essentially three reasons for this preference:

1. One advantage of using \( G_E \) and \( G_M \) instead of \( F_1 \) and \( F_2 \) is algebraic convenience. This is immediately apparent since \( G_E \) and \( G_M \) appear separately in the Rosenbluth equation, whereas if \( F_1 \) and \( F_2 \) are used the cross term \( F_1 \cdot F_2 \) appears. Thus, using \( G_E \) and \( G_M \) is of greater algebraic convenience when extracting form factors and errors out of the experimental data.

2. As was shown in Dr. Beckmann's lectures, the \( G_E \) term corresponds to the exchange of a virtual photon with longitudinal polarization, whereas the \( G_M \) term corresponds to the exchange of a virtual photon with transverse polarization. Longitudinal polarization implies zero angular momentum transferred along the direction of the virtual photon exchanged in the scattering process. Transverse polarization implies one unit of angular momentum transferred along that direction. When the exchanged photon carries one unit of angular momentum along \( q \) the nucleon spin must be flipped in the course of the scattering process with respect to \( q \). Thus, the \( G_M \) term is the spin-flip term in the Breit or brick-wall system.

3. According to Sachs, the form factors \( G_E \) and \( G_M \) may have a more fundamental significance than \( F_1 \) and \( F_2 \) when it comes to a physical interpretation of these form factors in terms of spatial distributions of charge and magnetization inside the nucleon. However, the physical interpretation of the form factors in terms of spatial distributions of charge and current "inside" the nucleon is by no means obvious and all attempts in this direction are more or less esoteric. In any case, the Sachs argument is highly controversial. As is known from atomic physics, form factors are introduced into non-relativistic problems as Fourier-transforms of a charge density. For example, scattering of low-energy electrons by atoms can be analysed in terms of a form factor which can be Fourier-transformed back to give the charge distribution in the atoms. One would like to follow the same procedure for electron-nucleon scattering, i.e., determine the Fourier-transforms of the measured form factors \( G_E(q^2) \) and \( G_M(q^2) \) in three-dimensional momentum space to find spatial distributions of charge and magnetization inside the nucleon. However, difficulties are caused by the recoiling nucleon. It is no problem to define spatial density distributions in three-dimensional space by choosing a co-ordinate system in which the fourth component \( q_4 = 1/2 \) of the four-momentum transfer vanishes. One such system would be the centre-of-mass system of target nucleon and electron, another the Breit or brick-wall system. At low momentum transfers the proton can be considered to be at rest.
during the collision. If this were true for all momentum transfers, then a real physical
meaning could be attached to the Fourier-transforms. However, at large momentum transfers
the recoiling proton becomes relativistic. Since its velocity is then comparable to the
speed of the probing electron, the motion of the proton during the scattering process can no
longer be neglected. Therefore, the relation of the three-dimensional Fourier-transforms to
any real physical spatial structure of the proton is quite unclear, and the Fourier-transforms
of $G_E(q^2)$ and $G_M(q^2)$ in three-dimensional space may have only pictorial usefulness. This
must not worry us since all that is needed to describe the electron-nucleon electromagnetic
interaction in the one-photon approximation are just the four nucleon form factors.

Since the Rosenbluth cross-section contains as the only adjustable parameters the
two form factors $G_E(q^2)$ and $G_M(q^2)$, two elastic e-p cross-section measurements at a fixed
$q^2$-value but different scattering angles are needed to determine both $G_E$ and $G_M$. This
requires that we programme the scattering angle and incident energy in such a way as to
satisfy the relation

$$q^2 = 4E E' \sin^2 \frac{\Theta}{2}.$$ 

For precise results one cross-section measurement should be made at a large scattering angle
and low incident energy where the $G_M^2$ term dominates the cross-section. For the second measurement a small scattering angle and a correspondingly high primary energy should be selected
to obtain the largest possible influence of the $G_E^2$ term on the cross-section. At a given
four-momentum transfer, cross-section measurements at more than two scattering angles are, in
principle, not needed but can serve to improve the accuracy of the form factors derived from the
data. Measurements at more than two scattering angles can also be useful to check at least to
some extent the validity of the one-photon exchange approximation.

What is the procedure used to derive $G_E$ and $G_M$ from the experimental data? We
notice that for a fixed $q^2$ the term in parentheses in the Rosenbluth equation is a linear func-
tion of $\tan^2 \frac{\Theta}{2}$. This suggests the "Rosenbluth plot" in which the experimental cross-section
$\frac{d\sigma}{d\Omega}$ is divided by the cross-section $(d\sigma/d\Omega)_{NS}$ and plotted versus $\tan^2 \frac{\Theta}{2}$. If the Rosenbluth
equation is valid, this plot should be a straight line with a slope of $2 \tau G_M^2$ and a positive
intercept at $\tan^2 \frac{\Theta}{2} = \frac{1}{\tau} (1 + \tau)$ which is equal to $G_E^2 / (1 + \tau)$.
Thus, the slope gives $G_M(q^2)$ and the intercept $G_E^2(q^2)$. However, even if a straight line with a positive intercept at $\tan^2 \frac{\theta}{2} = 1/2(1 + \tau)$ is found this does not necessarily prove the validity of the one-photon exchange approximation. Additional tests are necessary. Note furthermore that only the squares of the form factors can be determined in this way. However, the known values of the four nucleon form factors at zero four-momentum transfer plus the requirement that the form factors should be smooth functions of $q^2$ gives the sign of all nucleon form factors except for the electric form factor of the neutron $G_E^0$.

At $q^2 = 0$ the form factors $G_E^p$ and $G_M^p$ are normalized such that

$$G_E^p(0) = 1 \quad G_M^p(0) = \frac{\mu_p}{\mu_K} \approx 2.79$$

$$G_E^n(0) = 0 \quad G_M^n(0) = \frac{\mu_n}{\mu_K} \approx -1.91.$$  

Since the total charge of the neutron is zero, the sign of the electric neutron form factor cannot be obtained this way.

At sufficiently large scattering angles the second term in the Rosenbluth equation always dominates the cross-section. At a scattering angle of $180^\circ$ the cross-section becomes

$$\frac{d\sigma}{d\Omega}(180^\circ) = \frac{\alpha^2}{q^2} \left(\frac{E'}{E}\right)^2 \frac{1}{\tan^2 \frac{\theta}{2}} \cdot 2 \frac{q^2}{4M^2} \cdot \left[ \frac{0}{2} \cdot G_M^2 \right]$$

$$= \frac{\alpha^2}{2} \lambda_N \left(\frac{E'}{E}\right)^2 \cdot G_M^2 \approx 1.2 \cdot 10^{-12} \left(\frac{E'}{E}\right) \cdot \lambda_N \left[ \frac{\text{cm}^2}{\text{sr}} \right],$$

where $\lambda_N$ is the nucleon Compton wavelength. For primary electron energies $E \gg M$ the ratio $E'/E$ is approximately given by $M/2E$. Thus, $G_M^2$ can be determined directly from a $180^\circ$ scattering experiment. According to my knowledge one such experiment has, in fact, been performed. However, in general, $G_M^2$ is obtained by the outlined method which requires measurements at two scattering angles for a fixed $q^2$-value.

When we discuss the available results on nucleon form factors we shall see that the magnetic form factor of the proton is known to a much higher accuracy than the electric form factor, especially at momentum transfers above $1 \text{ (GeV/c)}^2$. Why is this so? At large angles, i.e.

$$\tan^2 \frac{\theta}{2} \gg \frac{(G_E^p/G_M^p)^2 + \tau}{2\tau(1 + \tau)}$$

the elastic scattering cross-section is essentially determined by the second term in the Rosenbluth equation

$$\frac{d\sigma}{d\Omega} \text{ (large angle)} \approx \frac{\alpha^2}{2} \lambda_N \left(\frac{E'}{E}\right)^2 \cdot G_M^2(q^2).$$
The only energy dependence of the factor in front of $G_E^2$ is through the factor $(E'/E)^2$ and there is no angular dependence of the coefficient of $G_M^2(q^2)$ at all in this approximation. Thus, a systematic error in angle and/or energy would not affect the $G_M$ value appreciably, but would lead to inaccurate $G_E$-values. This follows immediately from the $G_E^2$ term in the Rosenbluth cross-section which is

$$\frac{d\sigma}{d\Omega} \propto (G_E^2)-\text{contribution} = \frac{\alpha^2}{4} \frac{x}{x^2} \left( \frac{E'}{E} \right)^2 \frac{1}{4} \frac{1}{t g^2 \frac{\theta}{2} \tau (1 + \tau)} \cdot G_E^2 .$$

The coefficient of $G_E^2$ is very sensitive to the measurement of angle and energy. This is one of the reasons why $G_E^2$ is so poorly known compared with $G_M$. Secondly, it can be seen from the Rosenbluth equation that a good determination of $G_E^2$ requires a measurement at a small angle and correspondingly high incident energy, since at small angles the cross-section is most sensitive to $G_E$. The ratio of the coefficients of $G_E^2$ and $G_M^2$ in the Rosenbluth equation as a function of angle for fixed $q^2$ values is given in Fig. 1. We see that the influence of the $G_E^2$ term on the cross-section increases with decreasing scattering angle. But, assuming that the incident energy is available, we see that even at small angles, i.e.

$$t g^2 \frac{\theta}{2} \ll \left( \frac{G_E^2}{G_M^2} + \tau \right) \frac{1}{2 \tau (1 + \tau)} ,$$

the cross-section is not given by $G_E^2$ alone but by $G_E^2 + \tau G_M^2$. Since the relative contribution of the $G_E^2$ term cannot exceed the value $1/(1 + \tau)$ we also see that the influence of the $G_E^2$ term decreases with increasing $q^2$. The influence which $G_E^2$ and $G_M^2$ may have on the cross-section can also be judged from Fig. 2 where the ratio of the coefficients of $G_E^2$ and $G_M^2$, as given by the Rosenbluth equation, is plotted versus scattering angle for various fixed incident electron energies. Until recently, the required high energies for precise $G_E$ measurements at momentum transfers above about 1 (GeV/c)$^2$ were not available. But with the 6 GeV electron beams now available at CEA and DESY one may hope that measurements at small electron scattering angles will yield more precise $G_E$ values at momentum transfers above 1 (GeV/c)$^2$. Whether it really will be possible to derive more accurate $G_E$ values from high energy small-angle measurements depends to a large extent on the value of the ratio $G_E^2/G_M^2$. Either $G_E^2/G_M^2 \ll 1$. Then the contribution of the $G_E^2$ term to the cross-section will always be small. In that case one will hardly be able to measure $G_E$ precisely at large momentum transfers. Or $G_E^2/G_M^2 \gg 1$. Then the contribution of the $G_E^2$ term to the cross-section is appreciable at sufficiently small angles even at high momentum transfers.

Measurements at a fixed four-momentum transfer at very small angles and correspondingly high incident energies are also desirable in view of the fact that the obtainable statistical accuracy of the cross-section measurements increases with decreasing scattering angle. This is due to the fact that the elastic scattering cross-section at a fixed four-momentum transfer increases considerably with decreasing scattering angle. By measuring the cross-section at very small scattering angles and correspondingly high energies one might therefore be able to detect even a relatively small $G_E^2$ term contribution to the cross-section.

