INTERSTELLAR GRAINS IN ELLIPTICAL GALAXIES: GRAIN EVOLUTION

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ABSTRACT

We consider the lifecycle of dust introduced into the hot interstellar medium in isolated elliptical galaxies. Dust grains are ejected into galactic-scale cooling flows in large ellipticals by normal mass loss from evolving red giants. Newly introduced dust rapidly enters the hot interstellar plasma and is sputtered away by thermal collisions with ions during the slow migration toward the galactic center in the cooling flow. Before the grains are completely sputtered away, however, they emit prodigious amounts of infrared radiation which may contribute to the large far infrared luminosities observed in ellipticals. The infrared emission depends critically on the sputtering rate. Since our understanding of both the plasma and radiation environments in ellipticals is quite good, these galaxies provide an excellent venue for studying the physical processes of dust grains and perhaps also their composition and size distribution.

In order to study the global properties of grains in ellipticals we construct a new series of King-type galactic models which are consistent with the fundamental plane, galactic mass to light ratios and other relevant observational correlations. We describe a new “continuity” procedure to construct simple time-dependent gas dynamic models for cooling flows.

Although grains can flow a considerable distance from their radius of origin in the hot interstellar medium of some galaxies before being sputtered away, we show that the grain size distribution at every radius is accurately determined by assuming \textit{in situ} sputtering of dust grains, completely ignoring advection. This occurs since the stellar density profile is so steep that the majority of grains at any galactic radius is produced locally.

Although thermal sputtering destroys the grains, we show that the dominant source of grain heating is absorption of starlight; grain heating by collisions with energetic thermal electrons or X-ray absorption are negligible. Previous studies have claimed that the loss of thermal energy from a hot, dusty plasma is dominated by grain heating via electron-grain collisions and subsequent IR radiation. However, we show that when self-consistent grain sputtering is included the dust-to-gas ratio is reduced and radiative cooling, not electron-grain interactions dominates plasma cooling, even for the most massive ellipticals. This conclusion is insensitive to the grain size distribution assumed for the stellar ejecta.

1. INTRODUCTION

Elliptical galaxies, long thought to be simple ensembles of non-interacting stars, are now known to be complicated multicomponent systems of stars and gas. The interstellar medium (ISM) in ellipticals is dominated by hot gas which is slowly cooling by radiative losses; as a result the ISM gradually flows inward as a cooling flow. X-ray observations of optically bright ellipticals reveal gas masses of $\sim 10^8 - 10^{9} M_\odot$ at high temperatures $\sim 10^7$K (Forman et al. 1979; Nulsen, Stewart, & Fabian 1984; Forman, Jones, & Tucker 1985; Canizares, Fabbiano, & Trinchieri 1987). Radio and optical observations (e.g., 21 cm, H\alpha, and CO) indicate that elliptical galaxies typically contain $\sim 10^2 - 10^5 M_\odot$ of cold gas usually located in the core regions (see e.g. Kormendy & Djorgovski 1989 for a review).

In view of the high temperature of the ISM it is remarkable that elliptical galaxies also contain substantial amounts of dust. Optical images of $\sim 50\%$ of all ellipticals ex-
hibit dust lanes, disks, or patches (e.g., Sadler & Gerhard 1985; Kormendy & Stauffer 1987; Ebneter et al. 1988). Since dusty disks would be difficult to see when viewed face on, it is possible that all ellipticals contain appreciable dust. Further evidence for dust are the large far-infrared IRAS luminosities detected in approximately 50% of ellipticals. The ratio of flux energy $\nu F_\nu$ at 100$\mu$m to optical B-band emission has a cosmic spread, $\nu_{100} F(100\mu\text{m})/\nu_B F(\nu_B) = 0.006 - 0.088$ (Jura et al. 1987); for cD galaxies this ratio is about ten times larger (Bregman, McNamara, & O’Connell 1990).

The geometrical distribution of dust and the associated infrared emission in ellipticals may be either concentrated near the galactic core or distributed throughout the stellar system. Concentrated patches or disks of dusty gas observed near the galactic cores could arise either (i) from merger events with galaxies having large amounts of cool, dusty gas and/or (ii) from cold gas deposited by galactic cooling flows as the hot ISM slowly radiates away its thermal energy. These two possibilities may be distinguished observationally. Because of the destruction of grains in the hot ISM by sputtering, the dust to gas ratio is expected to be much lower, at least initially, in cool gas condensed from cooling flows. Moreover, gas deposited by galactic cooling flows should exhibit the same global rotational dynamics and sense of rotation as that of the stellar system (Kley & Mathews 1995).

In addition to the central dust patches, more widely distributed dust is continuously generated by stellar mass loss from evolving red giants throughout the galaxy. Knapp, Gunn, & Wynn-Williams (1992) have shown that the relatively hot dust that emits 12$\mu$m radiation in ellipticals is spatially distributed like the galactic stars and may therefore be circumstellar. However, because of the dynamical interaction of the dusty stellar ejecta with the hot ISM, the time that recently ejected gas and dust lingers near the parent star is limited. When the dusty stellar ejecta interacts with the hot $T \sim 10^7$K ISM, numerous hydrodynamic instabilities are expected (Mathews 1990) resulting in breakup and deceleration by drag forces as the ejecta trails off behind the star. The fragmented new gas is rapidly heated by thermal conduction as it merges with the hot ISM, even in the presence of a magnetic field. It is easy to show (Mathews 1990) that the cool stellar ejecta is separated from its parent star and melts into the ISM on time scales $\sim 10^4 - 10^5$ yrs, so fast that little or no grain destruction can occur until the dust is immersed in the hot ISM. Once in contact with the hot ISM of number density $n$, grains with radius $a$ are sputtered away by thermal collisions with protons and helium nuclei; the sputtering lifetime is $t_{sp} \approx 10^9 (a/\mu\text{m}) (n/10^{-3}\text{cm}^{-3})^{-1}$ yrs provided ISM temperatures are $T > 2 \times 10^6$ K (Tielens et al. 1994; Draine & Salpeter 1979; Seab 1987). However, during their short lifetimes as they sputter away and are advected inward by the galactic cooling flow, the grains can nevertheless emit a huge amount of far IR radiation (Dwek 1986; 1987). But the dust-to-gas ratio and IR emissivity from heated grains is greatly reduced by the short sputtering lifetimes so emission and sputtering must be treated in a self-consistent manner. Emission from this distributed dust may in fact contribute substantially to the observed IRAS luminosities. Unfortunately the spatial resolution of IRAS observations ($3'\times5'$ at 100$\mu$m) is insufficient to discern whether the emission comes from the dusty clouds near the galactic centers or is distributed throughout the galaxies.

In this and a subsequent paper we study the physical properties and emission from
the distributed dust component. The infrared emission from distributed dust associated with stellar mass loss must be present in every elliptical and is a non-trivial lower limit to the observed fluxes. Fortunately the dust environment – particularly the gas density and temperature in the ISM – is well known from successful hydrodynamic models of cooling flows (e.g. Loewenstein & Mathews 1987). The local gas density depends on the rate of mass loss from an aging stellar population $\alpha_s(t)\rho_s(r) \, \text{g cm}^{-3} \, \text{s}^{-1}$ which is well known and insensitive to the stellar IMF (Mathews 1989). While grains are destroyed by ion impacts from the hot plasma, we find that they are heated primarily by the local radiation field. The radiation within ellipticals is also very well known in both the optical-IR (Renzini & Buzzoni 1986; Bruzual 1985) and in the near UV as observed with the Hopkins UV Telescope (e.g. Ferguson & Davidsen 1993). Soft X-ray and far-UV radiation from the hot ISM can be determined either theoretically from an atomic emissivity code (Raymond 1991) or from direct observation (e.g. Trinchieri, Fabbiano, & Canizares 1986; Serlemitsos et al. 1993). *Our excellent knowledge of the radiation and plasma environment makes the interstellar medium in ellipticals an excellent laboratory for studying the physics of grain sputtering and emission.* In comparing theoretical and observed infrared emission the primary uncertainties are the initial distribution of dust grain sizes and the grain composition in stellar ejecta; indeed, information about these parameters can be found by detailed comparison of theoretical models with infrared observations. A detailed study of the expected IR spectrum from ellipticals and their emission into each IRAS band will be the subject of a subsequent paper.

In this paper we discuss the life cycle, thermal properties, and size distribution of dust grains ejected into elliptical galaxy cooling flows (ISM) by evolving stars. In order to predict the infrared luminosity due to dust emission from a galaxy of given $L_B$ it is necessary to know the sputtering rate, the grain heating and cooling rates, the temperature and infrared emissivity of grains of all sizes and at all galactic radii. Since we want to explore the variation of infrared to optical luminosities among ellipticals of various masses, we develop in §2 a scheme for generating models of the radial stellar distribution within elliptical galaxies that is consistent with the fundamental plane and other observed galactic properties. In §3 we describe a new procedure to develop simple time-dependent models for the cooling hydrodynamics in the model galaxies. These cooling flows are used to study the effects of sputtering and advection on grain size distributions in §4. In §5 we compare various grain heating mechanisms: by ambient radiation, by electron and ion impacts, and by X-ray absorption. We conclude that grain heating by starlight dominates all other heating sources. Grain size distributions for each model galaxy are presented in §6. In §7 we consider the effects of the presence of grains on the temperature of the ISM; we show that cooling of the hot ISM by thermal collisions with grains is generally small compared to optically thin radiative losses.

