SU(2)$_f$ Violation in the Nucleon’s Light Sea within the Framework of the Effective Chiral Quark Theory

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Abstract

The violation of SU(2)$_f$ symmetry in the nucleon’s light sea ($\bar{u} \neq \bar{d}$) is analyzed within the framework of the effective chiral quark field theory 'matched' to the QCD improved parton model: At some ad hoc 'matching' scale $Q_0^2$, effective chiral quark fields are associated with QCD valence quark-partons and a prediction on the SU(2)$_f$ violating nonsinglet structure function $x(\bar{d} - \bar{u})$ is deduced from chiral field theory. A simpler model is also presented, which essentially reproduces the chiral result. The sensitivity of $x(\bar{d} - \bar{u})$ to the choice of the 'matching' scale $Q_0^2$ is investigated. It is shown that the effective chiral quark field theory cannot explain the experimentally observed NA51 Drell-Yan measurement $\bar{u}/\bar{d} = 0.51 \pm 0.06$ at $x = 0.18$, $Q^2 \simeq 27\text{GeV}^2$, since the predicted shape of $x(\bar{d} - \bar{u})$ is concentrated in the very small $x$-region, with $Q_0^2$ restricted to perturbatively accessible values.
1 Introduction

If the nucleon’s light quark (i.e. \( m_q \approx 0 \)) sea could be regarded as purely radiatively generated by perturbative LO QCD Altarelli-Parisi evolution equations [1], one would expect, due to the gluon’s flavor blindness, exact SU(3)\(_f\) (i.e. \( u = d = s \) ; unless otherwise noted, partons refer to the proton) to hold. In the NLO evolution a tiny violation of \( \text{SU}(3)_f \) occurs through Fermi-statistic effects [2], which is much too small to account for the measured effect [3-6] and can be safely neglected here. It has been shown [7, 8], however, that the boundary conditions for a radiative generation of the nucleon’s sea and gluon distribution functions require, besides valence distributions \( xq_v \), a valence-like input for the gluon \( xg \) and the light sea \( xq \), as expected for perturbative (‘current’) partons. Since the input densities are nonperturbative quantities, light flavor symmetry in the sea is an intuitive assumption rather than a perturbatively explicitly computable prediction. Experimental indications [3, 4] of an \( \text{SU}(3)_f \) violation can be phenomenologically taken into account for the input functions \( \bar{s} < \bar{u} = \bar{d} \) [8, 9] (indeed even \( \bar{s} = 0 \) in [8]). It is assumed, that in the nonperturbative very low \( Q^2 \) region, where even the light masses are nonnegligible, the nonvalidity of light quark mass degeneracy (\( m_s > m_u \approx m_d \)) should be a plausible—although perturbatively uncomputable—reason for the suppression of \( \bar{s}s \) pairs. Up to this point, as a consequence of the approximate equality of up and down masses, \( \text{SU}(2)_f \) (i.e. \( u = d \)) is left to be a symmetry of the nucleon’s sea. Experimental evidence [5, 6] during the last few years, however, has given rise to the commonly accepted notion that the \( \text{SU}(2)_f \) symmetry of the sea is violated too. The experimental constraints should be recalled very briefly:

The NMC measurement [5] of the Gottfried sum [10]:

The Gottfried sum in terms of the proton’s and the neutron’s structure functions \( F_2^{p,n}(x, Q^2) \)

\[
S_G = \int_0^1 \frac{dx}{x} \left( F_2^p - F_2^n \right) \tag{1}
\]
can be expressed in the quark-parton model as:

\[
S_G = \frac{1}{3} \int_0^1 dx \ (u_v - d_v) + \frac{2}{3} \int_0^1 dx \ (\bar{u} - \bar{d})
\]

\[
= \frac{1}{3} + \frac{2}{3} \int_0^1 dx \ (\bar{u} - \bar{d}) .
\] (2)

(Isospin symmetry of the nucleon doublet \((p, n)\) will be assumed. A justifying discussion of isospin violating effects on \(S_G\) can be found in [11, 12].) The NMC’s most recent estimate [5] on \(S_G\) is

\[
S_G = 0.235 \pm 0.026 ,
\] (3)

measured at \(Q^2 = 4\text{GeV}^2\). As an integrated nonsinglet density, \(S_G\), Eqs.(1) and (2), essentially does not evolve in \(Q^2\) within perturbative QCD. Hence Eq.(3) is not restricted to the scale of the experiment. The NMC [5] measurement of \(S_G\) clearly seems to imply [13] that \(\int_0^1 dx \ (\bar{u} - \bar{d}) < 0\), i.e. \(\bar{d}(x, Q^2) > \bar{u}(x, Q^2)\). It has to be mentioned, however, that the NMC data alone cannot definitely discriminate against SU(2)\(_f\) symmetric sea distributions [14, 15].

