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BOSON SPECTRA AND CORRELATIONS IN SMALL THERMALIZED SYSTEMS

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ABSTRACT

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The problems of spectra and correlations in the small inhomogeneous matter that eventually formed in ultra-relativistic A+A collisions are considered. It is proved that if the particle transversal masses are large enough in comparison with the temperature, the *long*-, *side*- and *out*- interferometry radii coincide with the corresponding lengths of homogeneity in the radiating systems. The model-independent analysis of the momentum behavior of the interferometry radii for the sources with intensive transversal and longitudinal flows have been done. If the average length of homogeneity is smaller than the wave-length of the bosons, the Wigner function is changed essentially in comparing to the Bose-Einstein distribution. The structure of the correlation function for such quanta is also changed that leads to the correlations between oppositely charged particles and larger intercept for neutral bosons. These effects become strongly if the system has non-zero chemical potential or is out of chemical equilibrium.

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BOSON SPECTRA AND CORRELATIONS IN SMALL THERMALIZED SYSTEMS

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INTRODUCTION

The systems formed in ultra-relativistic nucleus-nucleus collisions have a dual nature. On the one hand they are quasi-macroscopic ones producing $10^3 \div 10^5$ hadrons and are eventually thermalized. The classical language of thermodynamics is used often to describe these systems. On the other hand, it follows from current interferometry data that typical effective sizes of the systems are a few fm even at the final stage of their evolution. The systems of such kinds have to be considered as quantum objects. They are dense, thermalized and involved in fast collective expansion due to contiguity with vacuum. So, at least locally, the systems can have very small lengths of homogeneity for density, temperature and collective velocities. The spectra and correlations in small thermal relativistic quantum-field systems are not trivial and can be understood on the base of the space-time scale. It includes the total geometrical lengths that the thermalized system occupies, $\bar{R}_i$, the local lengths of homogeneity (hydrodynamic lengths), $\bar{\lambda}_i$, and the wave-length of the quanta, $\lambda_p \propto 1/p^0$.

The statistical mechanics methods for processes of multiparticle production were initially applied by Fermi¹ who used the global equilibrium hadron system in volume V that means actually $\bar{R}_i = \bar{\lambda}_i$. The next step was made by Landau² who considered the hydrodynamic one-dimensional expansion of a pion gas that corresponds to the following relations between system lengths: $\bar{R}_T = \bar{\lambda}_T$, $\bar{R}_L \gg \bar{\lambda}_L$. The method of calculation of the single particle spectra in a such picture is well-known now^{3,4}. The role of the transversal hydrodynamic flows ($\bar{R}_T > \bar{\lambda}_T$, $\bar{R}_L \gg \bar{\lambda}_L$) in explanation of the changing slope of the different single particle spectra have been demonstrated recently⁵. The results concerning the particle spectra were based mostly on the calculations using the local Bose-Einstein distribution in expanding hadron or quark-gluon gas. It has been shown⁶ that the particle (Wigner) distributions are modified essentially compared with the Bose-Einstein one if $\lambda_p \gg \bar{\lambda}_L$ due to

specific behavior of relativistic quantum-field systems with small regions of homogeneity. We continue to discuss the effect for wide classes of the systems in this paper.

The smallness of the effective emitting region is the basic condition for the interferometry effect to be realized experimentally. One can expect that for interferometry analysis the hierarchy of the system lengths displays itself even stronger than for spectrometry. Firstly it has been shown⁷ that in contrast to small homogeneous systems, $\bar{\mathbf{R}} = \bar{\boldsymbol{\lambda}}$, when the interferometry radius $\mathbf{R}$ coincides with the geometrical radius of the system, $\bar{\mathbf{R}} = \mathbf{R}$, for longitudinally expanding system, $\bar{R}_L \gg \bar{\lambda}_L$, the longitudinal interferometry radius is defined by the length of homogeneity, $R_L \propto \bar{\lambda}_L$. No one correct analytical consideration was present up to now to generalize this statement for the case of three-dimensional expansion: $\bar{R}_T > \bar{\lambda}_T$, $\bar{R}_L \gg \bar{\lambda}_L$. This is also one of the aims of this paper. It has been shown⁶ that the structure of the correlation function is modified seriously if the length of homogeneity is smaller than the wavelength of the particles $\lambda_p \gg \bar{\lambda}_L$. This effect appears under the same condition as for single particle spectra and has the same nature. We shall discuss the possibility to reveal this effect experimentally for the system with non-zero chemical potential and chemically non-equilibrium systems.

LENGTH SCALES FOR A LOCALLY THERMALIZED SYSTEM

The peculiarities of the spectra and correlations as we will show are strongly dependent on the ratio between different space-time lengths inherent in the system. The local length of homogeneity $\bar{\lambda}(x_0)$ is defined by the behavior of Wigner function. It means the length within which the deviation of the Wigner function is relatively small and is about the function value⁸

$$\frac{|f(p, x_0 + \bar{\lambda}) - f(p, x_0)|}{f(p, x_0)} = 1 \quad (1)$$

So the length of homogeneity will be

$$(\bar{\lambda}_i(x, p))^{-1} = \left| \frac{\partial f(p, x)}{\partial x_i} \right| / f(p, x) \text{ or } (\bar{\lambda}_i(x_0))^{-2} = \left| \frac{\partial^2 f(p, x)}{\partial^2 x_i} \right| / 2f(p, x) \Big|_{x_0(p)} \quad (2)$$