2. SELF-CONSISTENT MODELS FOR ELLIPTICAL GALAXIES

For analytic simplicity and for comparison with previous theoretical studies we assume
spherical, King-type model galaxies having stellar density profiles given by

$$\rho_*(\xi) = \rho_{*0}(1 + \xi^2)^{-\frac{3}{2}}; \quad \xi \equiv r/r_c$$  \hspace{1cm} (1)

where $r_c$ is the stellar core radius. The stellar distribution is terminated at an outer boundary $r_t$, or $\xi_t = r_t/r_c$. From eq. (1), the stellar mass within radius $\xi$ in units of the stellar core mass $M_{*c} = 4\pi r_c^3\rho_{*0}/3$ is

$$\mu_{mass}(\xi) = 3 \left[ \ln[\xi + (1 + \xi^2)^{\frac{1}{2}}] - \xi(1 + \xi^2)^{-\frac{1}{2}} \right]$$  \hspace{1cm} (2)

and the mass surface density

$$\Sigma_*(\lambda) = \frac{2\rho_{*0}r_c}{(1 + \lambda^2)} \left( 1 - \frac{1 + \lambda^2}{1 + \xi_t^2} \right)^{\frac{1}{2}}$$  \hspace{1cm} (3)

depends on the projected radius $\lambda = R/r_c$.

The parameters that determine the form of the stellar distribution within each galaxy - $\rho_{*0}$, $r_c$ and $r_t$ - must be consistent with observational constraints on the properties of ellipticals, particularly the fundamental plane relation (e.g., Dressler et al. 1987; Djorgovski & Davis 1987). In Appendix A we describe the procedure we have used to derive the stellar properties of a one parameter family of galaxy models. Relevant properties of the three representative galaxies used in our study of dust evolution are listed in Table 1. Note that the listed ellipticals extend from the brightest only to those of intermediate luminosity ($L_B \lesssim 10^{10} L_\odot$) for which the presence cooling flows can be reasonably assured. The global X-ray emission from ellipticals of lower luminosity is dominated by stellar sources, evidently low mass X-ray binaries. In addition, low luminosity ellipticals have shallower potential wells and the ISM can become a galactic wind if the collective luminosity of Type Ia supernovae exceeds the rate of emission from the hot ISM. We assume that most or all moderate to massive ellipticals have cooling flows as evidenced by their large X-ray luminosities $L_x$. Recent observations of low iron abundances in several bright ellipticals (Serlemitsos et al. 1993; Forman et al. 1993; Mushotzky et al. 1995) indicate that the Type Ia supernova rate is sufficiently low to permit cooling flows in all galaxies considered here (Loewenstein & Mathews 1987; 1991).

In order to estimate the sputtering rate of grains in the ISM, it is necessary to have estimates of the gas temperature, density, and velocity in the cooling flows. We describe how the gas density and velocity are determined in §3. The temperature, ideally, should be determined by observations of the X-ray emitting gas in galaxies having a spread in luminosities. However, current temperature determinations are available only for a few galaxies (e.g., NGC 4472 and NGC 1399; Serlemitsos et al. 1993; NGC 4636: Mushotzky et al. 1995) and are restricted to the highest luminosity cases. We therefore adopt a theoretical approach for determining the temperatures. Gas ejected by stars in the galaxy is rapidly virialized to the mean stellar temperature $\langle T_s \rangle$ which characterizes the average stellar velocity dispersion (weighted by the stellar mass density) as stars orbit in the total
galactic potential. Assuming most of the interstellar gas comes from stars and that other sources of heating are small, the temperature of the gas should be very close to \( \langle T_s \rangle \). This has been confirmed by detailed hydrodynamic calculations. To estimate \( \langle T_s \rangle \) we adopt appropriate density structures for the galactic dark matter halos and solve the equation of stellar hydrodynamics. This is described in Appendix A. The temperatures listed in Table 1 are computed in this manner.

As a check of the above procedure, we plot in Figure 1 the \((L_B, \langle T_s \rangle)\)-relation implied by our procedure for determining the gas temperature. The location of several of our model galaxies are indicated by filled circles and the general relation is given by the solid line. Three galaxies with recent gas temperature determinations by ASCA, NGC 4472, NGC 4406, and NGC 4636 (Makishima 1994; Mushotzky et. al. 1995) are also plotted. The observed temperatures of these galaxies agree reasonably well with our predicted \((L_B, \langle T_s \rangle)\)-relation. A more accurate determination of the temperatures is not required for the considerations of this paper.

3. TIME-DEPENDENT “CONTINUITY” MODELS FOR GALACTIC COOLING FLOWS

For three nearby elliptical galaxies which were well resolved by the Einstein Observatory Imaging Proportional Counter (IPC), Trinchieri, Fabbiano, & Canizares (1986) found that the X-ray surface brightness distribution is very similar to that of the starlight, i.e. \( \Sigma_x(\lambda) \propto \Sigma_s(\lambda) \). Since the X-ray emissivity is proportional to the square of the gas density, it follows from eq. (1) and the radial constancy of the stellar mass to light ratio that the gas density varies as

\[
\rho(\xi) = \rho_o(1 + \xi^2)^{-\frac{a}{2}}.
\]  

We have implicitly assumed that the gas temperature is independent of galactic radius. Isothermality throughout most of the volume of the cooling flow is a general theoretical result (Loewenstein & Mathews 1987; 1991) and is supported by recent BBXRT and ASCA observations (Serlemitsos et al. 1993; Makishima 1994; Mushotzky et al. 1995). Within the galactic cores, however, the gas must cool rapidly and large deviations from isothermality are expected. The gas temperature may also decrease at the very largest radii since \( T_s(r_t) \rightarrow 0 \) if the stellar velocities become preferentially radial at large radii.

Since the gas density distribution \( \rho(\xi) \) of eq. (4) has the same core radius and total radius as the stellar distribution, \( \rho(\xi) \) is completely specified by the central density \( \rho_o \). This can be determined by relating the gas density to the X-ray luminosity in the Einstein IPC band (\( \sim 0.2 \) to \( \sim 4 \) keV),

\[
\left( \frac{\rho_o}{m_p} \right) = 0.165 \left( \frac{\Lambda_{\Delta E}}{10^{-23} \text{ergs cm}^3 \text{s}^{-1}} \right)^{-\frac{1}{2}} \left( \frac{L_X}{10^{41} \text{ergs s}^{-1}} \right)^{\frac{1}{2}} \left( \frac{r_c}{\text{kpc}} \right)^{-\frac{a}{2}} \left[ \ln(2\xi_t) - 1 \right]^{-\frac{1}{2}} \text{cm}^{-3}
\]  

where \( m_p \) is the proton mass. The atomic emissivity in the given energy band, \( \Lambda_{\Delta E} \), which we have determined from the code of Raymond (1991), depends only on the gas
temperature (although this dependence is weak for the temperatures of interest). The X-ray luminosity is observed to correlate with the B-band optical luminosity

\[ L_x = 10^{17.85} \left( \frac{L_B}{L_{\odot}} \right)^{2.18} h^{2.36} \text{ergs s}^{-1} \]  

(Donnelly, Faber, & O’Connell 1990) where \( h \) is the Hubble constant normalized to 100 km s\(^{-1}\) Mpc\(^{-1}\). We assume \( h = 0.75 \) throughout.

Having specified the gas density distribution, we need to determine the velocity of the gas in the cooling flow \( u(r) \) to complete the description of the model galaxies. Although the inward gas flow velocity in galactic cooling flows is highly subsonic, these flows are not in steady state since the source term for new gas expelled by evolving stars varies considerably during the time required for gas to flow across the galaxy. The specific mass loss rate from a single-burst population of evolving stars is \( \alpha_s(t) = \alpha_n(t/t_{\text{now}})^{-p} \) where \( \alpha_n \approx 1.6 \times 10^{-12} \text{yr}^{-1} \), \( p = 1.35 \) and \( t_{\text{now}} \) is the present time. The time variation of \( \alpha_s(t) \) is insensitive to the IMF assumed for the galaxy (Mathews 1989).

Cooling flows were not present at the very earliest times in the history of massive ellipticals. The significant iron abundance in clusters of galaxies suggests that a considerable amount of enriched mass processed in stars was ejected long ago from the elliptical galaxies (or their progenitors) by galactic winds driven by Type II supernovae (Mathews 1989; David, Forman, & Jones 1991). It is therefore reasonable to assume that the cooling flows observed in massive ellipticals today began at some time \( t_{\text{cf}} \) in the past when the galactic outflow subsided.

We now show that, given several simplifying assumptions, the velocity field of the cooling flow at the present time \( u(r, t_{\text{now}}) \) can be found from the continuity equation alone,

\[ \frac{\partial \rho}{\partial t} + \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 \rho u) = \alpha_s(t) \rho_s. \]  

(7)

The inward flow velocity \( u(r, t) \) is the only unknown dependent variable if we note that the stellar density \( \rho_s \) does not vary significantly with time, and require that the gas density at time \( t \) vary as

\[ \rho(\xi, t) = \left( \frac{t}{t_{\text{now}}} \right)^{-s} \rho(\xi), \]  

(8)

where \( \rho(\xi) \) is the gas density at \( t_{\text{now}} \) given by eq. (4) and \( s \approx p/2 \). Condition (8) is based both on the results of detailed hydrodynamical calculations (Loewenstein & Mathews 1987; David, Forman, & Jones 1991) and on the approximate considerations described in Appendix B. During the evolution of cooling flows the X-ray luminosity varies with the stellar mass loss rate, \( L_x \propto \alpha_s(t) \). But \( L_x \) at any time is proportional to the square of the gas density at any radius, \( L_x \sim \Lambda_\Delta E \rho^2 \). If the gas temperature (and also \( \Lambda_\Delta \)) remains roughly uniform, it follows from these two relations that \( \rho \propto \alpha_s(t)^{1/2} \propto r^{-p/2} \) as in eq. (8) (see Appendix B).
Substituting eqs. (1) and (8) into eq. (7), we get an equation containing only a spatial derivative,

\[-s\gamma \Delta (\xi) \theta^{-s-1} + \frac{\theta^{-s}}{\xi^2} \frac{\partial}{\partial \xi} \left[ \xi^2 \Delta (\xi) \eta (\xi, \theta) \right] = \theta^{-p} \Delta_s (\xi) \]  

(9)

where \( \theta = t/t_{\text{now}} \), \( \gamma = \rho_o/\alpha_n \rho_{\text{so}} t_{\text{now}} \) is a dimensionless parameter, and the fluid variables are expressed in non-dimensional form:

\[\Delta_s (\xi) = \frac{\rho_s (\xi)}{\rho_{s0}} = (1 + \xi^2)^{-\frac{s}{2}}, \]  

(10)

\[\Delta (\xi) = \frac{\rho (\xi)}{\rho_o} = (1 + \xi^2)^{-\frac{s}{2}}, \]  

(11)

and

\[\eta (\xi, \theta) = \frac{u (\xi, \theta)}{u_o}; \quad u_o = \frac{r_c \alpha_n \rho_{s0}}{\rho_o}. \]  

(12)

Eq. (9) is valid at all times after the onset of the cooling flow (\( t_{\text{cf}} \)). It must therefore be valid at the present time, where \( \theta = 1 \). On integrating over \( \xi \), we determine the velocity field at the present time,

\[\eta (\xi, 1) = -\frac{1}{\Delta \xi^2} \int_\xi^{\xi_i} \left[ \Delta_s + s\gamma \Delta \right] \xi^2 d\xi'. \]  

(13)

where no flow across the outer boundary of the galaxy is assumed.