The NA51 estimate of the ratio \(\bar{u}/\bar{d}\) at \(x = 0.18\), \(Q^2 \simeq 27\text{GeV}^2\) [6]:

The NA51 experiment, following an idea suggested in [16], has measured the ratio of cross sections for muon pair production through 450 GeV protons colliding with proton (\(\sigma^{pp}\)) and deuterium (\(\sigma^{pd}\)) targets at \(y = 0\). Assuming \(\sigma^{pd} = \frac{1}{2}(\sigma^{pp} + \sigma^{pn})\), the asymmetry \(A_{DY}\) is given by

\[
A_{DY} \equiv \frac{\sigma^{pp} - \sigma^{pn}}{\sigma^{pp} + \sigma^{pn}} ,
\] (4)

where the cross sections are to be taken differential in \(y\) and \(\sqrt{s} = \frac{M}{\sqrt{s}}\) (e.g. \(\sigma^{pp} \equiv \frac{d^2 \sigma^{pp} - n^+ n^-}{dy d\sqrt{s}} \bigg|_{y=0}\)) ; \(y\) and \(M\) being the muon-pair’s c.o.m. rapidity and its invariant mass respectively, \(\sqrt{s}\) being the hadronic c.o.m. energy. Using the LO Drell-Yan expressions for \(\sigma^{pp}\) and \(\sigma^{pn}\) (again assuming isospin symmetry to hold) and restricting to dominant contributions, where at least one valence quark participates in the partonic subprocess, one obtains [6, 16]

\[
A_{DY} = \frac{(4 \lambda_v - 1)(\lambda_s - 1) + (\lambda_v - 1)(4 \lambda_s - 1)}{(4 \lambda_v + 1)(\lambda_s + 1) + (\lambda_v + 1)(4 \lambda_s + 1)} ,
\] (5)
where \( \lambda_v \equiv u_v/d_v \), \( \lambda_s \equiv \bar{u}/\bar{d} \), all to be evaluated at \( x = \sqrt{t} \) and \( Q^2 \equiv M^2 \simeq 27\text{GeV}^2 \). Considering \( \lambda_v \) to be reasonably well known, NA51 [6] extracts from the measured quantity

\[
A_{DY} = -0.09 \pm 0.03
\]  

the ratio

\[
\bar{u}/\bar{d} = 0.51 \pm 0.06 \text{ at } x = 0.18, \quad Q^2 \simeq 27\text{GeV}^2
\]  

Up to now Eq.(7) has to be regarded as the most direct evidence that SU(2)\(_f\) is indeed violated in the nucleon’s light sea (i.e. \( \bar{u} \neq \bar{d} \)). It severely constrains the symmetry violating nonsinglet structure function \( x(\bar{d} - \bar{u})(x, Q^2) \), requiring a considerable amount of SU(2)\(_f\) violation in the medium \( x \)-region \( x \simeq 0.2 \) at \( Q^2 \simeq 27\text{GeV}^2 \).

For completeness it should be mentioned that the Fermilab experiment E772 has published [17] values of the ratio of yields of massive muon pairs in collision of 800 GeV protons with tungsten and isoscalar targets, sensitive to the difference (\( \bar{d} - \bar{u} \)). The data are, however, incapable to discriminate between SU(2)\(_f\) symmetric or asymmetric sea distributions and may only serve -once an asymmetry is established- as an upper bound on (\( \bar{d} - \bar{u} \)) at the 2\( \sigma \)-statistical error level [17-19]. They are not considered in the following.

Most recently suggested parton distributions [20-22] have phenomenologically taken into account the experimental evidence for \( \bar{d}(x, Q^2) > \bar{u}(x, Q^2) \) in the perturbative LO/NLO evolutions by allowing for a non zero input function (\( \bar{d} - \bar{u} \))(\( x, Q^2_0 \)) at some input scale \( Q^2 = Q^2_0 \). However, the question for a theoretical explanation of the violation of SU(2)\(_f\) symmetry in the nucleon’s light sea is of its own interest. A first attempt to understand theoretically \( \bar{d} \neq \bar{u} \) was formulated in [23] within an effective chiral quark field theory [24]. We will investigate and fix the boundary conditions of this approach and compare the SU(2)\(_f\) violation, which is predicted from this model, with experiment [6]. Also, a simpler model [25] will be explored, which essentially reproduces the original framework [23].
2 Theoretical Framework

