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How to measure which sort of particles was emitted earlier and which later

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Abstract

The method allowing to measure which sort of particles was emitted earlier and which later at time scales as small as 10^{-22} s is suggested.

Since the pioneering papers of Kopylov and Podgoretsky (see a review [1]) it is well-known that the study of directional dependence of particle correlations at small relative velocities can be used to extract the information on the form of the production region and the emission time. We show here that not only the magnitude of the mean difference of particle emission times can be determined but that for nonidentical particles, due to the effect of final state interaction (FSI), even the sign of this difference can be measured.

Let us start with the usual assumption of sufficiently small density of the produced particles in the momentum space, such that the correlation of two nonidentical particles with a small relative velocity is influenced by the effect of their mutual FSI only. Define the correlation function $R(p_1, p_2)$ of the two particles as the ratio of their differential production cross section to the one which would be observed in the case of absence of the effect of FSI. Following Kopylov and Podgoretsky we introduce the normalized probability $W_S(x_1, p_1; x_2, p_2)$ of the emission of two noninteracting particles, with total spin S and 4-momenta p_1 and p_2 , by the one-particle sources decaying at the space-time points $x_1 = \{t_1, \vec{r}_1\}$ and $x_2 = \{t_2, \vec{r}_2\}$. Assuming the momentum dependence of the emission probability inessential when varying the 4-momenta p_1 and p_2 by the amount characteristic for the correlation due to FSI, and taking into account that FSI leads to the substitution of the plane wave $e^{ip_1 x_1 + ip_2 x_2}$ by the nonsymmetrized Bethe-Salpeter amplitudes in the continuous spectrum of the two-particle states $\psi_{p_1 p_2}^{S(+)}(x_1, x_2)$, we get [2]

$$\begin{aligned} R(p_1, p_2) &= \sum_S \int d^4 x_1 d^4 x_2 W_S(x_1, p_1; x_2, p_2) |\psi_{p_1 p_2}^{S(+)}(x_1, x_2)|^2 \\ &= \sum_S \rho_S(p_1, p_2) \langle |\psi_{p_1 p_2}^{S(+)}(x)|^2 \rangle_S, \end{aligned} \quad (1)$$

Here the mean $\langle f \rangle_S = \int d^4 x_1 d^4 x_2 W_S f / \int d^4 x_1 d^4 x_2 W_S$ is a function of p_1 and p_2 , and

$$\rho_S(p_1, p_2) = \int d^4 x_1 d^4 x_2 W_S(x_1, p_1; x_2, p_2), \quad \sum_S \rho_S(p_1, p_2) = 1, \quad (2)$$

describes the population of the total spin- S states, e.g., $\rho_S = (2S+1)/[(2s_1+1)(2s_2+1)]$ for unpolarized particles with spins s_1 and s_2 . The amplitude $\psi_{p_1 p_2}^{S(+)}(x)$, depending only on the relative 4-coordinate $x \equiv \{t, \vec{r}\} = x_1 - x_2$, is obtained from the Bethe-Salpeter amplitude after separation of the two-particle c.m.s. motion: $\psi_{p_1 p_2}^{S(+)}(x_1, x_2) = e^{iP \cdot X} \psi_{p_1 p_2}^{S(+)}(x)$, where

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$X = [(p_1 P)x_1 + (p_2 P)x_2]/P^2$ is the c.m.s. 4-coordinate and $P \equiv 2p = p_1 + p_2$. At equal emission times $t^* = t_1^* - t_2^* = 0$ in the two-particle c.m.s. it coincides with a stationary solution of the scattering problem $\psi_{-\vec{k}^*}^{S(+)}(\vec{r}^*)$, $\vec{k}^* = \vec{p}_1^* = -\vec{p}_2^*$, having at large r^* the asymptotics of superposition of the plane and diverging spherical waves. It can be shown [2] that the amplitude $\psi_{p_1 p_2}^{S(+)}(x)$ can usually be substituted by this solution (*equal time approximation*).