Electron-proton scattering experiments are basically simple and straightforward when an intense beam of high-energy electrons is available. Such beams exist at the linacs at Stanford and Orsay and at the electron synchrotrons at Cornell, Harvard, Frascati and DESY.
In a typical arrangement, as shown in Fig. 3, the collimated and almost monochromatic electron beam first traverses a proton target. Its intensity is measured by a Faraday cup, a secondary emission monitor, or a total absorption ionization chamber. Typically, the energy spread in the beam is 1% or less. Targets of liquid or high pressure gaseous hydrogen as well as polyethylene have been used. Various types of magnetic spectrometers have been designed to analyse the momentum spectrum of electrons scattered at a given angle in a given solid angle \( \Delta \Omega \). If one studies elastic scattering the spectrometer is set to select elastically scattered electrons. One can, of course, also select the recoil protons, but when using this method at high energies it is, in general, more difficult to discriminate between elastic and inelastic events by momentum analysis. The Stanford and Orsay groups use 180° deflecting double-focusing spectrometers of the Siegbahn type weighing up to 200 tons. At Cornell, Harvard, and DESY a different type of spectrometer is used, consisting simply of one quadrupole magnet (Fig. 4) that focuses a point source of electrons - the target - to a line image. At this image, a long narrow scintillation counter is placed which, together with a lead obstacle located at the centre of the quadrupole, defines the accepted momentum and the momentum resolution of the spectrometer. The electrons finally pass through a detector system which serves to identify and count the electrons. In order to discriminate effectively against pions in experiments in which primary electrons with energies of a few GeV are used, it is necessary to employ a detector system consisting of scintillation counters, a gas Čerenkov counter and, in some cases, a shower detector. Measurements of elastic e-p scattering become more difficult the larger the momentum transfer, since the cross-section for elastic e-p scattering decreases rapidly with increasing momentum transfer. Furthermore, background discrimination becomes more and more difficult with increasing energy. The rapid decrease of the cross-section for elastic e-p scattering with increasing scattering angle is shown \(^3\) in Fig. 5. These cross-sections were computed under the assumption that \( G^p_E \) and \( G^p_N \) decrease as \( q^2 \) at four-momentum transfers above \( q^2 = 100 \, \text{fm}^{-2} \). As an example, the cross-section at an incident electron energy of 6 GeV and a scattering angle of 150°, corresponding to a four-momentum transfer of about 270 \( \text{fm}^{-2} \), is expected to be approximately \( 5 \times 10^{-35} \, \text{cm}^2/\text{sr} \). This value is about two orders of magnitude smaller than the smallest cross-section which has been measured thus far in electron scattering work at CEA, and is comparable to the order of magnitude of cross-sections for neutrino-induced reactions. With the available primary electron intensities and conventional magnetic spectrometers, such cross-sections will yield counting rates between 1 and 10 elastic events per day. On the other hand, the total number of pions emitted in the same solid angle can be roughly \( 10^5 \) times larger than the number of elastically scattered electrons. Momentum analysis of the scattered particles, detection of scattered electron and recoil proton in coincidence (Fig. 4), and sophisticated detector systems are used to reduce this background, but high-momentum transfer experiments remain difficult. The low counting rates can be overcome only by constructing high-current electron accelerators like the Stanford monster or possibly by building large solid-angle magnetic spectrometers utilizing spark chambers.

In order to extract a cross-section from the experimental momentum spectrum of scattered electrons, a number of corrections have to be applied, most important of which are the so-called radiative corrections. They arise from the emission of real photons during the scattering, and the emission and reabsorption of virtual photons. Owing to the infra-red character of these processes the contributions to the radiative corrections due to the emission
of soft photons are most important. In most experiments only the scattered electrons are
detected at certain selected angles and momentum analysed. The emitted photons are not de-
tected. Without radiative effects the spectrum of elastically scattered electrons would be
represented by a $\delta$-function. Strictly speaking, this statement applies only in the limiting
case of vanishing energy spread and angular divergence of the primary beam, zero target thick-
ness, and zero angular acceptance of the spectrometer. If one includes radiative effects the
scattered electrons will be more or less degraded in energy by emission of real photons, and
the spectrum of scattered electrons is no longer represented by a $\delta$-function.

\[
\frac{dN}{dE} \quad \text{E'} \quad \frac{dN}{dE} \quad \text{E'}
\]

Instead, a momentum distribution with a long tail towards low energies is observed. The
cross-section $d\sigma/d\Omega (\Delta E)$ is obtained by integration over the double differential cross-section
\[
d\sigma/d\Omega dE = \int_{E'-\Delta E}^{E'} \frac{d^2\sigma}{d\Omega dE} dE.
\]

We see from this that pure elastic scattering - elastic scattering without emission of real
photons - is an idealization which does not occur in nature. The cross-section with $\Delta E = 0$
would be zero. In actual experiments the scattered electrons have a distribution in energy
as shown

\[
\frac{dN}{dN} \quad \text{E'} \quad \frac{dN}{dN} \quad \text{E'}
\]

The typical shape of the low-energy part of the spectrum reflects the radiative losses during
the scattering (radiative corrections) and emission of real bremsstrahlung. Since scattered
electrons are only detected if the electrons have lost, by real photon emission, an amount of
energy less than $\Delta E$ fixed by the experimental conditions (e.g. momentum resolution of the
spectrometer or the energy cut-off) some "elastically" scattered electrons are lost. The
only measurable cross-section is the cross-section for "elastic" scattering which includes
the sum of the contributions from the radiative diagrams,

\[
k < \Delta E \\
\text{and} \\
k < \Delta E
\]
the electron vertex modification,

\[
\text{the vacuum polarization contribution,}
\]

and higher order diagrams. Radiation from the proton current must also be included at momentum transfers \( q^2 \gtrsim 1 \text{(GeV/c)}^2 \). In order to compute the elastic e-p scattering cross-section \( \frac{d\sigma}{d\Omega} \) from \( \frac{d\sigma}{d\Omega} (\Delta E) \) a correction must be applied, the magnitude of which will depend on the value of \( \Delta E \). The correction increases the cross-section which is represented by the shaded area under the curve on page 61. However, since all these diagrams are one-photon exchange diagrams the resulting cross-section is still completely determined by the two-proton form factors. Schwinger was the first to show that the measured cross-section \( \frac{d\sigma}{d\Omega} (\Delta E) \), represented by the shaded area under the curve, is related to the Rosenbluth cross-section for elastic e-p scattering by

\[
\frac{d\sigma}{d\Omega} (\Delta E) = \left( \frac{d\sigma}{d\Omega} \right)_R (1 - \delta_{\text{tot}}),
\]

where \( \delta_{\text{tot}} \) is a complicated function of \( E, \Delta E \) and \( \theta \),

\[
\delta_{\text{tot}} = \delta(E, \Delta E, \theta) > 0.
\]

The experimental conditions are usually such that application of the radiative corrections amounts to an increase of the measured cross-section by about 15\% to 25\%, depending to some extent on primary energy, scattering angle, and accepted momentum band. This is demonstrated in Fig. 6 which is based on some recent numerical calculations by Kohaupt\(^3\). It shows the radiative correction \( \delta_{\text{tot}} \) as function of scattering angle (or secondary energy) for 2, 4, and 6 GeV incident electron energy and various cut-off energies \( \Delta E \). In addition to the radiative corrections one has to correct for the radiation loss due to the emission of bremsstrahlung in the target which is proportional to the effective target thickness.

Thus far we have neglected the so-called two-photon exchange graphs. If two-photon exchange terms contribute significantly to the elastic e-p scattering cross-section it would certainly modify the interpretation of the data in terms of form factors.
What evidence exists to justify the one-photon exchange approximation? This question should definitely be considered, especially at large momentum transfers, since the form factors become progressively smaller as $q^2$ increases. Furthermore, resonance enhancement effects may tend to equalize the one-photon and two-photon exchange terms.

If one includes a two-photon exchange term in the scattering amplitude, the elastic e-p scattering cross-section is given by

$$\frac{d\sigma}{d\Omega} \propto |aA_4 + a^2A_5|^2.$$  

Since the one-photon exchange amplitude $A_4$ is real this yields

$$\frac{d\sigma}{d\Omega} \propto (aA_4 + a^2 \text{Re} A_5)^2 + (a^2 \text{Im} A_5)^2$$

$$\geq a^2 A_5^2 + 2a^3 A_4 \text{Re} A_5.$$  

No theoretical predictions can be made about the two-photon exchange scattering amplitude $A_5$ without at least some knowledge of the structure of the nucleon. A typical two-photon exchange process is represented by a graph according to which one photon is exchanged which excites the nucleon to an isobaric state $N^*$,

When exchanging the second photon, the nucleon returns to the ground state. If one cuts this graph vertically (cut 1), one has the product of Compton scattering of virtual photons by protons and Compton scattering of virtual photons by electrons. Cutting the diagram horizontally (cut 2) yields the product of two inelastic e-p scattering processes with excitation of an isobaric state $N^*$. Such models or cuts may enable us to estimate the contribution of such two-photon exchange graphs to the elastic e-p scattering cross-section. The result is that at electron energies up to at least 1 GeV the diagram corresponding to the excitation of the $\frac{3}{2}, \frac{5}{2}$ resonance is likely not to contribute more than about 1% to the total cross-section. This is due to the fact that the real part of $A_5$, as was discussed in Dr. Tripp’s lectures, changes sign while going through the resonance. Therefore, upon integrating the virtual photon spectrum over the $\frac{3}{2}, \frac{5}{2}$ or any higher resonance the interference term $2a^3 A_4 \text{Re} A_5$ is largely cancelled and its net contribution is correspondingly small.

Heavy meson resonant states might also contribute to two-photon exchange terms.
According to this two-photon exchange diagram the coupling between the two-photons and the nucleon is mediated by a meson resonance in the virtual meson cloud surrounding the bare nucleon. One could even think of a hypothetical direct coupling between electron and meson resonance. Unfortunately, it seems to be difficult to estimate at present in a reliable way the two-photon exchange scattering amplitude due to such a diagram. Therefore, it is important, especially at high momentum transfers, to consider the experimental evidence which exists to justify the one-photon exchange approximation.

The presence of two-photon exchange contributions might show up experimentally in three ways:

1. Deviations from the Rosenbluth plot which, if the one-photon exchange approximation is valid, should be a straight line with a positive intercept at \( \theta^2 = 1/2(1+\tau) \). As can be seen on Fig. 7 no evidence for a systematic departure from linearity or for a negative intercept at \( \theta^2 = 1/2(1+\tau) \) has been found in the investigated region of momentum transfers \( q^2 \leq 45 \text{ f}^2 \). However, two qualifications should be made. First it should be mentioned that two-photon exchange terms can be invented for which the Rosenbluth plot remains a straight line with a positive intercept at \( \theta^2 = 1/2(1+\tau) \). Notice that this would completely change the interpretation of the plot in terms of electric and magnetic form factors.

Secondly, as can be seen in Fig. 7, the linearity of the Rosenbluth plot has not been checked at laboratory angles below 90° for 25 f \( q^2 \leq 35 f^2 \). At \( q^2 = 40 f^2 \) and \( 45 f^2 \) the minimum laboratory angle is 120°, and the Rosenbluth plot for \( q^2 = 45 f^2 \) (\( \tau \approx \frac{1}{2} \)) is just compatible with a zero or slightly positive intercept at \( \theta^2 = 1/2 \). On the other hand, Gourdin and Martin 4) and Flamm and Kummer 5) have shown that, if the coupling through two-photon exchange is mediated by a hypothetical \( J^{PC} = 1^{++} \) or \( 2^{++} \) intermediate state, a deviation from the linear Rosenbluth plot is to be expected but should only be measurable at small scattering angles. As Gourdin and Martin have discussed in their paper, for a \( J^{PC} = 1^{++} \) intermediate state one arrives at a cross-section

\[
\frac{\text{d}\sigma}{\text{d}q^2} = \left( \frac{\text{d}^2\sigma}{\text{d}q^2} \right)_{\text{NS}} \left( \frac{C_2^2 + \tau C_4^2}{1 + \tau} \right) + \left( 2\tau C_2^2 + C(\tau) \left( 1 + \frac{1}{1 + \tau} \cot \theta^2 \right) \right) \frac{\theta^2}{2}
\]

where the additional term in the cross-section formula results from the interference term \( 2a^3 A_0 \text{Re} A_0 \). At angles corresponding to \( \theta^2 = 1 \) the deviation from the straight line behaviour is likely to be too small to be noticeable, although the cross-section itself may differ appreciably from the Rosenbluth value. To observe such a deviation which may exist at sufficiently large momentum transfers, one has to measure the cross-section at small angles and correspondingly high incident electron energies. Sufficiently high-energy beams were not available until recently but are now available at CEA and DESY. It is therefore not surprising that thus far no non-linearities have been detected. To summarize, we can conclude that due to a lack of small-angle measurements we do not have a good check that the real part of the two-photon exchange scattering amplitude is zero for \( q^2 \geq 1 \text{ (GeV/c)}^2 \).

