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BARYON-NUMBER DENSITY IN HADRONIC GAS MODELS AND
BARYON-ASYMMETRY IN THE EARLY UNIVERSE

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ABSTRACT

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Isothermal baryon-number fluctuations arising from a first-order Quark-Hadron phase-transition in the early Universe obtained from the ratio of the baryon-number densities in the quark-gluon plasma (QGP) phase and hadron-resonance gas (HRG) phase, depend sensitively on the finite-size volume corrections for the hadrons in the HRG phase. This problem is explored in detail by comparing the predictions of various existing models for the baryon-number density in the HRG phase and its dependence on the temperature and baryon chemical potential. These studies throw much light on obtaining a proper equation of state (EOS) for the nuclear matter at high temperature and density.

I. INTRODUCTION

Recently the physics of the quark-hadron phase transition has been of much interest [1,2]. One interesting suggestion from the astrophysical point of view is regarding large baryon-number density fluctuations which can arise from a QCD phase transition occurring in the early Universe [3]. Assuming that the quark-hadron phase transition is first-order, the new phase is not reached immediately. The system is cooled down through the critical temperature T_c , but overcooling occurs in which fluctuations create small volumes of the new phase. The nucleated bubbles of the hadronic phase then expand in the QGP phase and thus generate shock waves which then reheat the plasma so that no further nucleation occurs [4]. The cosmological implications of baryon-number density inhomogeneities produced in the cosmic fluid lie in creating inhomogeneities in the ratio of neutrons to protons and thus significantly alters the predictions of the standard scenario for primordial nucleosynthesis (PNS) [5-8]. Considerations of phase transition in the early Universe have been very useful in restricting various existing models for QGP and HRG systems. The parameter which plays a significant role for baryon number fluctuations in the cosmological quark-hadron phase transition is the contrast ratio "R" in the baryon number density (n_B) for the two phases, i.e., $R = n_B^{\text{QGP}}/n_B^{\text{HRG}}$ which is evaluated at $T = T_c$ with baryon chemical potential $\mu_B \ll T_c$. The early Universe evolves with a value of μ_B/T approximately equal to 10^{-10} . Several authors have calculated the value of R by assuming a QGP as an ideal thermodynamical gas of quarks and gluons and similarly the HRG was also treated as an ideal gas of hadrons. The purpose of this paper is to compare the predictions of various models for the baryon-number density in the HRG phase in order to investigate the dependence of R on the choice of a proper equation of state (EOS) of the hot and dense nuclear matter.

Turner noticed [9] that $R \gg 1$ for $T_c < 150$ MeV when low lying baryon states beyond the neutron and proton are included in HRG phase non-relativistically. The actual behaviour is more reliably computed when the physical volume occupied by each hadron is taken into consideration in the equation of state (EOS) for the hadron gas. The calculation of R including finite volume thermodynamics of a hadron gas was done by several authors [8]. It was found that the inclusion of the physical volume of hadrons changes not only the values of R but it gives an opposite trend for the dependence of R on the critical temperature. Murugesan *et al* [10] found that the incorporation of quantum statistics for the description of particles in both the phases and Hagedorn's pressure ensemble correction [11] for the finite-size of the hadrons in the HRG phase results into lowering of the value of T_c and $R \geq 10$ for $T_c \leq 125$ MeV. Recently we have shown [12] that the incorporation of hard-core volumes for the baryons in the Kuono-Takagi model [13] and treating QGP as weakly interacting plasma significantly alters the prediction for baryon-number density inhomogeneity in the early Universe. We found that $R > 10$ for T_c lying outside the range $150 \text{ MeV} < T_c < 260 \text{ MeV}$. Moreover, the value of R is larger than 1 even when T_c lies in the above range. We noticed that the actual difference lies in the calculation of n_B in the HRG phase. In fact, the values of n_B in the HRG phase as obtained by us in the Kuono-Takagi approach of finite-size corrections differ quite significantly from the values obtained by Turner [9] and Murugesan *et al* [10]. Furthermore, baryon-number density n_B first increases with increasing T_c and after reaching a maximum value it falls rapidly if many strange as well as nonstrange baryons and resonances are included in the HRG phase whereas in the other two models, n_B increases exponentially and never falls back. Kapusta and Olive have considered [14] the repulsive interactions between a pair of hadrons by parametrizing the mean field potential energy and

