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ANOMALOUS STRUCTURE OF WEAK INTERMEDIATE BOSONS

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Abstract

The unpolarized and polarized quark and gluon content of the weak intermediate bosons of the Standard Model is investigated at energy scale much higher than their masses. The evolution equations for polarized quark and gluon densities are constructed and solved asymptotically. Structure of Z and W bosons turns out to be much richer than that of photon. Strong flavour and spin dependence is observed.

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1 Introduction

The anomalous structure of the real or nearly real photon has been studied for more than ten years [1]. It is known to dominate at high Q^2 over the hadronic (vector meson) component and can be measured in lepton-lepton or lepton-hadron scattering. Conceptually this structure emerges as QCD collinear quark-gluon cascade initiated by a quark-antiquark pair produced by photon. At high momentum transfers, such that $Q^2 \gg M_{W,Z}^2$, analogous cascade can develop from W and Z bosons — this time initiated via weak interactions [2].

There are several important differences as compared to the photon. Firstly, there are strong polarization asymmetries caused by different couplings to left- and righthanded quarks. Secondly, the W and Z bosons are both massive and as such have an on-shell longitudinal component. The trace of it will be discussed below.

One has rather rarely an access to the source of real weak intermediate bosons. As in the case of photons, the QCD structure is expected to be seen in the processes where nearly real bosons are emitted by fermions. One may then make use of the equivalent boson approximation known since long for the photons [3] and calculated only recently for the W and Z [4]. “Nearly real” means in this case that the boson momentum squared is negligible as compared to the other scales present in a given process. Whereas this approximation causes little doubts in the case of photons, it requires more attention with the massive weak bosons.

In this paper we derive asymptotic ($Q^2 \rightarrow \infty$) results for unpolarized and polarized quark and gluon content of weak bosons. In the lowest order of QCD these results can be used to calculate any structure functions by means of parton model formulae. In the next section we present evolution equations for unpolarized and polarized parton densities of weak bosons. In Sec. 3 we calculate weak boson splitting functions. Using these results, we discuss and present numerical solutions to asymptotic evolution equations in Sec. 4. We summarize our results in Sec. 5.

2 Evolution equations for parton densities

In the standard model weak intermediate bosons are elementary (point-like) particles. Nevertheless, when observed by a very high Q^2 particle, they can reveal QCD structure by collinear quark-gluon Bremsstrahlung. In this sense they become “composite” objects containing following partons: elementary weak intermediate bosons, quarks, antiquarks and gluons.

General master equations for parton α density, $f_{\alpha B}(x, t)$, inside any composite weak intermediate boson B read

$$\frac{df_{\alpha B}(x, t)}{dt} = \sum_{\beta} \mathcal{P}_{\alpha\beta}(x; t) \otimes f_{\beta B}(x, t), \quad (1)$$

where $t = \ln(Q^2/\Lambda_{\text{QCD}}^2)$, $\mathcal{P}_{\alpha\beta}(x; t)$ are splitting functions, the convolution is defined as

$$(P \otimes f)(x) \equiv \int dx_1 dx_2 P(x_1) f(x_2) \delta(x - x_1 x_2) \quad (2)$$

and for weak bosons the indices α, β go over polarized quarks, antiquarks, gluons and point-like γ , W and Z.

In the following we will consider the leading order QCD and 1-st order electroweak case. Denoting weak intermediate bosons by upper case letters and QCD partons by lower case ones,

we have

$$f_{AB}(x, t) = \delta_{AB} \delta(1 - x), \quad (3)$$

$$\mathcal{P}_{iB}(x; t) \equiv \frac{\alpha_{\text{em}}}{2\pi} P_{iB}(x), \quad (4)$$

$$\mathcal{P}_{ik}(x; t) \equiv \frac{\alpha_s(t)}{2\pi} P_{ik}(x). \quad (5)$$

Substituting this into Eq.(1) we arrive at the following non-homogenous evolution equations for the QCD content of a weak intermediate boson:

$$\frac{df_{\sigma B}(x, t)}{dt} = \frac{\alpha_{\text{em}}}{2\pi} P_{\sigma B}(x) + \frac{\alpha_s(t)}{2\pi} \sum_{k, \rho} P_{\sigma k \rho}(x) \otimes f_{k \rho B}(x, t) \quad (6)$$

where σ and ρ denote parton polarizations.

