E L E M E N T A R Y _ P A R T I C L E _ P H Y S I C S

LECTURE COURSES GIVEN AT CERN IN 1961

by:
B.T. Feld

G E N E V A
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G E N E V A
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What is a "fundamental" particle?

In the absence of a theory of the so-called elementary or fundamental particles - i.e., a simple, closed set of assumptions from which it is possible to infer their existence and to compute their properties - it is necessary to have at least a working definition of the term. By what criteria, for instance, do we label the proton and neutron as fundamental and the hydrogen atom or helium nucleus (\( q \)-particle) as composite particles? We shall see that the answer to this question depends, to some extent, on the theoretical approach which we may adopt. However, we begin by applying two, not unrelated, criteria.

1) A fundamental particle is relatively stable: here the crux is the word "relatively". The proton, for instance, appears to be absolutely stable *) against the decays

\[ p \rightarrow e^+ + \gamma \text{ or } e^+ + \nu + \bar{\nu} \]  \hspace{1cm} (1.1)

which are permitted by all the conservation laws except the law of conservation of protons, which must be postulated to prevent reactions (1.1)

Free neutrons, on the other hand, undergo $\beta$-decay

$$n \rightarrow p + e^- + \bar{\nu}$$

(1.2)

with a half-life of about 12 minutes; this is so long that free neutrons may be regarded as essentially stable *) . The problem, clearly, is to make a sharp distinction **) between "unstable" and "metastable", a task which we shall attempt in Chapter 4.

2) A fundamental particle cannot be disintegrated. We refer here to endoergic reactions rather than to exoergic decays, which were covered by the preceding criterion. Thus, from the reaction

$$H + 13.7eV \rightarrow p + e^-$$

(1.4)

we conclude that, if anything, it is $p$ and $e^-$ which may be fundamental,

*) The distinction between neutrons and protons implied here is completely artificial, depending on a small, accidental, difference in mass. Actually, inside nuclear matter, where the energy and momentum balance may be maintained by the rest of the particles, reaction (1.2) and

$$p \rightarrow n + e^+ + \nu$$

(1.3)

are completely symmetrical. Note the conservation of nucleons, however, which still forbids reactions (1.1).

**) We emphasize that such distinctions are strongly theory-dependent. Thus, in the absence of a reliable atomic theory, it might well appear that "ortho-helium" and "para-helium" are different atomic species; we know that they are two states of the same species between which, however, electromagnetic transition are highly forbidden. It is not unlikely, and indeed there are growing indications, that some of the so-called fundamental particles are different states of the same species, in the sense of the two heliums, but with the transitions between them "forbidden" in terms of normal transitions involving the meson fields.
but certainly not $H^*$). Superficially, the possible reactions on nucleons

$$N + 137 \text{ MeV} \rightarrow N^* \pi^-$$  (1.5)

($N$ stands for nucleon; there are two types, proton, $p$, or neutron, $n$; $\pi^-$ is pi-meson) bear some resemblance to reaction (1.4); but since the same nucleon appears on both sides, we are clearly dealing with a creation rather than with a disintegration process. Indeed, such reactions are not unfamiliar. We may describe the process of bremsstrahlung (photon creation) by an electron as

$$e^- \text{ kinetic energy} \rightarrow e^- \gamma$$  (1.6)

It is this apparent analogy between creation and absorption processes for the electromagnetic and the meson fields which is the basis of the Yukawa approach to the nuclear forces.

However, the situation is not so simple as indicated by a comparison between reactions (1.4) and (1.5). During the process of creating (or materializing) a meson, it may also be possible to transform one "elementary" particle into another. Thus, we have the high-energy reaction

$$N + 910 \text{ MeV} \rightarrow \Lambda + K^-$$  (1.7)

in which a nucleon is transformed into a $\Lambda$-particle through the creation of a $K$-meson. We believe reaction (1.7) to be more closely analogous to (1.5) than to (1.4), but this again involves a theoretical concept of the meson field. Indeed, it is not yet possible, on the basis of any known information, to make a definite choice between the idea that both $N$ and $\Lambda$ are fundamental, and the hypothesis that $N$ is a compound system ($\Lambda + K$) or, indeed, that $\Lambda$ is a compound system ($N + K$). Such ambiguities may, in fact, be inherent in the concept of "elementary particle".

*) It helps, here, that we have a theory of the hydrogen atom, but it is not essential to the argument.
In the following, we attempt to define the particles and fields which comprise the fundamental particles and their interactions. Our model, as noted, is the quantum theory of fields, first developed by Dirac, Heisenberg and Pauli for electromagnetic phenomena, extended for the first time to include matter fields by Fermi in the theory of $\beta$-decay (weak interactions), and brought to fruition by Yukawa in the meson theory of nuclear forces (strong interactions).
Listing and classification

The name for a fundamental particle is usually derived from that property which was most conspicuous in its discovery, or which was considered its most distinctive feature at the time of discovery. Thus, absence of charge is reflected in the name neutron; the neutrino is distinguished, in its name, from the neutron by its smaller mass. Likewise, it is the mass which is reflected in the name meson - intermediate (in mass) between the electron and the nucleon.

But a given particle is a sum total of many properties. Accordingly, the name, which singles out one of these, is frequently misleading. Thus, among the mesons, the $\mu$-meson is of a species entirely different from the others, being much more similar to the electron in all properties except for mass.

Charge and mass are not the only particle properties. A particle has spin (intrinsic angular momentum), magnetic and electric moments, (the number depending on the magnitude of its spin), intrinsic parity, and additional distinguishing features. In fact, one of the problems before us is to define a complete set of attributes in terms of which the different particles may be codified.

*) Indeed, we henceforth reserve the name meson, to be defined in the following, to the $\pi$, $K$, and other (as yet undiscovered) like particles. We shall refer to the $\mu$ as the muon, and avoid the term meson in connection with this particle; it is actually a lepton (a name reserved for the light particles associated with each other, and with nucleons, through the $\beta$-decay interaction).

**) Not all moments are possible, owing to certain invariance rules. For example, provided the laws of nature are invariant with respect to "time reversal", there can be no static electric dipole moments.
Such properties are generally referred to in terms of "quantum numbers", or "eigenvalues", or "constants of the motion". Precisely, these have the following meaning: we consider a system in the state of total momentum zero (barycentric, or centre-of-momentum system). Let \( \psi \) be the wave function for this system. Then an operator \( \Omega \) defines a quantum number \( o \) if

\[
0 \psi = o \psi. \tag{2.1}
\]

Common examples of such operators are the Hamiltonian (total energy) which, when operating on the wave function for a system in its ground state and when applied in the barycentric system, yields the rest-mass \( m \),

\[
H \psi = mc^2 \psi, \tag{2.2}
\]

and the angular momentum operator which, in the same circumstance, yields the intrinsic spin \( S \),

\[
J^2 \psi = S(S+1)\hbar^2 \psi. \tag{2.3}
\]

Other constants of the motion are defined in similar fashion, although the corresponding operators may be rather less familiar. Thus, the isotopic spin operators are appropriate to groups of particles which have, except for electromagnetic effects, identical properties and interactions. For the description of such groups we may define an isotopic spin and its three components (of which the third is directly related to the electrical charge); these operators stand in the same relationship to each other as the angular momentum operator to its components. Other quantum numbers and their interpretation will be discussed as the need arises.

