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DEFINED ON THE MODULI SPACE OF MONOPOLES

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**QUANTUM CORRECTIONS TO THE METRIC
DEFINED ON THE MODULI SPACE OF MONOPOLES**

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ABSTRACT

We treat the collective coordinates of the moduli space of monopoles within the Dirac formalism. We show that the constraints derived from the collective coordinates must depend on the physical modes. In turn, the metric defined on the moduli space gets corrected at the quantum level. In particular, the metric of the 2-monopole loses the $O(3)$ invariance due to these corrections.

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The study of the spectrum of monopoles of the low energy effective theory of heterotic string theory compactified on the torus has enjoyed a revival since the work of Sen [1]. In this work, it was shown that certain magnitudes which are not renormalized by perturbation theory are invariant under S-duality. The study of the moduli space of monopoles has also received much attention by many authors [2][4][3]. I will make an attempt, for the first time, to include the quantum corrections in the description of the moduli space.

Monopole solutions are topological solutions of the static equations of motions of a Yang-Mills Lagrangian. In order to define them, one must make use of certain parameters. A solution carrying magnetic charge k will be described in terms of $4k$ parameters. In studying the spectrum defined by the quadratic Hamiltonian, we will find $4k$ zero modes. Each of these zero modes will be associated to a parameter. In order to integrate the zero modes and thus get an infrared divergent free theory we must raise the parameters to collective coordinates. This means that the parameters now depend on time. The fields will then depend on the collective coordinates. To be explicit, consider the monopole sector with charge k . Let

$$W_\mu^a(x_{int}, X_\alpha(t)), \quad \mu = 1, \dots, 4, \quad x \in R^4 \quad \alpha = 1, \dots, 4k, \quad a = 1, \dots, 3 \quad (1)$$

be the fields of the Yang-Mills action. The first three components of the field W are $SO(3)$ connections, while the fourth component, the Higgs field, transforms in the adjoint representation of $SO(3)$. The gauge I have used is the vanishing of the zeroth component of the gauge connection. The collective coordinates X_α are the coordinates of the moduli space $\mathcal{M}_k$ describing a given state of the monopole sector with charge k obtained from the heterotic string compactified on a torus.

The introduction of collective coordinates in addition to the original fields lead to an artificial enlargement of the space of variables. In order to recover the description of the theory one must introduce constraints. It will be this constraint along with a gauge fixing function that will allow us to integrate the zero modes associated to the collective coordinates. The correct treatment of this constraints in field theory can be done within the Dirac formalism of collective coordinates [5]. This treatment, as I shall show, will

recover the metric found by Gauntlett [4] to leading order. However, there are corrections to this metric which follow from the quantum sector of the theory, even if it has arbitrary supersymmetry. The constraint will depend on the physical quantum fluctuations about the classical solution and will be shown to yield the corrected metric when perturbative corrections are considered.

To include the effect of the constraints and the gauges it is better to work in the Hamiltonian formulation of the system. The conjugate momenta of the fields and the collective variables will be

$$\begin{aligned}\partial_{\dot{W}_\mu^a} L &= \dot{W}_\mu^a \equiv P_\mu^a, \quad [W_\mu^a, P_\nu^b] = i\delta_{\mu\nu}\delta_{ab}\delta(x-y) \\ \partial_{\dot{X}_\alpha} L &= \int d^3x \text{Tr } P_\mu \delta_\alpha W_\mu \equiv \Pi_\alpha, \quad [X_\alpha, \Pi_\beta] = i\delta_{\alpha\beta}.\end{aligned}\tag{2}$$

Here, I have used the definition

$$\frac{\partial \dot{W}}{\partial \dot{X}_\alpha} = \delta_\alpha W.\tag{3}$$

The first equation in (2) defines the momenta of the fields. The second equation are the constraints of the system since it relates the collective momenta to the intrinsic field. the commutation relations reminds us that the collective variables are of quantum nature. The variations $\delta_\alpha W$ define the metric of the moduli space [4]

$$g_{\alpha\beta} = \int d^3x \text{tr } \delta_\alpha W_\mu \delta_\beta W_\mu.\tag{4}$$

At the classical level, the system is naturally constrained since it is not affected by their quantum variables. However at the quantum level where the perturbative analysis takes place the constraints become relevant and proper treatment of them is crucial.