In the case of spin dependent parton densities it is convenient to use unpolarized and polarized functions ([5]):

$$P_{ik}(x) = P_{i+k+}(x) + P_{i-k+}(x), \quad (7)$$

$$\Delta P_{ik}(x) = P_{i+k+}(x) - P_{i-k+}(x), \quad (8)$$

$$f_{kB}(x, t) = f_{k+B}(x, t) + f_{k-B}(x, t), \quad (9)$$

$$\Delta f_{kB}(x, t) = f_{k+B}(x, t) - f_{k-B}(x, t). \quad (10)$$

Now unpolarized and polarized evolutions separate:

$$\frac{df_{iB}(x, t)}{dt} = \frac{\alpha_{\text{em}}}{2\pi} P_{iB}(x) + \frac{\alpha_s(t)}{2\pi} \sum_{k=q, \bar{q}, G} P_{ik}(x) \otimes f_{kB}(x, t), \quad (11)$$

$$\frac{d\Delta f_{iB}(x, t)}{dt} = \frac{\alpha_{\text{em}}}{2\pi} \Delta P_{iB}(x) + \frac{\alpha_s(t)}{2\pi} \sum_{k=q, \bar{q}, G} \Delta P_{ik}(x) \otimes \Delta f_{kB}(x, t). \quad (12)$$

These equations simplify considerably in the leading-log approximation where $\alpha_s(t)$ reads

$$\alpha_s(t) = \frac{2\pi}{bt}, \quad (13)$$

with $b = 11/2 - n_f/3$ for n_f flavours. The asymptotic (large t) solution to Eq.(12) can be now parametrized as

$$f_k(x, t) \simeq \frac{\alpha_{\text{em}}}{2\pi} f_k^{\text{as}}(x) t \quad (14)$$

resulting in purely integral equations

$$\begin{aligned} f_{iB}^{\text{as}}(x) &= P_{iB}(x) + b \sum_{k=q, \bar{q}, G} P_{ik}(x) \otimes f_{kB}^{\text{as}}(x), \\ \Delta f_{iB}^{\text{as}}(x) &= \Delta P_{iB}(x) + b \sum_{k=q, \bar{q}, G} \Delta P_{ik}(x) \otimes \Delta f_{kB}^{\text{as}}(x). \end{aligned} \quad (15)$$

In order to solve Eqs.(15) we need all splitting functions. We take the QCD ones from Ref.[5] and we calculate $P_{iB}, \Delta P_{iB}$ in the next section. $B = \gamma$ case has been considered by many authors (see eg. [1]). Our results concern W and Z bosons.

θ_W is the Weinberg angle and T_{3q} is the third weak isospin component of quark q . The $F_{\lambda\sigma}$ functions are given in Table I.

Table I. Coefficients $F_{\lambda\sigma}(x, \mathcal{L})$, of Eqs.(28 – 30).

	$\lambda = -$	$\lambda = +$	$\lambda = $
$\sigma = -$	$(1-x)^2(\mathcal{L}-2)$	$x^2(\mathcal{L}-2)$	$2x(1-x)$
$\sigma = +$	$x^2(\mathcal{L}-2)$	$(1-x)^2(\mathcal{L}-2)$	$2x(1-x)$
$\sigma = $	$2x(1-x)$	$2x(1-x)$	0