2. A second method of checking the presence or absence of two-photon exchange terms in elastic scattering is to compare the elastic scattering of positrons and electrons on protons. Since the difference between the cross-sections is caused by the interference term between the real single photon amplitude and the real part of the two-photon exchange amplitude, this
experiment again is sensitive to the real part of the amplitude of the two-photon exchange term. Measurements on electron-proton and positron-proton scattering were made at Stanford up to momentum transfers of 19.5 \( f^{-2} \). It was found that the ratio \( \sigma_+ - \sigma_-/\sigma_+ + \sigma_- \), which is directly proportional to \( a^2 A_1 \Re A_2 \), is equal to zero within the limits of error, implying that \( \Re A_2 \approx 0 \) for \( q^2 \leq 19.5 \ f^{-2} \). It would be worth while to check the validity of the one-photon exchange approximation by this method at much higher four-momentum transfers. But with the presently available positron beam intensities (5 \( \cdot 10^8 \) positrons per second) experiments of this kind can hardly be extended beyond momentum transfers of about 60 \( f^{-2} \).

3. A third way to test the validity of the one-photon exchange hypothesis is to measure the polarization of the recoil proton which should be zero in the absence of two-photon exchange contributions. This experiment is also sensitive directly to the amplitude of the two-photon exchange term. The polarization of the recoil proton is perpendicular to the scattering plane and is proportional to

\[
P \propto a^2 A_1 \Im A_2 .
\]

It is thus the imaginary part of the two-photon exchange term which gives rise to a polarization of the recoil proton in electron-proton scattering from unpolarized initial particles, whereas the difference between electron-proton and positron-proton cross-section was proportional to the real part of the two-photon amplitude. The two checks are thus not equivalent but supplement each other. Experiments to detect a possible polarization of the recoil proton in e-p scattering were performed at Orsay at a four-momentum transfer of 16.5 \( f^{-2} \) by Bizot et al.\(^7\) and at Frascati at a momentum transfer of 20 \( f^{-2} \) by di Giorgio et al.\( ^8 \). In both experiments the polarization of the recoil proton was studied by measuring the left-right asymmetry in the scattering of the recoil protons in the carbon plates of a spark chamber.

The result of both experiments was zero polarization within the limits of error. This means \( \Im A_2 \approx 0 \) at these four-momentum transfers and is in agreement with recent theoretical predictions\(^9\). It will be difficult to extend these measurements to higher four-momentum transfers because the efficiency for the polarization analysis decreases with the proton energy. A momentum transfer of 20 \( f^{-2} \) already corresponds to a recoil proton energy of 416 MeV.

Furthermore, the e-p cross-section drops rapidly with increasing momentum transfer. Hopefully, polarization measurements may be feasible with presently available beam intensities up to four-momentum transfers of about 40 \( f^{-2} \).

The present situation can be summarized in the following way. Although form factors are derived from electron-nucleon scattering work up to momentum transfers of 6.81 (GeV/c)\(^2 \) (8 175 \( f^{-2} \)) we have thus far no experimental justification that the one-photon exchange approximation is valid above four-momentum transfers of 20 \( f^{-2} \) or 0.8 (GeV/c)\(^2 \).

Next we shall discuss how one can obtain information on the electromagnetic structure of the neutron. Since we do not have dense enough free neutron targets, information on the neutron form factors is obtained mainly from high-energy electron-deuteron scattering experiments. Here one must distinguish between elastic e-d scattering and incoherent scattering of electrons from the loosely bound nucleons in the deuteron. Sometimes this second method is simply referred to as inelastic or quasi-elastic e-d scattering.

For elastic e-d scattering, the differential cross-section in the laboratory system is given by a modified Rosenbluth equation,
\frac{dg}{dt} = \left( \frac{d\sigma}{dt} \right)_{NS} \left[ A'(q^2) + B'(q^2) \tan^2 \theta/2 \right],

which again shows the linear dependence on $\tan^2 \theta/2$. In the case of e-d scattering, however, the coefficients $A'$ and $B'$ depend on the squares of the three deuteron form factors. These three deuteron form factors are the charge form factor $G^d_E(q^2)$, an electric quadrupole form factor $G^d_Q(q^2)$, and a magnetic dipole form factor $G^d_M(q^2)$. Just as in the case of the proton, the coefficients $A'$ and $B'$ can be obtained from the Rosenbluth plot. It turns out that $B'$ depends only on the square of the magnetic form factor

$$B' = \frac{2}{5} \eta (1+\eta) G^2_M(q^2), \quad \text{with } \eta = \frac{2}{4} \frac{q^2}{d^2}$$

$$A' = G^2_E(q^2) + \frac{8}{9} \eta G^2_Q(q^2) + \frac{2}{3} \eta (1+\eta) G^2_M(q^2).$$

Thus $G^2_E$ can be obtained directly from the slope. The coefficient $A'$ depends on all three deuteron form factors. Unfortunately it is not possible to separate the charge and quadrupole form factors experimentally, but at small momentum transfers ($q^2 \lesssim 2.5 \text{ fm}^{-2}$) the term containing the quadrupole form factor $G^2_Q(q^2)$ can be neglected in which case $G^2_E$ can be derived from the Rosenbluth plot. To illustrate the behaviour of the three deuteron form factors as a function of $q^2$ we show in Fig. 8 the form factors calculated by Glendenning and Kramer.\(^1\)

However, here we are not interested in the form factors of the deuteron but in the form factors of the neutron. To use elastic e-d scattering to study the electromagnetic structure of the neutron requires a thorough understanding of the connection between the nucleon form factors and those of the deuteron. In order to establish this connection, one must make a number of simplifying assumptions. The most important of these are the use of the non-relativistic wave function of the deuteron and the assumption that the interaction of the virtual photon with meson exchange currents between the nucleons can be neglected. If we accept these approximations the deuteron form factors can be expressed in a simple manner in terms of the nucleon form factors. For example, the electric deuteron form factor $G^d_E$ can be interpreted as

$$G^d_E = (G^p_E + G^n_E) \cdot F_D(q^2),$$

where $F_D(q^2)$, in the non-relativistic approximation, is the Fourier transform of $|\Psi_D|^2$. Here $\Psi_D$ is the deuteron ground state wave function and $|\Psi_D|^2$ would describe the charge distribution inside the deuteron if the nucleons had no charge structure but were point particles. Since upon squaring the nucleon isoscalar form factor $(G^p_E + G^n_E)$ one obtains a cross-term $G^p_E \cdot G^N_E$, in principle experiments on elastic e-d scattering even allow one to establish the sign of the electric form factor of the neutron.

The electromagnetic structure of the neutron can also be studied in experiments on inelastic electron-deuteron scattering,

$$e + d \rightarrow e' + n + p.$$

The method is based on the idea that because of the loose binding of the nucleons in the deuteron, the inelastic electron-deuteron cross-section can be written approximately as the sum of the elastic electron-proton and electron-neutron scattering cross-section. In other
words, the deuteron, because of its extended structure and weak binding, acts as a reasonably good source of "quasi-free" neutrons. The problem can be treated in the so-called "impulse approximation". In this method the nucleons are considered as free particles, with a distribution of momenta caused by their binding within the deuteron. The cross-sections for scattering from the two nucleons are averaged over the initial nucleon momenta to give the total differential cross-section. However, the actual analysis of these experiments is complicated by many problems related to the detailed structure of the deuteron and to the final-state interactions between the emergent nucleons. One can distinguish the process of quasi-elastic scattering from elastic e-d scattering because of the fact that in this type of inelastic scattering the recoiling nucleon takes away much more energy than does the recoiling deuteron in elastic e-d scattering. Although the electrons scattered by the loosely bound nucleons in the deuteron are spread out to some extent in energy because of the Fermi motion of the nucleons inside the deuteron, the separation between the elastic peak and the inelastic continuum is such that discrimination between elastic and inelastic events is possible by momentum analysis of the scattered electrons. Because of the extended structure of the deuteron, the cross-section for elastic e-d scattering is small compared to the cross-section for incoherent electron-nucleon scattering at momentum transfers \( q^2 \gg 1/a^2 \), where \( a \) is the deuteron rms radius.

By far the simplest way to investigate this type of scattering is to use the method in which only the scattered electron is detected. Since in first approximation the measured cross-section is given by the sum of the elastic electron-proton and electron-neutron cross-section, the elastic cross-section of the neutron is obtained by a simple subtraction of the e-p elastic cross-section. The e-p cross-section must therefore be measured simultaneously using a free proton target.

A more reliable method of determining the neutron form factors from inelastic e-d scattering is to detect both scattered electron and recoil neutron in coincidence. One selects the events with the recoil neutron momenta in a narrow cone around \( \hat{q} \), the three momentum transferred from the electron to the nucleons in the laboratory system,

![Diagram](image)

Since neutron detection is difficult it is experimentally often more convenient to replace the neutron counter by a high efficiency proton detector and register the number of coincidences. When using this method a scattered electron detected in coincidence is assumed to be an electron scattered by the neutron. The assumption is justified provided the proton counter covers a sufficiently large solid angle. At the same time, one also measures e-p coincidences from the deuteron target. The ratio of e-n to e-p scattering which is not so sensitive to experimental and theoretical uncertainties is then multiplied by the corresponding free proton cross-section to yield the neutron cross-section.

When discussing the experimental results we shall see that not only is the accuracy of the neutron form factors not as good as in the case of the proton but also there are obvious inconsistencies in the neutron data. This is especially the case for the electric form factor of the neutron. It is not too surprising in view of the following two facts:
1. Electron-deuteron scattering is generally dominated, especially at low momentum transfers, by the proton contribution. This is so since the magnetic moment of the neutron is smaller than the magnetic moment of the proton. Furthermore, \( G_E^N(q^2) < 1 \) at all momentum transfers. Thus, in most experiments one suffers from the fact that one must subtract two large and comparable cross-sections in order to find the small neutron cross-section. This implies large errors.

2. Although only loosely bound, the fact that the neutron instead of being free "sees" an extra proton introduces considerable theoretical uncertainties coming from meson exchange effects, the final state interactions between neutron and proton, and from the deuteron wave function which is not known precisely. Since the corresponding corrections cannot all be evaluated in a truly reliable way this introduces considerable uncertainties in the neutron form factors.

It thus appears that the present theoretical and experimental status of e-d scattering work is not sufficient to give truly reliable values of the neutron form factors.

We shall now discuss what is known experimentally about the behaviour of the nucleon form factors in the region of space-like momentum transfers.

A. Proton

Figure 9 shows a plot of \( G_E^P \) and \( G_M^P/(1 + \kappa_P) \) versus the four-momentum transfer \( q^2 \).

The most recent experimental results on proton form factors are not contained in Fig. 9 but are given in Fig. 10 which extends up to four-momentum transfers of 6.8 (GeV/c)^2 or 175 fm^{-2}. One notices immediately that \( G_E^P(q^2) \) is not accurately known above momentum transfers of 1 (GeV/c)^2.

What major conclusions can be drawn from these figures?

1. For space-like momentum transfers \( G_E^P \) and \( G_M^P \) are smoothly decreasing functions of \( q^2 \), implying that the proton has structure. It is interesting to notice that \( G_E^P(q^2) \) and \( G_M^P(q^2)/(1 + \kappa_P) \) follow the same curve to within the experimental error at least up to four-momentum transfers of 1 (GeV/c)^2. At higher momentum transfers, \( G_E^P(q^2) \) is not known well enough to prove or disprove this relationship. It is not known whether this empirical relation has any deeper meaning. At present we have no theoretical justification for this experimental result. We only know that the relation \( G_E^P = G_M^P/(1 + \kappa_P) \) can certainly not be valid at all four-momentum transfers since \( G_E^P \) and \( G_M^P \) must be equal to each other at a four-momentum transfer of \( q^2 = -4M^2(\tau = -1) \).

2. It is well known that the initial slope of the form factor curve versus \( q^2 \) is related to the rms radius of the corresponding spatial distribution. This is the reason why electron-nucleon scattering experiments at small momentum transfers are so important. From the slope of the proton form factor curves at \( q^2 = 0 \), one arrives at the conclusion that charge radius and magnetic radius of the proton are equal to within 2%. Both radii are 0.85 fm and are thus considerably smaller than the pion-Compton wavelength \( \lambda_{\pi} \approx 1.4 \) fm.