It is interesting to remember in this context that in 1976 Feynman and Field [13], considering a those days’ estimate on the Gottfried sum \( S_G = 0.27 < \frac{1}{3} \), did not expect the light sea to be SU(2)\(_f\) symmetric. Without specifying the scale \( Q^2 \), they suggested [13]:

\[
\begin{align*}
xu & = 0.17 (1-x)^{10} \\
x\bar{d} & = 0.17 (1-x)^7,
\end{align*}
\]

arguing that the origin of the asymmetry might be a suppression of \( uu \) pairs in comparison to \( dd \) pairs through Pauli’s exclusion principle due to the fact, that the proton consists of two up quarks and just one down quark. Though Eq.(8) has been superseded by the need for QCD improved, i.e. \( Q^2 \) dependent, parton distributions, the curious fact remains that it yields

\[
\bar{u}/\bar{d} (x = 0.18) = 0.55
\]

‘predicting’ the NA51 result (7) to be measured eighteen years later. If we follow Feynman’s intuition and modify the SU(2)\(_f\) symmetric light sea input of [8] e.g., \( xu = x\bar{d} = Ax^\gamma(1-x)^\delta \), according to \( xu = Ax^\gamma(1-x)^{\gamma+\delta} \), \( xd = Ax^\gamma(1-x)^\gamma \), and fix \( \gamma \) and \( \delta \) by the momentum sum rule and Eq.(3), we obtain a remarkable resemblance of the nonsinglet structure function \( x(\bar{d} - \bar{u}) \) to recent fits, e.g. the MRSA [21] parton distribution set (as can be seen in the inset of Fig.5 below). In any case an explanation like the Pauli-principle can only motivate such an ad hoc ansatz and maintains the desire for a theoretical, i.e. dynamical explanation.

Meson-cloud models (see e.g. [26-28] and references therein) can report some remarkable success. In these nuclear physical approaches, SU(2)\(_f\) violation in the nucleon’s light sea arises from virtual components of a nucleon’s Fock-state, consisting of a radiated meson (mainly a pion) and a recoil baryon. A more desireable understanding of SU(2)\(_f\) violation, since experimental evidence thereof comes from hard scattering processes, would, however, be on a more microscopic scale, within the QCD improved parton model, complemented with calculable nonperturbative effects. It has been suggested in
[29, 12, 19] to allow for a nonperturbative emission of $q\bar{q}$ bound states off quark-partons, modifying the Altarelli-Parisi evolution equations [1] through an addition of nonperturbative quark-bound state splitting functions in the low and medium $Q^2$-range. The NMC value of $S_G$ at $Q^2 = 4\text{GeV}^2$ is obtained dynamically through a nonperturbative evolution of the constituent quark model value of $\frac{1}{3}$ (Gottfried sum rule) [29], and the NA51 constraint (7) can be reproduced as well [19]. A result of these theoretical considerations is that the intermediate $Q^2$-range, in which nonperturbative evolution is expected to be of importance, extends to quite large scales (3 to 100 GeV$^2$ depending on the choice of the axial coupling parameter [29]), where the purely pointlike perturbative QCD evolution is usually assumed to be valid. A theoretical appealing model, proposed by Eichten, Hinchliffe and Quigg (EHQ) [23, 30], for explaining and calculating SU(2)$_R$ symmetry violating effects ($\bar{u} \neq \bar{d}$) is based on the effective chiral quark field theory [24]. It predicts SU(2)$_R$ violation in the nucleon's light sea through mesonic (pionic) radiation on a partonic level (i.e. pions are radiated off valence quarks). Once the nonperturbative symmetry violation is implemented, confidence in a pure pointlike, perturbative QCD evolution of the parton distributions in $Q^2$ is maintained. We will take a closer look at this approach, especially ask for its predictive power and compare it with the numerical NA51 constraint (7).

The effective chiral quark theory is comprehensively formulated and discussed in [24] and its implications for SU(2)$_R$ violation are derived in [23]. Therefore we shall only briefly recall those formulæ which will be needed for our analysis. Up to first order in the pion fields, which appear as members of an octet of pseudoscalar Goldstone bosons through chiral symmetry breaking, the effective SU(2)$_R$ interaction part of the Lagrangian, describing an effective quark – pion coupling (to be identified with valence-quark – pion coupling, see below) reads (Fig.1a)

$$\mathcal{L}_I = -\frac{g_A}{2f_\pi} \partial_\mu \bar{\psi} \gamma^\mu \gamma^5 \psi$$

(10)

where $g_A = 0.7524$, $f_\pi = 93\text{MeV}$, $\psi = \left(\begin{array}{c} u \\ d \end{array}\right)$, $\bar{\tau}$ is the Pauli-matrix isovector and as usual $\pi^0 = \pi^3; \pi^\pm = \frac{1}{\sqrt{2}}(\pi^1 \pm i\pi^2)$. Although not explicitly denoted, it should be clear that the effective chiral quark fields $u, \, d$ forming the doublet $\psi$ in Eq.(10) are not identical with
the corresponding fields appearing in the QCD Lagrangian. For an explicit calculation of SU(2)_I violating effects [23], however, the effective quarks are assumed to be represented by perturbative QCD valence quark densities \( q_v(x, Q_0^2) \), where \( Q_0^2 \) is some undetermined scale, which has to be fixed by an ad hoc assumption (and was chosen to be \( Q_0^2 = 5 \text{GeV}^2 \) in [23]).