The last expression corresponds to the length of homogeneity at the point $x_0(p)$ where the distribution function is maximal, i.e.,

$$\frac{\partial f(p, x)}{\partial x_i} \Big|_{x_0(p)} = 0 \quad (3)$$

In the framework of phenomenological approach the typical distribution function for bosons forming in ultra-relativistic A+A collisions is the Bose-Einstein distribution in the cylindrical tube with the Gaussian radius $\bar{R}_T$ where the system is considered on the set of "constant proper time" hypersurfaces, $\tau = \sqrt{t^2 - x_L^2}$, has the longitudinally boost-invariant expansion, $v_L = x_L/t$, and also has a transversal expansion, $u_T(r)$. In the standard design-

nations the distribution function at $m_T \beta \gg (\beta(r)$ is the inverse of temperature) has the form

$$f(x, p) = \frac{1}{(2\pi)^3} \exp \left[-\beta(m_T \text{ch}(y_L - y) \text{ch} y_T - \mathbf{p}_T \cdot \frac{\mathbf{r}}{r} \text{sh} y_T - \mu) \right] \exp \left[-\frac{r^2}{2\bar{R}_T^2} \right] \quad (4)$$

The first exponent is Bose-Einstein distribution $\exp(\beta p \cdot u(x) - \beta \mu) - 1)^{-1}$ where 4-velocity of the hadron gas u^μ is expressed via longitudinal $y_L(x_L)$ and transversal $y_T(r)$ rapidity; the second exponent is taking into account effectively the finite transversal size of the source. Let us consider the spectral length of homogeneity near the maximum of the distribution function in the longitudinal rest system of the particle, $P_L = 0$ or $y = 0$. Keeping in mind the interferometry analysis, we use longitudinal length of homogeneity $\bar{\lambda}_L(p)$, outward $\bar{\lambda}_O(p)$ along the $\mathbf{p}_T$ -direction and $\bar{\lambda}_S(p)$ which is orthogonal to $\mathbf{p}_T$ -direction. Let us suppose the gradients of temperature and chemical potential in transversal direction are much smaller than the corresponding gradient of velocity on the freeze-out surface. Then the analysis of the Eq.(3) shows that under the condition

$$\beta p_T \frac{dy_T}{dr}(r_0) \ll \frac{r_0}{\bar{R}_T^2} \quad (5)$$

the transversal rapidity at point of maximum is $y_T(r_0) \ll 1$ and the values we are interested in are

$$\bar{\lambda}_L(p) \approx \sqrt{\frac{2T}{m_T}} \tau, \quad \bar{\lambda}_S(p) \approx \bar{\lambda}_O(p) \approx \bar{R}_T \quad (6)$$

Under the condition

$$\beta m \frac{dy_T}{dr}(r_0) \gg \frac{r_0}{\bar{R}_T^2} \quad (7)$$

$\text{th} y_T \approx p_T/m_T$ at the point r_0 and the spectral lengths of homogeneity tend to the thermal ones (in the infinite tube, $\bar{R}_T \rightarrow \infty$) which are

$$\bar{\lambda}_{th,L}(p) = \frac{\sqrt{2Tm}}{m_T} \tau(r_0), \quad \bar{\lambda}_{th,S}(p) = \frac{\sqrt{2Tm}}{p_T} r_0, \quad \bar{\lambda}_{th,O}(p) = \sqrt{\frac{2T}{m}} \left(\frac{dy_T}{dr}(r_0) \right)^{-1} \quad (8)$$

In the first case the transversal lengths of homogeneity characterized by the thermal Bose-Einstein distribution are larger than the geometrical radius of the system, $\bar{\lambda}_{th,S} \bar{\lambda}_{th,O} \gg \bar{R}_T^2$, so the effective lengths of homogeneity approximately coincide with geometrical radius, see Eq.(6). In the second case the thermal lengths of homogeneity are smaller than the geometrical radius. In this case the effective lengths of homogeneity are the thermal ones, see Eq.(8). In these two limits, as we will show, the "interferometry microscope" measures the smaller of a thermal and a geometrical length. In our consideration we

suppose that the temperature and chemical potential produce the lengths of homogeneity that are much larger than the collective flows and the geometry produce.

It is important to note that the condition (5) cannot be satisfied properly even at small p_T for specific class of the "soft" transversal flows with logarithmic or more slow dependence of transversal rapidity on the radius. But the qualitative dependence of the lengths of homogeneity λ at large βm_T is preserved.