The quantum fluctuation about the monopole solution are

$$W_\mu = W_{\mu,c} + q_\mu \quad P_\mu = p_\mu\tag{5}$$

where p_μ is the conjugate of q_μ . $W_{\mu,c}$ denotes the classical component of the field which yields the monopole solution.

The constraint in terms of the fluctuations reads

$$f_\alpha = \int \text{Tr}(p_\mu \delta_\alpha W_{\mu,c} + p_\mu \delta_\alpha q_\mu) - \Pi_\alpha.\tag{6}$$

A comment is in place. The absence of the second term in the constraint (6) would imply that physical modes are independent of the collective coordinates. The constraint in this case would be

$$f_\alpha = \int Tr p_\mu \delta_\alpha W_{\mu,c} - \Pi_\alpha \quad (7)$$

After an appropriate canonical transformation, the constraint (7) would take the form

$$f_\alpha = \int Tr (p_{\mu,0} + p_{\mu,re}) \delta_\alpha W_{\mu,c} \quad (8)$$

where I have separated the momenta in zero modes and real (physical) modes. If the operator $p_{\mu,re}$ is independent of the collective coordinates, expanding the classical field in powers of the collective coordinates we find that all physical modes must vanish from the spectrum since they will not be annihilated by the constraint. But if both the fluctuations and the classical field have the same dependence on the collective coordinates it will be guaranteed that the massive fluctuations will remain in the spectrum by orthogonality of the normal modes. Therefore, we learn that the fluctuations must depend on the collective coordinates in the same manner as the classical field justifying the second term in (6).

The main purpose of the paper is to see how the second term in the r.h.s of (6) affects the metric defined on the moduli space. Since the quantum fluctuations are smaller than the classical component of the field they may be treated perturbatively. The first term in (6), as we shall see, defines the quantum zero modes. The second term in the expression may be treated perturbatively. The last term which is just the collective momentum can be also treated perturbatively in the adiabatic limit in which it is much smaller than the classical component of the field.

In defining the monopole solution we minimized the Hamiltonian. It then follows that there can be no linear term in the fluctuations which would be of the order of the monopole solution. If we take the quadratic Hamiltonian to be of $\mathcal{O}(1)$, then the cubic and higher order terms would be of order inverse powers in the classical component of the field. They may thus be treated perturbatively.

The quadratic Hamiltonian may be written as

$$H^{(2)} = \int d^3x \left\{ Tr \frac{1}{2} p_\mu p_\mu + q_\mu^a V_{ab}^{\mu\nu} q_\nu^b \right\} \quad (9)$$

where $V_{ab}^{\mu\nu}$ is a differential operator which only depends on the classical field configuration $W_{\mu,c}$. The completion relation which follows from the normal mode expansion of the fields and their commutation relations is [4]

$$\delta(x-y) \delta^{ab} \delta_{\mu\nu} = g^{\alpha\beta} \delta_\alpha W_\mu^a(x) \delta_\beta W_\nu^b(y) + \sum_i \psi_{i\mu}^a(x) \psi_{i\nu}^b(y), \quad (10)$$

where the eigenfunctions ψ 's represent finite frequency modes and $\delta_\alpha W$ the zero modes.

This relation can be inserted in the quadratic Hamiltonian

$$\begin{aligned} H^{(2)} &= H_{sp}^{(2)} + H_{re}^{(2)} \\ H_{sp}^{(2)} &= \frac{1}{2} f^{(1)\alpha} f_\alpha^{(1)} + g^{\alpha\beta} \int d^3x \delta_\alpha W_\mu^a(x) q_\mu^a(x) V_{ab}^{\mu\nu}(x) \int d^3y q_\nu^b(y) \delta_\beta W_\nu^b(y) \\ H_{re}^{(2)} &= \sum_{b\mu i} \omega_i (a_{b\mu i}^+ a_{b\mu i} + \frac{1}{2}). \end{aligned} \quad (11)$$

The Hamiltonian $H_{re}^{(2)}$ describes the massive excitations. Diagonalization of it in the usual way is relevant for the perturbative treatment. The two terms, which define the spurious Hamiltonian $H_{sp}^{(2)}$, are the ones which will yield the metric to leading order. We thus concentrate on them.