The simple two-body approach should be modified in the case of heavy-ion reactions, when the particles are produced in a strong Coulomb field of residual nuclei. In such a situation, instead of the two-particle Bethe-Salpeter amplitude $\psi_{p_1 p_2}^{S(+)}(x_1, x_2)$, the correlation function is determined by the amplitude $\psi_{p_1 p_2}^{S\{\alpha\}+}(x_1, x_2)$ representing the solution of a complicated multibody problem, taking into account interaction between the two particles and also their interaction with the residual system described by the quantum numbers $\{\alpha\}$. For the correlation function, defined as the ratio of the two-particle production cross section to the one in the case of absence of mutual FSI between the two particles ($\psi \rightarrow \tilde{\psi}$), instead of Eq. (1), we have

$$R(p_1, p_2) = \frac{\sum_S \rho_S(p_1, p_2) \langle |\psi_{p_1 p_2}^{S\{\alpha\}+}(x_1, x_2)|^2 \rangle_S}{\sum_S \rho_S(p_1, p_2) \langle |\tilde{\psi}_{p_1 p_2}^{S\{\alpha\}+}(x_1, x_2)|^2 \rangle_S}. \quad (3)$$

We will consider the case of particles emitted by sufficiently heavy compound nucleus N such that the recoil effects can be neglected and the c.m. of the system *particle + nucleus* can be identified with the rest frame of the nucleus, which we situate at the origin. Neglecting further the change of the nucleus electric charge during the process of particle emission, we approximate it by an effective charge Ze . Finally, neglecting the interaction of the particles 1 and 2 with other emitted particles, the quantum numbers $\{\alpha\}$ of the *residual system* are reduced to the effective charge number Z . In such a situation the Bethe-Salpeter amplitude corresponding to the case of "switched off" the interaction between the two particles is obtained by the substitution of the spatial parts of the plane waves $e^{ip_i x_i}$ by the usual Coulomb wave functions describing particle interaction with the Coulomb center: $e^{-i\vec{p}_i \vec{r}_i} \rightarrow e^{-i\vec{p}_i \vec{r}_i} \Phi_{\vec{p}_i}^{z_i Z}(\vec{r}_i)$. Thus

$$\tilde{\psi}_{p_1 p_2}^{SZ(+)}(x_1, x_2) = e^{ip_1 x_1 + ip_2 x_2} \Phi_{\vec{p}_1}^{z_1 Z}(\vec{r}_1) \Phi_{\vec{p}_2}^{z_2 Z}(\vec{r}_2) = e^{iPX} e^{-i\vec{k}^* \vec{r}^*} \Phi_{\vec{p}_1}^{z_1 Z}(\vec{r}_1) \Phi_{\vec{p}_2}^{z_2 Z}(\vec{r}_2). \quad (4)$$

The complete Bethe-Salpeter amplitude can be obtained in the *adiabatic* approximation [3], i.e. assuming relative motion of the two particles at characteristic distances much slower compared with their motion with respect to the Coulomb center, just by substituting the plane wave $e^{-i\vec{k}^* \vec{r}^*}$ in Eq. (4) by the Bethe-Salpeter amplitude $\psi_{p_1 p_2}^{S(+)}(x)$ describing the relative motion of isolated interacting particles:

$$\psi_{p_1 p_2}^{SZ(+)}(x_1, x_2) = e^{iPX} \psi_{p_1 p_2}^{S(+)}(x) \Phi_{\vec{p}_1}^{z_1 Z}(\vec{r}_1) \Phi_{\vec{p}_2}^{z_2 Z}(\vec{r}_2). \quad (5)$$

The analysis shows [4] that the *adiabatic* approximation can be used for sufficiently light particles or fragments.