In general a constant of the motion reflects some fundamental invariance principle. Thus, the homogeneity of space leads to the conservation of energy momentum (invariance against translation), of angular momentum (against rotation), and of parity (against reflection), although
the last is known not to be a property of all the fundamental interactions.

The general symmetry principles which lead to the other invariances are not quite so familiar or well-understood. The invariance of "natural laws" with respect to reversal of the time direction *) leads to dynamical effects which are generally described under the "principle of detailed balancing". Another type of possible invariance is with respect to the process of "charge conjugation", in which all particles are changed to their anti-particles. The consequences of such invariances are mainly dynamical with many striking examples, in the decays and interactions of the fundamental particles, which will be discussed in due course. But, as in the case of parity (space reflection invariance), charge conjugation invariance is not a universal property, also being violated in the weak (β-decay) interactions. In this instance, however, there seems to be a more general invariance which involves redefinition of the operation of reflection: if a mirror is defined as a device which changes all particles to antiparticles (charge conjugation) at the same time that it inverts all space co-ordinates (space reflection), then all known processes appear to be invariant with respect to this generalized type of reflection.

The β-decay situation represents a special case of a more general invariance property. This is described by the "TCP Theorem" **) which requires invariance with respect to the simultaneous combined operations of

*) Classically, this is equivalent to maintaining all positions fixed and reversing all velocity directions; simple classical systems are known to be exactly reversible under this operation, but the behaviour of quantum mechanical systems is, in this respect, more complex.

time-reversal, charge-conjugation, and space-reflection (parity). The TCP theorem is a consequence of invariance with respect to the transformations represented by the proper Lorentz group, and can be shown to apply to essentially all theories of possible physical interest. The TCP theorem, together with the above-described "reflection" invariance of the weak interactions, implies the invariance of all interactions with respect to time-reversal, a statement which appears still to be experimentally tenable after rather close scrutiny in recent years.

To return to the classification of the fundamental particles: the problem is to define the complete set of properties (constants, quantum numbers) which together serve for their identification. The existence of such constants is, it is conjectured, a consequence of the universal symmetries and invariance properties of natural laws, some of which have been discussed above. The main task of a theory of the fundamental particles as it would now appear, is to uncover the general symmetry and invariance properties which would account for those conservation principles now known or awaiting discovery.

A convenient way of classifying the known particles is in terms of their spins or, alternatively and equivalently, the statistics which they obey. Particles of half-integral spin are fermions, while those of integral spin are bosons. Fermions of spin $1/2$ (which is the spin of all the fundamental fermions whose spins are measured) are described by the Dirac equation, or a suitable modification thereof, which has a consequence that their creation or disappearance is always in a particle, anti-particle pair. Accordingly, one is immediately led to the law of conservation of fermions—the total number of fermions minus the total number of anti-fermions in an

*) The conservation of angular momentum alone requires their creation or destruction in even numbers.
isolated system is a constant. The law, as actually observed, is even more restrictive: there are two types of fermions - leptons and baryons *) - and the conservation law appears to hold separately for each type.

Bosons, having integral spin, are subject to no such restrictions with respect to their creation or destruction; it is this feature which gives rise to their simple characteristics as the carriers of a field (interaction). We divide the bosons into two types **) - photons and mesons, a distinction which is determined primarily by the interactions in which are involved: photons are the carriers of the electromagnetic field; mesons of the nuclear (strong) interactions. Other important differences arise out of the zero-rest-mass and spin 1 of the photon.

In Table 2.1 are listed the fundamental particles known at the time of writing, together with those of their properties which are believed to be most useful in distinguishing them. As indicated above, the particles are listed under four classifications: two types of bosons and two types of fermions. In addition to the listing of constants, a number of remarks are appropriate:

*) Leptons, as previously noted, are light fermions which interact with each other (and with baryons) through the weak $\beta$-decay interactions. Baryons are nucleonic-type fermions which interact strongly with each other through the mesonic fields. All particles interact via the electromagnetic and gravitational fields.

**) We omit discussion, here, of a possible "graviton", the carrier of the gravitational field, only because this ultra-weak interaction is so little understood.
A.1. Photons ($\gamma$): the photon, the quantum of the electromagnetic field, has zero rest-mass and spin 1. Owing to the zero rest-mass, with the consequence that it travels with the speed of light in all co-ordinate systems, its spin is always directed either along or opposite to its direction of motion. That is, of the three possible orientations of a spin 1, only two are available to the photon; this arises from the invariance of the state of polarization with respect to the Lorentz frame in which it is viewed *). A photon can carry different amounts of orbital angular momentum, which determines the multipolarity of the radiation, but the component of its angular momentum in the direction of its motion is always $\pm \hbar$.

A.2. Mesons: those may be defined as the carriers of the strong interaction fields. Two types have been observed, $\pi$ and $K$. However, the K-meson has the feature that it carries another property, "strangeness" or "hypercharge". This gives rise, among other properties, to a difference between $K$ and $\bar{K}$ (anti-$K$), which is a significant feature of the Yukawa interactions involving K-mesons. Thus, at low energies we have for K-mesons only the elastic scattering reaction,

$$K + N \rightarrow K' + N'.$$  \hspace{1cm} (2.4)

For $\bar{K}$, on the other hand, we have additional absorptive reactions **),

$$\bar{K} + N \rightarrow \Lambda + \pi$$  \hspace{1cm} (2.5)

without violating any conservation laws.

*) E.P. Wigner, Physics Today 1957.

**) Y stands for "hyperon", which includes the $\Lambda$ - and $\Sigma$ -particles, both of which are baryons.
B.1. Leptons: the light fermions all have spin $1/2$ and are described by the free particle Dirac equation, except for quite small corrections arising out of their interactions with the electromagnetic field. They are coupled to the other fundamental particles through the $\beta$-decay interactions, which are so weak as to have no observable influence on their static properties.

One lepton, the neutrino, having zero rest-mass, exhibits the same property as the photon with regard to its direction of polarisation. Such a particle has the possibility of also exhibiting "chirality" i.e., of having its spin orientation uniquely tied to its direction of motion - since no Lorentz transformation could alter this state of affairs. Such a situation would clearly violate the "law" of invariance of systems with respect to space reflection (parity), since the mirror image would reverse the orientation of the spin with respect to the direction of motion. Obnoxious as this situation may once have appeared, it turns out to be the case for the weak interactions. However, as noted in the preceding, if we redefine a mirror as that object which not only inverts space but also changes particles to anti-particles (i.e. $\gamma \rightarrow \bar{\nu}$, $e^- \rightarrow e^+$, $\mu^- \rightarrow \mu^+$), the weak interactions are invariant with respect to such reflection.