Now we consider the average length of homogeneity or hydrodynamic length $\bar{\lambda}(x)$. It is defined as the average of $\bar{\lambda}(x, p)$ over ensemble near point x :

$$\lambda^{-2}(x) = \left\langle \bar{\lambda}^{-2}(x, p) \right\rangle_p \quad (9)$$

Using the local equilibrium Bose-Einstein function with non-zero chemical potential, one can calculate the length of homogeneity $\bar{\lambda}(x, p)$:

$$\lambda^{-2}(x) = (\nabla_\mu u^\nu)^2 - \frac{3}{T^2} (\nabla T)^2 + \frac{3}{nT} \nabla T \cdot \nabla n - \frac{1}{n^2} (\nabla n)^2 \quad (10)$$

The last two terms appear only if chemical potential is not constant, $\mu(x) \neq \text{const}$. Here

$$\nabla_\mu = \frac{\partial}{\partial x^\mu} - u_\mu u^\nu \frac{\partial}{\partial x^\nu}$$

For one-dimensional boost-invariant hydrodynamic expansion the average length of homogeneity coincides with proper time

$$\lambda(x) = \tau \equiv \sqrt{t^2 - x_L^2} \quad (11)$$

The final length we are introducing is the wave-length of the quanta λ_p . It might be the Compton length for massive field or the de Broglie one for massless field. We will consider

$$\lambda_p = 1/p^0 \quad (12)$$

The locally equilibrium approach in zero- approximation (without dissipative effects) usually is based on the substitution $\beta = \text{const} \rightarrow \beta(x)$, $u = \text{const} \rightarrow u(x)$ in the Wigner function of global equilibrium system. Physically it is a good approach if quanta "feel" only the quasi-homogeneous part of a system. Under this condition the wave-length of the quanta has to be smaller than the length of homogeneity in a thermalized medium, $\lambda_p \ll \bar{\lambda}$. In the next section we limit ourselves just to this approximation. After that we consider the general case of an arbitrary ratio λ_p to $\bar{\lambda}$.

BOSE-EINSTEIN CORRELATIONS FOR THREE-DIMENSIONAL EXPANDING HADRON SOURCES

The description of the inclusive spectra and correlations for multiparticle production is based on the computation of the following type of averages

$$p^0 \frac{dN}{d\mathbf{p}} = \langle a_{\mathbf{p}}^+ a_{\mathbf{p}} \rangle, \quad p_1^0 p_2^0 \frac{dN}{d\mathbf{p}_1 d\mathbf{p}_2} = \langle a_{\mathbf{p}_1}^+ a_{\mathbf{p}_2}^+ a_{\mathbf{p}_1} a_{\mathbf{p}_2} \rangle, \text{ etc.} \quad (13)$$

where $a_{\mathbf{p}}^+$ and $a_{\mathbf{p}}$ are the creation and annihilation operators and brackets $\langle \dots \rangle$ mean the average over some density matrix corresponding to the state of the system on some space-time hypersurface. In S-matrix theory the state is $|out\rangle$ -state at $\sigma: t = \infty$. In statistical (thermodynamic) models of multiparticle production the density matrix is chosen to be the statistical operator ρ and σ is the freeze-out hypersurface or it corresponds to the earlier stage of an evolution if one studies the dilepton or photon production from hadron matter and/or quark-gluon plasma.

The double particle spectrum in (13) is usually considered under supposition that the four-operator averages can be written in terms of the irreducible two-operator ones

$$\langle a_{\mathbf{p}_1}^+ a_{\mathbf{p}_2}^+ a_{\mathbf{p}_1} a_{\mathbf{p}_2} \rangle = \langle a_{\mathbf{p}_1}^+ a_{\mathbf{p}_1} \rangle \langle a_{\mathbf{p}_2}^+ a_{\mathbf{p}_2} \rangle + \langle a_{\mathbf{p}_1}^+ a_{\mathbf{p}_2} \rangle \langle a_{\mathbf{p}_2}^+ a_{\mathbf{p}_1} \rangle \quad (14)$$

In this case the problem of multi-particle spectra is reduced to calculation of the $\langle a_{\mathbf{p}_1}^+ a_{\mathbf{p}_2} \rangle$ averages.

If one considers the identical particles 1 and 2 and uses the on-mass-shell approximation, the two-operator average can be expressed by means of the so-called single-particle Wigner function

$$f(x, p) = (2\pi)^{-3} \int d^4 u \delta(p \cdot u) e^{-iu \cdot x} \langle a^+(p - \frac{u}{2}) a(p + \frac{u}{2}) \rangle \quad (15)$$

where $p = (p_1 + p_2)/2$. Indeed, if the statistical operator ρ is defined on some hypersurface σ which can be closed by a plane surface $t = \text{const}$, it is possible to use the equation

$$\int d\sigma_\mu p^\mu e^{ik \cdot x} = (2\pi)^3 p^0 e^{ik^0 t} \delta^3(\mathbf{k}) \quad \text{if} \quad p \cdot k = 0 \quad (16)$$

that follows from the Gauss theorem. Then

$$\begin{aligned} \langle a^+(p_1) a(p_2) \rangle_\sigma &= \int d^4 u p^0 e^{i(\Delta p^0 - u^0)t} \delta(p \cdot u) \delta^3(\Delta \mathbf{p} - \mathbf{u}) \langle a^+(p - \frac{u}{2}) a(p + \frac{u}{2}) \rangle_\sigma \\ &= \int d\sigma_\mu p^\mu e^{i\Delta p \cdot x} f(x, p) \end{aligned} \quad (17)$$

If the analysis of the final spectra and correlations is carried out, one has to suppose the Wigner function $f(x, p)_\sigma$ to correspond to free (or weakly interacting) particles on the hypersurface σ . When σ is changed from event to event it is necessary to do the additional averages in σ in all final expressions.