Eq. (13) represents an approximation for the velocity field in a fully time dependent galactic cooling flow. Velocities computed from eq. (13) for the three galaxies of Table 1 are shown in Figure 2. The smallest galaxy has the smallest velocity at any given value of the normalized radius \( \xi/\xi_i \) because the value of \( \gamma \) is least for the smallest galaxy. The velocity profiles compare favorably with those derived from detailed hydrodynamical calculations (e.g. Loewenstein & Mathews 1987) as do the flow times predicted from the velocities (see Figure 3).

4. GRAIN SPUTTERING

Dust grains introduced into hot gas are eroded by collisions with energetic protons and helium ions in the gas. The rate at which dust particles are sputtered away has been computed in detail by Tielens et al. (1994) and by Draine & Salpeter (1979) for both graphite and silicate grains. We adopt the analytic form

\[\frac{da}{dt} = -h \left( \frac{\rho}{m_p} \right) \left[ \left( \frac{T_d}{T} \right)^w + 1 \right]^{-1} \]  

(14)

for the rate at which the radius of the dust grain \( a \) decreases with time in a hot plasma of temperature \( T \) and density \( \rho \). This relation is a good approximation to the detailed
calculations for both graphite and silicate when $h = 3.2 \times 10^{-18} \, \text{cm}^4 \, \text{s}^{-1}$, $\omega = 2.5$, and $T_d = 2 \times 10^6 \, \text{K}$.

Eq. (14) can be used to compute the local sputtering time for a grain, defined by

$$t_{sp} = a \left| \frac{da}{dt} \right|^{-1}$$  \hspace{1cm} (15)

which can be compared to the flow time to the core radius, defined by

$$t_{flow}(\xi) = r_c \int_1^\xi \frac{d\xi'}{|u(\xi')|}.$$  \hspace{1cm} (16)

These timescales are plotted in Figure 3 for the three galaxy models of Table 1. The sputtering time is computed assuming a large grain with a radius of 0.3$\mu$m. Smaller grains have correspondingly shorter sputtering times. Figure 3 indicates that sputtering generally occurs faster than the time for the grains to flow to the core in larger galaxies, but in smaller galaxies grains may be carried inward by the cooling flow for a significant distance before sputtering away. This suggests that the grain population at a given radius in the smaller galaxies will have an admixture of grains that originated further out in the galaxy. In contrast, the largest galaxy produces and sputters grains on the spot, except at the innermost radii where large grains may flow somewhat.

In the case of the largest galaxy (model a), the grain size distribution in the ISM can be determined approximately by balancing the local rate of grain input from stars and the rate of grain destruction by sputtering. That is,

$$\frac{\partial}{\partial a} \left( N \frac{da}{dt} \right) = S(a)$$  \hspace{1cm} (17)

where $N$ is the number density of grains per unit grain radius, $S$ represents the stellar source of grains, and $\dot{a} = da/dt$ is given by eq. (14). If, for example, we assume that the grain size distribution within recently introduced stellar ejecta varies as a power law, i.e. $S(a) = A a^{-\gamma}$, then eq. (17) is readily solved to give

$$N(a) = N(a_{\text{max}}) + \frac{A}{\bar{h} a^\gamma} \left( \frac{1}{1 - \gamma} \right) (a_{\text{max}}^{1-\gamma} - a^{1-\gamma})$$  \hspace{1cm} (18)

where $a_{\text{max}}$ is the adopted size of the largest grain. The constant $\bar{h}$ is related to $h$ of eq. (14) by

$$\bar{h} = h \frac{m_p}{m_p} \left[ \left( \frac{T_d}{T} \right)^{\omega} + 1 \right]^{-1}$$  \hspace{1cm} (19)

where $T$ is the (uniform) temperature of the hot ISM. Since we assume that there are no sources of grains bigger than $a_{\text{max}}$, $N(a_{\text{max}}) = 0$ in eq. (18). Thus, for small grains, the size distribution in the ISM goes approximately as a power law in the grain size, $N(a) \propto a^{-\gamma}$. 


but with an exponent that differs by unity from that of the source function. As \( a \) approaches \( a_{\text{max}} \), the size distribution steepens and falls towards zero. Note that because the sputtering rate depends only weakly on \( T \) for the temperatures under consideration, both \( \dot{a} \) and the grain size distribution are accurately determined despite uncertainties in our evaluation of the gas temperature (Figure 1).

Eq. (18) demonstrates that sputtering significantly modifies the grain size distribution from that in the stellar ejecta; smaller grains are relatively less numerous in the intergalactic medium compared to that of the stellar ejecta. The general case where inward advection of the dust becomes important for determining the grain size distribution will be treated in §6, however, the approximate distribution of eq. (18) applies for a large region of galaxy model a and the outer regions of galaxy models b and c. The predicted IR spectrum and bolometric IR luminosity are greatly modified due to the reduction in the dust-to-gas ratio and the altered grain size distribution as a result of sputtering. The relative underpopulation of smaller grains implies decreased emission at smaller IR wavelengths.

5. SOURCES FOR DUST HEATING

5.1 Heating by Ambient Starlight

Absorption of ambient optical radiation by grains is an important means of grain heating. In order to estimate this heating we must compute the mean intensity of starlight at any radius in the model galaxies. The mean intensity at radius \( r \) is given by

\[
J_{\nu}(r) = \frac{1}{4\pi} \int_{\Omega} I_{\nu}(r, \Omega) \ d\Omega = \frac{1}{2} \int_{-1}^{1} I_{\nu} \ d\mu_{\theta}
\]

where \( \mu_{\theta} = \cos \theta \), \( \Omega \) is the solid angle, and \( I_{\nu} \) is the specific intensity of starlight (ergs s\(^{-1}\) cm\(^{-2}\) ster\(^{-1}\) Hz\(^{-1}\)). To compute \( I_{\nu} \), we assume that the stellar emissivity \( j_{\nu} \) (ergs s\(^{-1}\) cm\(^{-3}\) Hz\(^{-1}\)) is proportional to the stellar density:

\[
j_{\nu}(\xi) = j_{o} \ (1 + \xi^{2})^{-\frac{3}{2}}
\]

where \( j_{o} \) is an as yet unspecified constant which carries all the frequency dependence of \( j_{\nu} \). We further assume that dust distributed throughout the galactic volume is optically thin to stellar light. (This latter assumption can be checked for consistency once the dust content of the interstellar gas is computed in §6.) The specific intensity \( I_{\nu}(\xi, \mu_{\theta}) \) in a direction specified by the azimuthal angle \( \mu_{\theta} \) is computed as an integral along a ray starting at the current location and extending to the outer boundary of the galaxy:

\[
I_{\nu}(\xi, \mu_{\theta}) = r_{c} \int_{0}^{l_{\text{max}}} j_{\nu}(\tilde{\xi}) \ dl.
\]

In this expression, \( l \) is a dimensionless length along the line of sight,

\[
\tilde{\xi}^{2} = \xi^{2} + l^{2} + 2\xi l \cos\theta,
\]
and
\[ l_{\text{max}} = -\xi \mu_{\theta} + \sqrt{\xi^2 \mu_{\theta}^2 + \xi_t^2 - \xi^2}. \]  

Eq. (22) can be evaluated to yield
\[
I_\nu (\xi, \mu_\theta) = j_0 \nu c \left[ \frac{l_{\text{max}} + \xi \mu_\theta}{(1 + \xi^2 - \xi^2 \mu_{\theta}^2) \sqrt{1 + \xi^2 + 2 \xi \mu_{\theta} l_{\text{max}} + l_{\text{max}}^2}} - \frac{\xi \mu_\theta}{(1 + \xi^2 - \xi^2 \mu_{\theta}^2) \sqrt{1 + \xi^2}} \right].
\]

The mean intensity of starlight is then given by eq. (20),
\[
J_\nu (\xi) = \frac{j_0 r c}{2 \xi \sqrt{1 + \xi^2} \sqrt{1 + \xi_t^2}} \left[ \bar{J}(0, -B) - \bar{J}(\xi, -B) + \bar{J}(\xi, B) - \bar{J}(0, B) \right]
\]
where the function \( \bar{J} \) is defined as
\[
\bar{J}(x, y) = \sqrt{A^2 + x^2} - y \ln \left( 2 \sqrt{A^2 + x^2 + 2x} \right) - \sqrt{A^2 + y^2} \ln \left( 2 \sqrt{A^2 + y^2} \sqrt{A^2 + x^2} - 2xy + 2A^2 \right). 
\]

In these last two equations, \( B^2 = 1 + \xi^2 \), and \( A^2 = \xi_t^2 - \xi^2 \). At the center of the galaxy, the mean intensity is given by
\[
J_\nu (0) = \frac{j_0 r c \xi_t}{\sqrt{1 + \xi_t^2}}
\]
and at the outer edge of the galaxy its value is
\[
J_\nu (\xi_t) = \frac{j_0 r c}{2 \xi_t \sqrt{1 + \xi_t^2}} \ln \left( 1 + \xi_t^2 \right).
\]

The local heating (ergs s\(^{-1}\)) of a spherical dust grain of radius \( a \) by ambient starlight is
\[
H_* = \int \left( \frac{4\pi J_\nu}{c} \right) eQ_\nu \pi a^2 d\nu.
\]

Here \( c \) is the speed of light, \( Q_\nu \) is the dust absorption efficiency (kindly provided by Dwek 1993), and the integration is over optical and ultraviolet frequencies.