Another feature of the lepton's decays (weak interactions) is the law of conservation of leptons. It appears that we may classify the leptons into two groups: the leptons $\ell$, comprising $e^-$, $\gamma^-$, $\mu^-$, and the anti-leptons $\bar{\ell}$, $e^+$, $\gamma^+$, $\mu^+$. In all interactions involving the creation or destruction of leptons, they are created or disappear in pairs, with the total number of $\ell$ minus the number of $\bar{\ell}$ remaining constant. Part of this is, of course, associated with the conservation of charge, a property of the electromagnetic interaction; part comes from the conservation of angular momentum, i.e., it is impossible to create an odd number of particles of spin $1/2$ having zero or integral total angular momentum. But the concept of conservation of leptons becomes meaningful only as a result of the property of chirality of the neutrino, which makes it possible to distinguish,
in unambiguous fashion, between \( \gamma \) and \( \bar{\gamma} \), through the direction of orientation of their spins. We return to this problem in the discussion of the weak interactions.

**B 2. Baryons:** in addition to the weakly interacting (and electromagnetically interacting, when charged) leptons, there is another class of spin 1/2 particles, including the nucleons and other, more-recently-discovered particles known as hyperons. As previously mentioned, there appears to be a law of conservation of nucleons - but this must be generalized to include the hyperons, i.e., all baryons, as a result of observation of a reaction of the form

\[
\gamma + N \rightarrow Y + K \quad (\text{strong}), \quad (2.6)
\]

\[
Y \rightarrow N + \bar{K} \quad (\text{weak}). \quad (2.7)
\]

Owing to the strong interaction of baryons with the meson fields, the properties of the baryons are rather far removed from those expected on the basis of the Dirac equation for "free" particles. Nevertheless, one important feature of the Dirac equation pertains; namely, the existence of anti-particles. For each baryon listed in Table 2.1 there is an anti-baryon whose properties, except for a change in sign of the charge and "strangeness", are the same as for the baryon.

A main feature of the baryons is their involvement in the strong Yukawa reactions,

\[
B \leftrightarrow B\,\gamma + \text{meson}, \quad (2.8)
\]

the discussion of which will occupy much of the rest of this work.
### Table 2.1
Properties of the fundamental particles

#### A. BOSONS

1. Photon: spin 1, zero rest-mass, no charge
2. Mesons: spin 0

<table>
<thead>
<tr>
<th>Type</th>
<th>Particle</th>
<th>Mass (MeV)</th>
<th>Parity</th>
<th>I-spin (T)</th>
<th>Strangeness (S)</th>
<th>Mean-life (sec)</th>
<th>Decay modes</th>
</tr>
</thead>
<tbody>
<tr>
<td>pion</td>
<td>$\pi^0$</td>
<td>135.0</td>
<td>-</td>
<td>1</td>
<td>0</td>
<td>$2 \times 10^{-16}$</td>
<td>$2\gamma, \gamma + e^+ e^-, 2e^+ 2e^-$</td>
</tr>
<tr>
<td></td>
<td>$\pi^\pm$</td>
<td>139.6</td>
<td></td>
<td></td>
<td></td>
<td>$2.55 \times 10^{-8}$</td>
<td>$\mu + \nu, e + \nu$</td>
</tr>
<tr>
<td>kaon</td>
<td>$K^0, \bar{K}^0$</td>
<td>497.8</td>
<td>(-?)</td>
<td>1/2</td>
<td>$\pm 1$</td>
<td>$\begin{cases} (\alpha^0) 1.00 \times 10^{-10} \ (\alpha^2) 6 \times 10^{-8} \end{cases}$</td>
<td>$2\pi$</td>
</tr>
<tr>
<td></td>
<td>$K^\pm$</td>
<td>493.9</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$3\pi, \mu + \nu + \pi, e + \nu + \pi$</td>
</tr>
</tbody>
</table>

#### B. Fermions

3. Baryons: spin 1/2

<table>
<thead>
<tr>
<th>Type</th>
<th>Particle</th>
<th>Mass (MeV)</th>
<th>Parity</th>
<th>I-spin (T)</th>
<th>Strangeness (S)</th>
<th>Mean-life (sec)</th>
<th>Decay modes</th>
</tr>
</thead>
<tbody>
<tr>
<td>nucleon, N</td>
<td>$p^+$</td>
<td>938.2</td>
<td>(+)</td>
<td>1/2</td>
<td>0</td>
<td>stable</td>
<td>$p + e^- + \bar{\nu}$</td>
</tr>
<tr>
<td></td>
<td>$n^0$</td>
<td>939.5</td>
<td></td>
<td></td>
<td></td>
<td>$1.1 \times 10^3$</td>
<td></td>
</tr>
<tr>
<td>hyperon, Y</td>
<td>$\Lambda^0$</td>
<td>1115.4</td>
<td>(+)</td>
<td>0</td>
<td>$-1$</td>
<td>$2.5 \times 10^{-10}$</td>
<td>$N + \pi$</td>
</tr>
<tr>
<td></td>
<td>$\Sigma^+$</td>
<td>1189.4</td>
<td></td>
<td></td>
<td></td>
<td>$0.61 \times 10^{-10}$</td>
<td>$N + \pi$</td>
</tr>
<tr>
<td></td>
<td>$\Sigma^0$</td>
<td>1191.5</td>
<td>?</td>
<td>1</td>
<td>$-1$</td>
<td>$&lt; 10^{-11}$</td>
<td>$\Lambda^0 + \gamma$</td>
</tr>
<tr>
<td></td>
<td>$\Sigma^-$</td>
<td>1196.0</td>
<td></td>
<td></td>
<td></td>
<td>$1.61 \times 10^{-10}$</td>
<td>$\pi + \pi^-$</td>
</tr>
<tr>
<td>cascade</td>
<td>$\Xi^-$</td>
<td>1319.4</td>
<td>?</td>
<td>1/2</td>
<td>$-2$</td>
<td>$1.3 \times 10^{-10}$</td>
<td>$\Lambda^0 + \pi^-$</td>
</tr>
<tr>
<td></td>
<td>$\Xi^0$ ($\sim \Xi^1$)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$&lt; 10^{-10}$</td>
<td>$\Lambda^0 + \pi^0$</td>
</tr>
</tbody>
</table>

4. Leptons: $\ell$: spin 1/2

<table>
<thead>
<tr>
<th>Type</th>
<th>Particle</th>
<th>Mass (MeV)</th>
<th>Parity</th>
<th>Mean-life</th>
<th>Decay modes</th>
</tr>
</thead>
<tbody>
<tr>
<td>neutrino</td>
<td>$\nu, \bar{\nu}$</td>
<td>0</td>
<td></td>
<td>stable</td>
<td></td>
</tr>
<tr>
<td>electron</td>
<td>$e^-, e^+$</td>
<td>0.511</td>
<td></td>
<td>stable</td>
<td></td>
</tr>
<tr>
<td>muon</td>
<td>$\mu^-, \mu^+$</td>
<td>105.66</td>
<td></td>
<td>$2.212 \times 10^{-6}$</td>
<td>$e + \nu + \bar{\nu}$</td>
</tr>
</tbody>
</table>
TABLE 2.1
CAPTIONS

a) Values are of \( mc^2 \) in MeV. Unless otherwise stated we adopt, throughout this paper, a system of units in which \( c=1 \).

