Transformer Coils for High Magnetic Fields

by

L. Hoffmann and V. Scheuing

Geneva
Propriété littéraire et scientifique réservée pour tous les pays du monde. Ce document ne peut être reproduit ou traduit en tout ou en partie sans l'autorisation écrite du Directeur général du CERN, titulaire du droit d'auteur. Dans les cas appropriés, et s'il s'agit d'utiliser le document à des fins non commerciales, cette autorisation sera volontiers accordée.

Le CERN ne revendique pas la propriété des inventions brevetables et dessins ou modèles susceptibles de dépôt qui pourraient être décrits dans le présent document; ceux-ci peuvent être librement utilisés par les instituts de recherche, les industriels et autres intéressés. Cependant, le CERN se réserve le droit de s'opposer à toute revendication qu'un usager pourrait faire de la propriété scientifique ou industrielle de toute invention et tout dessin ou modèle décrits dans le présent document.

© Copyright CERN, Genève, 1963

Literary and scientific copyrights reserved in all countries of the world. This report, or any part of it, may not be reprinted or translated without written permission of the copyright holder, the Director-General of CERN. However, permission will be freely granted for appropriate non-commercial use. If any patentable invention or registrable design is described in the report, CERN makes no claim to property rights in it but offers it for the free use of research institutions, manufacturers and others. CERN, however, may oppose any attempt by a user to claim any proprietary or patent rights in such inventions or designs as may be described in the present document.
TRANSFORMER COILS FOR HIGH MAGNETIC FIELDS

by

L. Hoffmann and V. Scheuring

G E N E V A
# CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>I. INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>II. PRINCIPLE OF TRANSFORMER COILS</td>
<td>2</td>
</tr>
<tr>
<td>III. ELECTRICAL CHARACTERISTICS</td>
<td>3</td>
</tr>
<tr>
<td>IV. THE FLUX CONCENTRATOR EFFECT</td>
<td>7</td>
</tr>
<tr>
<td>V. DESIGN OF COILS AND EXPERIMENTAL RESULTS</td>
<td>12</td>
</tr>
<tr>
<td>ACKNOWLEDGEMENTS</td>
<td>16</td>
</tr>
<tr>
<td>REFERENCES</td>
<td>16</td>
</tr>
<tr>
<td>FIGURE CAPTIONS</td>
<td>17</td>
</tr>
</tbody>
</table>

* * *
Transformer Coils for High Magnetic Fields

by

L. Hoffmann and V. Scheuing

* * *

I. Introduction

Pulsed coils producing high magnetic fields over a time interval of a few milliseconds are in general of the Bitter type (single conductor discs joined to form a multiturn spiral). If the internal diameter of the turns or the current is increased beyond a certain limit, the forces become big enough to break the usual insulation materials after some thousands of pulses.

Coils of the Bitter type which we used\(^1\) to produce 200 kgauss over 7.2 cm diameter for emulsion experiments could stand about 3,000 pulses. However, in some points the force exceeded the pressure strength of the insulation of vetronite and araldite, which is of the order of 2 tons/cm\(^2\).

To reach longer lifetimes or higher fields we directed our attention to other types of coils. In particular we have investigated coils of the transformer type with a shaped single turnblock as secondary. The primary windings fit from the outside into a helical groove cut in the secondary cylinder. This has the advantage of reducing the mechanical forces on the primary. By shaping the secondary, high fields can be obtained due to flux concentration. These coils seem to be useful to produce fields higher than 200 kgauss over small volumes, and particularly in cases where sparking has to be avoided inside the useful volume as well as implosion of the coil, as for example, in the case when one wishes to use a hydrogen target inside a coil.

The principles of the construction of this type of coil will be reported here and the experimental results obtained on models will be discussed. In Sections II and III the attempt is made of introducing some
theoretical considerations on the principle of the transformer coils and some approximative calculations are made in order to interpret finally in Section V the experimental results. In Section IV the gain of the field due to flux-concentration and due to the special current distribution is discussed.

II. PRINCIPLE OF TRANSFORMER COILS

Different types of transformer coils have been already constructed in several laboratories\textsuperscript{2,3}. There is first the simplest approach by inserting a secondary slug in ordinary multiturn coils. Kim and Flatner\textsuperscript{2} have used a slotted secondary cylinder in a tape wound coil. Howland and Foner\textsuperscript{3} have improved this method using primary wires in a helical groove of the secondary. In this case a gain of strength on the primary can be obtained and also a better coupling of the primary flux to the secondary is achieved. Also for our models we used first parallel wires as primary winding, but finally a helical spiral fitting inside the slotted secondary cylinder (Figs. 1a and 1b).

