Microscopic Description of Alpha Decay to Intruder 0_2^+ States in Pb, Po, Hg, and Pt Isotopes

D. S. Delion and A. Florescu

Institute of Atomic Physics, Bucharest Magurele, POB MG 6, Romania

M. Huyse and J. Wauters

Leuven Isotope Separation On-Line, Instituut voor Kern- en Stralingsfysika, and Katholieke Universiteit Leuven, Celestijnenlaan 200 D, B-3001 Leuven, Belgium

P. Van Duppen and ISOLDE Collaboration

CERN, CH-1211 Geneva 23, Switzerland

A. Insolia

Department of Physics, University of Catania and Istituto Nazionale di Fisica Nucleare, I-95129 Catania, Italy

R. J. Liotta

Royal Institute of Technology at Frescati, S-10405 Stockholm, Sweden

(Received 20 June 1994; revised manuscript received 21 December 1994)

A microscopic description of the α decay to intruder 0_2^+ states of Pb isotopes in terms of proton-neutron pairing vibrations is performed. The calculated hindrance factors with respect to α transitions into the ground state are in good agreement with the available experimental data. Predictions for further measurements are given. Hindrance factors corresponding to the decay towards Po, Hg, and Pt isotopes are calculated within a simple model. The calculated quantities agree well with the experimental data, indicating that α decay is a powerful tool to precisely determine the transition from spherical to deformed shapes in the region around $Z = 82$.

PACS numbers: 23.60.+e, 21.10.Re, 27.80.+w

There has been an increasing interest in experimental and theoretical studies of intruder 0_2^+ states in nuclei near the $Z = 82$ proton shell closure, where these states were experimentally observed in γ spectroscopy [1,2] and α -decay experiments [3]. Recently the hindrance factors of the α -decay transitions to the 0_2^+ states in the neutron deficient isotopes $^{190,192,194}\text{Pb}$ relative to the corresponding ground state to ground state transitions were reported [4]. These hindrance factors decrease with the energies of the 0_2^+ states when diminishing the number of neutrons.

Several theoretical descriptions have been proposed [5–8] to explain the structure of the light lead isotopes, in particular of the 0_2^+ states, suggesting the conclusion that there is a coexistence of spherical and deformed degrees of freedom in those isotopes. Some states can be described within a pure spherical basis [9] showing that in these cases the spherical and deformed degrees of freedom are decoupled. But it has not been possible so far to explain microscopically the mechanism that decreases the energies of the state 0_2^+ with decreasing neutron number. This state starts as a pure spherical excitation in ^{208}Pb [5] to become eventually the head of a deformed band.

In this Letter we will describe the 0_2^+ states as neutron superconducting excitations [9] coupled to two-particle two-hole normal proton pairing vibrations [5]. That is, we assume that deformation is induced by the two-particle two-hole proton excitations.

To evaluate the hindrance factors in the α -decay processes mentioned above, we start by analyzing the

width corresponding to the decay

$$B \rightarrow A + \alpha, \quad (1)$$

which we will evaluate by assuming that the absolute decay width is [10,11]

$$\Gamma(R) = P(R)\gamma^2(R), \quad (2)$$

where $P(R)$ is the penetration factor through the Coulomb barrier at the distance R between the center of mass of the daughter nucleus and the emitted α cluster and the reduced width γ is

$$\gamma^2(R) = \frac{\hbar^2 R}{2M} |F(R)|^2, \quad (3)$$

where the formation amplitude $F(R)$ is, with standard notation,

$$F_L(R) = \int d\xi_\alpha d\xi_A d\Omega \Phi_B^*(\xi_B) \{\phi_\alpha(\xi_\alpha) \Phi_A(\xi_A) Y_L \times (\Omega)\}_{J_B M_B}. \quad (4)$$

Equation (2) can be applied only if the formation of the α particle is adequately described well outside the nuclear surface where the α -decay process can be considered as a two-body problem. This is, the theory can be considered valid if $\Gamma(R)$ does not depend strongly on R for distances outside the nuclear surface. We will use this property to check the soundness of the calculations, as usual [11].