b) Parities of all particles are relative to the nucleons and \( \Lambda^0 \), all three assumed even (positive) by convention.

c) If more than one mode is possible, we indicate the decay types only. All decays conserve charge and "lepton number" as defined in footnote e.

d) Only the particles are listed. Anti-baryons are obtained by reversing the signs of the charge, parity, and strangeness quantum numbers; the rest of the properties are the same as those listed for the respective baryons.

e) Particle (\( \ell \)) and anti-particle (\( \bar{\ell} \)) are listed, respectively, first and second. Conservation of leptons (in decays) requires that the total number of leptons minus anti-leptons remains constant.

f) Note that the muon is here classified as a lepton, which it is, rather than a meson, which it is in name (and mass) only; we prefer to reserve the term meson for the bosons of finite rest-mass involved in the strong interactions.
Properties of the $\pi$-meson field

We are interested in the main feature of the Yukawa reaction

$$N \Rightarrow N + \pi$$

as described by the Yukawa equation for a pseudoscalar pion (gradient coupling)

$$\left( \Box^2 - k^2 \right) \varphi = -4\pi \frac{f}{k} \tau_3 \vec{\nabla} \cdot \rho_N (\vec{r}, \vec{s}, t_3)$$

$\varphi$, the "amplitude" of the meson field is regarded as a vector in the "isotopic spin space".

a) Static solution

$$\nabla^2 \varphi - k^2 \varphi = -4\pi \frac{f}{k} \tau_3 Q (\vec{r}, \vec{s}, t_3)$$

$$\varphi = \frac{\pm}{k} \frac{f}{t_3} \int \frac{e^{-k |\vec{r} - \vec{r}'|}}{|\vec{r} - \vec{r}'|} \vec{\nabla}' \rho_N (\vec{r}', t_3) d\vec{r}'$$

$$= \frac{\pm}{k} \frac{f}{t_3} \int \vec{\nabla}' e^{-k |\vec{r} - \vec{r}'|} d\vec{r}'$$

$$\Rightarrow \frac{\pm}{k} \frac{f}{t_3} \vec{\nabla} e^{-k r} \chi_N (\vec{s}, t_3)$$

for a point nucleon.
The spatial dependence of $\frac{Y}{r}$ is given by
\[
\frac{d}{dr} \left( \frac{e^{-Kr}}{r^2} \right) = - (1 + Kr) \frac{e^{-Kr}}{r^2} \equiv - Kr^2 Y'(Kr)
\]
We note that for $Kr \gg 1$, 
\[
Y'(Kr) \rightarrow \frac{e^{-Kr}}{Kr} = \frac{Y}{Kr}
\]
The conventional Yukawa function. For small Kr, the divergence is sharper than for $Y(Kr)$. The distance at which the amplitude falls off rapidly is known as the "range"
\[
R \sim K^{-1}.
\]
b) The free-wave solution: consider the plane wave solution in free space of the Yukawa equation
\[
\left( \Box^2 - K^2 \right) \psi = -\left( \nabla^2 - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} - K^2 \right) \psi = 0
\]
\[
\psi = e^{i \left( \vec{K} \cdot \vec{r} - \omega t \right)}
\]
\[
-K^2 + \frac{\omega^2}{c^2} = 0
\]
\[
K^2 + K^2 = \frac{\omega^2}{c^2}
\]
Now using the De Broglie relationships

\[ \hbar \mathbf{k} = p \]
\[ \hbar \omega = E \]

gives

\[ p^2 + \hbar^2 k^2 = \frac{E^2}{c^2} \]

leads to the association of \( k \) with a rest mass

\[ \hbar^2 k^2 = \mu^2 c^2 \]
\[ k = \frac{\mu c}{\hbar} \]

\( k^{-1} \) is the Compton wavelength of the carrier of the meson field, a meson of mass \( \mu \).

c) **Meaning of \( f \) and energy units:** treating \( \mathcal{L} \) in part a) as the strength of the field, the interaction energy with a second nucleon is given by

\[ \mathcal{L} \cdot \mathcal{L} = \left( \frac{e}{\hbar} \right)^2 \left( t_{\mathbf{1}} t_{\mathbf{2}} \right) \left( \bar{\mathbf{t}} \cdot \mathbf{v} \right) \left( \bar{\mathbf{t}} \cdot \mathbf{v} \right) \frac{e^{-kr}}{r} \]
\[ = f^2 k \left( t_{\mathbf{1}} t_{\mathbf{2}} \right) \left( \bar{t} \cdot \mathbf{v} \right) \left( \bar{t} \cdot \mathbf{v} \right) \frac{e^{-kr}}{kr} \]
\[ = \left( \frac{e^2}{\mu c} \right) \mu c^2 \sqrt[2]{(t, s, l, r)} \]
Note that the unit of energy is $\mu^2$ and the dimensionless strength constant (analogous to the electromagnetic fine-structure constant $\alpha = \frac{e^2}{\hbar c} = \frac{1}{137}$) is $(f^2/\mu)$. 

d) **Angular dependence of pion field**: suppose the source is a nucleon of spin up $\frac{\hbar}{2}$ or spin down $\frac{-\hbar}{2}$. The pion field becomes

$$\vec{\sigma} \cdot \vec{\nabla} \begin{pmatrix} \alpha \\ \beta \end{pmatrix} \frac{e^{-kr}}{r}$$

(We treat both possibilities simultaneously)

$$\frac{\vec{\nabla}}{r} = \sum_{\pm} \frac{1}{2} (\sigma_{\pm} \vec{\nabla}_{\pm} + \sigma_{0} \vec{\nabla}_{0})$$

$$\sigma_{\pm} = \sigma_{x} \pm \sigma_{y}$$

$$\nabla_{\pm} = \frac{\partial}{\partial x} \pm \frac{\partial}{\partial y}$$

then

$$\vec{\sigma} \cdot \vec{\nabla} \begin{pmatrix} \alpha \\ \beta \end{pmatrix} \frac{e^{-kr}}{r} = \begin{pmatrix} -\beta \sin \theta \cos \phi \Delta \alpha - \alpha \cos \theta \\ -\alpha \sin \theta \cos \phi \Delta \beta + \beta \cos \theta \end{pmatrix} k^2 Y_{(Kr)}$$

$$= \begin{pmatrix} \sqrt{\frac{2}{3}} \beta Y_0^0(\theta, \phi) & \sqrt{\frac{1}{3}} \alpha Y_1^1(\theta, \phi) \\ -\sqrt{\frac{1}{3}} \alpha Y_1^-1(\theta, \phi) + \sqrt{\frac{2}{3}} \beta Y_0^0(\theta, \phi) \end{pmatrix} k^2 Y_{(Kr)}$$

$$\chi = \sum_k \chi_k \begin{pmatrix} \sqrt{\frac{2}{3}} \beta Y_1^1(\theta, \phi) \\ -\sqrt{\frac{1}{3}} \alpha Y_1^-1(\theta, \phi) + \sqrt{\frac{2}{3}} \beta Y_0^0(\theta, \phi) \end{pmatrix} Y_{(Kr)}$$
Thus, the angular dependence is that of a spin 1/2 nucleon combined with \( l=1 \) pion into the \( p_{1/2} \) state.