The expressions for the single particle spectrum and the double particle correlation function follow immediately from Eqs. (13)-(15), (17)

$$p^0 \frac{dN}{d\mathbf{p}} = \int d\sigma_\mu p^\mu f(x, p) \quad (18)$$

$$C(p_1, p_2) = 1 + \left(p_1^0 p_2^0 \frac{dN}{d\mathbf{p}_1 d\mathbf{p}_2} \right)^{-1} \left| \int d\sigma_\mu p^\mu e^{i\Delta p \cdot x} f(x, P) \right|^2 \quad (19)$$

where $P = (p_1 + p_2)/2$, $\Delta p = p_1 - p_2$.

As we will show in the next section the Wigner function will be approximately equal to the local Bose-Einstein distribution, $f(x, p) = f_{BE}(x, p)$ if $\lambda_p \ll \lambda$. So, for longitudinal boost-invariant expansion with transversal rapidity distribution $y_T(r)$ and under the conditions: $P^0 \tau \gg 1$, $M_T \beta \gg 1$ (M_T is half sum of transversal masses) we will use the Wigner function as it is given by the Eq. (4).

The boost-invariant integral measure in Eq. (19) is $d^2 x_T dx_L / u^0 = \tau d^2 x_T dy_L$. One of the transversal axes, x_o , we direct along the vector $\mathbf{P}_T$. This direction is called the "outward one". The direction orthogonal to it, x_s , is the "sideward one". In this coordinate system the exponent in Eq. (19) can be rewritten as the following:

$$\exp[i\Delta p \cdot x] = \exp \left[-i \left(q_{out} \left(x_o - t \frac{P_T}{P_0} \right) + q_{side} x_s + q_{long} \left(x_L - t \frac{P_L}{P_0} \right) \right) \right] \quad (20)$$

To calculate the integral in the expression (19) for the correlation function we use the saddle point method at the large parameter $m_T \beta \gg 1$ in the Wigner function $f(x, p)$ (4). Then we have for each *long*-, *out*-, *side*- projection of the correlation function the structure of the integral we are interested in⁹:

$$F_\alpha(\Delta p) = \int \varphi(x) \exp[\alpha S(x)] e^{i\Delta p g(x)} dx \xrightarrow{\alpha \rightarrow \infty} \exp[\alpha S(x_0)] \sum_{k=0}^{\infty} c_k \alpha^{-k-\frac{1}{2}} \quad (21)$$

where $\alpha S(x) = \ln f(x)$, $\alpha = \beta m_T$, saddle point x_0 is defined from the equations $S'(x_0) = 0$ that is equivalent to Eqs. (3) and

$$c_k = \frac{\Gamma\left(k + \frac{1}{2}\right)}{(2k)!} \left(\frac{d}{dx}\right)^{2k} \left[\varphi(x) \left(\frac{S(x_0) - S(x)}{(x - x_0)^2} \right)^{-k-\frac{1}{2}} e^{i\Delta p g(x)} \right]_{x=x_0}$$

Leaving the maximal coefficient at each power of Δp in the expansion of $|F_\alpha(\Delta p)|^2$, one can sum series (21) up and find

$$|F_\alpha(\Delta p)| = \left[\frac{2\pi}{\alpha S''(x_0)} \right]^{\frac{1}{2}} \varphi(x_0) \exp[\alpha S(x_0)] \exp \left[\frac{\Delta p^2 (g'(x_0))^2}{2\alpha S''(x_0)} \right] \quad (22)$$

Due to $t = f(x_L)$ the correlation function is factorized in the frame of reference, where $P_L = 0$ only:

$$C(\Delta \mathbf{p}, P_L = 0) - 1 = (C(\Delta p_{out}) - 1)(C(\Delta p_{side}) - 1)(C(\Delta p_{long}) - 1) \quad (23)$$

We will use just this reference system. Taking into account that integral (21), (22) corresponds to single particle distribution (18) at $\Delta p = 0$ and the relationship between the Wigner function (4) and αS , $\alpha S(x) = \ln f(x)$, we find for each *i*- projection of the correlation function (19):

$$C(\Delta p_i) = 1 + \exp \left[-\frac{1}{2} (\Delta p_i)^2 R_i^2 \right] \quad \text{where } R_i^2 = \frac{2f(x_0)}{|\partial^2 f(x_0)/\partial x_i^2|} \left(\frac{\partial g_i(x_0)}{\partial x_i} \right)^2 \quad (24)$$

So we have

$$R_i^2(P) = \bar{\lambda}_i^2(x_0(P))(g'_i)^2 \quad (25)$$

The form of the functions g'_i is obvious from Eq.(20):

$$|g'_{long}| = 1, \quad |g'_{side}| = 1, \quad |g'_{out}| = 1 - \frac{P_T}{M_T} \frac{d\tau}{dr}(r_0) \quad (26)$$

The result (24) shows that the interferometry radii are actually the spectral lengths of homogeneity $\bar{\lambda}_i(x_0(p))$, see Eq. (2), that are corrected by the factor $g'(x_0)$. For the *side*- and *long*-radii is no correction. As for the *out*-radius, the correction takes into account the duration of radiation $\Delta\tau$ within the length of homogeneity $\bar{\lambda}_{out}(x_0)$. It appears due to the presence of the time-variable in the integral for the correlator (see Eq. (20)). Finally, if $r_0 = x_{out,0} \neq 0$, the interferometry radii in 3-dimensional expanding system in ($P_L = 0$)-frame of reference are:

$$R_L = \bar{\lambda}_L(p) = \tau \sqrt{\frac{2T}{M_T \text{ch } y_T}}, \quad R_S = \bar{\lambda}_S(p) = r_0 \sqrt{\frac{2T}{p_T \text{sh } y_T}} \quad (27)$$

$$R_O = \bar{\lambda}_O(p) + \frac{P_T}{M_T} \Delta\tau =$$

$$\sqrt{\frac{2T}{m}} \left(y_T'' \text{sh}(y_T - \eta_T) + (y_T')^2 \text{ch}(y_T - \eta_T) + T/m\bar{R}_T^2 \right)^{-\frac{1}{2}} \left(1 - \frac{P_T}{M_T} \frac{d\tau}{dr}(r_0) \right)$$

where η_T is the average transversal rapidity of a pair, $y_T = y_T(r_0)$, $\tau = \tau(r_0)$.