Let us observe that the splitting functions of the longitudinally polarized weak intermediate bosons ($\sigma = ||$) vanish in the lowest α_s order. That means that for $Q^2 \rightarrow \infty$ the longitudinal $W^\pm$ and Z^0 will contain negligible amounts of quarks and gluons as compared to the transverse bosons. Keeping this in mind we present the following results for transverse weak intermediate bosons only, defined as $B_\perp = (B_+ + B_-)/2$. This choice is also convenient for the comparison with the photon. Comparing the coefficients at logs in the Eqs.(28 – 30) and Eq.(27) we obtain the spin-dependent splitting functions of weak intermediate bosons. All non-zero splitting functions read

$$P_{q\pm\gamma}(x) = P_{q\pm\gamma}(x) = e_q^2 s(x), \quad (35)$$

$$P_{q\pm Z_\perp}(x) = P_{q\pm Z_\perp}(x) = z_{q\pm} s(x), \quad (36)$$

$$P_{d-W_\perp}(x) = P_{u-W_\perp}(x) = \frac{1}{2\sin^2\theta_W} s(x), \quad (37)$$

with $s(x) = [x^2 + (1-x)^2]/2$.

4 Asymptotic solutions to parton densities

In this section we present the solutions to Eqs.(15) for the parton densities inside weak intermediate bosons at asymptotically large Q^2 . Both equations have the same generic form

$$f_i(x) = a_i(x) + b \sum_k \int_x^1 \frac{dz}{z} P_{ik}\left(\frac{x}{z}\right) f_k(z) \quad (38)$$

and we integrate them numerically by a procedure described below.

Eq.(38) tells us that $f_i(x)$ is determined by the values of all $f_k(z)$ functions for $x < z < 1$. Thus, once we know $f_k(z)$ for $z \rightarrow 1$ we can find them for all values of z by iterative integration. The integration, however, must be taken carefully as, in general, the hadronic splitting functions are distributions of the following form [5]:

$$P_{ik}(x) = \hat{P}_{ik}(x) + \delta_{ik} \left[\frac{\varphi_k(x)}{(1-x)_+} + \omega_k \delta(1-x) \right], \quad (39)$$

where $\hat{P}_{ik}(x)$ is a regular function of x .

We take $f_k(z)$ to be smooth functions of z in any interval $[x_0, 1]$ with $0 < x_0 < 1$. Dividing the interval $[x_0, 1]$ into N equal parts of width $h = \frac{1-x_0}{N}$, we get the broken line approximation to $f_k(z)$:

$$\begin{aligned} x_n &= x_0 + nh, & n &= 0, 1, \dots, N \\ f_k &= f_k(x_n). \end{aligned} \quad (40)$$

The numerical solution is now given as the series of N points with accuracy increasing with N :

$$f_{kn} = a_{kn} + b \sum_j \int_{x_n}^1 \frac{dz}{z} P_{kj}\left(\frac{x_n}{z}\right) f_j(z) \quad (41)$$

$$= a_{kn} + b \sum_j \left(\int_{x_n}^{x_{n+1}} + \int_{x_{n+1}}^1 \right) \frac{dz}{z} P_{kj}\left(\frac{x_n}{z}\right) f_j(z) \quad (42)$$

$$\equiv a_{kn} + b \sum_j (J_{kj}^n + K_{kj}^n). \quad (43)$$

J_{kj}^n contain integrals with $1/(1-z)_+$ distributions and we calculate them by means of the following formula:

$$\int_x^1 \frac{dz}{z} \frac{\varphi(z)}{(1-z)_+} f(z) = \int_x^1 \frac{dz}{z} \frac{\varphi(z)}{(1-z)} [f(z) - f(1)] + f(1)\Phi(x), \quad (44)$$

where

$$\Phi(x) = \int_x^1 \frac{dz}{z} \varphi(z) (1-z)^{\epsilon-1} - \frac{1}{\epsilon} \varphi(1) (\epsilon \rightarrow 0). \quad (45)$$

Let us define

$$\xi_n = \frac{x_n}{x_{n+1}} \equiv \frac{x_n}{x_n + h}. \quad (46)$$

A straightforward calculation results in:

$$\begin{aligned} J_{kj}^n &= \delta_{kj} [(f_{k,n+1} - f_{kn})\bar{\Phi}_k(\xi_n) + f_{kn}\Phi_k(\xi_n) + \omega_k f_{kn}] \\ &+ \frac{h}{2} \left[\frac{1}{x_{n+1}} \hat{P}_{kj}(\xi_n) f_{j,n+1} + \frac{1}{x_n} \hat{P}_{kj}(1) f_{jn} \right], \end{aligned} \quad (47)$$

where

$$\bar{\Phi}_k(\xi) = \frac{1}{h} \int_x^{x+h} dz \varphi_k\left(\frac{x}{z}\right) = \frac{\xi}{1-\xi} \int_\xi^1 \frac{dz}{z^2} \varphi_k(z). \quad (48)$$

We see that J_{kj}^n is a linear combination of f_{jn} and $f_{j,n+1}$.