e) **Charge distribution of the pion field:** let

\[
\phi^* = \sqrt{\frac{1}{2}} \left( \psi_x + i \psi_y \right)
\]

create a \( \pi^+ \)

\[
\phi = \sqrt{\frac{1}{2}} \left( \psi_x - i \psi_y \right)
\]

create a \( \pi^- \)

\[\Phi_3\]

create a \( \pi^0 \)

Now, we may project out the pion charge

\[
\Phi \Phi^* = \int K \left\{ \begin{array}{c} \Upsilon' (Kr) \times \mathcal{S}_n \end{array} \right\}
\]

\([p]_h\) stands for an initial proton or neutron source.

\[
\begin{aligned}
t \cdot \phi &= \sqrt{2} \zeta_+ \Phi + \sqrt{2} \zeta_- \Phi^* + \zeta_3 \Phi_3 \\
\left\{ \begin{array}{c} [p]_h \end{array} \right\} &= \left\{ \begin{array}{c} \frac{-\sqrt{2}}{2} \pi_0 + p \pi^0 \\\n\frac{\sqrt{2}}{2} \pi_0 - h \pi^0 \end{array} \right\}
\end{aligned}
\]

Thus the \((N+\pi)\) charge state is that expected for the combination of a \( t=1/2 \) (nucleon) with a \( t=1 \) (pion) to give \( t=1/2 \) (physical nucleon).
Emission and absorption of mesons

In the preceding chapter we have considered some features of the virtual processes which follow from Yukawa's field-theoretic description of the nuclear interactions and which, in turn, determine the static properties of nucleons and mesons. We also noted that the time-dependent free space solutions of the field equation correspond to particles of finite rest-mass - i.e., mesons. In this chapter we shall examine, still qualitatively, some features of the emission and absorption of free mesons by nucleonic sources.

The meson field equations of Yukawa are based on a close analogy to electromagnetic theory. We may extend this analogy to radiation processes. To do this, we review briefly the emission of electromagnetic radiation by an excited system of charges. Consider an oscillating electric dipole of frequency \( \omega \) and amplitude \( p_0 = eR \). The classical rate of energy loss is

\[
- \frac{dW}{dt} = \frac{2}{3} \frac{e^2}{c^3} R^2 \omega^4
\]

(4.1)

This expression applies also to the quantum description of the electromagnetic field, in which the radiation is emitted in quanta of energy \( \hbar \omega \). The number of photons radiated per second is

\[
\lambda \frac{dW}{d\omega} = \frac{2}{3} \frac{e^2}{\hbar \omega} \left( \frac{c}{R} \right)^3 \omega^3
\]

(4.2)

An understanding of the origins of the factors in Eq. (4.2) will enable us to write down, by analogy, the corresponding expression for the rate of emission of mesons by an excited nucleonic source. Thus, \( \frac{e^2}{\hbar c} \) represents
the strength of the electromagnetic interaction, the probability of finding a photon in the field; \( R/c = \tau \) is a characteristic unit of time for the system, the time required for the (compound) system of charge and photon to separate in the absence of inhibiting factors. Finally, \( (vR/c)^3 = (R/\chi)^3 \) is a kinematical factor, which arises in part from the dependence of the final state phase-space density on the momentum \( \frac{p}{c} \) of the emitted photon and in part from the "angular momentum barrier" associated with the (dipole) emission of photons with \( \ell = 1 \). In general, for the emission of photons of angular momentum \( \ell \) (multipolarity = \( 2^\ell \)), the kinematical factor becomes \( (R/\chi)^{2\ell+1} \) (aside from numerical coefficients and assuming \( R \ll \chi \)).

If we consider the decay, through meson emission, of an excited nucleonic system

\[
N^* \longrightarrow N + \pi^0
\]  
(4.3)

each of the above factors has its mesonic analogue, and we may write for the rate of meson emission (neglecting numerical factors of order unity)

\[
\frac{6}{\hbar c} \langle \hbar c \rangle \zeta(\eta) v_\ell(\eta)
\]  
(4.4)

in which we have taken, for the size of the system, \( R = \frac{1}{\chi} \ll \hbar/mc \);

\[
\eta = R/\chi = p/mc
\]  
(4.5)

and the characteristic lifetime is

\[
\tau_0 = \langle \hbar c \rangle^{-1} = \frac{\hbar mc^2}{2} \sim 5 \times 10^{-24} \text{sec}.
\]  
(4.6)

We have, in Eq. (4.4), separated the kinematical term into a phase-space factor \( \zeta(\eta) \) and a barrier penetration factor \( v_\ell(\eta) \). The latter has the well-known asymptotic properties

\[\text{*) Blatt and Weisskopf, Theoretical Nuclear Physics.}\]  
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\[ v_1(\eta \ll 1) \sim \eta^{21/12} \ldots (21-1) \eta^2 \]  
(4.7a)

\[ v_1(\eta \gg 1) \sim 1. \]  
(4.7b)

Thus, if we consider an excited nucleonic system which emits a pion of \( \eta \sim 1 \) (\( \xi \gg 1 \)) in a state of \( l=1 \) (\( v_1 \approx 1/2 \)), then a coupling constant of order 1 leads to a decay mean life of \( \sim 10^{-23} \) sec. This value is quite commensurate with the observations on the "resonance" in the pion-nucleon interaction, whose "width" \( \pi \approx \frac{1}{2} \xi \) is \( \sim 500 \text{MeV} \approx mc^2/3 \).

Now, the same factors which determine the decay rate of an excited nucleon, Eq. (4.4) also governs the cross-section for the inverse pion absorption reaction

\[ \pi + N \rightarrow N^*. \]  
(4.8)

Although the computation of the rate of reaction (4.8) requires a knowledge of the properties of the excited nucleonic state \(^*\), we may obtain a rough estimate from the following argument: the rate of reaction (4.3) is related to the cross-section for its inverse (4.8) through

\[ \lambda = \frac{n v}{\tau_c} \]  
(4.9a)

in which \( n \) is the density of pions in the excited (compound) state \( N^* \), \( v \) is their velocity, and \( \tau_c \) is the cross-section for reaction (4.8) with, however, the condition that the incident pions are in the same state in which they are emitted in reaction (4.3). For the density of pions, we may write

\[ n \sim \frac{3}{4 \pi} \left( \frac{\xi}{v} \right)^3 \]  
(4.9b)

with which, combining Eqs. (4.9) with Eq. (4.4), we obtain

\[ \tau_c \sim \left( \frac{\xi}{2mc} \right)^2 \left( \frac{\eta}{\xi} \right) \gamma \left( \frac{\eta}{\xi} \right) \pi \left( \frac{h}{mc} \right)^2 \]  
(4.10a)