The correction to the *out*-radius is linear in the duration time $\Delta\tau$. It is typical in general when the radiation process continuous during the matter evolution. In this case there is the dependence between the radius and the freeze-out time $\tau(r)$ because at each time τ most of the particles that are leaving the system are near its surface. For each momentum p the intensity of the radiation is at its greatest near the point $r_0(p)$. In a more artificial picture from the point of view of the hydrodynamics evolution when the process of radiation in time is not correlated with its intensity in configuration space, the correction to the *out*-radius due to duration in time is square-law in $\Delta\tau$ ¹⁰:

$$R_O^2 = \bar{R}_T^2 + \frac{P_T}{M_T} \Delta\tau^2 \quad (28)$$

In our approach we have the result (28) in the case $\tau(r) = \text{const}$ after additional averaging in τ with the weight $\sim \exp(-\tau^2/2\Delta\tau^2)$.

If the thermal lengths of homogeneity are much larger than the geometrical sizes, $\bar{\lambda}_{th,S} \bar{\lambda}_{th,O} \gg \bar{R}_T^2$, see Eq.(5), we have the results (6) for spectral lengths of homogeneity

or for the interferometry radii (27). In this case *side*- and *out*- radii are approximately a constant, *long*- radius is decreasing as $1/\sqrt{M_T}$. For the intensive transversal flow (the large gradient of transversal velocity, see Eq. (7)), the spectral lengths of homogeneity are approximately the thermal ones (8). If the freeze-out proper time $\tau(r_0(p))$ is smooth enough in P_T , the *long*-radius defined by the Eqs.(8),(27) will decrease as $1/M_T$. instead of inverse of square root dependence for very soft transversal flows. As to the *side*- and *out*-radii, their behavior strongly depends on $r_0(p_T)$ and $y'_T(r_0(p_T))$ behavior. If these functions are rather smooth, the *side*-radius decreases as $1/p_T$ and the *out*-radius is approximately constant. It is easy to illustrate in this important example:

$$y_T(r) = \left(r^2/2\bar{R}_v^2\right)^n \quad (29)$$

Then $r_0^2, y' \propto \ln^n(M_T/m)$ that conforms to the above statement concerning the momentum dependence of the interferometry radii. The important analytically solved case is $n=1$. Then we have for single particle spectra the inverse power behavior at large m_T :

$$p_0 \frac{d^3 N}{d^3 p} \propto e^{-\beta m_T} \text{ if Eq. (5); } p_0 \frac{d^3 N}{d^3 p} \propto \left(\frac{2p_T}{m}\right)^{-2-\frac{R_v^2}{\bar{R}_T^2}} \text{ if Eq. (7)} \quad (30)$$

In the last case when conditions (7) is satisfied, i.e., $\beta m \bar{R}_T^2 \gg R_v^2$, the single particle spectra can have the power-law decreasing. The P_T -behavior of the correlation functions then changes dramatically in comparison with Eqs.(6), (27):

$$R_L = \tau \frac{\sqrt{2Tm}}{M_T}, \quad R_S = \frac{2\bar{R}_v \sqrt{Tm} \ln^{\frac{1}{2}}\left(\frac{M_T + P_T}{m}\right)}{P_T} \quad (31)$$

$$R_O = \frac{\bar{R}_v}{\ln^{\frac{1}{2}}\left(\frac{M_T + P_T}{m}\right)} \sqrt{\frac{T}{m}} \left(1 + \frac{P_T}{M_T} \left| \frac{d\tau}{dr}(r_0) \right| \right)$$

All this demonstrates the correlation between the single particle spectrum's behavior and momentum dependence of interferometry radii and how these peculiarities are connected with the length scale in the emitting system.

It is important to mention also the specific class of the transversal flows with logarithmic or more slow dependence of transversal rapidity on the radius, e.g., $v_T = \text{th} y_T = r / \sqrt{\bar{R}_v^2 + r^2}$. Under the conditions (7) when the interferometry radii are equal to the thermal lengths of homogeneity, the *long*-radius is decreasing as $1/M_T$, *side*-radius is constant, and *out*-radius grows when P_T increases. It allows one to distinguish the different types of flows.

The general consequence of the intensive transversal flows is that the longitudinal radius decreases stronger than $1/\sqrt{M_T}$ when P_T grows, and there are noticeable differ-

ence in slopes dR_i/dP_T : $dR_{long}/dP_T \leq dR_{side}/dP_T < dR_{out}/dP_T$. The last value is approximately zero or even positive for the systems with developed transversal flows.