As a consequence of the skin effect the current induced in the secondary cylinder has a path near the surface. In a simplified picture we can regard the secondary cylinder as consisting of a conducting sheet along the surface as indicated in Fig. 1b. By the change of the flux due to the primary current, the induced voltage at the ends of the outer circle B (Fig. 1b) is proportional to the change of the primary flux $\frac{\delta B}{\delta t}$, and the induced voltage at the ends of the inner circle A is proportional to $-\frac{\delta A}{\delta t}$. However $|\frac{\delta B}{\delta t}| > |\frac{\delta A}{\delta t}|$, therefore a current is flowing in the secondary, which is opposite to the direction of the primary current in the outside region, and in the same direction as the primary current in the inside sheet. The field due to the inner current is additive to the field produced by the primary current; the field due to the outer current is subtractive. In Fig. 2 the current and the field lines in the secondary are shown schematically.

This above consideration suggests that an optimum wall thickness of the cylinder exists. However, in reality this subject is more complicated,
especially by the fact that shaped secondary cylinders are used and the current density varies along the height of the cylinder. This will be discussed in more detail in Section IV.

The forces on the primary are reduced, if the outer current flows in the outermost parts of the secondary cylinder between the primary. Therefore the distance 'a' in Fig. 1a has to exceed the doubled skin depth δ. No axial or radial frame is needed if the secondary cylinder is strong enough to stand the forces acting on the slot and tending to expand the cylinder.

In all respects this type of coil becomes more advantageous as the skin depths are decreased. However, taking into account the fact that these coils should give a pulse length of the order of milliseconds, the frequency cannot be increased too much by the choice of inductance and capacity. It is helpful to use as secondary a material with low resistivity ρ as, for example, copper in order to reduce the skin depths.

III. ELECTRICAL CHARACTERISTICS

Calculations of the circuit characteristics can be based in a simple way on the equivalent representation of an imperfectly coupled transformer. However, this can be done only with assumptions discussed below. The exact description can be achieved by solving the system of two second-order differential equations.

1. Equivalent representation

Figure 3a shows schematically the transformer circuit as used by us. The electrical characteristics of the transformer coil are: self inductance $L_1$ and $L_2$, mutual inductance $M = k \sqrt{L_1 L_2}$ (k coupling constant), a.c. resistances $R_1$ and $R_2$.

$L_1$ and $R_1$ can be determined by replacing the secondary metal cylinder by an insulating material. Then the characteristics of the primary circuit can be measured on an oscillograph picture of the damped voltage
oscillation. The two characteristics are: the half period $T/2$ and the amplitude ratio $U/U_0$. From these quantities $L_1$ and $R_1$ can be calculated by the following relations:

$$\frac{U}{U_0} = e^{-\frac{R_1}{2L_1} \cdot \frac{T}{2}}; \quad (1a)$$

$$\frac{2\pi}{T} = \sqrt{\frac{1}{L_1} - \frac{R_1^2}{4L_2}} = \omega_1. \quad (1b)$$

To calculate the coupling between the primary and secondary circuit we make use of the equivalent circuit shown in Fig. 3b, with the inductance $L'$ and the resistance $R'$. This is a usual procedure for transformers; but this approach is exact only for an applied primary voltage of a sinusoidal shape and in a time interval after the time of the transient. We use this formulation because in our case the time of the transient is small compared to the rise time $T/4$ and the voltage follows the law of a damped oscillation given by the product of a sinus function and an exponential function. This method, as applied to our case, can further be justified by the agreement of the calculations with the measurements, for example, for the oscillation time $T$ and the primary current $i'$ as affected by the secondary circuit.

In these calculations the equivalent impedance $Z'$ is given by the sum of the impedance of the primary $Z_1$ and the coupled impedance:

$$Z' = Z_1 + \frac{\omega^2 L_2^2}{Z_2} = (R_1 + iX_1) + \left(\frac{\omega^2 L_2^2}{R_2 + iX_2}\right), \quad (2)$$

or

$$R' + iX' = \left(\frac{R_1 + R_2}{R_2^2 + X_2^2}\right) + i\left(\frac{X_1 - X_2}{R_2 + X_2^2}\right) \quad (3)$$

where $X = \omega L - 1/\omega C$ is the reactance of the circuits. From Eq. (3) it follows that

$$\frac{R_2}{X_2} = \frac{R' - R_1}{X_1 - X'}. \quad (4)$$
This ratio is given with the measured values \( R_1, R', L_1 \) and \( L' \). The result for our coils is always \( R_2 \ll X_2 \); therefore the following approximation for Eq. (3) can be made:

\[
X' = X_1 - \frac{\omega^2 L^2}{X_2} \quad (4a)
\]
and

\[
R' = R_1 + \frac{\omega^2 L^2}{X_2} \cdot R_2 \quad (4b)
\]

and using \( \mu = k \sqrt{L_1 \cdot L_2} \), one obtains for Eq. (4a):

\[
X_1 - X' = \omega L_1 \cdot k^2 \quad (5)
\]