Let us now demonstrate the sensitivity of the correlation function of two nonidentical particles to the sign of the difference of the emission times $t = t_1 - t_2$. Let us consider sufficiently small momentum k^* of the particles in their c.m.s. so that their strong interaction is dominated by s-wave and the wave function $\psi_{-\vec{k}^*}^{S(+)}(\vec{r}^*)$ in the absence of Coulomb interaction takes the form

$$\psi_{-\vec{k}^*}^{S(+)}(\vec{r}^*) = e^{-\vec{k}^* \vec{r}^*} + \phi_{\vec{k}^*}^S(r^*), \quad (6)$$

where the scattered wave $\phi_{k^*}^S(r^*)$ is independent of the directions of the vectors $\vec{k}^*$ and $\vec{r}^*$. Neglecting the interaction of the two particles with the residual system and assuming that the conditions of *equal time* approximation are fulfilled, we can write the correlation function in the form³

$$\begin{aligned} R(p_1, p_2) &= \sum_S \rho_S \langle |\psi_{-\vec{k}^*}^{S(+)}(\vec{r}^*)|^2 \rangle_S \\ &= 1 + \sum_S \rho_S \langle |\phi_{k^*}^S(r^*)|^2 + 2\text{Re}\phi_{k^*}^S(r^*) \cos \vec{k}^* \vec{r}^* - 2\text{Im}\phi_{k^*}^S(r^*) \sin \vec{k}^* \vec{r}^* \rangle_S. \end{aligned} \quad (7)$$

Consider now the behaviour of the vector $\vec{r}^*$ in the limit $|vt| \gg r$. Making the Lorentz transformation from the source rest frame to c.m.s. of the two particles: $r_L^* = \gamma(r_L - vt)$, $r_T^* = r_T$, we see that, in the considered limit, the vector $\vec{r}^* \doteq -\gamma\vec{v}t$ is nearly parallel or antiparallel to the vector of pair velocity $\vec{v}$, depending on the sign of the time difference t . Therefore, the correlation function is sensitive to $\text{sign}(t)$ due to the odd term $\sim \sin \vec{k}^* \vec{r}^*$. For charged particles there arise additional odd terms with respect to the argument $\vec{k}^* \vec{r}^*$ due to the confluent hypergeometrical function $F(\alpha, 1, z) = 1 + \alpha z + \alpha(\alpha + 1)(z/2!)^2 + \dots$ modifying the plane wave in Eq. (6):

$$\psi_{-\vec{k}^*}^{S(+)}(\vec{r}^*) = e^{i\delta} \sqrt{A_c(k^*a)} \{ e^{-\vec{k}^* \vec{r}^*} F[\frac{-i}{k^*a}, 1, i(\vec{k}^* \vec{r}^* + k^*r^*)] + \phi_{ck^*}^S(r^*) \}, \quad (8)$$

where $\delta = \arg\Gamma[1 + i/(k^*a)]$ and a is the Bohr radius of the two-particle system. The sensitivity of the correlation function to $\text{sign}(t)$ can be also modified due to the Coulomb interaction with the emitting nucleus (see Eq. (5)).

Thus we see that the sign of the mean time difference $\langle t \rangle$ can be determined provided the sign of the scalar product $\vec{k}^* \vec{v}$ is fixed. A straightforward way is to measure the correlation functions $R_+(\vec{k}^* \vec{v} \geq 0)$ and $R_-(\vec{k}^* \vec{v} < 0)$. Depending on $\langle t \rangle$, their ratio R_+/R_- should show a peak or a dip or oscillate in the region of small k^* and approach 1 both at $k^* \rightarrow 0$ and $k^* \rightarrow \infty$.

As the sign of the scalar product $\vec{k}^* \vec{v}$ is approximately equal to that of the difference of particle velocities $v_1 - v_2$ (this equality is exact for particles of equal masses), the sensitivity of the correlation functions R_+ and R_- to the sign of the difference of particle emission times has a simple explanation in terms of the classical trajectory approach (see, e.g., [5]). Clearly, the interaction between the particles in the case of earlier emission of faster particle will be different compared with the case of its later emission (the interaction time being longer in the latter case).