3. The asymptotic behaviour of the nucleon form factors gives information about the possible existence of a "hard core" inside the nucleon. Furthermore, experiments on the form factor behaviour at large \( q^2 \) values can serve to check some of the current theoretical predictions on the asymptotic behaviour of form factors.
Using spatial concepts, one could imagine a concentration of a certain fraction of the nucleon charge or current density in the centre of the nucleon. Depending on the size of that volume, one would speak of a "hard core" (i.e., point-like concentration) or a "soft core". As is obvious from Figs. 9 and 10 both proton form factors decrease steadily towards asymptotic values which do not seem to be significantly different from zero. This behaviour implies that the proton has no hard core. This rejection of the hard core is considered to be the most important result of recent e-p scattering work. However, a "soft core" is not excluded by the data. It must be pointed out in this connection that at the largest momentum transfer that has so far been investigated, $6.8 \ (\text{GeV/c})^2$, it has only been possible to establish upper limits for the proton form factors. This is explained by the fact that up to now measurements at such a large momentum transfer have only been made at one angle, namely the smallest angle which could be selected with the existing apparatus at CEA. At the larger angles the cross-sections were too small to yield reasonable counting rates. Upper limits on the form factors were obtained by attributing the entire cross-section either to the $G_E^p$ term, thus putting $G_E^p$ equal to zero, or vice versa. In Fig. 10 the upper limit for $G_M^p/(1 + x)$ is given as an "inverted ground" at a four-momentum transfer of $6.81 \ (\text{GeV/c})^2$.

It would be very interesting to know in detail how the form factors decrease towards zero at large momentum transfers because this would make it possible to test a number of predictions based on different theoretical ideas. Dr. Beckmann has outlined in his lectures that according to dispersion theoretical ideas the asymptotic behaviour of the form factors may be expected to be given by the power law $1/q^2$. According to some other speculations it should be possible to relate e-p scattering at high four-momentum transfers to p-p scattering measurements which extend up to momentum transfers of $244 \ (\text{GeV/c})^2$. Yang and Wu suggested that the rapid decrease of the differential cross-section for p-p scattering with increasing momentum transfer is due to the fact that with increasing momentum transfer it is increasingly unlikely that the protons stay intact during the collision. Since in e-p scattering only one proton is involved and since furthermore the electron is not surrounded by an extended cloud of strongly interacting particles, Yang and Wu suggest that the elastic e-p scattering cross-section should be proportional to the square root of the p-p scattering cross-section. This implies that the electromagnetic form factors of the nucleon could be proportional to the fourth root of the p-p scattering cross-section. According to Orear, the p-p scattering cross-section can be fitted quite well by

$$\frac{d\sigma}{dp^2} \bigg|_{\text{pp}} = A \cdot e^{-\frac{p \cdot \sin \theta}{0.75}},$$

where the transverse momentum transfer $p = p \cdot \sin \theta$ is expressed in units of inverse fermi. In order to establish the kinematical relationship between p-p and e-p scattering, Yang and Wu suggest that the transverse momentum transfer in p-p scattering should be replaced by $\sqrt{q^2}$ in e-p scattering. According to these speculations, the form factors should then decrease as $e^{-\sqrt{q^2}/3}$, where $\sqrt{q^2}$ is given in units of $\text{f}^{-1}$. As can be seen from Fig. 11 the experimental data can be fitted quite well by the exponential $e^{-\sqrt{q^2}/3}$ but the presently available experimental data are also compatible with a $1/q^2$ dependence in the asymptotic region.
B. Neutron

The neutron form factors are not as well known as the proton form factors mainly because of the difficulties in the theoretical interpretation of e-d scattering experiments. Elastic e-d scattering cross-sections have been measured up to momentum transfers of 0.3 (GeV/c)^2 or 8 f^{-2}. Inelastic e-d scattering data are available \(^{13}\) up to q^2-values of 6.8 (GeV/c)^2 or 175 f^{-2}. However, the measurements at q^2-values q^2 \geq 2.9 (GeV/c)^2 or 75 f^{-2} yield only upper limits on the neutron form factors since at these large momentum transfers measurements have thus far only been made at one scattering angle. Some of the more recent neutron form factor values derived from inelastic e-d scattering work are given in Fig. 12. In this figure, G^2_N/k_n and (G^2_E)^\perp are plotted versus the four-momentum transfer q^2. The reason for plotting (G^2_E)^\perp instead of G^2_E is that, because of the zero net charge of the neutron, the inelastic e-d scattering experiments do not allow us to determine the sign of G^2_E. Upper limits for the magnetic form factor of the neutron, including two standard deviation errors, are shown as inverted bars in Fig. 12. The plot extends up to a q^2 of 4 (GeV/c)^2, although upper limits on G^2_E are available up to momentum transfers of 6.8 (GeV/c)^2.

The available experimental information on the electromagnetic form factors of the neutron can be summarized as follows:

1. Both neutron form factors are smoothly varying functions of q^2. The relation G_E = G^\perp_M/k_n does not hold for the neutron in the range of four-momentum transfers where it was found to be valid in the case of the proton. For the neutron it is found that at least up to q^2 \approx 1 (GeV/c)^2,

   \[
   (G^2_E) \ll \left( \frac{\pi}{k_n} \right)^2 .
   \]

2. Just as in case of the proton, both form factors of the neutron seem to go towards asymptotic values at large momentum transfers which are not significantly different from zero. Therefore, a "hard neutron core" seems to be excluded. We do not know yet how rapidly the neutron form factors decrease with increasing momentum transfer. The magnetic form factor data which are thus far available in the asymptotic region can be fitted equally well by the power law 1/q^2 or by the exponential e^{-c\sqrt{q^2}}.

3. The precise behaviour of the electric form factor of the neutron is unknown. According to the data obtained in inelastic e-d scattering experiments shown in Fig. 12, (G^2_E)^\perp may be significantly different from zero and positive in the region of momentum transfers between 5 and 15 f^{-2}. Figure 13 gives some more neutron form factor data, especially in the region of small momentum transfers. The G^2_E values at q^2 \leq 8 f^{-2} were derived from elastic e-d scattering experiments and are compatible with the assumption G^2_E(q^2) = 0. This is not only inconsistent with the electric form factor data shown in Fig. 12, but is also clearly inconsistent with the results of experiments on the scattering of thermal neutrons by electrons bound in atoms. According to these experiments, \( (dG^2_E/dq^2)_0 = 0.021 \text{ f}^{-2} \), which implies that (G^2_E)^\perp \approx 4.4 \times 10^{-4} \cdot q^4 at small momentum transfers (q^4 is expressed in units of f^{-4}). In order to demonstrate our lack of knowledge of G^2_E, a third plot of (G^2_E)^\perp versus q^2 is shown in Fig. 14. These form factor data were derived by Hughes et al.\(^{14} \). He even obtained negative (G^2_E)^\perp values for the neutron, a result which obviously makes no sense since the electromagnetic form factors must be real functions of q^2 in the
region of space-like momentum transfers. This clearly demonstrates that in spite of all the
work which has been devoted in the last ten years towards the investigation of the neutron form
factors, we do not yet know the detailed behaviour of \( G_E^p \) as function of \( q^2 \). All we know is
that the electric form factor of the neutron is considerably smaller than unity at all momentum
transfers investigated thus far. This is not inconsistent with SU6, which predicts \(^{(15)}\) that the
ratio \( C_E^p(q^2)/C_E^n(q^2) \) should be equal to zero. Since it is experimentally well established that
\( G_E^p(q^2) \neq 0 \), SU6 predicts \( C_E^p(q^2) = 0 \). Thus far, the experiments have only shown that
\( C_E^p(q^2) \ll G_E^p(q^2) \) up to momentum transfers of 1 (GeV/c)\(^2\). Also at higher momentum transfers,
the SU6 prediction is not excluded by the presently available experimental data.

SU6 makes a second prediction, according to which the ratio of the magnetic form
factors of the neutron and proton should be equal to the ratio of their magnetic moments which
is minus 3/2. As can be concluded from Fig. 14, the prediction is very well fulfilled at least
up to four-momentum transfers of 30 f\(^{-2}\).

With the exception of the electric form factor of the neutron, all electromagnetic
form factors of the nucleon coincide, provided \( G_E^p \) and \( G_M^p, G_M^n \) are normalized to 1 at \( q^2 = 0 \).
This implies, as is summarized in Fig. 15, that the corresponding rms radii of proton and neu-
tron agree within the experimental errors. They turn out to be 0.85 f.

As was discussed in Dr. Beckmann's lectures, one can try to understand the \( q^2 \) dependence
of the electromagnetic form factors of the nucleon in terms of a vector-meson cloud sur-
rrounding the bare nucleon. The properties of the form factors in this model are thought to
arise chiefly from the electromagnetic coupling of the photon to these heavy virtual vector
mesons. This leads to the Clementel-Villu fit for the isoscalar and isovector form factors,

\[
G(q^2) = G(0) - \sum \frac{c_i}{1 + \frac{q^2}{m_i^2}}
\]

in which the constant term represents the contribution of a "hard core" and/or the contribution
of very heavy hadrons, i.e. the "soft core". According to our present ideas, the virtual mes-
on cloud is thought to be composed of the known vector mesons \( \rho, \phi \) and \( \omega \). These are con-
dered to be the resonances which dominate the behaviour of the electromagnetic form factors in
the space-like region but as long as one is not willing to treat the \( \rho \) mass as an adjustable
parameter these three known vector mesons are not adequate to get a more or less satisfac-
tory account of the nucleon form factors. If one treats the \( \rho \) mass as an adjustable para-
meter, a reasonably good fit of the experimental data up to at least \( q^2 = 30 \) f\(^{-2}\) is obtained
for an effective \( \rho \) mass of about 550 MeV\(^{(14)}\). An alternative procedure is to introduce at
least one more vector meson, called the \( \rho' \), with a fictitious mass of at least 950 MeV.
But although quite a few new meson resonances have been reported in the recent literature
none of these seems to have the proper quantum numbers to contribute to the isovector nucleon
form factor \( (J^{PC} = 1^{--}, I = 1) \).

The most recent attempt to fit the form-factor data with four-resonant states
\( (\rho, \varphi, \omega \) and the fictitious \( \rho' \) \) and no-core terms resulted in the expressions
\[
C_{ES} = \frac{1.24}{1 + \frac{q^2}{15.8}} - \frac{0.74}{1 + \frac{q^2}{26.7}} \\
C_{EV} = \frac{2.01}{1 + \frac{q^2}{14.5}} - \frac{1.51}{1 + \frac{q^2}{23.0}} \\
C_{MS} = \frac{1.12}{1 + \frac{q^2}{15.8}} - \frac{0.68}{1 + \frac{q^2}{26.7}} \\
C_{MV} = \frac{6.23}{1 + \frac{q^2}{14.5}} - \frac{3.87}{1 + \frac{q^2}{23.0}}
\]

where the mass values of the resonant states and \(q^2\) are expressed in units of \(f^{-2}\).

It may seem that there were a total number of eight adjustable parameters plus the mass value \(m_{\rho'}\) of the fictitious vector meson \(\rho'\) with isospin 1. This, however, is not true. One has two constraints for the condition \(C_E = C_M\) at \(q^2 = -4M^2\), four constraints from the normalization condition at \(q^2 = 0\) and one further constraint from the measured derivative of \(C_E\) with respect to \(q^2\) at \(q^2 = 0\). Thus, there remains only one free parameter plus the adjustable fictitious mass \(m_{\rho'}\). The best fit is obtained for \(m_{\rho'} = 945\ MeV \pm 23.0\ f^{-2}\) but the fit is far from good. The fit can be improved by replacing the actual \(\rho\) mass of \(760\ MeV \pm 14.5\ f^{-2}\) by a somewhat smaller effective \(\rho\) mass. If the fictitious isovector \(\rho'\) meson is omitted (one-pole approximation for the isovector form factor) the best fit is obtained for an effective \(\rho\) mass of about \(550\ MeV\). It is important to remember that one should not expect too good a fit since non-resonant pion states which are not included in the Clementel-Villi formula may contribute to the form factors. Furthermore, the Clementel-Villi form factor formula is obtained by replacing the spectral functions or weight functions of the vector resonant states by \(\delta\) functions. This is a good approximation for the \(\omega\) and \(\varphi\) meson but the approximation cannot be justified for the 100 MeV wide \(\rho\) state. These are some of the reasons why one should not attach too much importance to these fits. Remember that the more pole terms are included the more adjustable parameters are available. Every new vector meson brings into the fit a new pole term and its unknown coupling constants to the photon and the nucleon-antinucleon pair. The more resonant states are introduced, the better will be the fit, whether the theory is right or wrong.
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Figure Captions

Fig. 1 Ratio A/B of electric and magnetic coefficients in Rosenbluth formula
\[ \frac{\sigma}{\sigma^0} = A_E^2 + B_M^2 \] versus electron scattering angle for fixed \( q^2 \) values. Dotted curves indicate accessible region for various incident electron energies.