To evaluate \( H_* \), we must determine the coefficient \( j_0 \). Assuming that the stellar mass to light ratio is spatially constant, the ratio of the stellar mass density to the stellar emissivity in a given frequency interval is also constant by assumption (21). Therefore the
ratio of $\rho_{so}$ in eq. (1) to $j_o$ in eq. (21), when integrated over the B-band, is fixed by the stellar mass to B-band light ratio of eq. (A2),

$$\frac{j_o B}{\rho_{so}} = \left( \frac{M_{*1}}{L_B} \right)^{-1}.$$  \hspace{1cm} (31)

The determination of $j_o B$ allows the total heating due to starlight in the B-band to be computed. However, we require the heating due to all of the starlight, not just the light in the B-band. This is obtained by multiplying the heating due to starlight in the B-band with a bolometric correction factor given by

$$BC = \frac{\int_{\text{uv, optical}} F(\lambda) Q_\lambda d\lambda}{\int_{\text{B band}} F(\lambda) Q_\lambda d\lambda} \hspace{1cm} (32)$$

where $Q_\lambda(\sigma)$ is again the dust absorption efficiency at wavelength $\lambda$. The upper integral extends over all optical and ultraviolet wavelengths, the lower integral extends only over the B-band, and $F(\lambda)$ is the spectrum of the stellar light in ellipticals. We assume that $F(\lambda)$ does not depend on the radius $\xi$ and we use the analytic fit

$$\log F(\lambda) = \begin{cases} 
\log (\lambda) - 3.215 & \text{if } \lambda < 1000\text{Å} \\
-1.509 \log (\lambda) + 4.314 & \text{if } 1000\text{Å} \leq \lambda < 1800\text{Å} \\
-0.6 & \text{if } 1800\text{Å} \leq \lambda < 2123\text{Å} \\
4.078 \log (\lambda) - 14.17 & \text{if } 2123\text{Å} \leq \lambda < 4842\text{Å} \\
0.86 & \text{if } 4842\text{Å} \leq \lambda < 8913\text{Å} \\
-0.8812 \log (\lambda) + 4.341 & \text{if } 8913\text{Å} \leq \lambda < 16255\text{Å} \\
-2.837 \log (\lambda) + 12.58 & \text{if } \lambda > 16255\text{Å}
\end{cases} \hspace{1cm} (33)$$

where $\lambda$ is given in Å and $F$ is in arbitrary units; the absolute normalization of $F(\lambda)$ is given by eq. (31). Equation (33) gives a good fit ($\lesssim 15\%$ in the optical part) to the spectrum of stellar light in elliptical galaxies as determined by Bruzual (1985), Renzini & Buzzoni (1986), and Ferguson & Davidsen (1993).

5.2 Heating by Hot Electrons

While dust grains are sputtered by collisions with thermal protons and helium nuclei, collisional heating of grains is dominated by more frequent collisions with equally energetic but more rapidly moving electrons in the hot interstellar gas. At plasma temperatures $T \gtrsim 10^6\text{K}$ the electric charge of the grains has a negligible influence on these collision rates (Draine & Salpeter 1979). For larger grains the energy transferred from the colliding electron to the dust grain is small relative to the total thermal energy of the grain. Heating therefore occurs in a smooth fashion, requiring many electron impacts. For small grains, however, a single electron collision can introduce enough energy to significantly raise the temperature of the grain. For small grains, electron collisions are sufficiently infrequent
that substantial cooling by radiative losses may occur between successive impacts. Although small grains are stochastically heated (Dwek 1986), the time-averaged heating rate due to hot electrons is given by eq. (3) of Dwek (1986):

$$H_e = \pi a^2 \left( \frac{\rho}{m_p} \right) \int_0^\infty E f(E) v(E) \zeta(E) dE$$

where $E$ and $v(E)$ are the energy and velocity of the impinging electron, and $\rho$ is the gas density. The energy distribution of the electrons is Maxwellian, $f(E) = 2\pi^{-1/2} (kT)^{-3/2} E^{1/2} \exp(-E/kT)$, where $T$ is the gas temperature and $k$ is Boltzmann’s constant. The function $\zeta(E)$ gives the fraction of the energy of an incident electron that is absorbed by the grain (Dwek & Werner 1981),

$$\zeta(E) = \begin{cases} 1 & \text{if } E < E_* \\ 1 - \left[ 1 - (E_* / E)^{3/2} \right]^{2/3} & \text{if } E \geq E_* \end{cases}$$

where $E_*(\text{ergs}) = 3.7 \times 10^{-8} (a/\mu\text{m})^{2/3}$.

5.3 Heating by X–Rays

Some fraction of the thermal X–rays emitted by the hot interstellar gas in ellipticals is absorbed by the dust. Since the energy contained in each X–ray photon is comparable to the thermal energy of a grain in equilibrium with the stellar radiation field, X–rays also cause stochastic heating of the grain. Stochastic heating can also occur for the smaller grains when an ultraviolet stellar photon is absorbed (see e.g., Draine & Anderson 1985). Although stochastic temperature excursions are important in considering the predicted emission spectrum from the grains, we restrict our attention here to the average heating rate due to X–ray absorption.

The X–ray emissivity of the hot gas is given by

$$j_X(\xi, \nu) = \left[ \frac{\rho(\xi)}{m_p} \right]^2 \Lambda_X(T, \nu)$$

where $\Lambda_X$ is the atomic emissivity of the hot gas (ergs cm$^3$ s$^{-1}$ Hz$^{-1}$) and $T$ is the gas temperature. Since $\rho$ is given by eq. (4), $j_X$ can be represented by eq. (21) with the identification $j_o = (\rho_o/m_p)^2 \Lambda_X(T, \nu)$. The heating rate can then be computed using eq. (30) integrating over X–ray frequencies. We assume $Q_\nu = 1$ so that we actually compute an upper bound to the X–ray heating. The integral in eq. (30) reduces to an integral over only $j_o$ since the entire frequency dependence of $J_\nu$ is contained in $j_o \propto \Lambda_X(T, \nu)$.

5.4 Comparison of Heating Rates

The heating rates per grain for each of the three processes discussed above are plotted as a function of grain size in Figure 4 for the specific case of graphite grains at a radius
of 0.4 $\xi_4$ in galaxy model a. Heating due to starlight is orders of magnitude greater than that due to either electronic collisions or absorption of thermal X-rays. The dominance of heating by starlight absorption holds at all radii for all galaxy models in Table 1 and for both graphite and silicate grains. Heating rate variations with grain size for other radii and galactic models are qualitatively similar to those shown in Figure 4 and are not presented. These results are consistent with previous findings based on less quantitative treatments (de Jong 1986, Jura 1986) and can be expected from elementary considerations. First, for even the brightest ellipticals, the optical luminosity (B-band) is stronger than X-ray luminosity by a factor of $\sim 100$ (see eq. [6]) and this factor carries over to grain heating. Further, unless the energy lost from the hot interstellar gas through electronic heating of grains is vastly larger than that lost through plasma radiation, grain heating by electronic collisions must also be small (the various channels of energy loss for the hot gas are considered in §7).

Similar energetic considerations for cD galaxies or for elliptical galaxies at the centers of clusters of galaxies would be very interesting since it has been suggested that the principle grain heating mechanism is electronic collisions rather than optical heating (Bregman, McNamara, & O'Connell 1990; de Jong et al. 1990). Based on their far larger ratios of $L_x$ to $L_B$ (e.g. Edge & Stewart 1991), cD galaxies have a much greater amount of hot gas per star than do isolated elliptical galaxies; they also have much larger ratios of $L_{IR}$ to $L_x$ (Edge & Stewart 1991). The implication is that higher ratios of gas to stellar density in cD galaxies favor plasma cooling by electron-grain interactions. This will be considered in a subsequent paper.

6. EVOLUTION OF THE GRAIN SIZE DISTRIBUTION

The equation for evolution of grains in a hot flowing gas with sources is

$$\frac{\partial N(a, r, t)}{\partial t} + \frac{\partial}{\partial a} \left[ N(a, r, t) \frac{da}{dt} \right] + \vec{\nabla} \cdot [N(a, r, t) \vec{u}] = S(a, r, t).$$

(37)

In this equation, $N(a, r, t)$ is the grain size distribution at galactic radius $r$ and is given in units of cm$^{-3}$µm$^{-1}$, $\dot{a} = da/dt$ is the sputtering rate given by eq. (14) and $S$ is the source function for dust. Assuming spherical symmetry and steady state conditions, this reduces to

$$-\bar{h}\rho \frac{\partial N}{\partial a} + u \frac{\partial N}{\partial r} = S(a, r) - N \left( 2 \frac{u}{r} + \frac{du}{dr} \right).$$

(38)

where $u$ is the radial velocity and $\bar{h}$ is given by eq. (19). We recognize that the source of grains $S \propto a_s(t)$ is a strong function of time, but it will be apparent from our results below that the steady state approximation is sufficient to illustrate the effects of grain evolution. The quantity $S(a, r)$ in eq. (38) is therefore the source function for dust at the present time, $t_{now}$.

The differential equation (38) is quasi-linear and can be solved by the standard method of characteristics. The velocity of the gas $u(r)$ has already been determined by the cooling flow calculations of §3, but the source function $S$ and appropriate boundary conditions need
to be specified. Since we only consider dust ejected from evolving stars, $S$ is simply given by the rate of dust input from these stars. Unfortunately, there are presently few observational constraints on the grain content of red giant winds since dust absorption features are difficult to observe in the spectra of these cool stars. What constraints exist suggest that there are relatively few small grains ($a < 0.08\mu m$, Seab & Snow 1989). However, in view of the absence of information on the grain size distribution we assume the grain size distribution is similar to that in our Galaxy: a power law with an exponent given by the MRN (Mathis, Rumpl, & Nordsieck 1977) value of $g = 3.5$,

$$S(a, r) = A_n\rho_s \left(\frac{A}{\mu m_p}\right) a^{-g},$$

where $A$ is an as yet undetermined normalizing coefficient. We adopt a minimum grain size $a_{min} = 0.001\mu m$ and a maximum grain size of $a_{max} = 3.0\mu m$.