*) Blatt and Weisskopf, Theoretical Nuclear Physics
For an incident plane-wave, only a fraction (also dependent on \( \eta \)) of the beam is in the appropriate angular momentum states, and the actual absorption cross-section contains additional, energy-dependent factors. Appropriate expressions for the cross-section will be given and discussed subsequently; for the purposes of the present qualitative considerations we may write

\[
\mathcal{O} \sim \left( \frac{g^2}{\hbar \alpha} \right) F(\eta) \cdot \mathcal{I} \left( \frac{\hbar}{mc} \right)^2 \tag{4.10b}
\]

in which the factor \( F(\eta) \) is of the order of unity for \( \eta \gg 1 \). Thus, with \( (g^2/\hbar \alpha) \sim 1 \), \( F(\eta) \sim 1 \), we expect pion absorption cross-sections of order

\[
\mathcal{O}_o = \mathcal{I} \left( \frac{\hbar}{mc} \right)^2 \sim 60 \text{mb}. \tag{4.11}
\]

This is, indeed, the order of magnitude of observed pion absorption cross-sections at high energies (although perhaps too large by a factor of 2-5).

Thus we see that the Yukawa picture of the pion field associated with nucleons provides a reasonable qualitative description of the emission and absorption of pions. It then becomes of great interest and importance to investigate whether similar considerations may be applied with comparable success to processes involving other baryons and mesons (Table 2.1).

We consider first the K-mesons: assuming a Yukawa reaction of the form

\[
B \leftrightarrow B' + K, \tag{4.12}
\]

we may use an approach similar to the preceding in discussing the virtual and real processes involving K-mesons. Since the range associated with the K-meson field is small compared to that of the pions

\[
R_K \sim \frac{\hbar}{m_K} = 0.26 \frac{\hbar}{mc} = 0.39 \times 10^{-13} \text{ cm}, \tag{4.13}
\]
we expect the K-meson field to have an important effect only on reactions which penetrate relatively deeply into the nucleon — i.e., reactions involving relatively high energies. However, at sufficiently high energies the k-meson field should be excited, and the emission of k-mesons from highly excited nucleon systems should be characterized by rates of the order

$$\lambda_K = \left(\frac{\sigma_K}{m_0}\right)^2 \sum V(\eta_K) \approx 10^{24} \left(\frac{\sigma_K}{m_0}\right)^2 \sum v_K \text{sec}^{-1}. \quad (4.14)$$

We may estimate $\lambda_K$ from the observation that, for excitation of $\sim 1$ GeV, the cross-section for kaon production is $\sim 1/10$ as much as for pion production

$$\frac{\lambda_K}{\lambda_{\pi}} = \left(\frac{\sigma_K}{\sigma_{\pi}}\right)^2 \left(\frac{m_K}{m_\pi}\right)^2 \sum \frac{v_K}{v_\pi} \ll \frac{1}{10} \quad (4.15)$$

from which we conclude that the coupling constants of the kaon and pion fields cannot differ by many orders of magnitude.

So far, the kaon field seems to exhibit properties very similar to those of the pion field. However, serious difficulty is encountered when we attempt to understand the decay of the K-meson in terms of the Yukawa reactions. Since $m_K = 5.5 m_\pi$, we would expect to observe the decays

$$K \rightarrow 2 \pi \text{ or } 3 \pi \quad (4.16a)$$

as a result of the Yukawa reactions

$$k \rightarrow B + 3 \pi \rightarrow 2 \pi \text{ or } 3 \pi. \quad (4.16b)$$

Such decays are, indeed, observed; but the meanlives are $\sim 10^{-10} - 10^{-8}$ sec, while we would expect

$$\frac{\sigma_{\pi}}{\sigma_K} \approx \left(\frac{\sigma_K}{m_0}\right)^2 \left(\frac{\sigma_{\pi}}{m_0}\right)^2 \sum v_{\pi} \approx \left(\frac{\sigma_K}{m_0}\right)^2 \times 10^{23} \text{sec}^{-1}. \quad (4.17)$$
Thus to account for the K-meson decay rate, we require \((\frac{e^2}{\hbar c}) \sim 10^{-13}\) or less, which is completely incommensurate with the observations on K-meson production!

Other difficulties are encountered in attempting to understand the kaon by analogy with the pion. Thus, although kaons in all three charge states are observed, there is a very large charge asymmetry in their production; while \(K^+\)-mesons are produced with cross-sections commensurate with the strong Yukawa reaction (4.12), \(K^-\)-mesons are produced in far lesser abundance (by many orders of magnitude) in pion-nucleon and nucleon-nucleon reactions in the \(\sim 1\) GeV energy range. Such an asymmetry cannot be understood if the kaon is assumed to be a normal isotopic triplet.

Similar difficulties are encountered in attempting to understand the production and decay of the hyperons (\(\Lambda\) and \(\Sigma\), see Table 2.1). Since hyperons (say the \(\Lambda^0\)) are produced by the reaction *)

\[
\overline{K} + N \rightarrow \Lambda^0
\]

(4.18)

and decay by the inverse

\[
\Lambda^0 \rightarrow N + \overline{K}
\]

(4.19)

it is perhaps natural to assume that the \(\Lambda^0\) represents an excited nucleon state \((N^*)\), to which the previous discussion applies. For pion energies of \(\sim 1\) GeV \((\eta \sim 5)\), reaction (4.18) has a cross-section of \(\sim 0.5\) mb.

Then applying \(k_d\). (4.10b) we obtain from the production

\[
\left(\frac{e^2}{\hbar c}\right) F(\eta \sim 5) \sim 10^{-2}.
\]

(4.20a)

*) We omit from the right-hand side a second particle, presumably a meson, required to conserve momentum.
The decay mean life, on the other hand, is \(10^{-10}\) sec. Using Eq. (4.4) with \(\eta = 0.74, Q_{\Lambda} = 37\text{MeV}\)

\[
\sqrt{\frac{\eta}{2}} v_1(0.8) \approx 10^{-13},
\]

(Eq. 4.20b)

Eqs. (4.20a) and (4.20b) are incompatible unless \(v_1(0.74) \ll P(5)\). This is not inconceivable for sufficiently large \(l\). Thus for \(l=7\), Eq. (4.7a) gives \(v_1(0.74) = 0.8 \times 10^{12}\).

However, although one might attempt to explain the large discrepancy between the rates of production and decay of the \(\Lambda^{0}\) on the basis of the assumption of a large spin-value for the \(\Lambda^{0}\), this explanation does not hold up when subjected to closer scrutiny. Thus, studies of the properties of the \(\Lambda^{0}\) production reactions (to be discussed in a later section) indicate very strongly that the \(\Lambda^{0}\) has spin 1/2. Furthermore, the same explanation, when applied to the slow decay rate of the \(\Sigma\)-hyperons (meanlives also \(\sim 10^{-10}\) sec) requires still large \(l\)-values since the \(\Sigma\)-decay has a \(Q\)-value of \(\sim 115\) MeV, \(\eta = 1.4\). But the factor \(P(\eta)\) also contains \(v_1\) and \(v_1(5) \ll 1\) for the \(l\)-values required; it then becomes very difficult to understand the observation that the cross-section for production of \(\Sigma\)'s in the 1 GeV region is essentially the same as for the \(\Lambda^{0}\). Furthermore, the evidence is that the \(\Sigma\)'s also have spin 1/2.