The hindrance factor is defined as

$$F_H = \frac{|F(R; 0_1^+ \rightarrow 0_1^+)|^2}{|F(R; 0_1^+ \rightarrow 0_2^+)|^2}. \quad (5)$$

For the cases that will be considered in this Letter $L = J_B = 0$ and the integral in Eq. (4) can be evaluated in terms of a coherent superposition of harmonic oscillator wave functions [11].

We will describe the wave functions of the mother and daughter nuclei in terms of neutron BCS degrees

of freedom [9] and normal proton pairing vibrations [5]. Without the inclusion of neutron-proton interactions we found that the hindrance factors (5) are unsatisfactory.

In order to include the neutron-proton interaction we consider a general monopole Hamiltonian, i.e.,

$$H_{\pi\nu} = \sum_{j_k} (\epsilon_{\pi j_k} - \lambda_{\pi}) N_{\pi j_k} + \sum_{j_k} (\epsilon_{\nu j_l} - \lambda_{\nu}) N_{\nu j_l} - \frac{G_{\pi}}{4} \sum_{j_k j_{k'}} P_{\pi j_k}^{\dagger} P_{\pi j_{k'}} - \frac{G_{\nu}}{4} \sum_{j_l j_{l'}} P_{\nu j_l}^{\dagger} P_{\nu j_{l'}} - \frac{G_{\pi\nu}}{4} \sum_{j_k j_l} (P_{\pi j_k}^{\dagger} P_{\nu j_l} + P_{\nu j_l}^{\dagger} P_{\pi j_k}), \quad (6)$$

where λ_r is the Lagrange multiplier, $N_{\tau j_l}$ is the number operator, $P_{\pi j_k}^{\dagger} = \hat{j}_k (c_{\pi j_k}^{\dagger} c_{\pi j_k}^{\dagger})_0$ and G are the pairing strengths. We evaluated the BCS quasiparticle quantities and the corresponding quasirandom phase approximation (QRPA) creation operators by using the Hamiltonian Eq. (6). In the case of a normal system (e.g., ^{208}Pb) the BCS occupation probabilities U and V have sharp distributions and the system decouples into normal particle-particle, hole-hole, and particle-hole components [12]. In our case neutrons are moving in superconducting shells while protons are in normal states. To obtain normal proton pairing vibrations we neglect the terms of the form UV (particle-hole) in the case of neutron excitations. We have indeed found that these terms give negligible corrections to the α formation amplitude.

The random phase approximation (RPA) vacuum has the form [12]

$$|\text{RPA}\rangle = N \exp \left[\sum_{\tau=\pi,\nu} \sum_{j_m j_i} Z_{j_m j_i}^{\tau} B_{j_m}^{\dagger} B_{j_i} \right] Z_{j_m j_i}^{\tau} = \sum_k Y_{j_i}^k (X^k)_{j_m}^{-1},$$

where N is a normalization constant, X (Y) is the forward (backward) RPA amplitude, and $B_{\tau j_m}^{\dagger} = P_{\tau j_m}^{\dagger} / \hat{j}_m$. We use the letter m (i) to label particle (hole) states.

It is worthwhile to point out that the backward amplitudes Y are small numbers (in absolute value), as usual. With the wave functions thus calculated we evaluated the formation amplitudes [Eq. (6)]. Up to first order in XY one obtains for the $0_1^+ \rightarrow 0_1^+$ transition

$$\langle B | (c_{\pi j_m}^{\dagger} c_{\pi j_m}^{\dagger})_0 (c_{\nu j_n}^{\dagger} c_{\nu j_n}^{\dagger})_0 | A \rangle = N U_{\pi j_m}^A V_{\pi j_m}^B U_{\nu j_n}^A V_{\nu j_n}^B \left(1 - 2 \sum_{j_i} Z_{j_m j_i}^{\pi} - 2 \sum_{j_j} Z_{j_n j_j}^{\nu} \right), \quad (7)$$