From one of the simple Feynman diagrams (Fig.1a) the EHQ [23] probabilistic splitting function \( P_{\chi FT}(z) \), describing the decay of a valence quark ('valence quark' in the sense just explained above) into a virtual pion with momentum fraction \( z \) and a recoil quark with momentum fraction \( 1-z \) of the initial valence quark’s momentum is obtained to be

\[
P_{\chi FT}(z) = \frac{g_A^2 m^2}{8 f_{\pi}^2 \pi^2} \left\{ \ln \frac{\Lambda^2 + M_\pi^2}{\tau(z) + M_\pi^2} + M_\pi^2 \left[ \frac{1}{\Lambda^2 + M_\pi^2} - \frac{1}{\tau(z) + M_\pi^2} \right] \right\} \Theta(z_m - z),
\]

with \( \tau(z) = m^2 z^2/(1-z) \). \( P_{\chi FT}(z) \) refers to the radiation of a charged pion whereas an isospin factor of one half for \( \pi^0 \)-radiation must be added explicitly. The degenerate constituent \( u,d \) quark mass is taken to be \( m \simeq 350 \text{MeV} \) and the pion mass is \( M_\pi \simeq 140 \text{MeV} \). The cut-off parameter on the four-momentum squared of the radiated pion, \( \Lambda^2 \), will be taken to be \( \Lambda = 2350 \text{MeV} \), normalized to the NMC’s estimate on \( S_G \), Eq.(3) (see below). (Originally EHQ [23] used \( \Lambda = 1800 \text{MeV} \). The implications of the specific choice of \( \Lambda \) are not too serious in any case.) The maximal value of fractional momentum carried by the radiated pion due to the cut-off \( \Lambda \) is given by [23]

\[
z_m = \frac{-\Lambda^2 + \sqrt{\Lambda^2 + 4m^2}}{2m^2} \lesssim 1
\]

which is close to one (\( z_m = 0.98 \) for \( \Lambda = 2350 \text{MeV} \)). The functional shape of \( P_{\chi FT}(z) \) is shown in Fig.2.

We follow the ad hoc assumption of [23] and associate effective chiral quark fields with perturbative valence quark-partons at some scale \( Q_0^2 \). Then the chiral field theory (\( \chi FT \)) contributions to the nucleon’s parton distribution functions (Fig.1) are obtained [23] by
convolutions:

\[
\begin{align*}
    u(x, Q_0^2) &= \frac{1}{4} \int_x^1 dy \int_y^1 dz q_\pi^\nu \left( \frac{x}{yz}, Q_0^2 \right) P(z) (5u_\nu + d_\nu)(y, Q_0^2) \\
    &+ \int_x^1 dy \quad R \left( \frac{x}{y} \right) \left( \frac{1}{2}u_\nu + d_\nu \right)(y, Q_0^2) \\
    \bar{u}(x, Q_0^2) &= \frac{1}{4} \int_x^1 dy \int_y^1 dz q_\pi^\nu \left( \frac{x}{yz}, Q_0^2 \right) \frac{1}{2} q_\pi^\nu \left( \frac{x}{yz}, Q_0^2 \right) P(z) (u_\nu + 5d_\nu)(y, Q_0^2)
\end{align*}
\]