Fig. 2 Ratio A/B of electric and magnetic coefficients in Rosenbluth formula
\[ \frac{\sigma}{\sigma^0} = A_E^2 + B_M^2 \] versus electron scattering angle for fixed incident electron energies. Four momentum transfers can be derived from right hand diagram\(^2\).

Fig. 3 Typical experimental arrangement for the investigation of electron-nucleon scattering.

Fig. 4a and 4b The quadrupole spectrometers for the investigation of electron-nucleon scattering using an internal electron beam at DESY/Hamburg.

Fig. 5 Cross-sections for elastic e-p scattering. Above \( q^2 = 100 \) \( f^{-2} \) extrapolated cross-sections are given assuming that both proton form factors decrease as \( 1/q^2 \) with increasing momentum transfer\(^2\).

Fig. 6 Radiative corrections for elastic e-p scattering at small angles\(^3\).

Fig. 7 Experimental "Rosenbluth plots".

Fig. 8 The calculated deuteron form factors of Glendenning and Kramer\(^6\) versus the four-momentum transfer \( q^2 \).

Fig. 9 Proton form factors versus \( q^2 \) (N.F. Ramsey, Proc. of the Intern. Conf. on High Energy Physics, Dubna 1964).

Fig. 10 Proton form factors versus \( q^2 \) at large momentum transfers\(^4\).

Fig. 11 Proton form factors \( G_E^p \) and \( G_M^p \) versus \( \sqrt{q^2} \)\(^5\).

Fig. 12 Neutron form factors at large four-momentum transfers obtained from inelastic e-d scattering\(^4\).

Fig. 13 Neutron form factors at small four-momentum transfers.

Fig. 14 Neutron form factors versus \( q^2 \)\(^6\).

Fig. 15 Experimental derivatives of the nucleon form factors at \( q^2 = 0 \) and corresponding electric and magnetic rms-radii.
Ratio of Electric and Magnetic Coefficients in Rosenbluth Formula

\[ G = A G_E^2 + B G_M^2 \]

Fig. 1
Fig. 2
Fig. 3
Fig. 6

Fig. 7
Fig. 8

Fig. 9

- $G_{np}$/B(k)
- $G_{np}$ Janssen et al. (To be published)
- $G_{np}$
- $G_{np}(1+k)^2$ Chen et al. (To be published)
- $G_{np}$ Dunning et al.
- with $G_{np}$ used Corrected -PRL, 13500 (1983)
- in Table II
- $G_{np}$ Lehman et al.
- $G_{np}$ 
- $G_{np}$ Frauenjaque et al.
- $Q^2$=175 $G_{n}$<0.09 if $G_{n}$=0
- $Q^2$<0.45 if $G_{n}$=0

PROTON ELECTROMAGNETIC FORM FACTORS ($G_{np}$) and ($G_{np}$)

Electron-proton form factors.
Proton form factors.

Fig. 10

Fig. 11
EXPERIMENTAL DERIVATIVES OF THE FORM FACTORS
AT ZERO MOMENTUM TRANSFER AND CORRESPONDING RMS-RADI

PROTON

\[ \frac{dG_p}{dq^2}(0) = -\left(0.118 \pm 0.004\right) F^2 \]
\[ a_E = (0.84 \pm 0.05) F \]
\[ a_P = (0.85 \pm 0.02) F \]

NEUTRON

\[ \frac{dG_n}{dq^2}(0) = +\left(0.021 \pm 0.001\right) F^2 \]
\[ a_E = (0.355 \pm 0.001) F \]
\[ a_N = (0.90 \pm 0.16) F \]
VARIATION OF IONIZATION WITH VELOCITY

C. O'Ceallaigh,
Dublin Institute for Advanced Studies

I. INTRODUCTION AND SUMMARY OF THEORY

The aim of the talk will be to present an account of the current situation in the field of the variation of ionization with velocity in condensed media, based on work which involves the use of photographic emulsions. Several review articles have been published which cover the topic. They are listed in the bibliography, together with what is believed to be a fairly complete list of papers and communications. The fundamental theoretical papers associated, among others, with the names of Bethe, Bloch, Fermi, Halpern, Hall, and Sternheimer, may be found in a convenient collected form in the "Series of Selected Papers in Physics" under the title "Electron Transfer Processes". These have been printed by the Physical Society of Japan for the use of its domestic members only, but might be made available through the kind offices of a Member.

For obvious reasons, much experimental work has been carried out in nuclear emulsions, and the variation of the values of various parameters which measure energy loss by ionization has been studied over a very extensive interval of \( \gamma = (1 - \beta^2)^{-1/2} \), ranging from \( 1.1 \leq \gamma \leq 10^4 \). The parameter used is "grain density" \( \gamma \), defined and measured in various ways, blob density \( r \), mean gap length \( G \) and the Fowler-Perkins coefficient \( T = \frac{1}{\gamma} \). The study of the \( \gamma \) dependence of those parameters has been carried out with two distinct aims. These are: (1) calibration measurements with a view to obtaining a reliable tool for the estimation of the velocity of a particle of known charge from experimental estimates of ionization, and (2) an attempt to verify the theoretical expressions which predict the variation with \( \gamma \) of the so-called "restricted ionization loss". We must now define our terms of reference.

The collisions in which a moving charged particle parts with a quantity \( \Delta E \) of its energy to an electron in an atom of the absorber, may be divided into two categories (Uehling loc.cit. p.317):

a) close collisions in which the energy transfer exceeds a certain chosen threshold \( \Delta E_0 \), and

b) distant collisions in which \( \Delta E < \Delta E_0 \).

\( \Delta E_0 \gg \) binding energy of the electron, but it must be sufficiently small that for energy transfer of the order of \( \Delta E_0 \) or less, the effective collision parameter is large compared with atomic dimensions, so that we may regard the charged particle as being a point-particle.

For \( \Delta E_0 \) of the order \( 10^{-4} \) to \( 10^{-5} \) eV, both conditions are satisfied. For energy transfers \( > \Delta E_0 \), however, the energy transfer depends on the mass and spin of the particles.

In the case of grain-counts in emulsion, we may ignore the effect of close collisions since they will tend to give rise to energetic \( \delta \) rays, which, unless in the forward direction, are unlikely to be included in the measured value of the chosen parameter.
We may start with the fundamental formula of Bethe-Bloch which is valid for distant collisions defined as above, and reads

\[
-(dE/dx)_{\Delta E < T_0} = \frac{2m(Z_1 e^2)^2}{m_0 v^2} \ln \left(\frac{2m_0 v^2 T_0^{1/2}}{(1-\beta^2)^{1/2}}\right) - \beta^2
\]

(1)

where \(m_0/e\) are electronic mass/charge, \(Z_1\) is the charge of the ionizing particle, \(T_0\) is the maximum energy transfer in any admitted collision, and will be less than \(\Delta E_0\). \(\beta\) is a mean excitation potential of the atoms of the absorber. The other symbols have their usual meanings.

In order that (1) be valid, \(137\beta >> 2Z_1 Z_2\), where \(Z_2\) is the effective atomic number of the atoms in the absorber. Even for the singly-charged particles this condition is not satisfied by AgBr (\(Z_2 = 41\)), but it has been shown by Walske\(^4\) that the consequences of nonsatisfaction of this condition are negligible in the region with which we shall be concerned.

The fundamental formula (1) has been modified by Halpern and Hall, and by Sternheimer, to take into account the polarization of the medium by the field of the charged particle which limits the rate of energy loss in the extreme relativistic region, an effect first considered by Fermi\(^5\).

Sternheimer has expressed the effect of polarization by a simple modification to the fundamental formula (1) as follows:

\[
-(dE/dx)_{\Delta E < T_0} = \frac{2m(Z_1 e^2)^2}{m_0 v^2} \ln \left(\frac{2m_0 v^2 T_0^{1/2}}{(1-\beta^2)^{1/2}}\right) - \beta^2 - \delta(\beta)
\]

(2)

We may rewrite this expression in the form

\[
-(dE/dx)_{\Delta E < T_0} = P\beta^2 \left[Q + \ln(\gamma^2 - 1) - 1 + \gamma^{-2} - \delta(\beta)\right]
\]

(3)

where \(Q = \ln[2m_0 c^2 T_0/\beta^2]\) and \(P = 2m_0 Z_1^2 e^4/m_0 c^2\). In what follows, since we shall have to deal only with normalized measurements, we will not be concerned with the value of \(P\). The Sternheimer function \(\delta(\beta)\) will be such as to tend to cancel the logarithmic term \(\ln(\gamma^2 - 1)\) which causes the energy loss to diverge as \(\beta \to 1\). Sternheimer\(^6\) has given an analytic sufficient approximation to \(\delta(\beta)\) in the following form, in terms of \(2x = \log_{10}(\gamma^2 - 1) = (2.503)^{-1} \ln(\gamma^2 - 1)\), namely,

\[
a) \text{ for } x \leq x_0; \quad \delta(\beta) = 0,
\]

\[
b) \text{ for } x_0 \leq x \leq x_1; \quad \delta(\beta) = 2.303 \log_{10}(\gamma^2 - 1) + C + e(x_1 - x)^m,
\]

\[
c) \text{ for } x_1 \leq x; \quad \delta(\beta) = 2.303 \log_{10}(\gamma^2 - 1) + C.
\]

(4)

Inserting the above values of \(\delta(\beta)\) in (3), we have,

\[
x_0 \leq x \leq x_1; -(dE/dx)_{\Delta E < T_0} = P\beta^2 \left[Q - C - 1 + \gamma^{-2} - a(x_1 - x)^m\right]
\]

(5)

where \(-C = \ln(\gamma^2 - 1) + a(x_1 - x_0)^m\). Furthermore, Sternheimer shows that

\[
-C = 1 + 2\ln(\nu'/\nu_p)
\]

(6)
where \( \nu_m \) is the mean excitation frequency, and \( \nu_p = \left[ \frac{ne^2}{\pi \hbar m} \right]^{1/2} \) = plasma frequency of the absorbing medium.

II. EXPERIMENTAL WORK IN EMULSIONS

During the past 15 years, much experimental work has been carried out using photographic emulsions. The work may be considered under the following headings:

i) work based on a study of cosmic-ray particles, both primary and secondary, in plates exposed by means of high-flying balloons;

ii) work involving a comparison of the ionization of known particles produced by machines and injected into plates at known momenta.

The earlier work*) in category (i) need not be described in detail, except to observe that the existence of a rise to plateau from the minimum value was first established for photographic emulsion by Pickup and Vojvodic7), and the first satisfactory comparison between theory and experiment was made by Stillier and Shapiro8).

Later work was carried out by machine-produced particles, the pioneering work of this nature being that of Michaelis and Violet (1955)9), who injected electrons at energies corresponding to the expected values at minimum and plateau, namely 2.93 and 293 MeV and found a P/T ratio of 1.087 ± 0.01. We may note that this value is probably an underestimate because of the difficulty of excluding background grains when blob-counting on heavily scattered tracks.