In the standard MRN model, the value of $A$ is determined by extinction measurements of the Galactic interstellar medium. Since these data do not exist for our case, we determine $A$ based on the local metallicity of stars in elliptical galaxies. Consider the grain size distribution of an evolving star:

$$N_{ej}(a) = \rho_{ej} A_{ej} a^{-g}$$

where $\rho_{ej}$ is the gas density in the ejecta. The total mass density of grains in the ejecta is then

$$\rho_{dust} = \int_{a_{min}}^{a_{max}} \left(\frac{4}{3}\pi a^3 \rho_{grain}\right) \rho_{ej} A_{ej} a^{-g} da$$

where $\rho_{grain}$ is the mass density of an individual grain. We take $\rho_{grain} = 2.2 g \text{ cm}^{-3}$ for graphite and $\rho_{grain} = 3.0 g \text{ cm}^{-3}$ for silicate. The density of dust is related to the gas density by the dust to gas ratio $y_g$:

$$\rho_{dust} = y_g \rho_{ej}.$$  

Then, from eq. (41):

$$A_{ej} = \frac{y_g}{4\pi \rho_{grain}} \left(\int_{a_{min}}^{a_{max}} a^{3-g} da\right)^{-1}.$$  

The solar abundance by mass of both silicon and carbon is about 0.005. We therefore set

$$y_g = 0.005 \delta Z(r, L_B),$$

where $\delta$ is the fraction of either Si or C in the ejecta contained in grains, and $Z$ is the metallicity of the stars relative to solar. Based on optical line width measurements (Davies, Sadler, & Peletier 1993; Schombert et al. 1993; Gonzales et al. 1994), the metallicity is known to vary with radius and B–band luminosity of the galaxy approximately as

$$Z(r, L_B) = \left(\frac{L_B/10^{10} L_\odot}{1 + (r/r_c)^{0.3}}\right)^{0.301}$$

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where $r_c$ is the galaxy core radius. We therefore find that

$$\frac{A}{\mu m_p} = A_{ej} = \left( \frac{y_g}{\frac{4}{3} \pi \rho_{grain}} \right) \left( \frac{4-g}{a_{max}^{4-g} - a_{min}^{4-g}} \right)$$  \hspace{1cm} (46)$$

where $y_g$ is given by eq. (44). In deriving $S$ from (39) we have assumed that few grains are sputtered before the stellar ejecta merges into the hot ISM. We provide a brief argument in Appendix C to support this assumption.

It is useful to compare our values for the normalization $A$ with standard values associated with the MRN distribution. For carbon grains in a typical galaxy of $10^{11} L_\odot$, we find $A = 0.053$ at the core radius $r_c$. This assumes that all the carbon in the ejecta is incorporated into grains ($\delta_C = 1$). This is close to the standard MRN value of $A = 0.069$ (Draine & Anderson 1985) for graphite grains in the Galaxy. For silicate, again assuming $\delta_S = 1$, we get $A = 0.039$ which is similar to the MRN value $A = 0.078$. Although our adoption of the source function in eqs. (39 - 46) is approximate, a detailed comparison of infrared observations with computed IRAS spectra will lead to an improved understanding of the grain abundance and size distribution in the ejecta of red giants and planetary nebulae.

Regarding an appropriate boundary condition at large galactic radius for solving eq. (38), we note that a favorable aspect of our method is that the grain size distribution in the galaxy actually does not depend on the choice of the outer boundary condition although something has to be specified for the numerical calculation. This is seen by referring to the timescales of Figure 3. In all cases, towards the outer boundary, the flow time far exceeds the sputtering time of the largest assumed grain. Indeed, the flow time tends to infinity as the flow velocity decreases toward the outer boundary. The grains in the outer regions therefore sputter on the spot and the grain size distribution at small or intermediate galactic radii is insensitive to the distribution at larger radii. We may specify any grain size distribution at large radius to start the integration of the grain flow equation (eq. [38]). For consistency, we assume that the grain content at large radius is determined by the on the spot sputtering assumption of eq. (18) with the source function of eq. (39), i.e.

$$N(a, r) = \frac{\alpha_n \rho_s}{(g-1) \hbar p} \left( \frac{A}{\mu m_p} \right) (a_1^{-g} - a_{max}^{1-g}).$$  \hspace{1cm} (47)$$

Finally, we must specify that the number density of grains vanish for the biggest grains in the ISM $a_{max}$, $N(a_{max}, r) = 0$ at all $r$.

Figure 5a shows grain size distributions for graphite grains determined from eq. (38) for model galaxy a. The grain size distributions are given at the core radius $r_c$ (topmost curve), and at successively larger radii (lower curves). The first implication of this figure is that the grain size distribution is very close to that of in situ sputtering (eq. [18]). For small grains the distribution is essentially a power law that is shallower than that of the stellar ejecta by unity. At large grain sizes, the density steepens and drops to zero at the largest assumed grain size. We anticipated this result wherever the sputtering times are shorter than the flow times (Figure 3). What is somewhat surprising, however, is that the assumption of in situ sputtering can be used to accurately determine the size distribution
even where the sputtering time for the largest grains is comparable to or exceeds the flow times to the core. This can be understood since the flow of grains from upstream into a given volume $4\pi r^2 dr$ near the galactic core is significantly smaller than the amount of grains produced by stars within $4\pi r^2 dr$ – this is a result of the steep stellar density profile $\rho_s(r)$.

The grain size distributions for graphite in the smallest galaxy (model c) are shown in Figure 5b. In contrast to galaxy model a, the flow time to the core radius is comparable to the sputtering time for the largest grain over most of the galaxy. Again, the grain size distributions are well determined by assuming in situ sputtering. Similar results hold for galaxy model b which are not illustrated. Grain size distributions for silicate dust have identical shapes to those of graphite, although the normalizations are slightly different, so these results are not shown. A favorable consequence of the above results is that the grain size distribution in the ISM depends only on the ratio of the stellar density to the gas density (and on the rate of dust injection into the ISM). These quantities are reasonably well known. The grain size distribution does not depend on the more poorly known gas velocities in the cooling flow.

In order to gain further insight, we have varied $a_{\text{min}}$ and $a_{\text{max}}$ while keeping the slope of the grain distribution $g$ fixed. For example, if instead of $a_{\text{max}} = 0.3\mu$m we set $a_{\text{max}} = 1.0\mu$m in the stellar ejecta, the grain size distribution is modified in the manner shown in Figure 5c. Over the common range of grain sizes, the case with $a_{\text{max}} = 1.0\mu$m gives about half the grain density relative to the case with $a_{\text{max}} = 0.3\mu$m. Since the total mass of grains ejected by stars is fixed, extending the range over which grain masses are distributed requires a decrease in the density per unit grain size. That is, by increasing $a_{\text{max}}$ we decrease the normalization $A$ by a factor of $\sim 0.53$.

One final case we consider is motivated by the observational evidence that there may be fewer small grains in stellar ejecta than in the Galactic ISM. For example, consider a power law grain size distribution $S(a) \propto a^{-g}$ with $a_{\text{min}} = 0.08\mu$m $< a < a_{\text{max}} = 1.0\mu$m. In the cooling flow ISM, however, we continue to follow the evolution of grains smaller than $a_{\text{min}}$ since these smaller grains are produced by sputtering. The resulting grain size distribution is shown in Figure 5c as the dashed line. Since the range of grain sizes in the source function $S(a)$ is restricted at small grain radius $a$, the density per unit grain size is higher than in the case where $a_{\text{min}} = 0.001\mu$m and comparable to that of the first case considered (solid line of Figure 5c). For $a < a_{\text{min}}$ the grain size distribution in the ISM is essentially constant. Because there are no stellar sources of dust in this range, the only sources are the sputtered remnants of larger grains. Since the sputtering rate is independent of the grain size (eq. [14]), all grains get smaller at the same rate. Similar results are obtained for galaxy models b and c but are not illustrated.

7. ENERGY LOSS FROM THE INTERSTELLAR GAS

Given the grain size distributions of §6, we can compute the total rate of energy loss from the hot ISM by electron-grain collisions. This can be done at any given radius by simply convolving the heating rate per grain (eq. [34]) with the grain size distribution. This rate of energy loss from the plasma by electron-grain collisions can be compared with
the local rate of energy loss due to thermal emission,

\[
\frac{dE_X}{dt} = \left[ \frac{\rho(\xi)}{m_p} \right]^2 \Lambda_{tot}(T),
\]

where \( \Lambda_{tot}(T) \) is the bolometric atomic X-ray emissivity.