METHOD OF LOCALLY-EQUILIBRIUM STATISTICAL OPERATOR

The description of the inclusive spectra and correlations for multiparticle production is based on the computation of the averages (13) over states of a system which consider of some hypersurface σ . The operator ρ for local equilibrium systems is defined from the maximum entropy principle and has the form¹¹

$$\rho = e^{-S(\sigma)} = \frac{1}{Z} \exp \left[-\alpha \int d\sigma_\gamma \left(\beta_\nu \hat{T}^{\gamma\nu}(x) - \mu \beta \hat{J}^\gamma(x) - \mu_0 \beta \hat{J}_0^\gamma(x) \right) \right] \equiv \rho(\alpha)|_{\alpha=1} \quad (32)$$

where $\hat{T}^{\gamma\nu}(x)$ is the operator of the energy-momentum tensor, $\hat{J}^\gamma(x)$ is the current density operator, for the pion field it is $\hat{J}^\mu = i(\varphi^* \varphi^{;\mu} - \varphi^{*;\mu} \varphi)$, μ is the charge chemical potential, $\hat{J}_0^\mu$ is the particle flow density operator for chemically non-equilibrium systems, $\hat{J}_0^\mu = i(\varphi^{(-)} \cdot \varphi^{(+);\mu} - \varphi^{(+)} \cdot \varphi^{(-);\mu})$, (+) and (-) corresponds to positive- and negative-frequency field components, μ_0 is a chemical potential for number of the particles, $\beta^\nu = u^\nu / T$ (u^ν is hydrodynamic 4-velocity, T is the temperature), $\beta = \sqrt{\beta_\nu \beta^\nu}$. The basic model for the consideration is the scalar field.

The main idea of the calculation of the averages (13) is presented in Ref. [6]. Let us introduce the operators for all the field components of the pion triplet:

$$\mathbf{A}(p, \alpha) \equiv \begin{pmatrix} a^+(p, \alpha) \\ a(p, \alpha) \end{pmatrix} = Z^2 \rho(\alpha) \mathbf{A}(p) \rho(-\alpha); \quad \mathbf{A}(p) \equiv \begin{pmatrix} a^+(p) \\ a(p) \end{pmatrix} \quad (33)$$

Then

$$\mathbf{A}(p, \alpha = 0) = \mathbf{A}(p)$$

$$\langle \mathbf{A}(p) a(p') \rangle = \langle a(p') \mathbf{A}(p, \alpha = 1) \rangle, \quad \langle \mathbf{A}(p) a^+(p') \rangle = \langle a^+(p') \mathbf{A}(p, \alpha = 1) \rangle \quad (34)$$

and

$$\frac{\partial \mathbf{A}(p, \alpha)}{\partial \alpha} = [\mathbf{A}(p, \alpha), S(\sigma)] = \int d^3 \mathbf{k} \mathcal{K}(p, k) \mathbf{A}(k, \alpha) \mathcal{K} \quad (35)$$

where for π^0 -pions $\mathcal{K} = K$, for π^+ : $\mathcal{K} = K + iL$ and for π^- : $\mathcal{K} = K - iL$

$$K(p, k) = \begin{pmatrix} (G + iM_0) & \bar{G} \\ -\bar{G}^* & -(G^* - iM_0^*) \end{pmatrix}, \quad L(p, k) = \begin{pmatrix} M & \bar{M} \\ -\bar{M}^* & -M^* \end{pmatrix}$$

$$G(p, k) = \frac{-i}{2(2\pi)^3} \int d\sigma e^{i(k-p) \cdot x} \frac{\beta(x)}{k^0} \left[k \cdot u p \cdot u - (k \cdot p - m^2) / 2 \right]$$

$$\overline{G}(p, k) = \frac{1}{(2\pi)^3} \int d\sigma e^{i(k+p) \cdot x} \frac{\beta(x)}{k^0} \left[k \cdot u p \cdot u - (k \cdot p + m^2)/2 \right]$$

$$M(p, k) = \frac{-i}{2(2\pi)^3} \int d\sigma e^{i(k-p) \cdot x} \frac{\beta(x)\mu(x)}{k^0} [k \cdot u + p \cdot u]$$

$$\overline{M}(p, k) = \frac{-i}{2(2\pi)^3} \int d\sigma e^{i(k+p) \cdot x} \frac{\beta(x)\mu(x)}{k^0} [p \cdot u - k \cdot u]$$

$$M_0 = iM(\mu \rightarrow \mu_0)$$

The solution of the integro-differential equation with the Cauchy condition $\mathbf{A}(p, \alpha = 0) = \mathbf{A}(p)$ is

$$\mathbf{A}(p, \alpha = 1) = \mathbf{A}(p) + \sum_{n=1}^{\infty} \frac{1}{n!} \int d^3 \mathbf{k} \mathcal{K}_n(p, k) \mathbf{A}(k) \equiv \mathbf{A}(p) + \hat{\mathcal{K}}(p, k) \mathbf{A}(k) \quad (36)$$

here $\mathcal{K}_n(p, k)$ is the n-th iteration of matrix kernel $\mathcal{K}$.