\( X' \) and \( X_1 \) can be replaced by \( X' = \omega L' - 1/\omega C \) and \( X_1 = \omega L_1 - 1/\omega C \), which gives finally:

\[
L' = L_1(1-k^2) \text{ or } k^2 = 1 - \frac{L'}{L_1} \quad (6)
\]

From Eqs. (5) or (6) the coupling \( k \) between the primary and the secondary circuit can be calculated. With the assumption \( R_2 \ll L_2 \) the effect of the secondary is essentially to reduce the effective inductance in the primary. Therefore the oscillation time is also reduced to

\[
\frac{\omega}{\omega_1} \approx \sqrt{1-k^2} \quad (7)
\]
as follows from Eq. (6). The peak current is increased if the primary is coupled with the secondary and given by

\[
i' = U_0 \sqrt{\frac{C}{L'}} \cdot e^{\frac{R'}{2L'} \cdot \frac{\delta}{\omega}} \quad (8)
\]
with \( \tan \delta = 2\omega L'/R' \). The ratio of the current with and without secondary is, using \( R' \approx R_1 \),

\[
\frac{i'}{i_1} \approx \frac{1}{\sqrt{1-k^2}} \cdot e^{\left[ \frac{R_1}{2L_1 \cdot \omega_1} \cdot \left( \frac{\delta}{\delta_1 - (1-k^2)^{3/2}} \right) \right]} \quad (9)
\]
with \( \text{tg} \ \delta_1 = 2\omega_1 \frac{L_1}{R_1} \). The ratios given in Eqs. (7) and (9) as well as the current \( i' \) in Eq. (8) can also be measured.

2. Exact representation

The differential equations for the primary and secondary circuit are given by

\[
L_1 \ \ddot{q}_1 - M\dot{q}_2 + R_1 \ \dot{q}_1 + \frac{1}{C} q_1 = 0
\]

\[
L_2 \ \ddot{q}_2 - M\dot{q}_1 + R_2 \ \dot{q}_2 = 0
\]

with the initial conditions that for \( t = 0 \):

\( q_1 = q_0 = \text{total charge of the condenser bank} \);
\( \dot{q}_1 = 0 \) (\( \dot{q}_1 \) = current in the primary);
\( \dot{q}_2 = 0 \) (\( \dot{q}_2 \) = current in the secondary).

The system of two second-order equations can be reduced to one third-order equation:

\[
a \dddot{q}_1 + b\ddot{q}_1 + c\dot{q}_1 + dq_1 = 0
\]

with

\[
a = 1 - k^2,
\]

\[
b = \frac{R_1}{L_1} + \frac{R_2}{L_2},
\]

\[
c = \left( \frac{R_1}{L_1} \cdot \frac{R_2}{L_2} \right) - \frac{1}{L_1 C},
\]

\[
d = \left( \frac{R_2}{L_2} \cdot \frac{1}{L_1 C} \right).
\]

The solution is of the form \( A e^{-\lambda t} \), where \( \lambda \) satisfies the cubic equation

\[
a\lambda^3 + b\lambda^2 + c\lambda + d = 0.
\]

The solutions can be given as simple formulae only for special choices of \( L_1, R_1 \), etc., where some terms in the equations can be neglected. Unfortunately this is not the case for our coils. Therefore, we have to solve the equation for each coil separately, for the values \( L_1, R_1 \), etc.
In the case as given for all our coils one obtains one real and two complex conjugated $\lambda$'s. The general form of the currents is:

\[ i_1 = A_1 \cdot e^{-\alpha_1 t} + B_1 \cdot e^{-\beta_1 t} \cdot \sin(\omega t + \varphi_1) \]
\[ i_2 = A_2 \cdot e^{-\alpha_2 t} + B_2 \cdot e^{-\beta_2 t} \cdot \sin(\omega t + \varphi_2). \]

The integration constants can be calculated from the initial conditions.

IV. THE FLUX CONCENTRATOR EFFECT

The secondary cylinder of a transformer coil can easily be shaped as a flux concentrator\(^*\). We consider the effect of flux concentration especially for the case illustrated in Fig. 4. The secondary cylinder (Fig. 4a) is turned out from both sides (hatched pieces in Fig. 4b) so that a ring of the original diameter remains in the centre. The magnetic field is then increased in the centre of the coil.