The integrals K_{kj}^n in Eq.(43) depend only on $f_{j,n+1}, \dots, f_{j,N}$ and we calculate them numerically.

Now it is clear that for each n Eq.(43) is a system of linear equations for f_{jn} with coefficients depending on f_{jk} with $k > n$. Thus the only thing we need to start the iteration is to notice that all $f_j(x)$ must vanish at $x = 1$.

Inserting explicit expressions for all splitting functions in the above formulae we obtain numerical solutions for the unpolarized and polarized content of γ , $Z_\perp$ and $W_\perp$ bosons. The results are shown in Figures 2 and 3.

3 Intermediate boson splitting functions

In the first order in α_{em} the elementary weak intermediate bosons can split into quarks and antiquarks only. To keep track of all finite terms in the cross-sections we calculate a standard current matrix element squared for weak intermediate boson of momentum p and polarization σ going into a polarized quark-antiquark pair:

$$\mathcal{H}_{\mu\nu}(p, q; \sigma) = \frac{1}{4\pi} \int d\Gamma(p'_1) d\Gamma(p'_2) (2\pi)^4 \delta^4(p + q - p'_1 - p'_2) \times \langle p\sigma | J_\mu^\dagger(0) | \psi' \rangle \langle \psi' | J_\nu(0) | p\sigma \rangle, \quad (16)$$

where primed quantities refer to the final state.

The P_{iB} and ΔP_{iB} functions will be found as appropriate coefficients of large logs, as described below.

The type of the current corresponds to the virtual particle “probing” the weak intermediate boson structure. In Fig. 1 we show the lowest order diagrams with a gluon or photon as probes (QCD and electromagnetic currents, respectively).

Figure 1: Lowest order diagrams for $G^*/\gamma^* + Z^0/W^- \rightarrow q+\bar{q}$ scattering.

The current conservation (for massless quarks) allows for following helicity decomposition:

$$\tilde{H}_\lambda(p, q) = \epsilon_{(\lambda)}^{\mu*}(q) \mathcal{H}_{\mu\nu}(p, q) \epsilon_{(\lambda)}^\nu(q), \quad (17)$$

where $\epsilon_{(\lambda)}^\mu(q)$ are polarization vectors of a spin-1 boson with momentum $q^\mu = (q_0, 0, 0, q_z)$

$$\epsilon_L^\mu = \frac{1}{\sqrt{2}}(0, 1, -i, 0), \quad (18)$$

$$\epsilon_R^\mu = \frac{1}{\sqrt{2}}(0, 1, +i, 0), \quad (19)$$

$$\epsilon_\parallel^\mu(q) = \sqrt{\frac{1}{|q^2|}}(q_z, 0, 0, q_0). \quad (20)$$

The $\tilde{H}_\lambda$ functions intuitively correspond to polarized probes and are related to the standard F_k structure functions, as follows

$$F_1 = (\tilde{H}_L + \tilde{H}_R)/2, \quad (21)$$

$$F_2 = x(\tilde{H}_L + \tilde{H}_R + 2\tilde{H}_\parallel), \quad (22)$$

$$F_3 = \tilde{H}_L - \tilde{H}_R. \quad (23)$$

In the massless limit $\tilde{H}_\lambda$ contain collinear singularities which, in our case, get regularized by the finite weak intermediate boson mass. For the photon we can introduce a regularizing mass M_γ . The $\mathcal{O}(\alpha_{\text{em}}^1)$ result will have general form:

$$\tilde{H}_\lambda = \alpha_{\text{em}} \ln \frac{Q^2}{M_B^2} h_\lambda^{(1)} + \text{finite terms}, \quad (24)$$

where $Q^2 \equiv -q^2$.