In addition, the level of development was high, 26/100 \( \mu \) at plateau. The same is true but even to a greater degree (27–40 blobs/100 \( \mu \)), in the work of Fleming and Lord (1953)10), in which comparison was made between pions of various momenta and the tracks of electrons of average energy, 34 ± 10 MeV secondary to \( \mu \) decay. They obtained a result similar to that of Michaelis and Violet, but of less precision. Alexander and Johnston (1957)11), carried out an investigation using plates which had been exposed at the Bevatron and had been used for a careful investigation of the relative frequency of the decay modes of \( K^- \) mesons. The secondary particles of the two-body decay modes \( K_{\mu 2} \) and \( K_{\pi 2} \) were compared with those of beam pions, and an attempt was made to fit the curve obtained and to obtain an estimate of \( Q \), equation (5). This work was extended into the region beyond the minimum by Johnston, O'Ceallaigh, Prowse and Shaukat (1960)12) using machine-produced pions of various momenta which were compared with the tracks of electrons produced at the Caltech Electron-Synchrotron (Fig. 1). In addition, other plates were used which had been exposed to the 4.3 \( \pi^- \) beam at Berkeley, the electrons, in this case, being those from high-energy pairs, the end product of the decay of \( \pi^0 \) mesons produced in interactions by the \( \pi^- \) beam mesons. This procedure is a good guarantee that the beam tracks and the electrons were both registered during the period of the exposure and the results are unlikely to have been affected by the temperature differences and the occurrence of differential fading. The latter

*) See bibliography and references.
technique was used in the extensive investigation of Jongejans (1960)\textsuperscript{13} who used an exposure to $\pi^-$ mesons produced in the Bevatron within the momentum range 5.2 - 5.7 GeV/c. In the last two investigations an attempt has been made to compare the observed values of normalized grain-counts with the prediction of Sternheimer, and the results will be described in more detail later. A table, (3.1), summarizing most of the foregoing work has been drawn up by Venus\textsuperscript{14}, Thesis, Bristol 1964.

We may summarize, as follows, the conclusions to be drawn from the above investigations. Although there were many differences between the results, some of which might be explained on the basis of differences in procedure, there was fairly general agreement that the P/T ratio lay between 1.06 and 1.14, while the plateau appeared to have been reached at $10 < \gamma < 200$. The results of the experiments could be fitted by curves of the same general form as in Section III (Fig. 2), but there were wide discrepancies between the values of the parameters in equations (7) and (8) which could be derived from the various attempts at fitting.

In the meantime, some doubts had arisen as to whether the shape of the experimental curve and the value of the observed P/T ratio was, in fact, dependent on the degree of development and also on the type of emulsion employed, in so far as the size of the undeveloped crystals varied from emulsion to emulsion. It was felt also that, in many cases, the temperatures of the various emulsions had not been sufficiently well controlled to avoid criticism on the grounds that the sensitivity of the emulsion might be different at the times of injection of the various particles, and might even vary from region to region in the emulsion stacks. Finally, the question of the possible effects of differential fading gave rise to some uneasiness.

Accordingly, a series of more carefully controlled exposures was devised which will now, be described in summary. Two features have characterized these experiments. They have been dogged by misfortune due to malfunction of the machines and exposure gear at critical junctures. The results have become more difficult to interpret and, in fact, the more refinement has been introduced, the greater has been the apparent inconsistency of the results: Some of the trouble would appear to have arisen from the fact that the attempts to achieve temperature control by the refrigeration of the stacks had been nullified by uneven and imperfect warming up of the pellets due to the poor thermal conductivity of emulsion. The steps used to control fading have probably been more successful. They were, a) the enclosure of the assembled stacks in envelopes of light-tight plastic which obviates changes in humidity and, b) the allowance of an adequate interval between exposure and development, so that the effects of differential fading could be neglected.

Stillier (1963)\textsuperscript{15}, exposed a composite stack to a $\pi^-$ beam of nominal energy 450 MeV at the Brookhaven Cosmotron and, subsequently, to $e^-$ beams of nominal energies 1000, 450, 210, and 100 MeV at the Cornell Synchrotron. The emulsions used, together with the undeveloped crystal sizes for each (in microns), were as follows: Ilford G.5 (0.27), K.5 (0.20), L.4 (0.14), Nikfi Br (0.20), Kodak NTB 4 (> 0.27). It was found that the P/T ratio for blobs was 1.07 for G.5, 1.09 for K.5, and 1.13 for L.4. This result was in some contradiction with the earlier results of Stillier and Shapiro, which gave P/T = 1.14 for G.5 developed to a similar degree, namely, light to moderate. Evidence was put forward for the existence of an increase of P/T with decreasing crystal size. At the same time, it was felt that the emulsion packets
had not been given an adequate time to warm up to a constant temperature before exposure to the electron beams. This explanation for the low G.5 value seems plausible at first sight. On closer scrutiny, however, it would appear that the evidence for the low P/T for G.5 and for variation of P/T with crystal size could be acceptable only on the hypothesis that sensitivity decreases with rising temperature in the range -20°C to +20°C. This assumption appears to be in disagreement with available evidence that the sensitivity gradient is positive in the range temperatures with which it would have been concerned. It would appear from Fig. 1 of their text and the description of their procedure that, at all relevant times, the G.5 pellicles must have been at a higher temperature than those either of K.5 and L.4.

The results of an extensive experiment by Buskirk et al.\textsuperscript{(6)} using blob-counts in Ilford K.5 plates exposed to 16 GeV/c π⁻ at CERN, and using the technique of comparison with high-energy electrons from pairs caused by π⁰ mesons produced in interactions by the primary particles, are in excellent agreement with those of Jongeijans and with Johnston et al., who used G.5, and yield the same P/T ratio of ~ 1.14.

A collaborative experiment to investigate the ionization of different emulsions exposed simultaneously to high-energy scattered-out protons and high-energy electrons at CERN, was carried out in 1961 by the Bristol-Dublin group\textsuperscript{(7)}. The stack of plates were very carefully maintained at constant temperature during exposure, and hermetically sealed in a light-tight container to prevent variations in humidity. The emulsions used were G.5, K.5, and hypersensitized G.5 and L.4, and different procedures were used to produce degrees of development varying from light to heavy. The whole exposure was carried through within a period of eight hours, and the exact times of exposure to the various beams were recorded as follows:

<table>
<thead>
<tr>
<th>Exposure</th>
<th>Type of particle</th>
<th>( \gamma )</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Protons 24 GeV/c</td>
<td>26.6</td>
<td>7.30</td>
</tr>
<tr>
<td>2</td>
<td>Protons 12 GeV/c</td>
<td>13.8</td>
<td>12.10</td>
</tr>
<tr>
<td>3</td>
<td>Protons 8 GeV/c</td>
<td>9.5</td>
<td>12.50</td>
</tr>
<tr>
<td>4</td>
<td>Positive Particles 5 GeV/c</td>
<td>5.4 - 35.8</td>
<td>14.30</td>
</tr>
<tr>
<td>5</td>
<td>Photons, etc.</td>
<td>( 3 \times 10^4 )</td>
<td>14.55 - 15.15</td>
</tr>
<tr>
<td>6</td>
<td>Protons 24 GeV/c</td>
<td>26.6</td>
<td>15.30</td>
</tr>
</tbody>
</table>

The exposures 1 and 6 were designed to test whether effects had occurred which could be attributed to changes of sensitivity during the time of exposure due to possible temperature changes, to fading, or possibly to a combination of both. The results suggest that \( g_6 \) was, on the average, \( 0.9 \pm 0.5\% \) higher than \( g_1 \). This is reasonably satisfactory. Certain anomalies appeared with the hypersensitized plates which need not be discussed here. While the work is still in progress, statistically significant evidence appears to have been found for a decrease in the ratio
P/T with increasing degree of development. The variation appears to be well represented by a straight line of negative slope.

If this observation is correct, it would help to explain the discrepancies between the low P/T ratios found, for example, by Michaelis and Violet, and Fleming and Lord whose plates were heavily developed, and those of others who employed plates developed to more orthodox levels.

III. PROCEDURE FOR COMPARING EXPERIMENTAL MEASUREMENTS WITH THEORY OF BETHE-BLOCH AND STERNHEIMER

Clearly, it is of interest to see if it is possible to fit the march of the ionization curve with an expression of the type (1) Bethe-Bloch (B.B.) or, in the appropriate energy region, by the modified type proposed by Sternheimer (S.). A procedure elaborated by the present author will now be described.

In experimental work it is necessary to normalize ionization measurements to some standard point. In this work we choose the limit of (4) as \( \beta \to 1 \), namely (4c) the plateau value, as the standard point, and express ionization measurements at lower velocities in terms of the observed ionization at plateau. We leave open, for the moment, the question of what ionization parameter \( (g) \) measures the restricted energy loss, but suppose it to be known, and that \( - \begin{pmatrix} dE/dx \end{pmatrix}_{\Delta E < T_0} = Kg. \) As usual, we set \( Kg(\gamma)/Kg(\text{plateau}) = g. \)

For \( x = x_1 \), Sternheimer’s expression (5) may be written

\[
- \begin{pmatrix} dE/dx \end{pmatrix}_{\Delta E < T_0} = \frac{Bv^2}{\gamma - 1} \left[ Q - C - 1 \right] \simeq P [Q - C - 1] = PB^{-1} \tag{2a}
\]

since for \( x = x_1 \), \( \gamma^2 > 4 \times 10^4 \). Hence, we have the two following expressions for restricted energy loss normalized to plateau:

**Bethe-Bloch (\( x \leq x_0 \))**

\[
* \quad g_{B,B.} = \frac{Bv^2}{\gamma - 1} \left[ Q - 1 + \gamma^{-2} + \ln (\gamma^2 - 1) \right] \tag{7}
\]

and

**Sternheimer (\( x_0 \leq x \leq x_1 \))**

\[
* \quad g_S = \frac{Bv^2}{\gamma - 1} \left[ Q - C - 1 + \gamma^{-2} - a(x_1 - x)^m \right] \tag{8}
\]

and finally, of course,

\[
* \quad g = 1 \quad \text{for} \quad x > x_1
\]

where \( B = (Q - C - 1)^{-1} \), and \( -C = \ln (\gamma_0^2 - 1) + a(x_1 - x_0)^m \).
Since there is no practicable experimental method of determining \( x_0 \), we must assume the value given by Sternheimer, namely, \( x_0 = 0.23 \) noting, however, that its theoretical value is not critically determined in Sternheimer's treatment.

For \( x = x_0 \), therefore, (7) and (8) have the same value, so that \( \frac{d}{dx} \left( \frac{S_{B_s, B_s}}{S_s} \right) = 0 \). In addition, however, since the curve must be continuous at the point \( x = x_0 \), we have the following constraint

\[
\lim_{x \to x_0} \frac{\Delta S_{B_s, B_s}}{\Delta x} = \lim_{x \to x_0} \frac{\Delta S_s}{\Delta x}.
\]

One rather suspects that the necessity of satisfaction of (9) has been overlooked by previous workers in the field.

Differentiating (7) and (8), we find

\[
d/dx \left( \frac{S_{B_s, B_s}}{S_s} \right) = \frac{-2B/M}{\gamma^2 - 1} \left[ Q + \ln(\gamma^2 - 1) - \gamma^2 \right]
\]

and

\[
d/dx \left( \frac{S_{B_s, B_s}}{S_s} \right) = \frac{-2B/M}{\gamma^2 - 1} \left[ Q - C - a(x_1 - x) \ln \left( 1 + \frac{\gamma^2 M}{(x_1 - x)} \right) \right]
\]

\( N^{-1} = \log_{10} e. \)

Proceeding to the limit \( x \to x_0 \) and equating, we find the important constraint condition

\[
2/M = ma(x_1 - x_0)^{N^{-1}}.
\]

Furthermore, from (10) and (11) we find the following conditions for the existence of a minimum value of \( g \), namely,

\textbf{Bethe-Bloch}

\[
Q = \gamma_m^2 - \ln(\gamma_m^2 - 1)
\]

\textbf{Sternheimer}

\[
Q - C = a(x_1 - x_m)^m \left[ 1 + \frac{\gamma_m M}{(x_1 - x_m)} \right]
\]

where \( x_m \) and \( \gamma_m \) are the values of those parameters at minimum. For \( x_0 \to x_m \) it is easy to show that (14) reduces to (13).

Now, let us consider the results of any experimental determination of \( g \) as a function of \( x = \frac{1}{2} \log_{10} (\gamma^2 - 1) \). By definition, \( g \) is supposed to be linearly proportional to the restricted energy loss. We may expect, therefore, if the Sternheimer corrections to the B.B. theory are valid, that the experimental values of \( g \) will follow a curve of the form shown in

\[
\text{(13)}
\]

\[
\text{(14)}
\]
Fig. 2, where the dotted portion is given by (7), and the full line portion is given by (8), with choice of the Sternheimer parameters which is appropriate to the medium, in this case, emulsion.