The quadratic spurious Hamiltonian must be a physical operator to leading order. It then follows that it must commute with the constraint

$$[f_\alpha^{(1)}, H] = 0. \quad (12)$$

Here

$$f_\alpha^{(1)} = \int Tr p_\mu \delta_\alpha W_{\mu,c} \quad (13)$$

Thus, the second term in (11) must vanish as a consequence of the equations of motion of the quadratic Hamiltonian. Otherwise the total quadratic Hamiltonian would not be physical. Thus, the quadratic spurious Hamiltonian reads

$$H_{sp} = \frac{1}{2} f^{(1)\alpha} f_\alpha^{(1)}. \quad (14)$$

The modes obtained by this Hamiltonian are indeed zero modes and lead to infrared divergences when calculating perturbative corrections. We make use of the Dirac treatment of collective coordinates [5] in order to take care of the infrared divergences.

The procedure I will use requires us to take the constraint

$$f_\gamma = \sum_{a,\mu} \int p_\mu^a \delta_\gamma W_\mu^a + \frac{1}{2} \int \{p_\mu^a, \delta_\gamma q_\mu^a\} - \Pi_\gamma$$

to the linear form

$$f_\gamma = \sum_{a,\mu} \int p_\mu^a \delta_\gamma W_\mu^a.$$

In order to do this Gervais and coworker as well as Tomboulis made use of a transformation in soliton models which is not canonical as it does not preserve the Dirac brackets. However, to the order we are interested in, the physical magnitudes will not be affected by this non canonical transformation [6].

The momentum then takes the form

$$\begin{aligned} p_\nu^a \rightarrow p_\nu^a + g^{\alpha\beta} \delta_\alpha W_\nu^a & \left((\Pi_\beta - \frac{1}{2} \sum_{a_1, \nu_1} \int \{p_{\nu_1}^{a_1}, \delta_\beta q_{\nu_1}^{a_1}\} - \right. \\ & \frac{1}{2} \sum_{a_2, \nu_2} \int \{ \delta_\beta q_{\nu_2}^{a_2} \delta_\epsilon W_{\nu_2}^{a_2} g^{\epsilon\kappa}, (\Pi_\kappa - \frac{1}{2} \sum_{a_3, \nu_3} \int \{p_{\nu_3}^{a_3}, \delta_\kappa q_{\nu_3}^{a_3}\} \} + \\ & \frac{1}{4} \sum_{a_4, \nu_4} \int \{ \delta_\beta q_{\nu_4}^{a_4} \delta_\epsilon W_{\nu_4}^{a_4} g^{\epsilon\kappa}, \sum_{a_5, \nu_5} \int \{ \delta_\kappa q_{\nu_5}^{a_5} \delta_\rho W_{\nu_5}^{a_5} g^{\rho\sigma}, (\Pi_\sigma - \frac{1}{2} \sum_{a_6, \nu_6} \int \{p_{\nu_6}^{a_6}, \delta_\sigma q_{\nu_6}^{a_6}\} \} \} \\ & \left. + \dots \right) \end{aligned} \quad (15)$$

with

$$i[p_\mu^a(x), q_\nu^b(y)] \quad (16)$$

remaining invariant. It is here that canonicity is lost because the transformation does not act on the coordinates q , therefore, the commutator must be modified. The effect of this noncanonical transformation has been tested in the sine-Gordon model [6] with the use of the BRST formalism which does not make use of such a transformation. There it was shown that to the order we are interested the noncanonicity of the transformation does not affect the results we shall obtain.

The ellipsis in the expansion (15) denote terms which are needed to take the constraint to the linear form, and although they may be found explicitly they contribute only to higher orders than the ones we are looking for.

We can take the gauge to be

$$G^\gamma = g^{\gamma\beta} \sum_{a,\mu} \int q_\mu^a \delta_\beta W_\mu^a.$$

Thus

$$[G^\gamma, f_\alpha] = i g_\alpha^\gamma. \quad (17)$$

Thus I may enforce the constraint and the gauge if I make use of the Dirac brackets. Therefore, the zero modes may be integrated leading to an infrared free model.