while for the $0_1^+ \rightarrow 0_2^+$ transition one has

$$\begin{aligned} \langle B | (c_{\pi j_m}^{\dagger} c_{\pi j_m}^{\dagger})_0 (c_{\nu j_n}^{\dagger} c_{\nu j_n}^{\dagger})_0 \Gamma_{k_1}^{\dagger}(-2) \Gamma_{k_2}^{\dagger}(2) | A \rangle = & 2 N U_{\pi j_m}^A V_{\pi j_m}^B U_{\nu j_n}^A V_{\nu j_n}^B \\ & \times \left[\sum_{j_{m'}} X_{\pi j_{m'}}^{k_1} X_{\pi j_{m'}}^{k_2} + \sum_{j_{l'}} Y_{\pi j_{l'}}^{k_1} X_{\pi j_{m'}}^{k_2} - 2 \sum_{j_{m'} j_{l'}} X_{\pi j_{m'}}^{k_1} Y_{\pi j_{l'}}^{k_2} Z_{j_{m'} j_{l'}}^{\pi} \right. \\ & \left. + \sum_{j_{n'}} X_{\nu j_{n'}}^{k_1} X_{\nu j_{n'}}^{k_2} + \sum_{j_{j'}} Y_{\nu j_{j'}}^{k_1} X_{\nu j_{n'}}^{k_2} - 2 \sum_{j_{n'} j_{j'}} X_{\nu j_{n'}}^{k_1} Y_{\nu j_{j'}}^{k_2} Z_{j_{n'} j_{j'}}^{\nu} \right], \quad (8) \end{aligned}$$

where $\Gamma_{k_1}(-2) [\Gamma_{k_2}(2)]$ is the two-hole [two-particle] RPA proton-neutron phonon.

We found that the matrix Z has negative matrix elements, thus [according to Eq. (7)] increasing the formation amplitude and the corresponding decay probability to the ground states.

The energies of the states 0_2^+ decrease as the neutron number decrease, in agreement with available experimental data. As a result of this, the backward amplitudes Y and the overlap [Eq. (8)] increase. That is, the hindrance factor [Eq. (5)] decreases, also in agreement with experiment [4].

In our calculations we used for the mean field a Woods-Saxon potential with the so-called universal parametrization [13]. The Coulomb barrier was assumed to be spherical. Only two major shells around the Fermi surface were included in the calculations. We adjusted the

pairing strength $G_{\nu} = 0.1$ MeV to reproduce the experimental neutron gaps in all $^{190,192,194}\text{Pb}$ and $^{194,196,198}\text{Po}$ isotopes. In the same fashion, we adjusted $G_{\pi} = 0.16$ MeV in order to reproduce the experimental proton gaps in the Po isotopes. Finally, the strength $G_{\pi\nu} = 0.015$ MeV was determined by reproducing the F_H of the first of the three experimental decays in Fig. 1 (i.e., the one at 0.61 MeV). One then obtains the F_H as a function of the excitation energies of the 0_2^+ states for heavier Pb isotopes as shown in Fig. 1. The agreement with experiment is excellent, giving credibility to the predicted values for higher excitation energies, i.e., in the heavier lead isotopes. The relative decay widths calculated within this formalism also agree well with experiments, as expected [11]. Calculations performed within an extended basis, including up to six major shells above the Fermi surface, reproduce even the absolute decay widths, also as expected [11,14].

FIG. 1. Hindrance factor as a function of the energy of the states 0_2^+ in Pb isotopes. The experimental data (squares) are taken from Ref. [4]. The dashed line corresponds to values of F_H calculated without including the neutron-proton interaction.

Notice that this model includes only one free parameter, namely, $G_{\pi\nu}$.