they have also noticed $R \simeq 7$ for $160 \text{ MeV} < T_c < 240 \text{ MeV}$ and almost similar behaviour was obtained. These studies amply demonstrate the effects of considering the repulsive interactions in the form of hard-core volume correction for many baryonic resonances present in the early Universe which was at very high temperature. However, the equation of state used for hadronic gas in the Kuono-Takagi model is thermodynamically inconsistent. The baryon-number density cannot be derived from a partition function by a statistical relation of the type $n_B = (\lambda_B/V) \frac{\partial \ln Z}{\partial \lambda_B}$ where λ_B is the baryon-fugacity. Recently attempts have been made to develop models which take into account the finite-size corrections in a thermodynamically consistent manner [15,16]. The purpose of this paper is to compare the predictions of Kuono-Takagi model for baryon-number density n_B in the HRG phase with those of thermodynamically consistent approaches so that the conclusions reached in the earlier paper [12] can be justified. Moreover, we hope that our analysis will also throw enough light on the proper choice of the equation of state (EOS) for a dense and hot hadronic gas (HRG).

III. METHOD OF CALCULATION

Assuming HRG phase as consisting of non-interacting, point like baryons and mesons, all thermodynamical quantities can be deduced from the grand canonical partition function by using quantum statistical thermodynamics :

$$T \ln Z_i^O(T, \mu, V) = \frac{g_i V}{6\pi^2} \int_0^\infty \frac{dk k^4}{(k^2 + m_i^2)^{1/2}} \frac{1}{\exp \left[\frac{\sqrt{k^2 + m_i^2} - \mu_i}{T} \right] + \theta_i} \quad (1)$$

The superscript 'O' indicates that the hadron is treated as

point like particle, g_i is the spin-isospin degeneracy factor, θ_i is +1 for baryon and -1 for meson, respectively. We can obtain from (1) the baryon-number density of i -th baryon as

$$n_i^0(T, \mu_i) = T \left[\frac{\partial \ln Z_i}{\partial \mu_i} \right]_{T, V} \quad (2)$$

This point particle model allows the system to be compressed to arbitrary high densities which, however, contradicts the fact that the baryons experience a strong repulsive forces and this prevents unrestricted compression of nuclear matter. Thus at a large baryon density, a strong short-range repulsion sets in between two baryons. For meson-meson, meson-baryon and baryon-antibaryon, interactions are generally attractive leading to resonance production. Therefore we can account for such strong repulsion by considering baryons as hard spheres having a hard core volume v_B . Thus the net baryon-number density was first derived by Cleymans et al [17,18] by considering the 'excluded volume' in the Vander Waals' type correction :

$$n_B = \frac{n_B^0}{1 + \sum_B n_B^0 v_B} \quad (3)$$

In (3), summation extends over all kinds of baryons present in the system. In this model baryons and antibaryons are mixed and only the net baryon-number density was used.

In another approach, the finite size correction factor has been taken proportional to the energy density instead of baryon-number density. Hagedorn et al calculated the net baryon density in the HRG phase as

$$n_B = H_C n_B^0 \quad (4)$$

where $H_C = 1/(1 + \frac{\epsilon^0}{4B})$ is known as Hagedorn's pressure ensemble

correction factor and B is the bag constant for the vacuum pressure.

Considering the repulsive interaction between a pair of baryons as well as a pair of antibaryons separately, Kuono and Takagi modified the equation (3) by the following relation for the net baryon density :

$$n_B = \frac{n_B^0}{1 + \sum_B n_B^0 v_B} - \frac{n_{\bar{B}}^0}{1 + \sum_{\bar{B}} n_{\bar{B}}^0 v_B} \quad (5)$$

In the early Universe case, the temperature was large and hence there were a large number of baryons as well as antibaryons and pions, although the net baryon density n_B was still very small. Thus Kuono-Takagi model appears more suitable in this context.

However, the above approaches lack thermodynamical consistency because n_B can not be evaluated from the partition function as given in (2). In a cosmological context, the derivation of the thermodynamical quantities from the thermodynamical potential is more appropriate since the total volume of a comoving region is given while the pressure depends on the evolution. Recently two approaches have appeared in the literature which account for the finite-size corrections in a thermodynamically consistent way.