with \( P(z) = P_{\chi FT}(z) \) [in the following two other splitting functions will be considered as well]. The first term contributing to \( u_\pi \) in (13) is due to the Sullivan-process [31], where the pion's partonic valence content \( q_\pi^\nu \) (\( \pi^+ \rightarrow ud \), \( \pi^- \rightarrow d\bar{u} \), \( \pi^0 \rightarrow u\bar{u}, d\bar{d} \) in Fig.1b) is seen by a virtual photon in DIS, while the second term stems from the recoil quark (Fig.1b), i.e. \( R(z) \equiv P(1 - z) \). The integer and fractional factors appearing in Eq.(13) can be easily traced back [23] to an isospin factor of one half at the \( q_\pi \pi^0 \) vertices and another one half for \( \pi^0 \) Sullivan contributions, because \( q_\pi^\nu \pi^0 = \frac{1}{2} q_\pi^\nu \equiv q_\pi^\nu \). Down flavor contributions \( (d_x, \bar{d}_x) \) are obtained by SU(2)_L symmetry (i.e. \( u \leftrightarrow d \)) which, though violated in the sea, holds of course as a symmetry of the interaction. The scale \( Q_0^2 \) occurring in Eq.(13), as already pointed out, has to be chosen by an ad hoc assumption. Since the chiral quark theory [24] is considered as a theory of effective nonrelativistic quarks, we suppose a true 'matching' between effective, nonrelativistic chiral quarks and fundamental QCD valence quarks would only be valid at very low scales, i.e. bound state scales \( \gtrsim \Lambda_{QCD}^2 \), where one would intuitively expect \( \Sigma_x f dx q_v \rightarrow 1 \) [32]. Unfortunately at such low scales perturbative stability breaks down [7, 8] and the parton densities \( q_v, q_\pi^\nu \) in Eq.(13) are meaningless. We will nevertheless perform an explicit calculation of SU(2)_L violation based on Eq.(13), assuming the 'matching' scale \( Q_0^2 \) to lie in the perturbative region [23], or more precisely, we shall preferably choose \( Q_0^2 \) around the lower bound of the perturbatively accessible region, i.e. \( Q_0^2 \simeq 0.3 \text{GeV}^2 \) according to [8].

In the outlined \( \chi FT \) framework, the Gottfried integral in Eq.(2) can be derived [23] from Eq.(13)

\[
S_G = \frac{1 - 2P(n = 1)}{3},
\]

(14)
where $P(n = 1)$ denotes the first Mellin $n$-moment of $P(z)$, the $n^\text{th}$ moment of a function $f(x)$ being defined as

$$f(n) \equiv \int_0^1 dx \ x^{n-1} \ f(x) \ .$$  

(15)

To see that Eq.(13) directly leads to Eq.(14) for $S_G$ as given in Eq.(2), it should be noted that in Eq.(2) $\bar{u} - \bar{d} = \bar{u}_\chi - \bar{d}_\chi$ ($\bar{q} = \bar{q}_{SU(2)} + \bar{q}_\chi$ with $\bar{q}_{SU(2)}$ being SU$(2)_F$ symmetric [23]), and for the $n = 1$ moments $u_v = 2$, $d_v = 1$, $q^v_\chi = 1$, $P = R$. Equation (14) allows for a normalization to the NMC [5] experimental value in Eq.(3) by adjusting $\Lambda$ as mentioned above.

We now ask for the predictive power of the $\chi FT$ contributions in (13) to the nucleon's parton distribution functions. As they have to be added to some SU$(2)_F$ symmetric nonperturbative ad hoc distributions [23], only the difference between $\bar{d}_\chi$ and $\bar{u}_\chi$ can be regarded as a true prediction:

$$(\bar{d} - \bar{u})_\chi(x, Q^2_0) = \int_x^1 \frac{dy}{y} \int_{y^2}^1 \frac{dz}{z} \ q^v_\chi \left( \frac{x}{y^2}, Q^2_0 \right) \ P(z) \ (u_v - d_v)(y, Q^2_0) \ .$$  

(16)

This equation connects the SU$(2)_F$ violation of the nucleon’s sea to the flavor structure of a nucleon’s valence content [23], i.e. SU$(2)_F$ is broken in the sea, simply because the nucleon is not a flavor isosinglet ($u_v \neq d_v$). Since an analysis of the $\chi FT$ prediction will include perturbative evolution with $Q^2$ in Mellin space [1, 7], the Mellin-transformed Eq.(16) reads:

$$(\bar{d} - \bar{u})_\chi(n) = q^v_\chi(n) \ P(n) \ (u_v - d_v)(n) \ .$$  

(17)

For our subsequent discussion it suffices to Mellin-transform the parton distributions at $Q^2_0$, i.e. $q(n) \equiv \int dx \ x^{n-1} \ q(x, Q^2_0)$. Since the $n^\text{th}$ Mellin moment of $P_{\chi FT}(z)$ in (11) cannot be calculated analytically, we have parametrized $P_{\chi FT}(z)$ by a function $P_{\text{par}}(z)$ with simple analytical moments

$$P_{\text{par}}(z) = c \ z \ (1 - z^\alpha) \ ,$$  

(18)

with $c = 0.4720$, $\alpha = 3.3333$. Despite its functional simplicity, $P_{\text{par}}(z)$, shown in Fig.2, gives agreement in the $\chi FT$ contributions (13) to better than 1% whenever they are nonnegligible (indeed around their maximum the discrepancy is $\sim$ 0.02%).
Furthermore, since the $\chi FT$ contributions do not appear to be very sensitive to functional subtleties of the splitting function $P_{\chi FT}(z)$, we consider for comparison the simpler scenario of an SU(2)$_f$ respecting effective Yukawa interaction of an isovector of pseudoscalars coupled to a light quark doublet [25]

$$L^\text{YUK}_I = ig \bar{\phi} \cdot \bar{\psi} \gamma^\tau \gamma^5 \psi \quad .$$