The good agreement with experiment found above encourages us to treat even the other cases reported in Ref. [4], i.e., the α decay to the 0_2^+ intruder states of $^{196,198}\text{Po}$, $^{182,184}\text{Hg}$, and $^{176,178,180}\text{Pt}$. The striking feature in these cases is the sudden appearance of large hindrance factors as the neutron number decreases, as seen in Tables I and II. In fact, the hindrance factor corresponding to the decay of ^{200}Rn in Table I is the largest ever observed for a $\Delta I = 0$ transition. Previous studies [8,15] suggest that the ground and 0_2^+ states in all these nuclei are a combination of a pure neutron hole state and a pure neutron hole times two-particle two-hole proton state. For instance, without the neutron-proton interaction, which is the main agent to induce deformations, the states 0^+ in Po can be described as $|\text{Po(g.s.)}\rangle \approx |\pi^2\nu^{-n}\rangle$, where n is the number of neutron holes, and $|\text{Po}(0_2^+)\rangle \approx |\pi^4\pi^{-2}\nu^{-n}\rangle$. There is a large overlap between the neutron and proton wave functions in these states, since they occupy the same or similar orbitals. Thus the neutron-proton interaction, which brings down the excitation energies of the 0_2^+ states [8], becomes very effective and the core may rapidly become deformed. The main difficulty in describing this transition from spherical to deformed shape lies in the treatment of the correlations leading to the admixture between the two states 0^+ . We will just assume that the 0^+ wave functions are a combination of those two basis states. Using known experimental data in this region, we will try to find out the values of the admixture amplitudes that reproduce the experimental F_H . Our aim is to see whether there is a signature that clearly indicates the differences and similarities among those hindrance factors. As in Ref. [15], we will denote by $|u\rangle$ ($|\nu\rangle$) the uncorrelated wave function corresponding to

the state 0_1^+ (0_2^+) in the mother nucleus, while the corresponding wave functions in the daughter nucleus are $|x\rangle$ (0_1^+) and $|y\rangle$ (0_2^+). The correlated wave functions in the mother nucleus then are $|0_1^+\rangle_m = a|u\rangle + b|\nu\rangle$ and $|0_2^+\rangle_m = -b|u\rangle + a|\nu\rangle$, while in the daughter nucleus it is $|0_1^+\rangle_d = c|x\rangle + d|y\rangle$ and $|0_2^+\rangle_d = -d|x\rangle + c|y\rangle$, where the normalization conditions $a^2 + b^2 = c^2 + d^2 = 1$ are assumed. The hindrance factor in Eq. (5) then becomes

$$F_H = \left(\frac{d\langle 0_1^+ | T | 0_1^+ \rangle_m}{d\langle 0_2^+ | T | 0_1^+ \rangle_m} \right)^2 = \left(\frac{acT_1 + adT_2 + bcT_3 + bdT_4}{-adT_1 + acT_2 - bdT_3 + bcT_4} \right)^2, \quad (9)$$

where T_1 is the $0_1^+ \rightarrow 0_1^+$ uncorrelated transition matrix element, i.e., the matrix element corresponding to the transition $|u\rangle \rightarrow |x\rangle$. In the same fashion, T_2 corresponds to $0_1^+ \rightarrow 0_2^+$, T_3 to $0_2^+ \rightarrow 0_1^+$, and T_4 to $0_2^+ \rightarrow 0_2^+$. We have estimated the values of these matrix elements for the case of the decay of Po and found that $T_1 \approx T_2 \approx T_4$ while $T_3 \approx -\kappa T_1$ is smaller (i.e., $0 < \kappa < 1$). We assume that these relations are also valid for the decay of the Rn isotopes. For the proton-hole isotopes we found, instead, that $T_1 \approx T_3 \approx T_4$ and $T_2 \approx +\kappa T_1$, where again κ is a positive small number. The resulting expressions for the hindrance factors are very simple and can readily be obtained from Eq. (9). Within our previous calculation we estimated the values of κ to be 0.32, 0.17, and 0.10 for the decays of $^{194,196,198}\text{Po}$, respectively. With these and the amplitudes a and c extracted from our microscopic approach above, we obtained the hindrance factors shown in Table I by applying the very simple Eq. (9). The agreement between theory and experiment is excellent. Moreover, we have also checked that the calculated F_H are not strongly dependent upon reasonable variations of κ , a , and c , as can easily be verified by using Eq. (9).