The explanation of the paradox encountered above must therefore be sought in other directions. The nature of the solution was suggested by Pais (*); it lies in the principle of "associated production". In essence, this explanation requires that the new particles (K-mesons and hyperons) be produced in pairs only, through a strong Yukawa reaction of the form

\[
N \rightarrow Y + K
\]

(Eq. 4.21)

(Y stands for \(\Lambda^{0}\) or \(\Sigma\)); there are simply no strong interactions like

(4.16b), (4.18) and (4.19). The decays of the individual particles, however, violate the selection rules which govern reactions (4.21) and, therefore, cannot proceed through any of the strong Yukawa reactions. Thus these particles may be considered as "metastable" with respect to the strong interactions and accordingly their decays, while energetically allowed, are relatively very slow.

In the next chapter we consider some details of reactions (4.21) and of the new conservation laws (the selection rules and their associated quantum numbers) required to account for this "strange" behaviour of the "strange particles".
CHAPTER 5

Classification schemes for mesons and baryons

The concept of associated production provides a framework for the interpretation of the strong (production) and weak (decay) interactions of the heavy mesons and baryons. The requirement of associated production represents an additional selection rule operating on the reactions in question; as in the case of the older selection rules, discussed in the preceding, one would expect that it should follow as a consequence of a general conservation law and that it should be possible to express its effects in terms of some appropriate new quantum number(s).

A. The Gell-Mann - Nishijima scheme

A system of classification of the strongly-interacting particles, appropriate to these ends and requiring only one new quantum number, was first advanced by Gell-Mann *) and by Nakano and Nishijima **). The scheme is based on an extension of the conventional connection between a particle's charge, its isotopic spin, and its nucleonic number; for pions and nucleons, this relation is

\[ Q = t_3 \frac{\eta}{2} \]  

(5.1a)

with \( \eta = 1, 0, -1 \) for nucleons, pions, and antinucleons, respectively. The generalization consists in postulating an additional quantum number, \( S \) (called by Gell-Mann the "strangeness" number), capable of assuming positive or negative integral values, whose role in the classification scheme derives from a generalization of Eq. (5.1a)

*) M. Gell-Mann, Phys. Rev. 92, 833 (1953).

\[ Q = t_3 + \frac{\eta}{2} + \frac{s}{2}. \]  

Pions and nucleons have, perforce, \( S = 0 \). The new particles have \( S \neq 0 \); they may be classified into nucleonic types, or baryons (\( \eta = 1 \)), and mesonic (\( \eta = 0 \)), depending on the number of nucleons produced in their decays, since conservation of baryons is retained as an absolute selection rule.

The values of \( S \), associated with the observed particles, follow from the consideration of their properties: their production reactions, for which it is assumed that \( \Delta S = 0 \), and their decays which, if slow, imply \( \Delta S \neq 0 \). A further assumption, in classifying the particles, is that members of a given isotopic spin multiplet have approximately the same mass values, since these values are assumed to reflect, primarily, the nature of the strong interactions, presumed to be charge-independent. Relatively small mass differences within a given multiplet are to be understood as arising from the electromagnetic, charge-dependent reactions; the magnitudes of such electromagnetic mass differences are expected to be comparable with those observed among the nucleons and pions — order of a few MeV.

On the basis of such criteria we may distinguish three baryons (see Table 2.1):

1) the \( \Lambda^0 \), which stands alone and must therefore be a singlet (\( t = 0 \)) with \( \eta = 1 \) (\( \Lambda^0 \to N + \pi \)) and hence, from Eq. (5.1b), has \( S = 1 \).

2) the \( \Sigma \)'s, which have production and decay properties very similar to those of the \( \Lambda^0 \) and therefore, presumably, also \( S = -1 \); for these, Eq. (5.1b) requires integral \( t_3 \), and therefore integral \( t \). To accommodate charged particles we require, at least, \( t = 1 \), with the \( \Sigma^{+,-,0} \) having, respectively \( t_3 = +1, 0, -1 \).
3) Finally, there is the cascade particle (Ξ). Its slow decay 
(\Xi^- \rightarrow \Lambda^0 + \pi^-) implies |S| \geq 1, presumably S = -2. Eq. (5.1b) 
with M = 1 requires half-integral t for even S; the simplest choice 
is t = \frac{1}{2}, giving a \Xi^0, \Xi^- doublet.

The relationships among the particle masses and their quantum numbers 
are shown in Fig. 5.1. We may note that for each baryon we expect a cor-
responding anti-baryon, related to the baryon by the inversion of the signs 
of M, S, t, and (therefore) Q in Eq. (5.1b). The production of an 
anti-baryon requires the simultaneous production of a baryon to conserve M, 
and hence the energy equivalent of two baryon masses; production of a heavy 
baryon, on the other hand, requires only the energy necessary for the con-
version of a nucleon plus the associated creation of a K-meson, an energy 
expenditure which is much more easily attained either with existing particle 
accelerators or in the cosmic radiation.

For the mesons, Eq. (5.1b) with M = 0 implies that odd-S goes with 
half-integral t, and even-S with integral t. Thus, S = +1 may be 
associated with a doublet meson K^+(t_\frac{1}{2}) and K^0(t_\frac{-1}{2}). The antiparticle 
(\bar{K}), with S = -1 is also an isotopic doublet \bar{K}^0(t_\frac{1}{2}) and \bar{K}^- = K^+(t_\frac{-1}{2}).

As seen in Fig. 5.1, the Gell-Mann - Nishijima scheme contains a 
natural place for each of the observed particles. Actually, it goes further 
in that it provides the basis for interpreting all the observations on their 
production and decay. At the time of its suggestion, only the charged mem-
bers of the \Sigma and \Xi multiplets had been observed; since the electro-
magnetic decay of a \Sigma^0

\Sigma^0 \rightarrow \Lambda^0 + \gamma

(5.2)
satisfies all the known conservation laws, including strangeness, its decay 
is expected to be too rapid to permit the direct observation of the \Sigma^0.
The predictions, both as to its production and decay, have since been confirmed.

Another prediction of the scheme is the $\Xi^0$, whose expected decay into neutral particles only,

$$\Xi^0 \rightarrow \Lambda^0 + \eta^0$$ (5.3)

coupled with the greater difficulty of its production (see below), renders it rather more difficult to observe than the others. This particle has also been found.