(19)

If we identify the pseudoscalar fields with pions, we can calculate the pionic fluctuations in (13) by using the well known splitting function $P_{YUK}(z)$ from pseudoscalar gluon theories [25] at the UV fixed point $\alpha^* = g^2/4\pi$

$$P_{YUK}(z) = \frac{\alpha^*}{2\pi} z \ t \quad ,$$

(20)

where $t \equiv \ln(Q^2_0/\mu^2_0)$ with $\mu^2_0$ being some normalization point. We effectively normalize $\frac{\alpha^*}{2\pi} t \equiv G = 0.295$ to $S_G$ in (3). We also show the effective probabilistic Yukawa splitting function $P_{YUK}(z)$ in Fig.2.

Although the effective chiral quarks refer to constituent valence quarks at some non-perturbative bound state scale ($\simeq \Lambda_{QCD}$), one makes the ad hoc assumption [23] in Eq.(13) that they can be represented by perturbatively accessible parton distributions, in order to allow for explicit calculations. This implies that $Q^2_0$ in (13) should be chosen as low as possible as allowed by the reliability in the perturbative evolution of $q_v, q^v_\nu$: The stability of perturbative LO/NLO Altarelli-Parisi [1] evolution has been demonstrated within a model of radiatively generated parton distributions [8, 20] to hold down to a rather low scale $Q^2 = \mu^2 \simeq 0.3\text{GeV}^2$, at which the valence quarks carry $\sim 60\%$ of the nucleon’s momentum. Within the effective LO approach discussed thus far, we choose, for definiteness, the GRV protonic [20] and pionic [33] LO valence distributions, evaluated at the input scale, i.e. $Q^2_0 = \mu^2_{LO} = 0.23\text{GeV}^2$. (The use of NLO distributions, although not consistent with an effective LO approach, gives similar results.) Before we perform a calculation of SU(2)$_f$ violation (16), we have to check how the input valence densities are modified through pion radiation effects at $\mu^2_{LO}$. Since the proton structure function $F_2$ is $u_v$ dominated, $F_2 \sim \frac{4}{9} x u_v$, in the medium and large $x$ range, no large modifications are acceptable.
result in (7), $x(d - \bar{u})$ has to be evolved to the relevant experimental scale $Q^2 = 27 \text{GeV}^2$, which is shown by the solid curve in Fig.4: It peaks around $x \approx 3 \times 10^{-2}$ and becomes small in the medium $x$-range $\sim 0.2$ in disagreement with experiment [6]. The decrease in the medium $x$-range can be compared with the shape which results from a recent global fit to deep inelastic and related data, including the NA51 experiment, namely the MRSA (NLO) [21] parton distribution set which expands to larger $x$ (dotted curve in Fig.4). The far too small $\chi FT$ prediction of SU(2)$_F$ violation in the experimentally relevant medium $x$-region cannot be attributed to the particular choice of parton distributions, as can be seen from the long dashed curve in Fig.4 which corresponds to the original EHQ asymmetry [23], 4-flavor LO evolved to 27 GeV$^2$ [34]. The latter one is even smaller in the medium $x$-region, relevant for present experimental data [6], which is partly due to the fact, that the initial SU(2)$_F$ violating effects (16) take place at an essentially larger scale, i.e. $Q^2_0 = 5 \text{GeV}^2 \gg \mu^2_{LO}$ in Eq.(13) and hence in Eq.(16) according to the original EHQ model [23]. The fact that the maximum of the asymmetry $x(d - \bar{u})$ at a fixed scale $Q^2$ tends to smaller values of $x$ for a larger input scale $Q^2_0$ is a direct consequence of Eq.(16): Two nonsinglet densities, evolving to smaller $x$ with higher $Q^2$, on the r.h.s are convoluted at $Q^2_0$ to just one evolving non singlet density on the l.h.s. This effect can be clearly inferred from the short dashed curve in Fig.4, which results from a calculation identical to the one leading to the solid curve, but with $Q^2_0 = 5 \text{GeV}^2$. This might suggest, if one considers the discrepancy between the solid ($\chi FT$, $Q^2_0 = \mu^2_{LO}$) and the dotted curve (MRSA fit to data) that the 'matching' scale $Q^2_0$ should, as discussed above, be assumed to be even smaller than $\mu^2$. Unfortunately this region is definitely not accessible perturbatively [7, 8, 20] and a connection of calculations valid at such low scales [24] with present measurements [6] is not possible. Since, as just demonstrated, an increase of $Q^2_0$ even worsens the result, the calculation presented here (i.e. the solid curve of Fig.4) has to be regarded to supply the maximal amount of SU(2)$_F$ violation in the medium $x$-range, which can be obtained within the framework of the effective chiral quark field theory [24] 'matched' to the QCD improved parton model.