We will first discuss the effect of flux concentration as given by the following picture: the flux entering the shaped secondary cylinder can be compressed and the secondary acts as a shield due to the "directed eddy current". Therefore the flux $\Phi$ inside the coil is constant and the following geometrical relations hold:

\[ \Phi = 2\pi \cdot r^2 \cdot B_r = 2\pi \cdot r_0^2 \cdot B_{r_0}; \]

where $B_r$ and $B_{r_0}$ are the axial fields in the centre of cylindrical coils of a variable radius $r$ and a fixed radius $r_0$, both penetrated by the same total flux $\Phi$ (for the geometrical notation see Fig. 1a). From this model one can derive in a simple way the relative gain $F$ of the field by flux concentration in the centre of the coil. We assume a constant current for the cases with and without flux concentrator, and furthermore

\(^*\) In the following we regard as flux concentrator the inner ring of the secondary (with the length $t$ in Fig. 1a) in contrast to some authors who define the total inserted slug as the flux concentrator.
a constant current density. The current is proportional to \( \partial B \, dl \)
and therefore the following equation can be derived:

\[
B_R L = \frac{B_{r_0}}{r_0} l + 2 \int_0^{l/2} \frac{B_{r_0}}{r_0} \frac{F_0}{(r_0 + kx)^2} \, dx + \frac{B_{r_0}}{r_0} \frac{F_0^2}{R} g,
\]

where \( B_R \) is the axial field in the centre of a cylindrical coil of the
radius \( R \). With the notation

\[
\frac{F_0}{R} = \rho, \quad \frac{l}{L} = \lambda, \quad \frac{B_{r_0}}{L} = \epsilon, \quad \frac{g}{L} = \gamma
\]

the result can be written in the following form:

\[
F = \frac{B_R}{B_{r_0}} = \frac{1}{\lambda + \epsilon + \gamma \rho},
\]

with \( \lambda + \epsilon + \gamma = 1 \), in the limit

\[
\lambda \to 0 \quad \frac{B_R}{B_{r_0}} = \frac{R^2}{r_0^2}.
\]

This result for the axial field in the centre is exact only for a long
coil \( (L \gg R, l \gg r) \). In our cases this is not fulfilled; it is better
to consider the ratio \( F' = B_{r_0} \cdot \cos \alpha_f / B_R \cdot \cos \alpha_r \), which is still not
exact but is a sufficient approximation for our purpose, provided \( \lambda \geq 0.2 \).

With

\[
\cos \alpha_f = \frac{\frac{l}{2}}{\sqrt{r_0^2 + \frac{l^2}{4}}},
\]

for the flux concentrator ring, and

\[
\cos \alpha_r = \frac{\frac{L}{2}}{\sqrt{r^2 + \frac{L^2}{4}}},
\]

for a cylinder with a radius \( r_0 \leq r \leq R \), one obtains:
\[ F' = \frac{\lambda}{\lambda + \varepsilon \rho + \rho} \cdot \sqrt{\frac{r^2 + \left( \frac{L}{2} \right)^2}{r_0^2 + \left( \frac{L}{2} \right)^2}}. \]  

(14)

\( F' \) and \( F'' \), the ratios of the field in the centre of the flux concentrator with respect to the field in a cylinder with a radius of \( r \) equal to \( R \) and \( r_0 \) are given as a function of \( \lambda \) in Fig. 5. The calculated values (for \( r_0 = 1.5 \text{ cm}, \rho = 2 \)) show the increase of \( F' \) with decreasing \( \lambda \) from \( \lambda = 1 \) up to \( \lambda = \frac{1}{2} \). For \( \lambda < \frac{1}{2} \) the assumption for the use of Eqs. 12 and 13 is no longer justified. For \( \lambda \approx 0.2 \) the calculated values of \( F' \) are lower than the values obtained from experiment. Therefore we now consider other effects contributing to a change of the field in the centre.

i) The current distribution is changed and no longer uniform. The higher current density in the flux concentrator ring favours the field in the centre.

ii) Due to the different current path the resistance and the inductance in the secondary are changed. The resistance of the secondary circuit is in general slightly increased. Besides the effect on the secondary current, the primary current is also influenced due to the coupling of primary and secondary circuits [e.g. Eq. (2)]. However, by increasing the primary current (raising the primary voltage) this effect can be compensated.