In one model proposed by two of us [15] the finite-size effect has been incorporated in the very definition of the partition function for i -th hadron species

$$T \ln Z_i = \frac{g_i}{6\pi^2} \int_{V_i}^{V - \sum_j N_j V_j} dV' \int_0^\infty \frac{dk k^4}{(k^2 + m_i^2)^{1/2}} \frac{1}{\lambda_i^{-1} \exp [\beta E(k, m_i)] + 1} \quad (6)$$

Similar expression can be obtained for antibaryons by replacing the fugacity $\lambda_{\bar{B}} = \lambda_B^{-1}$ where $\lambda_B = \exp(\mu_B/T)$. Here V_j is the hard-core volume and N_j the total number of baryons of j th type, $\beta = 1/T$, $E(k, m_i)$ is the energy of the particle with momentum k and mass m_i , V is the given total volume of the system and g_i is the degeneracy factor for baryons of type i . Thus $\sum_j N_j V_j$ is the total excluded volume from all j -th baryon species. Replacing I as the momentum space part of the integral in Eq. (6), we obtain using large volume limit (i.e. $\frac{V_i}{V} \rightarrow 0$) as well as Boltzmann approximation.

$$\text{Trn } Z_i = V(1 - \sum_j N_j V_j) I_i \lambda_i \quad (7)$$

where n_j is the baryon-number density of j th baryon species. Use of Eq. (2) can give in the required baryon number density of finite size baryons. Thus for a multi-component system we get

$$n_i = (1 - \sum_j N_j V_j) I_i \lambda_i - I_i \lambda_i^2 \frac{\partial}{\partial \lambda_i} (\sum_j N_j V_j) \quad (8)$$

Solution of Eq. (8) can be obtained using the "Method of parametric line" and we finally get

$$R = \sum_j N_j V_j = \frac{1}{G(t)} \int_{-\infty}^t \left[\frac{1}{-t'} + \frac{I_2 V_2}{a_2 - I_2 V_2 t'} + \dots \right] G(t') dt' \quad (9)$$

where

$$G(t) = \frac{\exp(t)}{(a_2 - I_2 V_2 t)(a_3 - I_3 V_3 t) \dots} \quad (10)$$

We can get $t = 1/\lambda_1 I_1 V_1$ by setting $a_1 = 0$. The remaining a 's are obtained from

$$\lambda_j = \frac{1}{a_j - I_j V_j t}$$

Rischke *et al* have also proposed a thermodynamically consistent model in which baryon chemical potential μ_B in the evaluation of the quantities for finite-size particles, is replaced as $\tilde{\mu}_B = \mu_B - V_O p^{\text{ex}}(\beta, \mu_B)$ where p^{ex} is the pressure taking into consideration the effect of excluded volume and is given by $p^{\text{ex}}(\beta, \mu_B) = p^O(\beta, \tilde{\mu}_B)$ where p^O is the pressure of point like particles with modified chemical potential $\tilde{\mu}$. Thus the baryon-number density can be calculated from :

$$n_B^{\text{ex}}(\beta, \mu_B) = \frac{\partial p^{\text{ex}}(\beta, \mu_B)}{\partial \mu_B} \quad (11)$$

In other words

$$n_B^{\text{ex}}(\beta, \mu_B) = \frac{\partial p^O(\beta, \tilde{\mu}_B)}{\partial \mu_B} \frac{\partial \tilde{\mu}_B}{\partial \mu_B} \quad (12)$$

Thus we get finally

$$n_B^{\text{ex}}(\beta, \mu_B) = \frac{n_B^O(\beta, \tilde{\mu}_B)}{1 + V_O n_B^O(\beta, \tilde{\mu}_B)} \quad (13)$$

which is similar to Eq. (3).

We get similar expression for antibaryon number density and hence net baryon number density can be calculated.

For making a detailed comparison of the above models, we take HRG phase consisting of seven baryon resonances and their anti-particles upto the mass 1400 MeV as given in Table 1. Our purpose here is to search some distinguishing features of the above models and their ranges of applicability. Thus our analysis can lead us to a proper equation of state (EOS) for dense and hot nuclear matter.