Cooling rates for the hot gas in the largest galaxy (model a) are shown as a function of galactic radius in Figure 6. In all cases, the energy loss rate from the hot plasma due to grain heating is significantly smaller than that of the thermal emission. This result is very different from that of Dwek (1987) who found that dust heating accounted for as much as two orders of magnitude more energy loss from hot, dusty plasmas than thermal emission. Dwek then applied these results to supernova remnants. The difference between our results and those of Dwek’s is our inclusion of sputtering losses. To estimate the magnitude of the reduction of the dust-to-gas ratio due to sputtering, assume that all grains have the same radius \( a \) and mass \( m_g \) and that they are created and destroyed at the same rate. In this case the number density of grains \( N \) is related to the dust-to-gas ratio in the stellar ejecta \( y_g \) by \( N m_g/t_{sp} = \alpha_n \rho_s y_g \), where \( t_{sp} \approx am_p/h_p \) is the local sputtering time in plasma of density \( \rho \) (eqs. [14] and [15]). When sputtering is included the dust-to-gas ratio is \( y_{gs} = m_g N/\rho \approx y_g t_{sp} \alpha_n \rho_s /\rho \). It follows that sputtering reduces the dust-to-gas ratio by a factor \( y_{gs}/y_g \approx t_{sp} \alpha_n \rho_s /\rho \approx (am_p \alpha_n /h) \rho_s /\rho^2 \). Consider for example stellar and gas densities near the core radius of galaxy model a, \( \rho_s \approx 3 \times 10^{-21} \text{ g cm}^{-3} \) and \( \rho/m_p \approx 0.1 \text{ cm}^{-3} \). At this location the dust-to-gas ratio is reduced by \( y_{gs}/y_g \approx 3 \times 10^{-3} \) for grains of size \( a = 0.01 \mu m \); this is essentially the factor that accounts for the difference in our plasma cooling rates and those of Dwek (1987) who assumed an “extended” MRN distribution with an unsputtered dust-to-gas mass ratio. By allowing the MRN distribution in the stellar ejecta to be sputtered, we find that the plasma energy loss to grain heating is less, not very much greater, than radiative losses. We also find that the space density of the largest grains is preferentially decreased; these are the grains most responsible for energy loss from the hot gas (see below). Self-consistent grain sputtering therefore plays a decisive role in correctly assessing the significance of dust emission for the hot ISM in galactic cooling flows.

It is evident from Figure 6 that the heating of graphite grains (short dashed–dotted line) accounts for a little more than half of the total plasma cooling rate whereas the less numerous silicate grains (long dashed–dotted line) contribute somewhat less than half of the total. We also find that the various assumptions made concerning the grain size distribution of the stellar ejecta do not result in large differences in the rate of heat loss from the hot ISM. This is easy to understand by considering the dependence of the plasma cooling rate on the assumed grain distribution of the stellar ejecta. The heating rate for the dust is

\[
\frac{dE_{dust}}{dt} = \int_{a_{min}}^{a_{max}} N(a,r) H_e da \propto \left( \frac{1}{a_{max}^{4-\gamma} - a_{min}^{4-\gamma}} \right) \int_{a_{min}}^{a_{max}} a^{1-\gamma} a^2 da \sim \text{Constant.} \]

The term in parenthesis is due to the normalization of the grain size distribution (eq. [46]), the first factor in the second integrand is roughly the power law of the grain size
distribution in the hot ISM, and the second factor in the second integral is approximately proportional to the cross-section of the grain for electron-grain collisions. The plasma cooling rate due to electron-grain collisions is therefore insensitive to the assumed value of the slope \( g \) or the upper and lower cutoffs in the grain size distribution in the stellar ejecta. Eq. (49) shows that the plasma cooling rate depends only on the total mass of grains ejected by stars into the ISM and the ratio of the stellar mass to gas mass. Since these are well constrained by observations, the rate of plasma cooling by this process is well determined regardless of specific assumptions made about the grain size distribution in the stellar ejecta.

According to the results plotted in Figure 6, the ratio of energy loss from the hot ISM by dust heating to that by X-ray emission is relatively independent of galactic radius \( r \). The heating of individual grains by hot ISM electrons is proportional to the local gas density, \( H_e \propto \rho \) (eq. [34]). But the local grain density depends on the ratio of stellar to gas density and the stellar metallicity, \( N(a, r) \propto Z_{\rho_s} / \rho \); therefore \( N H_e \propto Z_{\rho_s} \). By comparison the rate that the hot ISM loses energy by thermal radiation \( dE_X / dt \propto \rho^2 \propto \rho_s \) (eqs. [1], [4], and [48]). Therefore the ratio of thermal losses by grain heating to optically thin emission varies slowly with galactic radius, \( N H_e / (dE_X / dt) \propto Z(r) \propto r^{-0.3} \), which accounts for the small rate of divergence between the two rates plotted in Figure 6. For the largest galaxy, for example, \((r_t / r_c)^{0.3} \sim 150^{-0.3} = 4.6\) is the ratio of the two heating rates at the core radius and at the outer radius, again consistent with Figure 6.

The loss of thermal energy from the hot ISM by grain heating in the two smaller galaxies \( b \) and \( c \) is even less important relative to radiative losses than in model \( a \). This can be understood because \( N H_e / (dE_X / dt) \propto Z \propto L_B^{0.3} \) (eq. [45]). The larger ratio of sputtering to flow times in smaller galaxies (Figure 3) plays a very minor role in this ratio; the relative contribution of advected dust to the total dust content at any radius is small because of the steep radial dependence of \( \rho_s(r) \).

8. CONCLUSIONS

We have investigated the evolution and energetics of dust embedded in the hot ISM of isolated elliptical galaxies. For this purpose we constructed a series of single-parameter galaxy models assuming King type profiles for the stellar density which are consistent with the fundamental plane relation, the mass to light ratio, core radius relation, and a virial condition for isothermal cores. We consider three representative model ellipticals with \( L_B \) spanning a range from \( \sim 10^{10} L_\odot \) to \( \sim 10^{11} L_\odot \) which are expected to contain galactic cooling flows.

The galaxy models were used to construct approximate isothermal – but fully time-dependent – cooling flow models for the hot ISM. The radial gas density variation is known if the X-ray surface brightness distributions of all ellipticals resemble those resolved by \textit{Einstein}. With this assumption we can solve for the velocity field using the continuity equation alone. These “continuity” models compare well with detailed hydrodynamical cooling flow calculations. The isothermal temperature of the gas is taken to be the same as the mean stellar temperature of the galaxy. This latter temperature is determined by solving the equation of stellar hydrodynamics in the presence of a massive dark halo. These
temperatures agree reasonably well with observed temperatures for galaxies of comparable luminosity (see Figure 1). A more detailed agreement of predicted and observed temperatures is not required for the purposes of this paper because the results depend only weakly on the assumed temperature of the ISM.

Having specified the plasma environment within the model galaxies, the radiation environment from the near IR through the UV is taken from observations. With this information we can compute the heating rate of interstellar grains due to absorption of stellar light, impacts with energetic electrons from the hot ISM, and absorption of thermal X-rays from the hot ISM. In spite of the efficiency of sputtering in grain destruction, the heating of grains in all model galaxies is dominated by absorption of ambient starlight by over an order of magnitude. This result applies for both graphite and silicate grains. Although this result was anticipated by de Jong (1986) and Jura (1986), our detailed calculation of the heating rates is relevant to the well constrained radiation-plasma environment of elliptical galaxies. In future work, we will consider the same issue for cD galaxies and giant ellipticals at the centers of clusters of galaxies where both the plasma environment and IR emission characteristics appear to be distinct from isolated ellipticals (see §5.4).

We have computed the grain size distribution in three representative galactic cooling flows including the effects of grain sputtering, advection by the cooling flow, and local stellar sources of dust. In this paper we have not considered external sources of grains – for example by recent mergers of gas-rich galaxies with ellipticals – or dust that may form within gas deposited in the centers of the cooling flows. In this sense our estimate of the amount of dust present in ellipticals is a lower limit. We assumed that the grain size distribution introduced into the cooling flow at every radius from evolving stars is given by a power law having the MRN index of $g = 3.5$, normalized according to the observed local metallicity of the parent stars. No significant loss of grains is expected during the brief transition from red giant winds or planetary nebulae into the ISM. The ratio of grain sputtering time to radial flow time varies within each galactic cooling flow and among galaxies of different total mass. When the sputtering time is very short we find that the grain size distribution for small grains will be a shallower power law with index $1 - g$, but we find that this same solution is still an excellent approximation even when the sputtering time is so long that the grains can move inward during their lifetimes across an appreciable part of the galaxy. This last more surprising result can be understood from the very steep radial gradient of the stellar sources of grains; grains advected from larger radii in the galaxy are far less numerous compared to those produced locally at every radius in the cooling flow. This latter property implies that the grain size distribution does not depend on the fluid velocities in the cooling flow. Rather, the distribution depends only on the better known ratio of the stellar density to the gas density.

The heating of grains by absorption of hot thermal electrons is a possible channel of energy loss for the hot gas in cooling flows. Dwek (1987) suggested that this mode of energy loss may far exceed that due to emission of thermal X-rays in hot plasma environments similar to that of the ISM of ellipticals. However, we find that this channel for energy loss is in fact much smaller than that due to thermal emission in galactic cooling flows. While Dwek’s result was based on assuming that the grains (with an MNR distribution)
are unsputtered while they reside in the hot plasma, in our model galaxies the dust-to-gas ratio and the distribution of grain sizes are considerably modified by sputtering. The total plasma cooling by a combination of graphite and silicate grains is a factor $\gtrsim 5$ below that of optically thin radiative losses in all galaxies we consider. Our results for the magnitude of plasma cooling by thermal interactions with grains are very insensitive to the assumed upper and lower grain size cutoffs as well as the slope $g$ of the grain distribution in the stellar ejecta.

In this paper we have surveyed the relevant physics of distributed grains in elliptical galaxies. A second paper, currently in preparation, will describe in detail the temperatures of grains at every radius in elliptical galaxy cooling flows and the far infrared emission into each of the IRAS bands. We will determine the radial distribution of IR emission for comparison with future, high resolution observations. We are also interested in estimating the IRAS luminosities from more centrally concentrated dusty patches and lanes in ellipticals for comparison with the more distributed dust.

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APPENDIX A
SELF-CONSISTENT GALAXY MODELS

The fundamental plane for elliptical galaxies

\[ L_B = 3.6 \times 10^3 \, R_e \sigma_{*o}^{1.3} h^{-1} \]  \hspace{1cm} (A1)

relates the B-band luminosity \( L_B \) (in units of \( L_{\odot}B \)), the effective radius \( R_e \) (in parsecs) and the central stellar velocity dispersion \( \sigma_{*o} \) (in km s\(^{-1}\)); \( h = H/100 \, \text{Mpc km} \, \text{s}^{-1} \) is the Hubble parameter (Dressler et al. 1987; Djorgovski & Davis 1987; Ciotti et al. 1991). The stellar luminosity \( L_B \) can be directly related to the total stellar mass \( M_{st} \) (in units of \( M_{\odot} \)) by the stellar mass to light ratio:

\[ M_{st}/L_B = 2.98 \times 10^{-3} \, L_B^{0.35} h^{-1.7} \]  \hspace{1cm} (A2)

based on the recent study of van der Marel (1991).