Far more important is the effect giving a field increase by the changed current distribution. The current near the ends of the shaped cylinder (Fig. 4b) is forced to flow in a larger circle. Therefore the length of the current path outside of the flux concentrator ring is increased and moreover the induced voltage \( (\Phi_B - \Phi_A) \) is decreased. The lower voltage in this part of the secondary as well as the larger resistance results in a relatively higher current-density in the middle part with respect to both ends of the cylinder. However, also the remaining part of the current flowing along a larger circle outside the flux concentrator ring contributes more to the axial field in the centre than the same current flowing on a smaller path at the same axial distance from the centre. This
is demonstrated in Fig. 6. Due to Biot-Savart's law the field vector element $h_a$ produced by a current element in the smaller circle (a) is larger, but the component in the $z$ direction $h_{az}$ contributes to the axial field less than that from the larger circle (b). From the expression for the axial field distribution

$$H_z(z, r = 0) \text{ proportional to } i \frac{r^2}{(r^2 + z^2)^{3/2}}$$  \hspace{1cm} (15)$$

one can derive the optimal radius in circles to maximize the axial field in the centre produced by a circular current path at $z = z_0$:

$$R_{opt} = \sqrt{2} \cdot z_0,$$

corresponding to an angle of about $55^\circ$ (Fig. 9).

In Fig. 7 the calculated field distributions along the middle axis ($z$) are plotted for linear current circles with the radii $R = 1$, $R = \sqrt{2}$, $R = 2$ cm. Figure 7 demonstrates again that for points far on the axis the large circles are more favourable. At $z - z_0 = 1$ cm the optimum radius is $R = \sqrt{2}$ cm in agreement with the calculation mentioned above.

Figure 8 shows the field distributions calculated as due to the cylindrical current sheets shown in Figs. 8a and 8b. In all four cases (Fig. 8) the same total current is assumed but different current densities. Therefore the area below the curves is always equal:

$$A = \int_{-\infty}^{+\infty} H_z(z) \, dz = i_{tot} = \text{constant}.$$  

The comparison between the curve 'a' and 'b1' in Fig. 8 shows the effect of the larger circle for the end parts of the slug; the comparison between 'b1', 'b2', and 'b3' shows the effect of the current concentration in the middle part of the ring.

In our coils the current flowing back along the outside of the secondary cylinder gives a negative contribution to the central field. We consider for this case a secondary cylinder without flux concentrator.
(Fig. 9) and assume uniform current distribution. For the current elements A and A' near the 55° cone in Fig. 9, the positive contribution from point A to the field in the centre and the negative contribution from point A' are about equal because they have about the same angular distance from the optimum 55° angle. For BB' the positive current element at point B contributes more to the centre field because it is nearer to the maximum angle of 55°. The opposite is true for the current elements at CC'. This means that inside and outside current together contribute inside the 55° cone (hatched in Fig. 9) negatively to the axial field of the primary in the centre. Therefore a long secondary cylinder without flux concentrator ring can result in a central field lower than the field of the primary alone. This is borne out by our experiments; an example is given in the next paragraph (Table 1, row "without flux concentrator").

### Table 1

<table>
<thead>
<tr>
<th>Flux concentrator length</th>
<th>Angle</th>
<th>Peak current</th>
<th>$B_{\text{max}}$</th>
<th>$B_{\text{max}}$</th>
<th>$T_2$</th>
<th>k</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
<td>(5)</td>
</tr>
<tr>
<td>6 cm</td>
<td>45°</td>
<td>21.5 kA</td>
<td>23.1 kG kV</td>
<td>1.07 kG kA</td>
<td>590 μs</td>
<td>0.5</td>
</tr>
<tr>
<td></td>
<td>55°</td>
<td>21.2</td>
<td>23.1</td>
<td>1.08</td>
<td>595</td>
<td>0.5</td>
</tr>
<tr>
<td>4 cm</td>
<td>35°</td>
<td>21.5</td>
<td>25</td>
<td>1.16</td>
<td>595</td>
<td>0.55</td>
</tr>
<tr>
<td></td>
<td>45°</td>
<td>21.6</td>
<td>26</td>
<td>1.21</td>
<td>610</td>
<td>0.6</td>
</tr>
<tr>
<td></td>
<td>55°</td>
<td>20.9</td>
<td>26.3</td>
<td>1.34</td>
<td>620</td>
<td>0.6</td>
</tr>
<tr>
<td></td>
<td>65°</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3 cm</td>
<td>45°</td>
<td>21</td>
<td>24.6</td>
<td>1.17</td>
<td>615</td>
<td>0.6</td>
</tr>
<tr>
<td></td>
<td>55°</td>
<td>20.8</td>
<td>28.1</td>
<td>1.40</td>
<td>620</td>
<td>0.6</td>
</tr>
<tr>
<td></td>
<td>65°</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>78°</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2 cm</td>
<td>45°</td>
<td>20.3</td>
<td>29</td>
<td>1.43</td>
<td>630</td>
<td>0.5</td>
</tr>
<tr>
<td></td>
<td>55°</td>
<td>20.1</td>
<td>28.6</td>
<td>1.42</td>
<td>640</td>
<td>0.5</td>
</tr>
<tr>
<td></td>
<td>65°</td>
<td>19.9</td>
<td>28.1</td>
<td>1.41</td>
<td>650</td>
<td>0.5</td>
</tr>
<tr>
<td>without flux concentrator</td>
<td></td>
<td>18.7</td>
<td>12.6</td>
<td>0.68</td>
<td>720</td>
<td>0.25</td>
</tr>
<tr>
<td>primary without secondary</td>
<td></td>
<td>18.8</td>
<td>14.1</td>
<td>0.75</td>
<td>730</td>
<td></td>
</tr>
</tbody>
</table>