The Hamiltonian has a contribution from the term $\frac{1}{2} \sum_{a,\mu} \int p_\mu^a p_\mu^a$ which is transformed to

$$\begin{aligned} & \frac{1}{2} \sum_{a,\nu} \int p_\nu^a p_\nu^a + \frac{1}{2} g^{\gamma\beta} \Pi_\gamma \Pi_\beta - g^{\gamma\beta} \Pi_\beta \frac{1}{2} \sum_{a_1,\nu_1} \int \{p_{\nu_1}^{a_1} \delta_\gamma q_{\nu_1}^{a_1}\} \\ & - g^{\gamma\beta} g^{\epsilon\kappa} \Pi_\kappa \Pi_\gamma \sum_{a_2,\nu_2} \int \delta_\beta q_{\nu_2}^{a_2} \delta_\epsilon W_{\nu_2}^{a_2} \\ & + g^{\gamma\beta} g^{\epsilon\kappa} g^{\rho\sigma} \Pi_\beta \Pi_\sigma \sum_{a_3,\nu_3,a_4,\nu_4} \int \delta_\gamma q_{\nu_3}^{a_3} \delta_\epsilon W_{\nu_3}^{a_3} \int \delta_\kappa q_{\nu_4}^{a_4} \delta_\rho W_{\nu_4}^{a_4} \\ & + \frac{1}{2} g^{\gamma\beta} g^{\epsilon\kappa} g^{\rho\sigma} \Pi_\kappa \Pi_\sigma \sum_{a_5,\nu_5,a_6,\nu_6} \int \delta_\gamma q_{\nu_5}^{a_5} \delta_\epsilon W_{\nu_5}^{a_5} \int \delta_\beta q_{\nu_6}^{a_6} \delta_\rho W_{\nu_6}^{a_6} \end{aligned} \quad (18)$$

The first term is the kinetic term for the fluctuations about the classical field configuration.

The second term is nothing but the the Hamiltonian obtained from the Lagrangian

$$L_{eff} = \frac{1}{2} g_{\alpha\beta} \dot{X}^\alpha \dot{X}^\beta. \quad (19)$$

It agrees with the Lagrangian obtained by Gauntlett [4]. The remaining terms along with the cubic term in the fluctuations in the expansion of the potential which has the form

$$\frac{1}{3!} \sum_{a,b,c,\mu,\nu,\rho} \int V_{abc}^{\mu\nu\rho} q_\mu^a q_\nu^b q_\rho^c. \quad (20)$$

will correct physical magnitudes. Some of the corrections will depend on the collective momenta. In particular, the correction to the energy of the fluctuations will yield a correction to the metric of the moduli space.

This shows that when fluctuations of the fields become relevant with respect to the metric they affect it. This of course we must expect since each collective momenta is proportional to the zero mode which in turn is coupled at the perturbative level to physical modes. This correction carries its effects onto the almost complex structure on the moduli space and onto its Christoffel connection which in turn defines the coupling of the fermionic collective degrees of freedom. The terms (18) and (20) are the ones responsible for the corrections to the metric.

The metric has the following form to $\mathcal{O}([g^{\alpha\beta}]^2)$

$$\begin{aligned}
& g^{\gamma\beta} + \frac{1}{4} g^{\rho\beta} g^{\sigma\kappa} g^{\epsilon\gamma} \sum_{n,a,\mu} \int \frac{\delta_\kappa \psi_{n\mu}^a}{\omega_n} \delta_\epsilon W_\mu^a \int \delta_\sigma \psi_{n\mu}^a \delta_\rho W_\mu^a \\
& + \frac{1}{4} g^{\alpha\gamma} g^{\epsilon\beta} \sum_{n,m,a,\mu} \left\{ \left(-\frac{1}{\omega_n} + \frac{1}{\omega_n + \omega_m} \right) \int \psi_{n\mu}^a \delta_\alpha \psi_{m\mu}^a \int \psi_{n\mu}^a \delta_\epsilon \psi_{m\mu}^a \right. \\
& \quad \left. - \frac{\int \psi_{n\mu}^a \delta_\alpha \psi_{m\mu}^a \int \psi_{m\mu}^a \delta_\epsilon \psi_{n\mu}^a}{\omega_n + \omega_m} \right\} \\
& + \frac{1}{4} g^{\kappa\gamma} g^{\epsilon\beta} \sum_{n,m,a,b,\mu,\nu} \int \delta_\kappa \psi_{n\mu}^a \delta_\epsilon W_\mu^a \int V_{abb}^{\mu\nu} \frac{\psi_{n\mu}^a (\psi_{m\nu}^b)^2}{\omega_n^2 \omega_m} \\
& + \frac{1}{2} g^{\rho\gamma} g^{\sigma\beta} g^{\epsilon\kappa} \sum_{n,a,\mu} \int \frac{\delta_\sigma \psi_{n\mu}^a}{\omega_n} \delta_\epsilon W_\mu^a \int \delta_\kappa \psi_{n\mu}^a \delta_\rho W_\mu^a
\end{aligned} \tag{21}$$