Next we calculated the F_H corresponding to the decay of Rn isotopes. Here we used the same values of κ as above. The daughter nuclei are now the mother nuclei in the Po decays analyzed above. Therefore, the values of c in $^{200,202}\text{Rn}$ are the same as the values of a in $^{196,198}\text{Po}$, respectively, as seen in Table I. To calculate the corresponding values of a we fitted the F_H of ^{202}Rn and obtained $a = -0.89$ (this is the only free parameter used in the calculations of Tables I and II). Then we assumed

TABLE I. Calculated hindrance factor (F_H) in proton-particle nuclei. The number a (c) is the ground state wave function amplitude corresponding to the uncorrelated basis state 0_1^+ in the mother (daughter) nucleus. The component b (d) is given by the normalization condition.

Mother	Daughter	F_H ($F_{H,\text{expt}}$)	κ	a	c
^{194}Po	^{190}Pb	1.9 (1.1 ± 0.1)	0.32	0.90	0.91
^{196}Po	^{192}Pb	2.5 (2.5 ± 0.1)	0.17	0.94	0.92
^{198}Po	^{194}Pb	2.8 (2.8 ± 0.5)	0.10	0.98	0.94
^{200}Rn	^{196}Po	600 (79 ± 7)	0.10	-0.87	0.94
^{202}Rn	^{198}Po	19 (19 ± 6)	0.10	-0.89	0.98

TABLE II. Same as Table I for proton-hole nuclei. The values of c for the decay of Hg isotopes were taken from Ref. [16].

Mother	Daughter	F_H ($F_{H\text{-expt}}$)	κ	a	c
^{186}Pb	^{182}Hg	21 (21 ± 4)	0.32	0.86	0.74
^{188}Pb	^{184}Hg	21 (21 ± 3)	0.32	0.88	0.75
^{180}Hg	^{176}Pt	47 (17 ± 5)	0.32	0.67	0.92
^{182}Hg	^{178}Pt	11 (3.5 ± 0.6)	0.32	0.74	0.71
^{184}Hg	^{180}Pt	1.5 (2.4 ± 0.4)	0.32	0.75	0.62

that the dependence of a upon the neutron number was the same as in Po isotopes, again in agreement with previous experience [8], and therefore we used $a = -0.87$ for ^{200}Rn , obtaining a tremendous increase in the value of F_H , as seen in Table I. This is exactly what one sees experimentally. In fact, the F_H is now extremely sensitive to small variations of a , for instance, for $a = -0.81$ one gets $F_H = 110$ and for $a = -0.85$ one obtains $F_H = 5300$. For $a = -0.8519$ the value of F_H diverges. We are thus confronted with a typical phase transition phenomenon, where calculated properties are unstable to small variations of the initial conditions.

The proton-hole nuclei in Table II are light and therefore we used κ as in light lead isotopes. For the decay of $^{186,188}\text{Pb}$ we extrapolated the values of a from the values of c in the decay of Po in Table I. We also assumed that the structure of the 0^+ states in the two proton-hole Hg nuclei is the same as in the two proton-particle Po isotopes and obtained the values of c shown in Table II. The values of the F_H calculated by applying Eq. (9) is again in excellent agreement with the corresponding experimental values. For the decay of the Hg isotopes in Table II we took the values of c from Ref. [16] while the values of a are the same as the values of c in the Pb decay above (since the same Hg isotopes are mother in Hg decay and daughter in Pb decay). Therefore, there is not any free parameter in this case. The calculated F_H follow the same tendency as the experimental one, particularly the transition from ^{182}Hg to ^{180}Hg , but the agreement with experiment is not as striking as before. Again in this case we found that the calculated F_H is very sensitive to small variations in the wave function amplitude c . It is worthwhile to point out the values of a and c can be extracted from $E0$ transition probabilities [17] and from energy systematics of the high spin members of rotational-like bands [16]. But our conclusions regarding the sensitivity of the F_H as a function of the mass number are independent of the exact values of the parameters a , c , and κ . That is, one does not need to invoke large deformations to explain the hindrance factors in this region around $Z = 82$. This is in agreement with previous studies, where the dependence of α -decay rates upon deformation was found to be weak [18].