Procedure for fitting

Using our experimental points which give $g$ as a function of $x$, we may derive good estimates of $g_0$ and also of $g_m$, which is the reciprocal of the plateau/trough ratio ($P/T$). From the march of the curve we may obtain a sufficient estimate of $x_1$ and an adequate, if approximate, estimate of $x_m$. We know that the dotted portion of the curve must satisfy (7) which involves only two parameters $B$ and $Q$ which, however, are not independent.

As far as the B.B. region is concerned, we may proceed to estimate the pair of values of $B$ and $Q$ which give a best fit to such experimental values of $g$ as are available in the region. Alternatively, as is quicker, we may assume first a sequence of values for $Q$ and find the corresponding values of $B$ which fit the observed experimental value of $g_0$. We may then examine in what manner that choice affects the fitting in the Bethe-Bloch (dotted) region. In this preliminary discussion I use two values only of $Q$, namely $Q = 12$ and $Q = 17$. Since $B^{-1} = Q - C - 1$ we obtain through this procedure a value of $-C$ corresponding to each choice. From (12), and the definition of $-C$, we find that value of the exponent $m$ which ensures continuity at the point 0 using the expression

$$m = \frac{2/(M(x_1 - x_0))}{-C - \ln(18 - 1)}.$$  

We substitute in (15) the value of $x_0 = 0.23$ given by Sternheimer for emulsion, and of $x_1$ as estimated from the behaviour of the experimental points for large values of $x$. Inserting in (12) the value of $m$ thus obtained, we find the corresponding value of the parameter $a$. We are now in possession of all the elements of (8), and we plot that Sternheimer curve which joins smoothly with the B.B. curve at the point 0 and yields the experimentally observed value of $g_0$.

In summary, we have assumed that the parameter used to estimate ionization is linearly proportional to the restricted energy loss. We have anchored the fitting curves at two points, namely at 0 and $x_1$, and we may consider how they agree with the experimentally observed values of $g$ in the B.B. and S. regions for two choices of the value of $Q$. Choice of $Q$ fixes $-C$ and $B$ because we require that $g_0$ agrees with the experimentally observed ratio which is $\sim 1$ for $x_0 = 0.23$.

We may make use of the foregoing to consider the fitting of the experimental results of Johnston et al., and Jongejans by means of theoretical expressions of the type (7) and (8). The experimental points and fitting curves are shown in Fig. 3. Those of Jongejans are distinguished by black squares labelled J. They chose as $g$, the parameter $g = (G + a_0)^{-1}$. The results of Johnston et al. are denoted by open circles and are expressed in terms of $g = \Gamma = (G)^{-1}$. Those labelled A are derived from the earlier work of Alexander and Johnston, and were recalculated in terms of $\Gamma$, in the belief (perhaps mistaken) that $\Gamma$ is a better measure of restricted energy loss than is $(G + a_0)^{-1}$. The effect of the different choice of parameter is negligible in the S region. The agreement between the results of these investigations is very striking and, furthermore, they agree equally well with those of Buskirk et al.
We choose $x_0 = 0.23$, as given by Sternheimer, and note that in all three investigations the value of $g_0$ lies between 0.99 and 1.00. Two values for $Q$ are chosen, namely $Q = 12$ (full curves) and $Q = 17$ (dotted curves). We also include, for purposes of comparison, the continuation of the corresponding B.B. curves which ignores the effect of polarization. Two values of $x_1$ have been taken, namely $x_1 = 2.0$ and $x_1 = 2.5$, and the curves are labelled accordingly. The values of the various parameters are listed in Table II.

<table>
<thead>
<tr>
<th>Q</th>
<th>B</th>
<th>$g_0$</th>
<th>$-C$</th>
<th>$x_1$</th>
<th>$m$</th>
<th>$s_m$</th>
<th>P/T</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td>0.05968</td>
<td>0.99</td>
<td>5.755</td>
<td>2.5</td>
<td>0.6492</td>
<td>2.226</td>
<td>0.866</td>
</tr>
<tr>
<td>12</td>
<td>0.05968</td>
<td>0.99</td>
<td>5.755</td>
<td>2.0</td>
<td>1.743</td>
<td>1.736</td>
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</tr>
<tr>
<td>17</td>
<td>0.04288</td>
<td>1.00</td>
<td>7.321</td>
<td>2.5</td>
<td>1.593</td>
<td>1.669</td>
<td>0.854</td>
</tr>
<tr>
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<td>1.00</td>
<td>7.321</td>
<td>2.0</td>
<td>2.978</td>
<td>1.302</td>
<td>0.855</td>
</tr>
</tbody>
</table>

Scrutiny of Fig. 3 and Table II leads to the following conclusions. Choice of $Q = 12$ and $x_1 = 2.5$ gives excellent fitting in the S. region but, in the B.B. region, the experimental points tend to lie well above the curve. For $Q = 17$, the fit in the B.B. region is satisfactory, while in the S. region the fit is decidedly less good. It will be noted that because the curves are anchored at the point $g_0 \simeq 1$, the fit in the region $x_0 \leq x \leq x_m$ is very weakly dependent on choice of $Q$ and of $x_1$. It follows, as a corollary, that it is futile to use points in that region to obtain estimates of $Q$ as was done by Alexander and Johnston.

Now, we do not possess very reliable information on the value of $\Gamma$ for $x < 0.1$, but consideration of older results based on material from the Sardinian Flights of 1954 yields an estimate of $\Gamma = 1.575$ for $\gamma = 1.360$. This point, which we will call 'a', if reliable, lies well above that predicted by the choice $Q = 12$ and is in much better agreement with the curve for $Q = 17$.

Nevertheless, we must reject the choice $Q = 17$ for the following reasons. Since $Q = \ln \left(2\pi e^2 T_0 / I^2 \right)$, $I$, the effective excitation potential = $h\nu_m'$, and the plasma energy $= h\nu_p = h\left(ne^2 / mme\right)^{1/2}$, a function only of the electron density of the absorbing medium, we may obtain from (6) the following relationships:

$$-C = 1 + 2\ln I - 2\ln \left(h\nu_p\right)$$  \hspace{1cm} (16a)

and,

$$B^{-1} = Q - C - 1 = \ln T_0 + \ln 10^6 - 2\ln \left(h\nu_p\right) .$$  \hspace{1cm} (16b)
If we take $h v_p = 5.33 \times 10^4$ eV, the value given for AgBr by Sternheimer and substitute values from Table II, we obtain the following pairs of values of $T_0$ and $I$:

**Table III**

<table>
<thead>
<tr>
<th>$Q$</th>
<th>$C$</th>
<th>$T_0$ (eV)</th>
<th>$I$ (eV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td>5.755</td>
<td>$5.32 \times 10^4$</td>
<td>546.2</td>
</tr>
<tr>
<td>17</td>
<td>7.321</td>
<td>$3.81 \times 10^7$</td>
<td>1258</td>
</tr>
</tbody>
</table>

The question of the appropriate value of the mean excitation potential is discussed exhaustively in the review article of Uehling pp.333-5. An estimate of that value is obtained from experiments on the energy loss in various media of protons of known speed. The experiments of Bakker and Segrè led to an expression of the type $I = 9.4Z$, where $Z$ is the effective value of $Z$ for the relevant absorbing medium. Work by Barkas\(^{18}\), using photographic emulsion, gave the higher value $I = (12.1 \pm 0.2)Z$, and calculations by Caldwell, based on the experimental results of Sachs and Richardson, yielded an average value of $13 \bar{Z}$. We adopt the results of Barkas which for AgBr ($Z = 41$) predict the value $I = 496$ eV. Now, according to Sacton, the ranges of $\delta$ rays for the energies with which we are concerned are approximately as follows:

**Table IV**

<table>
<thead>
<tr>
<th>Energy of electron (keV)</th>
<th>Range ($\mu$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>0.3</td>
</tr>
<tr>
<td>30</td>
<td>7</td>
</tr>
<tr>
<td>500</td>
<td>300</td>
</tr>
</tbody>
</table>

Thus, a value of $T_0$ in excess of 30 keV would lead to the production of a $\delta$ ray of > 7 $\mu$. Since $\delta$ rays at higher energy tend to be projected initially in the forward direction, it is quite certain, as will be discussed later, that a proportion of the measured ionization consists of a contribution from $\delta$ rays superimposed on the primary ionization. Clearly, however, we may reject without hesitation an estimate of $T_0 = 5.81 \times 10^7$ corresponding to $Q = 17$ in Table III. On the other hand, the assumption $Q = 12$ leads to an acceptable pair of values $I = 546$ keV and $T_0 = 53$ keV, to a good fit in the S. region, but to an unsatisfactory fit in the B.B. region.
Since we have excluded the possibility of fitting the S. and the B,B. regions by the above methods, we may set out the logic of the fitting procedure in a slightly different manner.

Let us adopt the value given by Barkas\textsuperscript{18}, I = 496 eV. This fixes automatically the value of $-C$ from (6), or from (16a). For I = 496 eV we must have $-C = 5,461$. We must also have,

$$
* g_0 = (Q - C - 1)^{-1} \beta_0^{-2} \left[ Q - 1 + \gamma_0^{-2} + \ln(\gamma_0^{-2} - 1) \right]
$$

(17)

where $\gamma_0$ is the value of $\gamma$ corresponding to $x_0 = 0.23$. The value of $g_0$ is rather well determined. Substituting the values of $\gamma_0$ and $g_0$ in (17) we obtain a value of $Q$, and from this, using the expression (16b), a corresponding value of $T_0$. We find:

Table V

<table>
<thead>
<tr>
<th>$I = 496$ eV; $-C = 5,461$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$g_0$ (Exp.)</td>
</tr>
<tr>
<td>1.00</td>
</tr>
<tr>
<td>0.99</td>
</tr>
</tbody>
</table>

Table V is, of course, an extension of Table III, although the logical steps involved in setting it out may appear to be slightly different. From the foregoing it is clear that the values of $T_0$ thus obtained are perfectly satisfactory, but, since $Q = 12$ has been shown to lead to poor agreement with experiment in the B,B. region, $Q = 11.6$ and $Q = 11.2$ must lead to worse results in that region.

So far, we have made the common assumption that $g = \Gamma = (\vec{b})^{-1}$ is linearly proportioned to the restricted energy loss so that we could write r.e.l. = $K\Gamma$. A priori, however, there seems to be no very cogent reason why this assumption should be strictly valid. We must remember that the mechanism of production of development centres in silver halide crystals is exceedingly complex and very imperfectly understood. There is some evidence that the probability of creating the condition of developability in a crystal is proportioned to the average amount of energy deposited in it. Nevertheless, the energy loss in an absorber of small dimensions is a stochastic process, as shown by the classical experiments on the absorption of electrons in thin foils.

Barkas\textsuperscript{19} has suggested that the probability of the creation of developability depends on an expression of the type

$$
\eta_0 \int_0^{w_0} w \ p(w) \ E(w) \ dw
$$
where \( \eta_0 \) is the electron density in the crystal, \( p(w) \) is the probability of an energy transfer by the ionizing particle within the range \( w \sim \gamma \delta w \), \( E(w) \) is a function which describes the efficiency of utilization of energy for the purpose of creating developability. \( \omega_0 \) is the energy above which the electron will escape from the crystal. If an energy transfer greater than \( \omega_0 \) occurs, a \( \delta \) ray is produced which may render developable crystals not penetrated by the primary ionizing particle. As has been remarked for higher velocities, the \( \delta \) rays will tend to be emitted along the direction of the track. Thus, some of the grains rendered developable by operation of this effect will be indistinguishable from the primary grains through which the ionizing particle has passed. The magnitude of that effect has been studied by Patrick and Barkas. On the basis of certain simple assumptions they find, for singly-charged particles, that the secondary grain density \( g_s \) is given by an expression of the form

\[
g_s = A\beta^{-2}.
\]  

Thus,

\[
g_e = g = g_s = s + A\beta^{-2}.
\]

where \( g_e \) is the total grain density as observed by experiment, and \( g \) is the primary grain density which may be expected to be linearly proportional to the r.e.l. and thus obey expressions of the type (4). Patrick and Barkas find \( A = 3.9/100 \mu \) a surprisingly high figure, one which would imply that \( \sim 25\% \) of the observed grain density at minimum is secondary in origin.