---

* $C = 6250 \mu F.$

7162/NF/امگ
V. DESIGN OF COILS AND EXPERIMENTAL RESULTS

Several transformer coils with different characteristics have been constructed. The most important geometrical, mechanical and electrical properties of these coils are summarized in Tables 1 and 2.

Table 2

<table>
<thead>
<tr>
<th>Coil</th>
<th>Primary *) (section of winding)</th>
<th>Secondary</th>
<th>Geometrical characteristics (cm)</th>
<th>Electrical characteristics</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>$\frac{I_{\text{peak}}}{U_0}$</td>
<td>$\frac{B_{\text{max}}}{U_0}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>kA/kV</td>
<td>kG/kV</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$2(R+L)$</td>
<td>L</td>
</tr>
<tr>
<td>I a)</td>
<td>Copper wires (0.22 cm²)</td>
<td>Copper</td>
<td>10</td>
<td>12</td>
</tr>
<tr>
<td></td>
<td>(shaped)</td>
<td>cylinder</td>
<td></td>
<td></td>
</tr>
<tr>
<td>b)</td>
<td></td>
<td>shaped</td>
<td>3</td>
<td>6</td>
</tr>
<tr>
<td>II a)</td>
<td>Copper wires (0.12 cm²)</td>
<td>brass</td>
<td>12</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td>(cylinder)</td>
<td>cylinder</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>shaped</td>
<td>4</td>
<td>8</td>
</tr>
<tr>
<td></td>
<td></td>
<td>shaped</td>
<td>1</td>
<td>8</td>
</tr>
<tr>
<td>b)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>c)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>III</td>
<td>Copper wires (0.12 cm²)</td>
<td>Copper</td>
<td>10</td>
<td>7.5</td>
</tr>
<tr>
<td></td>
<td>(cylinder)</td>
<td>cylinder</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>shaped</td>
<td>1</td>
<td>7.5</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>I b)</td>
<td>Copper wires (0.22 cm²)</td>
<td>Copper</td>
<td>10</td>
<td>12</td>
</tr>
<tr>
<td></td>
<td>(cylinder)</td>
<td>cylinder</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>shaped</td>
<td>1</td>
<td>6</td>
</tr>
<tr>
<td>c)</td>
<td>Copper tube $\varnothing_{\text{ext}} 4 \text{ mm} \varnothing_{\text{int}} 2 \text{ mm}$</td>
<td>Copper</td>
<td>10</td>
<td>12</td>
</tr>
<tr>
<td></td>
<td>(shaped)</td>
<td>cylinder</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>shaped</td>
<td>1</td>
<td>6</td>
</tr>
<tr>
<td>IV a)</td>
<td>Copper tube $\varnothing_{\text{ext}} 5 \text{ mm} \varnothing_{\text{int}} 3 \text{ mm}$</td>
<td>Copper</td>
<td>12</td>
<td>12</td>
</tr>
<tr>
<td></td>
<td>(aluminium spiral) (0.55 cm²)</td>
<td>cylinder</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>shaped</td>
<td>4</td>
<td>6</td>
</tr>
<tr>
<td>b)</td>
<td>Aluminium spiral (1.7 cm²)</td>
<td>Copper</td>
<td>15</td>
<td>12</td>
</tr>
<tr>
<td></td>
<td>(shaped)</td>
<td>cylinder</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>shaped</td>
<td>15</td>
<td>12</td>
</tr>
</tbody>
</table>

*) All primaries were fitting in a groove of about 1.2 cm depth, except for coil V. In this case the groove was 3.5 cm deep.

7162/NI/mag
The primaries of a part of these coils have been made from cords of twisted copper wire (1 mm Ø), the number of wires varying between 8 and 28. In a few cases copper tubes were used as primaries. Although fields of 100 kgauss could be reached with these models of primaries consisting of wires or tubes in general they were used only for studying field distributions, flux concentrator efficiency, etc. The coils made especially for the production of high fields have been constructed with a primary made of one helix turned from a metal cylinder (Fig. 1a, left side). Both types of primaries fit into a groove on the outside surface of the secondary cylinder. The material used for secondary and primary was copper, aluminium and in one case brass. Copper has the advantage of low electrical resistance and small skin depth. When aluminium was used the insulation between the primary and the secondary could be made of a thin aluminium oxide layer of about 50 μ thickness, produced by anodic oxidation on the surface of the aluminium. The insulation strength of the aluminium oxide was 2 - 3 kV if primary and secondary were oxidized. Vetronite covering the groove was additionally used for all materials of the secondary. Primary and secondary were poured together with resin. Table 1 shows some properties of three models of transformer coils, which have been constructed to investigate the effect of flux concentration. The secondaries of these coils consisted of aluminium, the primaries of copper wires. The geometrical dimensions were (Fig. 1a): L = 12 cm; R = 3.5 cm; r_o = 1.5 cm; c = 1 cm.