In conclusion, we have shown in this Letter that the α -decay hindrance factors recently reported [4] can be well reproduced by including two-particle two-hole

proton correlations and the neutron-proton interaction. We performed this task for the two-proton Po isotopes in the framework of a QRPA for neutrons and normal pairing vibrations for protons using a generalization of the standard monopole pairing interaction. For the other cases we described the states 0^+ by means of a very simple two-component wave function. We found that one can reproduce the experimental tendencies within this model. The calculations show that the analysis of hindrance factors in α -decay processes is a powerful tool to determine precisely the transition from a coexistent regime of spherical and deformed degrees of freedom, where the different configurations do not mix, to a regime where the nuclear ground state becomes deformed. This is consistent with the change in the ground state proton configuration in light Pt isotopes proposed in Ref. [8].

We would like to express our gratitude to J. Blomqvist for illuminating discussions.

-
- [1] P. Van Duppen, E. Coenen, K. Deneffe, M. Huyse, K. Heyde, and P. Van Isacker, Phys. Rev. Lett. **52**, 1974 (1984).
 - [2] J. Kantele, M. Luontama, W. Trzaska, R. Julin, A. Passoja, and K. Heyde, Phys. Lett. **171B**, 151 (1986).
 - [3] P. Van Duppen, E. Coenen, K. Deneffe, M. Huyse, and J.L. Wood, Phys. Lett. **154B**, 354 (1985).
 - [4] J. Wauters *et al.*, Phys. Rev. Lett. **72**, 1329 (1994), and private communication.
 - [5] P. Curutchet *et al.*, Phys. Lett. B **208**, 331 (1988).
 - [6] K. Heyde, J. Jolie, J. Moreau, J. Ryckebusch, M. Waroquier, P. Van Duppen, M. Huyse, and J.L. Wood, Nucl. Phys. A **466**, 189 (1987).
 - [7] V.G. Soloviev, Z. Phys. A **334**, 143 (1989); V.G. Soloviev, *Theory of Atomic Nuclei. Quasiparticles and Phonons* (Energoizdat, Moscow, 1989).
 - [8] J.L. Wood, K. Heyde, W. Nazarewicz, M. Huyse, and P. Van Duppen, Phys. Rep. **215**, 101 (1992).
 - [9] C. Pomar, J. Blomqvist, R.J. Liotta, and A. Insolia, Nucl. Phys. A **515**, 381 (1990).
 - [10] R.G. Thomas, Prog. Theor. Phys. **12**, 253 (1954).
 - [11] A. Insolia, P. Curutchet, R.J. Liotta, and D.S. Delion, Phys. Rev. C **44**, 545 (1991); D.S. Delion, A. Insolia, and R.J. Liotta, Phys. Rev. C **46**, 1346 (1992).
 - [12] P. Ring and P. Schuck, *The Nuclear Many-Body Problem* (Springer-Verlag, New York, 1980).
 - [13] R. Bengtsson, J. Dudek, W. Nazarewicz, and P. Olanders, Phys. Scr. **39**, 196 (1989); S. Cwiok, J. Dudek, W. Nazarewicz, J. Skalski, and T. Werner, Comput. Phys. Commun. **46**, 379 (1987).
 - [14] D. Delion, A. Insolia, and R.J. Liotta, Phys. Rev. C **46**, 884 (1992); **49**, 3024 (1994).
 - [15] J. Wauters *et al.*, Phys. Rev. C **50**, 2768 (1994).
 - [16] G.D. Dracoulis, A.E. Stuchbery, A.P. Byrne, A.R. Poletti, J. Gerl, and R.A. Bark, J. Phys. G **12**, L97 (1986).
 - [17] P. Dendooven *et al.*, Phys. Lett. B **226**, 27 (1989).
 - [18] J. Wauters *et al.*, Z. Phys. A **339**, 533 (1991).