The above considerations demonstrate that it is unreasonable, at first sight, to suppose that the measured grain density \( g_e \), estimated by \( \Gamma_e \) in the present experiments, is linearly proportional to the r.e.l. The next logical step is to examine the consequence of assuming

\[
r.e.l. = C[g_e - g_s] = K[\Gamma_e - \Gamma_s] = KT,
\]

where \( \Gamma_s \) is given by (18). From (18) and (20), we may exhibit the relationship between \( \Gamma \) and \( \Gamma_e \). We have

\[
\Gamma = \frac{\Gamma_e - a\beta^{-2}}{1 - a},
\]

where \( a = \lambda/(\Gamma_p \epsilon) \), and \( \Gamma_p \epsilon \) is the experimental value of \( \Gamma \) at plateau. Now, since \( \Gamma_p \epsilon \) depends on the conditions of development, it is reasonable to suppose that \( \lambda \) does so also. For moderately developed emulsions, \( \Gamma_p \epsilon \sim 20/100 \mu \) and we have \( a = 3.9/20 = 0.195 \). Using that value of \( a \), we may obtain from (21) a new experimental curve \( \hat{\Gamma} \). As before, we take \( I = 496 \text{ eV} \), whence we have \( -C = 5.461 \). Using these values we proceed to fit a B.B.S.-S.-type curve to the experimental values of \( \hat{\Gamma} \) at the points \( x_0 \) and \( x_1 \). From (17) and (21) we have

\[
\hat{\Gamma}_0 = \frac{(\hat{\Gamma}_0 - 0.195\beta\epsilon^{-2})}{1 - 0.195} = (Q' - C - 1)^{-1} \beta\epsilon^{-2} \left[ Q' - 1 + \gamma \epsilon^{-2} + \ln (\gamma \epsilon - 1) \right]
\]
whereas, on the assumption that \( \langle T_e \rangle \) is linearly proportional to the r.e.l. we had

\[
\langle T_e \rangle = (Q - C - 1) \beta \frac{\alpha - 1}{\alpha + Q} \left[ Q - 1 + \gamma \right] \ln (\gamma - 1)
\]

(17)

Taking \( \langle T_e \rangle = 0.99 \), we find for \( \alpha = 0.195 \), \( \beta = 0.9036 \) and, hence, \( Q' = 8.134 \). We may also show that a rather simple relationship exists between \( Q \) and \( Q' \), namely,

\[
\frac{Q' - C - 1}{Q - C - 1} = 1 - \alpha
\]

or,

\[
Q' = Q - \alpha (Q - C - 1)
\]

(23)

(24)

Equation (23) or (24) may be verified by setting \( Q = 11.185 \) (Table IV), \( \alpha = 0.195 \) and \( -C = 5.461 \).

We may now compare the fit of the curve based on the above values of \( Q' \) and \( \alpha \) with the experimental values of \( T_e \) computed from the experimental values of \( T_e \) by means of (21). The results are set out in Table VI. The brackets denote a forced fit at the points \( x_0 \) and \( x_1 \), and the values at minimum \( (g_m) \) have been evaluated on the assumption \( x_m = 0.6 \), which is sufficiently accurate for our purpose.

Table VI

<table>
<thead>
<tr>
<th>HYPOTHESIS</th>
<th>BETHE-BLOCH REGION</th>
<th>STERNHEIMER REGION</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( g_a )</td>
<td>( g_{A_1} )</td>
</tr>
<tr>
<td>1. r.e.l. = ( k' T_e = f g )</td>
<td>1.573</td>
<td>1.161</td>
</tr>
<tr>
<td>2. THEORY, ( Q = 11.185 )</td>
<td>1.470</td>
<td>1.134</td>
</tr>
<tr>
<td>3. r.e.l. = ( C_1(T_e - T_g) )</td>
<td>1.427</td>
<td>1.059</td>
</tr>
<tr>
<td>4. THEORY, ( Q' = 8.134 )</td>
<td>1.300</td>
<td>1.021</td>
</tr>
<tr>
<td>5. r.e.l. = ( C_2(T_e)^0.85 )</td>
<td>1.470</td>
<td>1.135</td>
</tr>
</tbody>
</table>
It will be observed immediately that as far as the B.B. region is concerned, the agreement between the numbers in rows 3 and 4 is, if anything, a little worse than between those in rows 1 and 2, while in the S region the fit of 3 by 4 appears to be distinctly worse between minimum and plateau. In fact, in the B.B. region, the ratios of the numbers in rows 1 and 2 for $\hat{x}_{\beta}$, $\hat{x}_{\mu}$, and $\hat{x}_{0}$ are little different from the corresponding numbers in rows 3 and 4. In other words, the apparent discrepancy between experiment and theory, if we assume that $K_{e} = r.e.l.$, is scarcely altered by substituting (20) with the assumption of Patrick and Barkas that $g_{0} = 3.9 \beta^{2}$. Indeed, this fact may be shown analytically using (22) and (24). Throughout the B.B. region, we must have

$$\frac{\hat{t}_{e}}{t} = \frac{Q' - C - 1}{Q - C - 1} \left[ \frac{Q - 1 + \gamma^{2} + \ln(\gamma^{2} - 1)}{Q' - 1 + \gamma^{2} + \ln(\gamma' - 1)} \right] = \frac{Q - 1 + \phi(\gamma)}{Q - 1 + \phi(\gamma) + (1 + \alpha) \alpha + \alpha \phi(\gamma)} , \quad (25)$$

where $\phi(\gamma) = \gamma^{2} + \ln(\gamma^{2} - 1)$. For $Q = 11$, $-C = 5.4$ and $\alpha \geq 0.2$, $\hat{t}_{e}$ will depend but little on $\gamma'$ or on $\alpha$ within the region with which we are concerned, a fact which we have seen from study of the numbers in Table VI.

We may conclude, from the foregoing analysis, that if the available experimental values of $(\hat{x}_{a})_{e}$ and $(\hat{x}_{\mu})_{e}$ in Table VI are reliable, the above findings would constitute evidence against the hypothesis of Patrick and Barkas that the velocity dependence of $g_{0}$ is given by $A\beta^{2}$. On the other hand, if the experimental values at the above points are in error and are too high, so that the apparent bad fit in the B.B. region is illusory, we would be forced to conclude from (25) that either $K_{e}$ or $C_{e}(T_{e} - T_{0})$ is a sufficient measure of the r.e.l. In those circumstances, if the theory of Patrick and Barkas is to be relied upon, then two important conclusions follow. Firstly, the lower value of $Q'$ for the fitting curve implies a lower value of $T_{0}$. For $g_{0} = 0.904$, and hence $Q' = 8.134$, we find $T_{0} = 0.84$ keV. This seems to be a very low value as will be seen by reference to Table IV. Secondly, the true value of the $P/T$ ratio, from rows 3 and 4 of Table VI, would be $\sim 1.216$.

On the other hand, it is possible to obtain a much better over-all fit to the numbers in the second row of Table IV by the hypothesis admittedly empirical, that $\Gamma = (T_{e})^{P}$, so that we may write,

$$\text{r.e.l.} = C_{e}(T_{e})^{P} . \quad (26)$$

Since $\Gamma$ must be less than $T_{e}$, the value of the exponent $p$ must be less than unity. As far as the S region is concerned, since $\Gamma_{e} > 0.870$ the effect will be slight, being nil at the point $x_{i}$ and very approximately so at the point $x_{0}$. At minimum, the effect will be to make $\Gamma > T_{e}$, but the change will be $< 2\%$. On the other hand, in the B.B. region the effect will be to make $\Gamma < T_{e}$, and by a percentage which increases as $x$ decreases. Choosing $p = 0.85$, we obtain the results shown in row 5 of Table VI. They will be seen to be very well fitted throughout the range by the Bethe-Bloch-Sternheimer curve for $Q = 11.185$. As has been discussed already, the corresponding value of $T_{0}$ (Table IV) is acceptable, and the true $P/T$ ratio will be 1.126. We may conclude, therefore, that if the experimental values of the $\hat{t}_{e}$ in the B.B. region are reliable, the hypothesis $\Gamma = (T_{e})^{0.85}$ allows of an excellent fit to the...
observed data over the entire region $\gamma > 1.1$. On the other hand, use of the expression (20) of Patrick and Barkas leads to unsatisfactory fitting and to an unduly low estimate of $T_0$.

It seems clear that there is need for further careful work in the B.B. region. Furthermore, the results of such measurements should throw useful light on the problem of the magnitude and velocity dependence of the secondary ionization.

Work of group at the Lebedev Institute, Moscow

Finally, we must consider the work of Zhdanov\textsuperscript{21}) and his collaborators carried out in the above Institute. From certain calculations of Tsytovich it would appear that considerations involving second-order approximations of perturbation theory lead to the conclusion that radiative corrections may become important at values of $\gamma > 10^5$. Hence, according to that author, the first-order approximations which take account of the effects of polarization by previous workers may fail in the extreme relativistic region.

According to Tsytovich\textsuperscript{22}), the radiative correction may be written

$$\frac{W - W_0}{W_0} = \frac{-e^2}{\pi \hbar c} \Delta (E/mc^2).$$

(27)

$W$ is the ionization loss taking account of radiative loss, $W_0$ is the ionization loss predicted by first-order theory, and $\Delta (\gamma)$ is a monotonically increasing function which tends asymptotically to $\Delta$, as $\gamma \rightarrow \infty$. Provided $1 < \Delta^{1/2} \ll (\hbar c)^{1/2}/e^2$, we may set

$$\Delta = 2 \ln^2 \xi,$$

(28)

where $\xi$ is a function of the electron density, and of the natural frequencies of the absorbing medium. In practice, it is found that the asymptotic value of $\Delta$ is attained for $\gamma > 1/\xi$, and that for AgBr, $100 \leq \xi^{-1} \leq 200$.

These conclusions were subjected to an experimental test by Zhdanov and his collaborators using stacks of Ilford G.5 and Nikfi R.10 exposed to a beam of 19.6 GeV/c protons produced by the CERN PS. Electron pairs produced through the decay of $\pi^0$ mesons from the interactions of the primary were subdivided into various energy groups by multiple scattering. Typical results were shown in Fig. 4 [From Proceedings of the Conference on Corpuscular Photography (IV) held in Munich in 1963], and these are interpreted as furnishing evidence for the correctness of the theoretical ideas of Tsytovich.

There would appear to be little evidence for the existence of the Tsytovich effect from the work of Jogejans, Johnston et al., Buskirk or Stiller, all of which workers have many points in the region $\gamma > 200^a$). It would appear prudent, therefore, to regard the matter as being open until a decisive experiment has been carried out.

\textsuperscript{*} Similar results have been found by D.P. Dubey for $20 < \gamma < 5 \times 10^3$ (private communication), and also for $293 < \gamma < 1953$ by A.J. Herz and B. Stiller, Proc. Vth Int.Conf. on Nuclear Photography, Vol. II, Geneva (1964).
Such an experimental programme is already in progress. A collaboration has been set up between Dublin (DIAS), Munich (MPI), and Washington (ONR) for the purpose of comparison of the ionization of electrons of various energies in the range $150 < \gamma < 10^5$ produced at the new electron synchrotron which has recently come into operation at Hamburg (DESY). Through the kindness of the Directorate of that machine, two trial exposures with G.5 emulsions have already been made, and the developed plates appear to be satisfactory. There is every reason to hope, therefore, that the problem of the existence or importance of the Tsytovich effect will be resolved in the near future.
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O. Halpem and H. Hall, Phys.Rev. 73, 477 (1948).
Figure captions

Fig. 1: Exposure details for the experiment carried out by Johnston et al.¹²).

Fig. 2: Normalized grain density γ² versus the function x[ = ½ log₂(γ² - 1)] according to the theories of Bethe-Bloch (dotted portion) and Sternheimer (full line).

Fig. 3: Comparison of experimental data with theory.

Fig. 4: Relative blob density-momentum dependence of electron tracks (nₑ, nₚ - blob density of electron and proton tracks of the primary beam) on the value pβ c/me². Curve I is a theoretical ionization-momentum dependence according to Barkas's tables; curves II and III are theoretical curves with the asymptotic value of the radiation correction (for different values).
FITTING OF EXPERIMENTAL $g^*$ BY MEANS OF EXPRESSIONS OF BETHE-BLOCH AND STERNHEIMER

$X = \frac{1}{2} \log_{10}(\gamma^2 - 1)$

Fig. 3
Fig. 4