The flux concentrator length l and the angle α (Fig. 10) have been varied as indicated in the first column of Table 1. In Fig. 10 the field distributions along the central axis are shown for three coil shapes; in all cases is l = 4 cm, but the angles are different. The second column of Table 1 gives the peak current normalized for voltage as calculated from the amplitude ratio U/U_0 of the voltage and the half period T/2 (column 5); the current decreases slowly with decreasing flux-concentrator length due to the change of the electrical characteristics. Columns 3 and 4 show the field strengths as measured by the Hall effect and normalized for voltage and current respectively. The central field as a function of the angle is plotted in Fig. 11. In this figure it is shown that steep angles α (Fig. 10) are more favourable for relatively long flux concentrators, but flat angles

---

*) The distance between primary windings and secondary is kept small as well as the width of the slot in the secondary cylinder in order to reduce energy losses.
are more favourable for short flux concentrators. For a flux-concentrator length equal to the inner diameter the optimum angle is about 70° for our choice of the geometrical dimensions. Column 6 of Table 1 gives the coupling constant, as calculated by formula (6).

Figure 12 shows the axial field distribution for \( r = \text{const} = 0 \) and \( r = \text{const} = 1 \text{ cm} \). As indicated by Fig. 12 there exists in the centre of the coil a small increase of the field by approaching the wall of the flux concentrator ring. In Fig. 13 the measured ratio of the radial field component to the axial field component is shown for different points near to the wall of a secondary cylinder, but outside the flux concentrator ring. The radial component never reaches zero for these points and already increases 1 cm before the end of the coil as well as for points approaching the flux concentrator. This suggests that the field lines in the schema of Fig. 2 are idealized and field lines are "leaking" at the end of the cylinder. However, a "leakage" of field lines through the flux concentrator cannot be seen in Fig. 13. The steep increase of the radial component for points near the flux concentrator indicates that the "leakage" through the flux concentrator is small.

Table 2 shows the characteristics of various coils with different geometrical and electrical properties. The geometrical dimensions as well as the material of the coils are indicated. If a shaped flux concentrator was used, the angle \( \alpha \) of the flux concentrator for the coils listed in Table 2 was always 45°. The primaries were varied but the number of windings for all primaries was \( n = 1/\text{cm} \). The electrical characteristics are given in the same order as in Table 1, but for two different values of the capacity.

The main conclusions which can be drawn from the results in Tables 1 and 2 are summarized as follows.

1) The primary field and the coupling \( k \) has to be maximized in order to optimize the field in the secondary. The a.c. resistance of \( R_1 \) should be chosen as small as possible; for example see case IV a and b in Table 2.
ii) The field in the centre can be increased using a shaped flux concentrator. The increase exceeds slightly the value $F'$ given by formula (14) for reasons mentioned in Section IV. This is demonstrated in Figs. 14 and 15 and by comparison of the values of the maximum field of coil I (Fig. 15) with the calculated values of $F'$ in Fig. 5.

iii) The optimum field in the centre depends on the length $l$ and the angle $\alpha$ of the flux concentrator, where $\alpha$ is correlated to $l$ and $r_0$ (Table 1 and Fig. 11).

iv) For decreasing frequencies (corresponding to a range of the capacity between $C = 12,500$ and $75,000 \ \mu F$) the ratio $B/I$ is nearly constant (Table 2, case I), which shows that the field is not sensitive to the current path changed by the skin depth in this range. However, the ratio $B/U_0$ increases slower than $\sqrt{T}$. This is due to the damping factor

$$e^{-\frac{R'}{2L'} \cdot \frac{T}{2\pi} \delta}$$

in Eq. (8) for the peak current. $R'$ is decreased proportionally to $\sqrt{T}$, but $T$ is increased proportionally to $\sqrt{T}$ and also $\delta$ is increased. Therefore current and field increase slower than $\sqrt{T}$ (Fig. 16).