There is no reason to think these quantum corrections will vanish. In fact, we know that for the metric components of the translational collective coordinates it should not vanish since they would account for the covariance of the zero point energy in (11). Under a boost of momentum Π of the system with monopole energy M and zero point energy ΔM the energy must vary as

$$((M + \Delta M)^2 + \Pi^2)^{\frac{1}{2}} = M + \Delta M + \frac{\Pi^2}{2M} - \Pi^2 \frac{\Delta M}{2M^2} + \dots \tag{22}$$

The fraction in the last term in (22) should agree with the corrected translational components of metric in (21) as it happens in other models [5].

The above corrections will be absent in supersymmetric systems since all quantum corrections will be at loop level. Nevertheless, there will be higher order corrections which will include tree diagrams coming from the tadpole in (18), which is quadratic in

the collective momenta. Therefore, there will be an infinite set of corrections given in powers of the collective momenta and many of them (those contributing at tree level) will survive in supersymmetric systems, including those with $N = 4$ supersymmetry. The metric (21) can be written in matrix form as

$$g + gT(g)g \tag{23}$$

where g is the zeroth order metric of $\mathcal{O}(W_c^{-2})$, and T is a matrix of $\mathcal{O}(1)$.

In particular, these quantum corrections will correct the metric found by Witten [7] and Atiyah and Hitchin [8] in the case in which $k = 2$. Thus, the correction to the metric also corrects the harmonic 2-form found by Sen [9]. In this case, three coordinates are used to label the space position of the monopole solution, one for the charge. The remaining four label the internal coordinates. One will be the “radial” coordinate, the other three will be Euler angles. The metric of the internal coordinates at the classical level possess $O(3)$ invariance. The metric will not be diagonal. However, it is possible to find a transformation among the internal coordinates which will take the leading internal metric to a diagonal form

$$ds^2 = d^2(\zeta)d\zeta^2 + a^2(\zeta)\sigma_1^2 + b^2(\zeta)\sigma_2^2 + c^2(\zeta)\sigma_3^2 \tag{24}$$

The condition for $O(3)$ invariance is

$$d^2 - (abc)^2 = 0 \tag{25}$$

The correction to the metric will include terms which will also be off the diagonal and will include infinite contributions all at different orders of the collective momenta. It is easy to see that for arbitrary momenta the metric at the perturbative level will lose the $O(3)$ symmetry of the the internal metric of the 2-monopole solution. Using (23) and (24), a generic T with no nonvanishing components (since there are no reasons for their vanishing) we find the corrected metric. After making use of first order perturbation theory to transform the coordinates to new ones in which the corrected metric takes a

diagonal form we find that the the corrected diagonal metric will must satisfy the condition (25) which reduces to the vanishing of

$$d^2T_{11} - (a^2T_{22} + b^2T_{33} + c^2T_{44}). \quad (26)$$

Variations of (26) with respect to a, b, c which satisfy (25) must also vanish because such variations would arise due to collective coordinate redefinitions in (19) which will not lead to a loss of $O(3)$ invariance. Variation of (26) with respect to rescaling of the metric due to variations of the coupling constant must also vanish since $O(3)$ invariance will not be affected. This leads to a total of five equations for four unknowns and cannot be solved because the equations are linearly independent. $O(3)$ invariance is thus lost at the quantum level because the necessary conditions is not satisfied at the perturbative level. Similar analysis in higher orders of perturbation theory at tree level in systems with arbitrary supersymmetry also shows the loss of $O(3)$ invariance for systems with such symmetries.

Only a small subset, the Poincare collective coordinates and a $U(1)$ generator, the symmetry generators of the system will not affect the $O(3)$ invariance because their contributions can be taken into account with a redefinition of the fields [10] This cannot be done for the remaining collective coordinates since they generate transformations outside the group of Poincare and $U(1)$ transformations.

A possible explanation for this loss of symmetry is that the coupling of the zero modes to the physical modes via the constraint may lead to the lifting of flat directions in the potential of the collective coordinates at the quantum level. Perhaps, the dimension of the quantum moduli space is smaller than the dimension of the classical moduli space and only includes the collective coordinates of the Poincare group. however, this is only speculation. Further work in this direction will be reported elsewhere.

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