The integral equations for $\langle a^+(p_1) a(p_2) \rangle$ and $\langle a(p_1) a(p_2) \rangle = \langle a^+(p_1) a^+(p_2) \rangle^*$ then follow from (34) and are

$$\hat{\mathcal{K}}(p, k) \langle a(p') \mathbf{A}(k) \rangle = \begin{pmatrix} -p^0 \delta^3(\mathbf{p} - \mathbf{p}') \\ 0 \end{pmatrix}, \quad \hat{\mathcal{K}}(p, k) \langle a^+(p') \mathbf{A}(k) \rangle = \begin{pmatrix} 0 \\ p^0 \delta^3(\mathbf{p} - \mathbf{p}') \end{pmatrix} \quad (37)$$

The matrix integral equation (37) is the basic one for calculation of the multiparticle spectra (13).

THERMAL WICK'S THEOREM FOR LOCALLY EQUILIBRIUM SYSTEMS

Basing on the Eqs. (34), (35), (37) one can reach the following results (see Ref.[6] for details)

$$\langle A(p_1) A(p_2) \dots A(p_n) \rangle = \sum \prod_{j' > j} \langle A(p_j) A(p_{j'}) \rangle, \quad (n = 2k) \quad (38)$$

The main peculiarity as compared with the classical result is the presence of additional terms likes $\langle a^+(p_1) a^+(p_2) \rangle$ and $\langle a(p_1) a(p_2) \rangle$ in the expansion. These non-zero terms arise because of the space-time finiteness of the homogeneity regions in locally- equilibrium systems.

BOSON SPECTRA AND CORRELATIONS IN LOCALLY-EQUILIBRIUM SYSTEMS

We consider here a physically important model of one-dimensional boost-invariant hydrodynamic expansion. We shall suppose that the temperature decreases with the radius as $\beta(r) = \beta_0 \exp[r^2/2R_\beta^2]$. The solution of the integral equation (37) gives the following results for the Wigner function of the locally thermalized pion field (the method of calculation is given in [6]). In the reference system where particle momentum (or average momentum of pair) is $p = (p_0, p_T, 0, 0)$

$$\frac{dN}{d^3x d^3p} = f(x, p)_\sigma = \frac{\langle a^+ a \rangle(t=0)}{(2\pi)^3} \quad (39)$$

where $\langle a^+ a \rangle(t)$ is the Fourier transformation of $\langle a^+(p_1) a(p_2) \rangle$, and

$$\begin{aligned} \langle a_{\pi^-} a_{\pi^+} \rangle &= \frac{(2\pi)^3}{2\omega} \overline{G^*} [f^+ + f^-], \quad \langle a_{\pi^-} a_{\pi^+} \rangle = \frac{(2\pi)^3}{2\omega} \overline{G^*} [f^+ + f^-] \\ f^+(t, \rho) &= \frac{(2\pi)^{-3}}{\exp[\beta(\rho)(\omega(t) - \mu)] - 1}, \quad f^- = f^+(\mu \rightarrow -\mu) \end{aligned} \quad (40)$$

where

$$G(t) = -\frac{\text{ch}(\pi t)}{\pi \tau} \int_0^\infty dz \sqrt{(2m_T \tau)^2 + z^2} K_{i2t}(z);$$

$$\overline{G}(t) = \frac{i m_T}{2} \int_{-\infty}^\infty dz \frac{e^{i2tz}}{\text{ch} z} H_1^{(2)}(2m_T \tau \text{ch} z)$$

$$\omega(t) \equiv \sqrt{(G(t) + \mu_0)^2 - \overline{G}^*(t) \overline{G}(t)}$$

Here $K_\nu(z)$ is a modified Bessel function and $H_1^{(2)}(z)$ is Hankel function of second order. The value τ is the proper time of the expansion, m_T is transversal mass. The chemical potential μ_0 shows the rate of deviation of the pion gas from chemical equilibrium.

The ratio of the Wigner function of bosons in expanding matter to the Bose-Einstein distribution is shown in Fig. 1. The main result is that the particle density as well as the momentum spectrum has an enhancement at small p^0 in comparison with the Bose-Einstein distribution at $p^0 \tau \leq 1$. This enhancement becomes stronger when the systems are far from chemical equilibrium ($\mu \rightarrow \mu_c$) as it can be shown from (40). The critical value μ_c for Bose condensation reduces as compared with standard one $\mu_c = m$ and according to (40) is $\mu_c = m(m\tau)$ for $m\tau \ll 1$.

Figure 1. The ratio of the single particle Wigner function of bosons in expanding matter to the Bose-Einstein heat distribution.

5. TWO- PARTICLE CORRELATIONS

The calculation of two- (many-) particle inclusive spectra is based on the generalized Wick's theorem (38). For true neutral identical particles it gives

$$p_1^0 p_2^0 \frac{dN_{inc}}{d\mathbf{p}_1 d\mathbf{p}_2} = \langle a^+(p_1) a(p_1) \rangle \langle a^+(p_2) a(p_2) \rangle + \langle a^+(p_1) a(p_2) \rangle \langle a^+(p_2) a(p_1) \rangle + \langle a(p_1) a(p_2) \rangle \langle a^+(p_2) a^+(p_1) \rangle$$

The last term is not equal zero due to inhomogeneity of the system. It brings the additional contribution in the correlation function of neutral bosons, e.g., of $\pi^0 \pi^0$ -pairs and due

Figure 2. The additional contribution to the correlation peak for neutral and oppositely charged bosons

to isotopic invariance the correlation appears for oppositely charged pions also. The dependence of $\text{Max } C_{\text{add}} = C_{\text{add}}(\mathbf{p}_1 = \mathbf{p}_2 = 0)$ on $m\tau$ for $\mu_0 = \mu = 0$ is demonstrated in Fig.2. For $\pi^0\pi^0$ -pairs the departure from standard maximum value 2 of the correlation function is noticeable for τ around 1 fm and is then 0.25. The maximal value of the correlation function for massless boson will be 3. Notice that parameter $m\tau = \lambda_p/\bar{\lambda}$.