The result of an identical calculation ($Q^2_0 = \mu^2_{LO}$) with the above introduced splitting
function $P_{YUK}(z)$ in Eq.(20) is shown for comparison in Fig.5 by the dotted-dashed curve. It almost exactly reproduces the original shape (solid line) predicted from $\chi FT$. The convolutions in (13) turn out to be rather insensitive to the details of the underlying quark-pion dynamics. Even an extreme situation, in which a valence quark would always transmit all its momentum to the radiated pion, represented by a toy splitting function $\sim \delta(1 - z)$, does not change the situation dramatically, as shown by the dotted curve in Fig.5. We can therefore trace back the discrepancy between the $\chi FT$ prediction and a fit [21] to the fact that we should—strictly speaking—perform effective chiral quark theory [24] calculations only around bound state scales. If, in order to perform an explicit calculation in (13), effective chiral quark theory [24] is 'matched' to the QCD improved parton model around the lowest perturbative scales of its applicability $\sim \mu^2$ [7, 8, 20], agreeement with experiment [6] is lost. On the other hand, it is interesting to note, that an intuitive ansatz according to Feynman and Field [13] reproduces remarkably well the tendency of the data, represented by the MRSA fit [21], as shown in the inset of Fig.5.

In order to compare our results from $\chi FT$ directly with experiment [6] (and not with a fit to experiment [21]), we have to take into account that the NA51 [6] result in (7) is sensitive not to the difference $\bar{d} - \bar{u}$ alone but also to the sum $\bar{u} + \bar{d}$:

$$\frac{\bar{u}}{\bar{d}} = \frac{(\bar{u} + \bar{d}) - (\bar{d} - \bar{u})}{(\bar{u} + \bar{d}) + (\bar{d} - \bar{u})} .$$  \hspace{1cm} (23)

Recent parton distributions [20-22], which represent all presently available deep inelastic and related data, require $x(\bar{u} + \bar{d}) \simeq 0.125$ at $x = 0.18, Q^2 = 27\text{GeV}^2$. Using, according to (16), $x(\bar{d} - \bar{u})_x = 0.010$ at $x = 0.18, Q^2 = 27\text{GeV}^2$ leads to the chiral estimate:

$$\left(\frac{\bar{u}}{\bar{d}}\right)_x = 0.85 \text{ at } x = 0.18, Q^2 = 27\text{GeV}^2 .$$  \hspace{1cm} (24)

Ignoring even the experimental knowledge of the absolute magnitude of the nucleon's SU(2) f sea as obtained from fits, a lower limit on the chiral estimate (24) can be set by self-consistency of the model: The least amount of light sea to be considered is the one generated by the $\chi FT$ contributions in (13) ($\bar{u} + \bar{d} \geq \bar{u}_x + \bar{d}_x$), i.e. $x(\bar{u} + \bar{d})_x = 0.070$ at $x = 0.18, Q^2 = 27\text{GeV}^2$. This implies, via Eq.(23), the lower bound

$$\left(\frac{\bar{u}}{\bar{d}}\right)_x \geq 0.75 \text{ at } x = 0.18, Q^2 = 27\text{GeV}^2 .$$  \hspace{1cm} (25)
which is more than $3\sigma$ away from the NA51 [6] result in (7).

4 Conclusions

We have critically analyzed SU(2)$_f$ violation in the nucleon’s sea, as inferred from experiment [5, 6], within the framework of the effective chiral quark theory [24] ‘matched’ to the QCD improved parton model. In the effective chiral quark theory [24], a nucleon consists of effective nonrelativistic quark fields, surrounded by virtual pseudoscalar (Goldstone) bosons (mainly pions), coupled to the quarks via chiral symmetry breaking. The pion cloud induces an effective light antiquark content of the nucleon, which is SU(2)$_f$ asymmetric, simply because the nucleon is not an isosinglet [23], i.e. $u_v \neq d_v$. The amount of SU(2)$_f$ violation, which is almost a parameter-free prediction from $\chi FT$ for the effective antiquark content [23], agrees with the integrated effect of SU(2)$_f$ violation, as extracted for the nucleon’s light perturbative antiquark content from the NMC measurement [5] of the Gottfried sum. This makes it tempting to assume that effective chiral quark theory, which is valid around a nonperturbative bound state scale of about $\Lambda_{QCD}^2$, can be associated with perturbative valence quark densities at some ad hoc chosen perturbative scale $Q_0^2$ [23]. We therefore assumed, in order to allow for explicit calculations, that the nonperturbative results from $\chi FT$ can be applied to the QCD improved parton model. Our detailed analysis revealed that the best possible ‘matching’ of perturbative QCD at $Q_0^2$ to the nonrelativistic chiral quark model is to associate perturbative valence densities with effective chiral quark fields at the lowest perturbatively possible scale. Unfortunately it was found that even if $Q_0^2$ is decreased to the lower bound of applicability of perturbative QCD, $Q_0^2 \approx 0.3$GeV$^2$ [7, 8, 20], the experimental NA51 measurement [6] of $u/d$ in Eq.(7) can not be accounted for, since $\chi FT$ predicts an SU(2)$_f$ violation concentrated at too small values of $x$. 