The effective radius \( R_e \) that contains half of the stellar mass or light in projection can be found from eq. (3) in terms of \( r_c \) and \( r_t \). The resulting implicit equation for the effective radius \( \lambda_e = R_e/r_c \) is approximated very well by

\[ \lambda_e = 0.857 \, \xi_{t}^{1/2} \]  \hspace{1cm} (A3)

where \( \xi_t = r_t/r_c \). In addition, the core radius \( r_c \), in parsecs, increases with galactic luminosity according to

\[ r_c = 6.84 \times 10^{-11} \, L_B^{1.20} h^{1.4} \]  \hspace{1cm} (A4)

(Lauer 1989). This relation is consistent with more recent galactic distances based on the \( D_n - \sigma_* \) method (Donnelly, Faber, & O’Connell 1990).

Finally, the core radius, central density, and central velocity dispersion are related via the standard core relation for isothermal spheres (Binney & Tremaine 1987),

\[ r_c = \sqrt{\frac{9 \sigma_{*o}^2}{4 \pi G \rho_{*o}}} . \]  \hspace{1cm} (A5)

By elimination among eqs. (A1)-(A5), all quantities of interest can be expressed in terms of a single parameter, most conveniently chosen to be the total radius \( \xi_t \). For a given \( \xi_t \), the optical luminosity (in \( L_{\odot}B \)) is given by

\[ L_B = 8.92 \times 10^{11} \, \xi_t^{-1.682} \mu_{mass}(\xi_t)^{2.186} h^{-2} \]  \hspace{1cm} (A6)

where \( \mu_{mass} \) is given by eq. (2) of the text. The central stellar density is then given by

\[ \rho_{*o} = 4.41 \times 10^{10} \, \xi_t^{-0.769} L_B^{-2.708} h^{-3.416} \, \text{g cm}^{-3} . \]  \hspace{1cm} (A7)
The corresponding core radius and total stellar mass are found from eqs. (A4) and (A2), respectively. Table 1 lists the parameters of a representative sample of the galaxy models generated by the above procedure. We assume $h = 0.75$ throughout.

We now estimate the mean stellar temperature for our galaxy models. Gas ejected from stars is heated to the virial temperature by dissipation. The gas can be further heated by Type Ia supernovae if the supernova rate is sufficiently large. The observed supernova rate is usually given in units of $\mathrm{SNU} = 1$ supernova every 100 years in a galaxy of $L_B = 10^{10} L_{\odot}$. For a typical large elliptical galaxy supernova heating dominates heating by stellar motions if $\mathrm{SNU} \gtrsim 0.1$. However, recent observations of NGC 1399 and NGC 4472 by Serlemitsos et al. (1993) and Forman et al. (1993) indicate iron abundances in the cooling flow gas that are near solar, similar to that of the stellar ejecta. The implied value of $\mathrm{SNU}$ is less than 0.05 when compared with the models of Loewenstein and Mathews (1991). If supernova heating is unimportant as seems likely, the gas temperature should be close to the mean stellar temperature $\langle T_s \rangle$. We determine approximate values for $\langle T_s \rangle$ by solving the equation of stellar hydrodynamics for the velocity dispersion (Mathews 1988) with plausible assumptions for the dark halo mass distribution and stellar velocity ellipsoids. The stellar temperature is then determined from the velocity dispersion by

$$
\sigma_s^2(\xi) = \frac{3kT_s(\xi)}{\mu m_p} = \frac{3}{2} \frac{G M_c}{r_c} \tau(\xi),
$$

where $\mu m_p$ is the mean mass per particle ($\mu$ is taken to be 0.5 and $m_p$ is the proton mass), $M_c = 4\pi r_c^3 \rho_s/3$ is the galactic core mass, and $\tau$ is a dimensionless temperature.

We assume that dark halos in ellipticals are pseudo-isothermal

$$
\rho_h(\xi) = \rho_{ho}[1 + (\xi/\beta)^2]^{-1}
$$

with a universal ratio of halo to stellar core radii $\beta = r_{ch}/r_c = 10$ and having the same outer radius $\xi_t$ as the stellar distribution. The total integrated masses of the dark halos is assumed to be larger than $M_{st}$ by a factor $\varpi = 10$. This implies values of $\rho_{ho}$ listed in Table 1. For the stellar velocity ellipsoid we adopt a radial dependence given by

$$
U(\xi) = 1 - \frac{v_{tr}^2}{v_r^2} = \left(\frac{\xi}{\xi_t}\right)^2
$$

(Mathews 1988) where $v_{tr}^2$ and $v_r^2$ are the transverse and radial velocity dispersions respectively.

The stellar temperatures weighted by stellar mass and listed in Table 1 are computed from the temperature distribution $T_s(\xi)$ using

$$
\langle T_s \rangle = T_{so} \tau = T_{so} \int_0^{\xi_t} \frac{\tau(\xi) \rho_s(\xi) \xi^2 d\xi}{\int_0^{\xi_t} \rho_s(\xi) \xi^2 d\xi} ; \quad T_{so} \equiv \frac{\mu m_p}{2k} \frac{G M_c}{r_c}.
$$
We find that the average temperatures computed this way imply the relation \( \langle T_s \rangle \propto L_B^{0.54} \) or equivalently, \( L_B \propto \langle \sigma_s \rangle^{3.7} \). Since this is close to the observed Faber–Jackson law, we conclude that the assumed properties for the massive halo and velocity ellipsoid are reasonable. In addition, recently measured temperatures from ASCA (Serlemitsos, et al. 1993; Mushotzky et al. 1995) agree well with eq. (A1) and our relation \( \langle T_s \rangle \propto L_B^{0.54} \) (Figure 1).

**APPENDIX B**

**THE \( L_x, L_B \) RELATION**

Canizares, Fabbiano & Trinchieri (1987) showed that the X-ray luminosity in the *Einstein* IPC band (\( \Delta E \sim 0.5 - 4.5 \text{ keV} \)) scales with optical luminosity as \( L_x \propto L_B^{1.5-1.7} \). Later Donnelly, Faber, & O’Connell (1990) found \( L_x \propto L_B^{2.0\pm0.1} \) using a smaller sample but with better galactic distances (\( D_n - \sigma_s \) method). Sarazin (1987) showed that \( L_x \propto \dot{M}_s / \sigma_s^2 \) provided the cooling flow gas is heated by compression in the gravitational field; from this and the Faber-Jackson relation it follows that \( L_x \propto L_B^{1.5} \). In the following we present an alternate derivation of this result.

Suppose that the hot interstellar gas in all bright ellipticals has a homologously similar distribution. If \( \bar{n} \) is the mean plasma density then the rate that energy is emitted into the *Einstein* band per unit volume is proportional to \( \bar{n}^2 \Lambda_{\Delta E} \) (\( \Lambda_{\Delta E} \) is the atomic emissivity in the *Einstein* band). The specific emissivity is \( \propto \bar{n} \Lambda_{\Delta E} / \mu m_p \), where \( \mu m_p \) is the mean mass per particle. The total observed X-ray luminosity should be \( L_x \propto \bar{n} \Lambda_{\Delta E} M_g / \mu m_p \) where \( M_g \) is the total mass of hot gas. If \( \langle \alpha_s \rangle \) is the specific mass loss rate from an evolving population of old stars (Mathews 1989) averaged over a cooling flow time (several Gyr), then by continuity of gas flow \( M_g / t_{cool} \approx \langle \alpha_s \rangle M_{st} \). The cooling time is given by \( t_{cool} \approx 5 kT / 2 \Lambda_{\Delta E} \bar{n} \) where \( \Lambda_{\Delta E} \) is the bolometric cooling function.

Combining these results, we expect

\[
L_x \propto \langle \alpha_s \rangle \left( \Lambda_{\Delta E} / \Lambda_{tot} \right) M_{st} T \propto \langle \alpha_s \rangle \left( \Lambda_{\Delta E} / \Lambda_{tot} \right) M_{st} L_B^{2/\eta}, \tag{B1}
\]

where we have used the Faber-Jackson law \( L_B \propto \sigma_s^4 \) and the virial condition that \( \sigma_s^2 \propto \langle T_s \rangle \approx T \).

If the stellar population in all ellipticals is dominated by equally old stellar populations, \( \alpha_s(t_n) \) should not vary with \( L_B \). However, eq. (B1) involves \( \langle \alpha_s(t) \rangle \) which is averaged over the time for gas to flow through the central regions (\( \xi \lesssim \lambda e \)) where most of the X-rays are produced. Typically this time is very short (\( \lesssim 10^8 \text{ yrs} \); see Fig. 2) so we conclude that \( \langle \alpha_s(t) \rangle \approx \alpha_s(t_n) \) does not vary appreciably with \( L_B \) as long as all ellipticals have about the same age. For plasmas with solar abundance, \( \Lambda_{\Delta E} / \Lambda_{tot} \approx 0.6 \) and is independent of \( T \) provided \( T \gtrsim 3 \times 10^6 \text{ K} \); \( \Lambda_{\Delta E} / \Lambda_{tot} \) should not vary with \( L_B \) even for the least luminous galaxy in Table 1. However, the stellar mass to light ratio varies as \( M_{st} / L_B \propto L_B^{\nu} \) where \( \nu \approx 0.2 \) (Dressler et al. 1987; Djorgovski & Davies 1987; Bender, Burstein, & Faber 1992; Renzini & Ciotti 1993) or \( \nu \approx 0.35 \pm 0.05 \) (van den Marel 1991). Using this latter value and \( \eta = 4 \) we find \( L_x \propto L_B^{1.9} \), very close to the observed relation.
Finally we note from eq. (B1) that $L_x(t) \propto \langle a_s(t) \rangle \propto t^{-1.35}$ which accounts for the slow secular decrease in X-ray luminosity during the cooling flow evolution of ellipticals (Loewenstein & Mathews 1991; Ciotti et al. 1991).