IV. RESULTS AND CONCLUSION

We have plotted in Fig. 1 the variation of the net baryon-number density with temperature at three fixed values of the baryon chemical potential μ_B as 50, 500 and 1000 MeV, respectively. Instead of using full quantum statistics for the particle distribution, we have considered Boltzmann approximation in our calculation. We have shown a comparative plot of Kuono-Takagi model (as shown by curves A, A', A'') with those of Saeed Uddin-Singh model (curves B, B', B'') and Rischke *et al* model (curves C, C', C''). Although the detailed features of the curves in all the above models remain the same, the numerical differences are quite significant in the prediction of the three models. However, at low chemical potential (i.e. $\mu_B \simeq 50$ MeV), the curves appear to overlap on each other and the differences almost disappear. The main feature of the curves common in all these models is an exponentially rising trend of the net baryon density with increasing temperature, reaching a maximum at a certain temperature and then a very rapid fall as temperature increases still further. This behaviour differs significantly from that of point like particles (as shown in Ref. 12) which shows an exponentially rising behaviour with temperature. The baryon contrast ratio R as obtained in our earlier paper will always show a second rise after reaching a minimum at around $T_C \simeq 200$ MeV and thus the incorporation of finite size effect in various ways does not alter our conclusions. We also notice from Fig. 1, a maximum in the net baryon number density at a temperature around 200 MeV and this value does not change even if the baryon chemical potential changes drastically. This is a strange feature common to all the models.

When we calculate the values of R at μ_B/T around 10^{-10} as was the case existing in the early Universe, we find strange results. The model of Rischke *et al* [16] is not

appropriate because $\tilde{\mu}_B = \mu_B - V_O p^{\text{ex}}(\beta, \mu_B)$ as well as n_B gives a negative result. In the model of Uddin and Singh [15], although we do not get such adverse result but in order to get a meaningful prediction, we need a computer with very high numerical accuracy. Thus the range of the applicability also draws a line in between these models. At such lower values of the baryon chemical potential μ_B , the model of Rischke *et al* clearly fails while the other two models remain still reliable.

In Fig. 2, we have shown the variation of the net baryon density in the HRG phase with baryon chemical potential μ_B at a fixed temperature. We again find a common feature that the baryon-number density exponentially rises in the beginning and then it gradually becomes independent of the chemical potential. Moreover, the curves at different temperatures indicate that the baryon-number density becomes independent of temperature as well as chemical potential beyond $\mu_B \gtrsim 1200$ MeV and this strange feature is again common to all the models. According to Gottlieb *et al* [19], we can define the baryon number susceptibility as $\chi = \partial n_b / \partial \mu_B$. Lattice calculations reveal that T lying in the vicinity of T_c , $\chi = 0.07$ when strongly interacting matter turns from a QGP to an hadron gas (HRG). From Fig. 2, we also find that at large values of temperature and chemical potential, the value of susceptibility is very small and is almost compatible with the above value in the Kuono-Takagi as well as in Uddin-Singh model.

In conclusion, the detailed, comparative studies of the various models existing for the HRG phase reveal that apart from some small numerical differences we find almost similar predictions for the baryon-number density in thermodynamically inconsistent as well as consistent approaches. However, at low values of μ_B as is the situation in the early Universe case, the models of Kuono-Takagi and Uddin-Singh remain

useful. We thus conclude that a proper choice of an EOS for a dense and hot nuclear matter lies in the Uddin-Singh model which also provides a thermodynamically consistent description for such a matter.

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Table 1 : Baryon resonances in HRG phase

Baryons	Mass in MeV	Spin	Isospin	Degeneracy factor
N	940	$\frac{1}{2}$	$\frac{1}{2}$	4
Δ	1236	$\frac{3}{2}$	$\frac{3}{2}$	16
Λ	1120	$\frac{1}{2}$	0	2
Σ	1190	$\frac{1}{2}$	1	6
Y_1^*	1385	$\frac{3}{2}$	1	12
Ξ	1320	$\frac{1}{2}$	$\frac{1}{2}$	4
Y_0^*	1405	$\frac{1}{2}$	0	2

FIGURE CAPTION

FIGURE 1 : Variation of net baryon density n_b with temperature at various fixed values of chemical potential $\mu_B = 50, 500$ and 1000 MeV. The solid curves are the predictions of Kuono-Takagi model [13], the dashed-dotted curves of Uddin and Singh model [15] and the dashed curves represent the predictions of Rischke et al model [16].

FIGURE 2 : Variation of the net baryon density n_b with baryon chemical potential μ_B at fixed values of temperature $T = 150$ and 200 MeV. The different cases are same as in Fig. 1. The solid curves are the predictions of Kuono-Takagi model [13], the dashed-dotted curves of Uddin and Singh model [15] and the dashed curves represent the predictions of Rischke et al model [16].

FIG -1

Fig. -2