13
Acknowledgements

I am grateful to E. Reya and M. Glück for advice and useful discussions, and to E. Reya and M. Stratmann for carefully reading the manuscript. This work has been supported in part by the 'Bundesministerium für Forschung und Technologie', Bonn.
References


[34] In [23] the EHQ parton distributions are derived from $\chi FT$ at the input scale $Q^2 = Q_0^2 = 5 \text{GeV}^2$, which is relevant for the NMC [5] measurement of the violation of the Gottfried sum rule. If $x(u - d)$ is not evolved to $Q^2 = 27 \text{GeV}^2$, the scale of the NA51 [6] experiment, the long dashed curve in Fig.4 increases by about 20% in the experimentally relevant medium $x$-region.
Figure Captions

Fig. 1 (a) SU(2)$_l$ conserving valence quark-pion couplings, induced by $\chi FT$. (b) Sullivan process (left) and recoil quark (right) contribution to a nucleon's structure functions.

Fig. 2 Probabilistic $q_v \to \pi$ splitting functions: The solid curve represents the original EHQ [23] function (modulo an adjusted cut-off $\Lambda$) obtained from $\chi FT$. Also shown is a parametrized version (long dashed curve) which was used to evolve our chiral results from $Q_0^2$ to larger $Q^2$, and a function corresponding to an effective $q_v \pi$ pseudoscalar Yukawa coupling [25] (dotted-dashed curve).

Fig. 3 The solid curves show the GRV LO [20] perturbative QCD valence distributions $q_v$ at the input scale $\mu^2_{LO} = 0.23$GeV$^2$. If they are naively associated with effective chiral quarks, they are (unacceptably) 'reduced' to the short dashed curves, $q_v^-$, by pion radiation effects ($q_v^\pm, q_v^-$). The 'larger' distributions $q_v^+$ are constructed according to Eq.(22), as to reproduce the original perturbative input $q_v$ after reduction by pion radiation ($q_v^+ = q_v$). If the 'matching' scale (between effective and valence quarks) $Q_0^2$ is identified with the GRV LO input scale $\mu^2_{LO}$, SU(2)$_l$ violation within $\chi FT$ has to be calculated from the 'large' distributions $q_v^+$.

Fig. 4 $x(d-\bar{u})$ at $Q^2 = 27$GeV$^2$: The solid curve corresponds to the $\chi FT$ prediction, if the 'matching' scale (between effective and valence quarks) $Q_0^2$ in Eq.(16) is identified with the perturbatively lowest possible GRV input scale $\mu^2_{LO}$ [20] and $x(d-\bar{u})$ is evolved to 27GeV$^2$. For illustration the unevolved asymmetry at the input scale $Q^2 = Q_0^2 = \mu^2_{LO}$ is shown by the dotted-dashed curve. The short dashed curve illustrates the effect, if $Q_0^2$ is increased to 5GeV$^2$. The long dashed curve is the original EHQ [23] suggestion (evolved to $Q^2 = 27$GeV$^2$ [34]) and the recent MRSA fit [21] is shown for comparison by the dotted curve. The latter one is clearly seen to expand to larger $x$, as required by the NA51 Drell-Yan experiment [6]. The NA51 point was obtained from the experimental result [6] $\bar{u}/\bar{d} = 0.51 \pm 0.06$ supplemented by $x(\bar{u} + \bar{d}) \simeq 0.125$ at $x = 0.18, Q^2 = 27$GeV$^2$ as obtained from all recent parton
distributions [20-22].

**Fig. 5** Same as Fig.4 but with different splitting functions (for all curves $Q_0^2 = \mu^2_L$):

The simpler $q_v \pi$ pseudoscalar Yukawa coupling [25] is seen to reproduce almost exactly the results of chiral dynamics. Even an extreme toy model with a splitting function $\sim \delta(1 - z)$ doesn’t change the results dramatically. For comparison, the shape obtained from an intuitive 'Feynman-Field' ansatz is included in the inset, remarkably resembling the MRSA fit [21] to data.
Fig. 1
Fig. 4
Fig. 5