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The Criterion for the Coupling States between Cavities with Losses

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Abstract

For a coupled two cavity system it is important to know whether the two cavities are coupled, critically coupled or under coupled with given coupling coefficient k and quality factor Q_0 . In this paper we will give a quantitative criterion to make the judgement. It is found that $kQ_0 > 2$, $kQ_0 = 2$ and $kQ_0 < 2$ correspond to the three cases stated above. It is well known that for a coupled two cavity system one single cavity mode normally will split into two modes. The phase shifts of the two resonant modes, however, are not exactly zero or π if the cavities have finite losses, and therefore, the ratio of the field amplitudes in the two cavities is different from one even the two cavities are identical in dimension and coupled. In this paper we derived the analytical formulae for the phase shifts, resonant frequencies and field ratio in a coupled two cavity system as a function of cavity quality factor Q_0 and coupling coefficient k . The detailed discussion about a coupled two cavity system has been easily extended to a multicavity standing wave structure.

Multicavity standing wave structure is the essential component in a standing wave linear accelerator. To understand better this kind of structure some relevant very basic physical pictures have to be clarified. The most basic element is a coupled two cavity system as shown in Fig. 1 which is the basic component of a multicavity standing wave structure. It is well known that two coupled cavities without losses have two resonant modes, zero and π , which are splitted from the corresponding single cavity

Figure 1: A two coupled cavity system is the fundamental component of linear accelerator structure.

mode, and the amplitude of the fields in the both cavities are same. In reality, however, cavities have losses, and to maintain the electromagnetic field in the standing wave structure the power flow in the structure should equal to the total ohm loss rate, that is to say

$$\frac{dW}{dt} = -\frac{\omega}{Q_0}W + \frac{W}{L}v_g = 0 \quad (1)$$

where W is the total energy stored in the structure, ω is the resonant angular frequency, L is the structure length and v_g is the group velocity at ω . From eq. 1 we know that

$$v_g = \frac{\omega}{Q_0}L \quad (2)$$

It is necessary to mention that for a travelling wave structure one has always

$$v_g > \frac{\omega}{Q_0}L \quad (3)$$

and the power out of the structure is

$$P_{out} = \left(\frac{v_g}{L} - \frac{\omega}{Q_0}\right)W \quad (4)$$

The dispersion relation for a periodic structure shown in Fig. 1 is described by the following equation

$$\omega^2 = \omega_{\pi/2}^2(1 - k\cos(\beta D)) \quad (5)$$

where k is the coupling coefficient, $\beta = 2\pi/\lambda$ and λ is the wavelength at the working frequency. Now we consider a coupled two cavity system shown in Fig. 1 and assuming that this dispersion relation still holds. To satisfy eq. 3 one has

$$1 - k\cos(\theta) = \frac{kQ_0}{2}|\sin(\theta)| \quad (6)$$

where $\theta = \beta D$. Since, usually, $k \ll 1$ and $Q_0 \gg 1$, eq. 6 can be simplified as

$$1 = \frac{kQ_0}{2}|\sin(\theta)| \quad (7)$$

$$\theta_1 = \arcsin\left(\frac{kQ_0}{2}\right) \quad (8)$$

$$\theta_2 = \pi - \theta_1 \quad (9)$$

The solutions of θ in eq. 7 can be found also in Fig. 2. It is obvious to see that there are two solutions θ_1 and θ_2 if $kQ_0 > 2$ and $\theta_1 \approx 0$, $\theta_2 \approx \pi$ when $kQ_0 \approx \infty$. When $kQ_0 = 2$, there will be only one solution $\theta_1 = \theta_2 = \pi/2$ and this case is called critical coupling. When $kQ_0 < 2$,

Figure 2: Solutions of θ .

however, there will be no solution for θ and the two cavities can be considered to be uncoupled (independent). Since $\theta_{1,2}$ depend on kQ_0 one can easily find the relation between resonant frequency and the kQ_0 .

$$\omega_{\theta_{1,2}}^2 = \omega_{\pi/2}^2 (1 - k \cos(\theta_{1,2})) \quad (10)$$

In practice the quality factor Q_0 can be damped by waveguide or beam loading, and in consequence, the resultant resonant frequency will change and can be found by eq. 10. As far as the ratio of the fields in two cavities, one has

$$R = \cos(\theta_{1,2}) = (1 - (\frac{2}{kQ_0})^2)^{1/2} \quad (11)$$

If we consider the fundamental mode in a two coupled system shown in Fig. 1 the coupling coefficient k can be expressed analytically as [1][2]

$$k = \frac{4a^3}{3\pi DR^2 J_1(u_{01})} e^{-\alpha_e d} \quad (12)$$

where u_{01} is the first root of Bessel function $J_1(x)$, and $\alpha_e = 2\pi((\lambda/2.62a)^2 - 1)^{1/2}/\lambda$. Eq. 11 corresponding to a S-band two cavity coupled system has been calculated from eq. 11 and 12, as shown in Fig. 3.

The discussion above can be easily extended to a multicavity coupled standing wave system. If one cascades N fundamental components shown in Fig. 1, eq. 6 and eq. 7 are replaced by

$$1 - k \cos(N\theta) = \frac{kQ_0}{2} |\sin(N\theta)| \quad (13)$$

$$1 = \frac{kQ_0}{2} |\sin(N\theta)| \quad (14)$$

If $N = 2$ the solution for θ can be found in Fig. 4. If $kQ_0 \approx \infty$ there will be only three solutions: $\theta_1 = 0$, $\theta_2 = \pi/2$ and $\theta_3 = \pi$, otherwise, there are four solutions of θ as shown in Fig. 4 which are not commonly recognized.

This paper gives a practical criterion to determine the coupling state of a multicavity standing wave structure:

Figure 3: The theoretical value of the ratio R

Figure 4: Solutions of θ .

coupled ($kQ_0 > 2$), critically coupled ($kQ_0 = 2$) or uncoupled ($kQ_0 < 2$). For a coupled multicavity standing wave structure the resonant modes (θ) can be found by solving eq. 13 and the corresponding resonant frequency can be found from eq. 5. The relation between the resonant frequency and kQ_0 is well established. This information is very important for the building of damped or heavily beam loaded multicavity standing wave structures.

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