Finally it should be remarked that the advantage of transformer coils is the fact that high magnetic fields of milliseconds time duration can be produced with lower forces on the multiturn primary compared to the ordinary multiturn coils. The secondary cylinder can be constructed with sufficient strength in order to avoid the use of a frame. Therefore a long lifetime of these coils can be expected. On the other hand the transformer coil is evidently less efficient if one is primarily interested in the efficiency of power consumption. The transformer coil can be regarded as a compromise between the multiturn coil and a single turn cylinder combining to a certain extent both advantages of these coils, that is, field pulses of long duration and high coil strength.
ACKNOWLEDGEMENTS

We would like to express our thanks to Mr. A. Cyvoct and Mr. R. Lorenzi for their assistance and continuous help with the measurements and the construction of the coils. We thank Mr. J. Burtcher who assisted with some of the calculations and Mr. K. Roberts and the SC workshop for the construction of models.

We enjoyed the interest of our colleagues of the Emulsion Group and of Mr. M. Morpurgo.

* * *

REFERENCES


* * *
FIGURE CAPTIONS

Fig. 1a  Section through a transformer coil with a shaped secondary. The primary windings are indicated on the left side as a helix screwed in the groove of the secondary, on the right side as wires wound in the groove.

Fig. 1b  Top view of the secondary of the coil shown in Fig. 1a. The arrows indicate the current path at the ends of the secondary cylinder.

Fig. 2  Schematic representation of the current and field lines in a secondary of a transformer coil.

Fig. 3a  Principle circuit for a transformer coil as used in an oscillation circuit with capacity C.

Fig. 3b  Equivalent representation of the circuit shown in Fig. 3a.

Fig. 4a  Secondary of a transformer coil with a cylindrical slug.

Fig. 4b  Secondary of a transformer coil with a slug shaped as flux concentrator.

Fig. 5  The gain of axial field strength in the centre of a shaped transformer coil calculated as a function of the relative flux concentrator length λ. F refers to an infinitely long coil. F_y and F_z represent the gain of field strength with respect to a cylindrical coil with the radii R and r_c respectively. The values L, R and r_c are chosen for the geometry of the secondary of coils I and IV (Table 2).

Fig. 6  Elements of the field vectors and their axial components as produced by a current element of the circles a and b.

Fig. 7  Calculated field distributions of three current circles with radii R = 1, \(\sqrt{2}\) and 2 cm.

Fig. 8  Calculated axial field distributions produced by the cylindrical current sheets shown in Figs. 8a and 8b. The current densities correspond for:

'a':  \(j_0 = 1\)  (Fig. 8a)
'b_1': \( j_1 : j_0 = 1 : 1 \)  
'\text{b}_2': \( j_1 : j_0 = 2 : 1 \)  
'\text{b}_3': \( j_1 : j_0 = 7 : 1 \)  
(Fig 8b)

In all four cases the total current is \( i_{\text{tot}} = \int_{-\infty}^{+\infty} H_z \, dz = \text{const.} \)

Fig. 9 The contribution of currents of different paths and different directions to the field in the centre. The direction of the current in the circle through A, B and C is opposite to the direction of the current in the circle through A', B' and C'. The field contribution of both current circles together inside the 55° cone (C, C') is opposite to the contribution of B, B' outside the 55° cone and is compensated for A, A'.

Fig. 10 Measured axial field distributions for transformer coils with flux concentrators of various angles, but equal length.

Fig. 11 Measured peak fields (in the centre of the coil) as a function of flux concentrator angle. The parameter V indicates the ratio of the length to the diameter of the flux concentrator.

Fig. 12 Measured axial field distribution at radial distances \( r = 0 \) and \( r = 1 \) cm.

Fig. 13 Ratio of the radial to the axial field component as a function of the distance \( z \) for \( r = \text{const.} \). The radial distance corresponds to 1 cm from the inner cylinder surface. The parameter \( \ell \) indicates the different lengths of the flux concentrator.

Fig. 14 Measured axial field distribution for coil II (Table 2) for different lengths \( \ell \) of the flux concentrator.

Fig. 15 Measured axial field distribution for coil I (Table 2) with flux concentrator length \( \ell = 1 \) and 3 cm, and without flux concentrator.

Fig. 16 Measured field increase as function of the capacity for coil I (Table 2).

* * *
Fig. 2

leakage induct.

\[ M = k\sqrt{L_1 L_2} \]

\[ (k-1)L_1 \quad (k-1)L_2 \]

Fig. 3a

Fig. 3b
Fig. 4a

Fig. 4b

\[ \frac{B_{r0}}{B_r} \]

\[ F \]

\[ F_1' \text{ for } r=R \]
\[ F_2' \text{ for } r=r_0 \]

\[ F_1' \]

\[ F_2' \]

\[ \lambda \]

Fig. 5
Fig. 10
Fig. 11

\[ V = \frac{\text{length of flux concentrator}}{\text{diameter of flux concentrator}} \]

- Field of the primary
- Field without flux concentrator