The factorization theorem (see e.g. [6]) tells us that

$$h_\lambda^{(1)} = \frac{1}{2\pi} \sum_k P_{kB} \otimes h_\lambda^{(0),k}, \quad (25)$$

where $h_\lambda^{(0),k}$ are $\mathcal{O}(\alpha_{\text{em}}^0)$ results for the k -th parton current matrix elements. The summation goes over all parton species and helicities.

In the following we will calculate the QCD current matrix elements — polarized gluons being the probing bosons. The advantage of this choice is that gluons couple to quarks only (graphs a) and b) in Fig. 1). Explicit expressions for $h_\lambda^{(0),k}$ read:

$$h_L^{(0),q+} = h_R^{(0),q-} = h_L^{(0),q+} = h_R^{(0),q-} = 4\pi\alpha_s\delta(1-x), \text{ all other} = 0. \quad (26)$$

Inserting above into Eq.(25) we get

$$\begin{aligned} h_L^{(1)} &= 2\alpha_s\alpha_{\text{em}} \ln \frac{Q^2}{M_B^2} [P_{q+B}(x) + P_{\bar{q}+B}(x)], \\ h_R^{(1)} &= 2\alpha_s\alpha_{\text{em}} \ln \frac{Q^2}{M_B^2} [P_{q-B}(x) + P_{\bar{q}-B}(x)], \end{aligned} \quad (27)$$

where $\bar{q}_-$ and $\bar{q}_+$ are antiparticles to q_+ and q_- , respectively.

As there are only two possible polarized final states: $q_- + \bar{q}_+$ and $q_+ + \bar{q}_-$, the contributions from the terms in square brackets of Eq.(27) get separated.

The direct $\mathcal{O}(\alpha_{\text{em}}^1)$ calculation of the Feynman diagrams in Fig. 1a) and b) for $\tilde{H}_\lambda$ with polarized final state ($q_\pm + \bar{q}_\mp$) gives

$$\tilde{H}_\lambda^{\gamma^*}(x, Q^2) = 2\alpha_s\alpha_{\text{em}}e_q^2 F_{\lambda\sigma}(x, \ln(Q^2/M_\gamma^2)), \quad (28)$$

$$\tilde{H}_\lambda^{Z^0}(x, Q^2) = 2\alpha_s\alpha_{\text{em}}z_{q\pm} F_{\lambda\sigma}(x, \ln(Q^2/M_Z^2)), \quad (29)$$

$$\tilde{H}_\lambda^{W^\pm}(x, Q^2) = 2\alpha_s\alpha_{\text{em}}w_{q\pm} F_{\lambda\sigma}(x, \ln(Q^2/M_W^2)), \quad (30)$$

where e_q is the quark (antiquark) charge in the units of e ,

$$z_{q+} = e_q^2 \tan^2 \theta_W, \quad (31)$$

$$z_{q-} = e_q^2 \tan^2 \theta_W \left(\frac{T_{3q}}{e_q \sin^2 \theta_W} - 1 \right)^2, \quad (32)$$

$$w_{q+} = 0, \quad (33)$$

$$w_{q-} = \frac{1/2 - T_{3q}}{\sin^2 \theta_W}, \quad (34)$$

5 Summary

In the paper we have considered the QCD structure of weak intermediate bosons. The evolution equations for the spin-averaged and spin-dependent quark and gluon distributions have been constructed and solved asymptotically. As a result one observes strong spin and flavour dependence resulting from the nature of weak couplings. We have presented only the results concerning transverse boson degrees of freedom. The longitudinal ones do not develop the QCD structure in the leading order. They may however cause problems if treated naively [2]. The structure of resolved W and Z bosons can influence substantially processes at very high energies. As an example one can quote the Higgs boson production. The contribution from W-W or Z-Z scattering, dominating the process [7], will be modified by the developed QCD structure, in analogy to the photon case [8].

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Figure 2: u-type and d-type quark distribution functions in γ , W_L^- and Z_L^0

Figure 3: Gluon distribution functions in γ , $W_{\perp}^{-}$ and $Z_{\perp}$