APPENDIX C

DUST GRAINS IN STELLAR EJECTA ENTERING THE COOLING FLOW

The complex evolution of gas ejected from red giants as winds or planetary nebulae as it interacts with ambient hot cooling flows has been discussed by Mathews (1990). A typical evolutionary sequence is: attainment of hydrostatic equilibrium with the hot gas, disruption by Rayleigh-Taylor instabilities into many small clouds that extend back along the stellar orbit as the orbital motion inherited from the parent star is dissipated, and conductive evaporation into the hot ICM after coming to rest. Gas ejected from stars interacts only with the hot gas (not with other stellar ejecta) and can be kept ionized by galactic UV radiation prior to conductive evaporation. However, gas ejected from stars must be conductively heated to cooling flow temperatures $T \sim 10^7$ K in less than $\sim 10^6$ yrs or the collective emission line luminosity would exceed that observed in elliptical galaxies. Conduction of thermal energy from the hot ISM into the stellar ejecta can occur by magnetic reconnection as soon as the debris of the stellar ejecta slows below the local Alfvén velocity.

In this paper we assume that dust grains survive this transient evolution from stellar ejecta into the cooling flow gas. The most convincing argument for this is to adopt the opposite point of view, a worst case scenario in which the grains are destroyed before encountering the hot ISM. Suppose, for example, that gas ejected from a star remains intact (i.e. no instabilities!) and that the density and temperature in the stellar ejecta are sufficient to allow sputtering destruction of grains before the gas density drops to ambient ISM values. Since sputtering is very slow unless $T \gtrsim 10^6$ K, we suppose that the stellar ejecta, while still rather dense, is heated by conduction to the ambient temperature $T_a$ of the local cooling flow. In this (very unlikely) circumstance the grains could sputter away before entering the hot ISM. We now consider the self-consistency of this worst case evolution.

For plasma temperatures $T \gtrsim 10^6$ K the sputtering time $t_{sp}$ for grains of size $a = 10^{-5}$ cm depends mostly on the plasma density in the cloud $n_c$, $t_{sp} = a/\dot{a} \approx 1 \times 10^5 a_{-5} n_c^{-1}$ yrs, where $a_{-5}$ is the grain size expressed in units of $10^{-5}$ cm ($n_c$ is in units of cm$^{-3}$). A spherical cloud of radius $r_c$ with the mass of an ejected stellar envelope, $M_e = 0.2 M_c$ and temperature $T_c$ will expand into the low pressure ISM in approximately the sound crossing time, $t_{sc} = r_c/c_s \approx 2.5 \times 10^5 T_{c7}^{-1/2} n_c^{-1/3}$ yrs ($T_{c7}$ is the temperature expressed in units of $10^7$ K) where the cloud is assumed to have been heated to temperatures comparable with the local ISM. In order for the grains to sputter away before entering the local ISM, we require $t_{sc} > t_{sp}$ or $n_c > 246 a_{-5}^{3/2} T_{c7}^{3/4}$ cm$^{-3}$. Since the internal pressure in such a cloud, $n_c T_c \sim 10^9$ cm$^{-3}$ K, far exceeds that in typical cooling flows, $n_a T_a \sim 10^4$ cm$^{-3}$ K, the spherical cloud of stellar ejecta will indeed expand.
However, the thermal energy necessary to heat the stellar ejecta to temperature $T_c$ must come from the local ISM. The radius of a region in the local ISM that contains the same thermal energy as the cloud is $r_a = r_c \left( n_c T_c / n_a T_a \right)^{1/3} \approx 3.8 \times 10^{19} T_{c7}^{1/3} (n_a T_a)^{1/3} / 10^4 \text{cm}^{-3}$, where $(n_a T_a)_4$ is $(n_a T_a)$ given in units of $10^4 \text{cm}^{-3} \text{K}$. Finally, thermal heat must be conductively transported into the stellar ejecta before the cloud can expand under its own pressure. Therefore in order to destroy the dust before it enters the local ISM we require that heat is conducted into the spherical cloud with a characteristic velocity $u_{fl} > r_a / t_{sc} \approx 4.8 \times 10^3 n_c^{1/3} T_{c7}^{5/6} (n_a T_a)_4^{-1/3} \text{km s}^{-1} > 3 \times 10^4 a_5^{1/2} T_{c7}^{13/12} (n_a T_a)_4^{-1/3} \text{km s}^{-1}$. Since this lower limit exceeds the velocity of thermal electrons in a gas at $T_a \sim 10^7 \text{K}$, $v_e \approx 2 \times 10^4 T_{7}^{-1/2} \text{km s}^{-1}$, we conclude that this hypothetical, maximally pessimistic evolutionary model is impossible. Dust grains do in fact largely survive the transition from stellar ejecta into the local cooling flow ISM as we have assumed here.
### Table 1

**Galaxy Model Parameters\(^a\)**

<table>
<thead>
<tr>
<th>Model</th>
<th>(L_B(10^{10} L_\odot))</th>
<th>(r_c (\text{kpc}))</th>
<th>(r_t (\text{kpc}))</th>
<th>(\rho_{so} \text{(^b)})</th>
<th>(\rho_{o} \text{(^b)})</th>
<th>(\rho_{ho} \text{(^b)})</th>
<th>(\langle T_s \rangle (10^7 \text{K}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>10.6</td>
<td>0.774</td>
<td>123.</td>
<td>3.37</td>
<td>2.42 \times 10^{-4}</td>
<td>1.12 \times 10^{-2}</td>
<td>0.922</td>
</tr>
<tr>
<td>b</td>
<td>3.31</td>
<td>0.192</td>
<td>76.4</td>
<td>38.6</td>
<td>5.06 \times 10^{-4}</td>
<td>5.74 \times 10^{-2}</td>
<td>0.468</td>
</tr>
<tr>
<td>c</td>
<td>0.976</td>
<td>4.44 \times 10^{-2}</td>
<td>44.4</td>
<td>518.</td>
<td>1.47 \times 10^{-3}</td>
<td>3.48 \times 10^{-1}</td>
<td>0.252</td>
</tr>
</tbody>
</table>

\(^a\) Parameters for the galaxy models given in the first column are listed. From the second to the eighth column, respectively, these are: the B-band luminosity, the core radius, the total radius, the central stellar density, the central gas density, the central dark matter halo density, and the mean temperature of the stellar distribution.

\(^b\) All densities are given in units of \(10^{-21} \text{ g cm}^{-3}\).
REFERENCES

Binney, J., & Tremaine, S. 1987, Galactic Dynamics (Princeton University Press), 228
Dwek, E. 1993, private communication
Makishima, K. 1994, talk presented at New Horizons of X-ray Astronomy, Tokyo, Japan
Raymond, J. C. 1991, private communication
Figure 1: The \((L_B,T_e)\)-relation for the model galaxies of Appendix A is shown. The filled circles (connected by the solid line) give results for some of our models. The open circles give the observed \(T\) and \(L_B\) for NGC 4636, NGC 4406, and NGC 4472.

Figure 2: The solid line gives the inward velocity of the gas in the cooling flow of galaxy a (see Table 1) plotted as a function of the radius normalized to the outer radius. The dashed and dashed–dotted lines give the velocities for galaxies b and c, respectively.

Figure 3: The sputtering time (defined by eq. [15] of the text) and flow time (eq. [16]) are shown by dashed and solid lines, respectively. The heaviest lines give the results of galaxy a, the second heaviest lines correspond to galaxy b, and the lightest lines correspond to galaxy c.

Figure 4: The heating rate (plotted versus grain size) due to absorption of ambient starlight is given as the solid line, the heating due to collisions with energetic electrons is given by the dashed–dotted line, and an upper bound for heating due to absorption of thermal X-rays is given by the dashed line. The indicated rates are computed at a radius of 0.4 times the outer radius of galaxy a (see Table 1).

Figure 5a: The grain size distribution (graphite) at various galactic radii are shown for model galaxy a. The grain size distribution extends from \(a_{\text{min}} = 0.001 \mu m\) to \(a_{\text{max}} = 0.3 \mu m\). The topmost curve gives the number density of grains per unit grain size at the core radius \(r_c\). The ten lower curves starting from the top give the grain size distributions at 0.1, 0.2,..., and 1.0 times the outer radius \(r_t\).

Figure 5b: The grain size distributions (graphite) for model galaxy c are shown at various radii. The topmost curve applies at the core radius, and the ten successively lower curves correspond to 0.1, 0.2,..., and 1.0 times the outer radius.

Figure 5c: The grain size distributions (graphite) at 0.5 times the outer radius for galaxy model a are shown. The solid line gives the case where we assume \(a_{\text{min}} = 0.001 \mu m\), \(a_{\text{max}} = 0.3 \mu m\), and that the grain size distribution in the stellar ejecta is given by eq. (39). The dashed dotted line corresponds to the same case as the solid line except we assume \(a_{\text{max}} = 1.0 \mu m\). The dashed line assumes the same power law in the stellar ejecta, but that \(a_{\text{min}} = 0.08 \mu m\) and \(a_{\text{max}} = 1.0 \mu m\). The grain size distribution in the ISM is, however, computed for smaller grains than \(a_{\text{min}}\).

Figure 6: The rate of energy loss from the hot ISM due to heating of grains is plotted as a function of radius for galaxy model a. The light solid line gives the total heating rate assuming \(a_{\text{min}} = 0.001 \mu m\) and \(a_{\text{max}} = 0.3 \mu m\). The contributions to the grain heating rate from graphite and silicate grains for this case are given by the short dashed–dotted and the long dashed–dotted lines, respectively. The short dashed line gives the total heating rate in the case where \(a_{\text{min}} = 0.001 \mu m\) and \(a_{\text{max}} = 1.0 \mu m\). The long dashed line corresponds to taking \(a_{\text{min}} = 0.08 \mu m\) and \(a_{\text{max}} = 1.0 \mu m\). The total heating rates given by the light solid line, the short dashed line, and long dashed line are nearly identical. The rate of energy loss due to X-ray emission is given by the heavy solid line.