In order to demonstrate the influence of chemical potential on the behaviour of the correlation function at large chemical potentials in "standard" region $M_T\tau \gg$ we show its approximation in the reference system where $P = (P_0, P_T, 0, 0)$

$$C(p_1, p_2) = 1 + \alpha_1 e^{-\frac{R_{L1}^2 q_L^2}{2}} e^{-\frac{R_{T1}^2 (\mathbf{p}_{1T} - \mathbf{p}_{2T})^2}{2}} + \alpha_2 e^{-\frac{R_{L2}^2 q_L^2}{2}} e^{-\frac{R_{T2}^2 (\mathbf{p}_{1T} + \mathbf{p}_{2T})^2}{2}} \quad (41)$$

We will use the designations from the second section. Under the condition $\beta_0(M_T - \mu_0 \pm \mu) \gg$ we have

• For $\pi^-\pi^-$ -pairs: $\alpha_1 =$, $\alpha_2 = 0$, $R_{L1}^2 = \frac{2\tau^2}{\beta_0 M_T} = \bar{\lambda}_L^2$,

$$R_{T1}^2 = \bar{R}_T^2 \quad \text{if} \quad \bar{R}_T^2 \ll R_\beta^2 \quad \text{and} \quad R_{T1}^2 = \frac{2R_\beta^2}{\beta_0(M_T - \mu_0)} = \bar{\lambda}_T^2 \quad \text{if} \quad \bar{R}_T^2 \gg R_\beta^2$$

• For $\pi^+\pi^-$ -pairs: $\alpha_1 = 0$, $\alpha_2 = \left(1 - \frac{\mu_0}{M_T}\right)^{-2} \left(1 + \left(\frac{2M_T\tau}{\text{ch}(\beta_0\mu)}\right)^2\right)^{-1} \left(1 + \left(\frac{2\tau}{\beta_0}\right)^2\right)^{-\frac{1}{2}}$

$$R_{L2}^2 = \frac{2\tau^2\beta_0 M_T}{(\beta_0 M_T)^2 + (2M_T\tau)^2}, \quad R_{T2}^2 = \bar{R}_T^2 \quad \text{or} \quad R_{T2}^2 = \frac{2R_\beta^2}{\beta_0(M_T - \mu_0)} = \bar{\lambda}_T^2 \quad \text{under the}$$

same conditions as for $\pi^-\pi^-$.

• For $\pi^0\pi^0$ -pairs the coefficient α and the corresponding radii R_{T1} and R_{L1} are the same as for $\pi^-\pi^-$ -pairs, the coefficient α_2 and the radii R_{T2} and R_{L2} are as for $\pi^+\pi^-$ -pairs with $\mu = 0$ in all expressions.

If the chemical potentials are near the critical values, $M_T \approx m$, $\mu + \mu_0 \approx M_T$ we have

• For $\pi^-\pi^-$ -pairs: $\alpha_1 = 1$, $\alpha_2 = 0$, $R_{L1}^2 \propto \sqrt{1 - \frac{\mu}{M_T - \mu_0}}$, $R_{T1}^2 = \bar{R}_T^2$ or $= 2R_\beta^2$

The transversal interferometry radius here is the smallest of two: $\bar{R}_T$ or $\bar{R}_\beta$

• For $\pi^+\pi^-$ -pairs: $\alpha_1 = 0$, $\alpha_2 = 1$, $R_{L2}^2 \propto \sqrt{1 - \frac{\mu}{M_T - \mu_0}}$, $R_{T2}^2 = \bar{R}_T^2$ or $= 2R_\beta^2$

• For $\pi^0\pi^0$ -pairs the coefficient α and the corresponding radii R_{T1} and R_{L1} are the same as for $\pi^-\pi^-$ -pairs, the coefficient α_2 and the radii R_{T2} and R_{L2} are as for $\pi^+\pi^-$ -pairs with $\mu = 0$ in all expressions.

As the result we can see that even at large $m\tau$ the additional terms in the correlator of oppositely charged bosons could be experimentally noticeable if the system is far from chemical equilibrium or has the large chemical potential: $|\mu| + \mu_0 \rightarrow m$.

CONCLUSIONS

We show that the experimentally observed particle spectra and correlation are very sensitive to the space-time structure of the emitting matter. The ratios between local lengths of homogeneity of the system, its geometrical sizes and wave-lengths of the particles define the peculiarities of the spectra and correlations. This is reflected in different behavior of the interferometry radii as depending on momentum regions. In particular, the strong transversal expansion leads to more quick than inverse of $\sqrt{m_T}$ decreasing of the longitudinal interferometry radius. At the same time the side-radius decreases more quickly than the out-radius when transversal momentum increases. Even the structure of the correlation function depends on ratio of the quanta's wave-length to the homogeneity length. The simple analytical approximation for observed values allows to clear up experimentally the space-time structure of the systems formed in ultra-relativistic nucleus-nucleus collisions and the details of its evolution.

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