For quantitative estimates we assume that, in the case of absence of FSI, the compound nucleus isotropically emits unpolarized particles with the energies distributed according to Maxwellian law with the temperature $T = 4$ MeV. The distribution of the 4-coordinates of the particle sources is approximated by Gaussian law

$$W_S(x_1, p_1; x_2, p_2) \propto \rho_S(p_1, p_2) \exp\left(-\frac{\vec{r}_1^2}{2r_0^2} - \frac{(t_1 - \langle t \rangle)^2}{2\tau_0^2}\right) \exp\left(-\frac{\vec{r}_2^2}{2r_0^2} - \frac{t_2^2}{2\tau_0^2}\right), \quad (9)$$

with the parameter $r_0 = 3.5$ fm roughly corresponding to the mass number of the emitting nucleus $A = 120$. Requiring the same dispersion of the difference $t = t_1 - t_2$ of the

³The correlation function of two noninteracting particles was first introduced in ref. [2] in a symmetrized form, so that the odd terms in variable $\vec{k}^* \vec{r}^*$ were omitted.

emission times for the Gaussian law and the exponential decay law, the parameter τ_0 can be identified with the emitter lifetime τ . For evaporation processes τ is typically several hundreds fm/c (leading to $\langle r^* \rangle \simeq v\tau$ of several tens fm). For the effective charge number of the residual nucleus we put $Z = 51$, which leads to about twice as large mean kinetic energy of the emitted protons or deuterons as compared with that of neutrons. The mean velocity of pp, pd or np pairs at small values of k^* is $\langle v \rangle \sim 0.15$. The above parameters roughly describe particle emission in the reaction $^{40}\text{Ar} + ^{108}\text{Ag}$ at 44 MeV/nucleon [5]. Instead of the 6-dimensional correlation function $R(p_1, p_2)$ we calculate the 1-dimensional one $R^Z(k^*)$ corresponding to Eq. (3) with the nominator and denominator integrated over 1-particle spectra $d^3\sigma_i(\vec{p}_i)/d^3\vec{p}_i$. To reveal the effect of the nucleus Coulomb field, we calculate also the correlation functions $R^{Z=0}(k^*)$ and $R^{Z''}(k^*)$, the latter taking into account the effect of the nucleus Coulomb field on one-particle spectra but not on particle correlations.

Our calculations show that the correlation functions R_+ , R_- and their ratio are substantially sensitive to $\langle t \rangle$ provided that $\langle t \rangle^2$ is comparable to or higher than the dispersions $\langle t_i^2 - \langle t_i \rangle^2 \rangle$. In Figs. 1 and 2 we present the results for pd and np systems assuming $\tau_0 = 50$ fm/c and $\langle t \rangle = -100$ fm/c. We see that in both cases the ratio R_+/R_- strongly deviates from unity in the region of small k^* achieving maximum value of ~ 2.0 (1.3) at $k^* \simeq 10$ (35) MeV/c for pd (np) system. This maximum would be replaced by a minimum in the case of reversed sign of the mean difference of the emission times $\langle t \rangle$. We can also see that the ratio R_+/R_- is only slightly affected by the Coulomb field of the residual nucleus.

In conclusion we summarize our results. It is well known that the directional dependence of two-particle correlations can be used to estimate the form of the production region and the magnitude of the mean difference of the times of particle emission. We have shown that also the sign of this difference can be measured. Thus a new possibility is open to determine which sort of particles (e.g., protons or neutrons, pions or kaons) was produced earlier and which later. For this we suggest to study the correlation functions of two nonidentical particles separately for the angles between the relative velocity $\vec{k}^*/\mu$ and the total pair velocity $\vec{v}$ in the emitter rest frame less and greater than 90° .

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Figure 1: The pd correlation functions $R_+(\vec{v}\vec{k}^* \geq 0)$, $R_-(\vec{v}\vec{k}^* < 0)$ and their ratio calculated for the effective charge numbers of the emitting nucleus $Z = 51$, " Z " = 51 (only one-particle spectra are influenced by the nucleus charge) and $Z = 0$. The particles are assumed to be unpolarized and emitted isotropically according to Maxwellian law with the temperature 4 MeV. The distribution of space-time coordinates of the particle sources is approximated by Gaussian law with $\langle \frac{1}{3}r_i^2 \rangle^{1/2} \equiv r_0 = 3.5$ fm, $\langle t_i^2 - \langle t_i \rangle^2 \rangle^{1/2} \equiv \tau_0 = 50$ fm/c and $\langle t_p - t_d \rangle = -100$ fm/c.

Figure 2: The same as in Fig. 1 for the np system. In particular, $\langle t_n - t_p \rangle = -100$ fm/c.