Beyond the classification of the particles, the scheme specifies the types of associated production reactions which are permitted and, furthermore, those which are not. Thus, the simplest type of allowed ($\Delta S = 0$) reaction involving strange particles are

$$\pi \ (\text{or } \gamma) + N \rightarrow Y + K$$ (5.4a)

and

$$N + N \rightarrow N + Y + K$$ (5.4b)

where $Y$ stands for a $\Lambda^0$ or $\Sigma$ hyperon, ($S = 1$) and $K$ for a member of the $K^{+}C$ doublet ($S = +1$). It is significant that a $\bar{K}$ ($S = -1$) cannot be produced in reactions (5.4); the simplest strong reactions in which it can be produced are of the form

$$\pi (\gamma) + N \rightarrow N + K + \bar{K}$$ (5.5a)

or

$$N + N \rightarrow N + N + K + \bar{K},$$ (5.5b)
reactions which require more energy than (5.4) and are, accordingly, less probable with the energies available at most accelerators. This is in accord with the relatively large $K^+/K^-$ ratios observed at accelerators. Similar considerations account for the relative scarcity of observed $\Xi$'s, since their production requires reactions like

$$\eta(\eta') N \rightarrow \Xi + 2K$$  \hspace{1cm} (5.6a)

or

$$N+N \rightarrow N+\Xi + 2K.$$  \hspace{1cm} (5.6b)

Significantly, all the nucleon-nucleon reactions for the production of strange particles (5.4b, 5.5b, 5.6b) have more particles in the final state and require higher projectile energies than the corresponding reactions with boson projectiles (5.4a, 5.5a, 5.6a). In this regard, the absence of the simple associated production reaction

$$N+N \rightarrow \Lambda+\bar{\Lambda}, \hspace{0.5cm} (\Delta S=0)$$ \hspace{1cm} (5.7a)

but observation of

$$N+\bar{N} \rightarrow \Lambda+\bar{\Lambda}, \hspace{0.5cm} (\Delta S=0)$$ \hspace{1cm} (5.7b)

is an important consequence of the strangeness scheme of Gell-Mann and Nishijima.

As for the slow decays, all the observations are consistent with the selection rule $\Delta S = \pm 1$. The absence of the decay $\Xi^- \rightarrow n + \pi^-$ indicates that decays with $\Delta S = \pm 2$ are even more strongly forbidden. The effects of these and other selection rules on the decay modes and lifetimes will be discussed in the next chapter.
B. Goldhaber's model of the hyperons

Before going on to the consideration of field-theoretic approaches to the interpretation and elucidation of the properties of the resonances and baryons, it is of interest to consider an interpretation suggested by Goldhaber.* According to this model, it is necessary to introduce only one new type of particle - the $K$-meson and its antiparticle, the $\bar{K}$. The new baryons are interpreted as compound systems of a nucleon and one or more $K$-mesons: $\Lambda$ and $\Sigma$ are compounded of a nucleon and one $K$; $\Xi$ is a nucleon and two $K$.

Charge independence requires that the hyperon charge states are the appropriate combinations of a nucleon ($t=\frac{1}{2}$) and a $K$($t'=-\frac{1}{2}$); $T=0$ for the $\Lambda^0$,

$$\chi(\Lambda^0) = \sqrt{1/2} (pK^- + \bar{n}^0)$$  \hspace{1cm} (5.8)

and $T=1$ for the $\Sigma^\pm$:

$$\chi(\Sigma^+) = pK^-$$  \hspace{1cm} (5.9a)

$$\chi(\Sigma^0) = \sqrt{1/2} (pK^- + \bar{n}^0)$$  \hspace{1cm} (5.9b)

$$\chi(\Sigma^-) = nK^-$$  \hspace{1cm} (5.9c)

The masses of the $\Lambda^0$ and $\Sigma$ are, on this model,

$$H_\chi = M_N + M_K - \epsilon_\chi = M_N + \epsilon_\chi$$  \hspace{1cm} (5.10)

in which $\epsilon_\chi$ is the binding energy of the compound system. Assuming a charge-independent interaction potential of the form

$$H = H_\perp \delta_{\perp} t'$$  \hspace{1cm} (5.11a)

in which the $H$'s contain all the spacial and spin dependences of the interaction, we have

$$\int Y \equiv \frac{M_K}{k} \xi_Y = A + B \cdot t \cdot t' \quad (5.12a)$$

in which

$$A = \frac{M_K}{k} \int \phi_k^* H_1 \phi_k \overline{d \Sigma}$$

$$B = \int \phi_k^* H_2 \phi_k \overline{d \Sigma} \quad (5.12c)$$

and $\phi_k$ represents the appropriate wave function for the bound solution of the Schrödinger equation

$$H \phi_k = -\xi \phi_k. \quad (5.11b)$$

To fit the masses of the $\Lambda$ and $\Sigma$ ($\delta_\Lambda = 176$ MeV, $\delta_\Sigma = 253$ MeV. Table 2.1 neglecting electromagnetic difference) we require

$$A = 234 \text{ MeV} \ (M_K - A \approx 260 \text{ MeV})$$

$$B \approx 77 \text{ MeV}. \quad (5.12d)$$

From the masses, alone, we learn very little more about the assumed interaction, Eq. (5.11a), than that $H_1$ is attractive, $H_2$ repulsive and considerably weaker than $H_1$ (provided they have roughly the same space and spin-dependence). Critical tests of the utility of the model arise in attempts to apply it to the interpretation of strong interactions involving hyperons and K-mesons. Thus, the associated production reactions require a mechanism which can be represented by the two-stage processes *)

*) Note that conservations of parity and angular momentum do not permit the reaction $\Xi \rightarrow K + \bar{K}$, since the combination of (spin-0) $K + \bar{K}$ in a state of (orbital) angular momentum 0 has positive parity, while the pion has negative intrinsic parity. We shall have more to say concerning this process in a following chapter.
\[
\gamma + N \rightarrow (N + \overline{K} + K) \rightarrow Y + K \tag{5.13a}
\]

or

\[
\pi + N \rightarrow (N + \bar{\pi} + \bar{K} + K) \rightarrow Y + K. \tag{5.13b}
\]

These are relatively complicated processes and their computation requires detailed assumptions concerning the nature of the \(K-\overline{K}\) production mechanism. Christy *) has considered some of the details of such interactions, on the basis of the Goldhaber model, and is able to obtain rough, qualitative agreement with some of the observations.

In particular the scattering of \(K\)-mesons and the scattering and absorption of \(\overline{K}\)-mesons by nucleons provides a direct test of the utility of the model, since these follow directly from application of the same interaction Hamiltonian, Eq. (5.11), as is assumed to account for the binding of the \(\overline{K}\) to the nucleon **). We reserve the details of such computations for later sections, in which we shall consider this approach, among others, to the understanding of the strong interactions; we note here only that the Goldhaber model does provide a useful empirical framework for the interpretation of the interactions in question.

If the model is to have a general validity, it should also be capable of accounting for the properties of the cascade particles (\(\Xi\)) as compound states of a nucleon and two \(\overline{K}\)-mesons. In order to take this next step, however, it is necessary to make some more specific assumptions concerning the nature of the primary \(N-\overline{K}\) interaction. In particular, since the \(\overline{K}\) has spin 0, the compound \(\Lambda\) or \(\Sigma^-\) (hyperon) state, having spin 1/2, can consist of a \(N\) plus a \(\overline{K}\) in an s-orbit or alternatively, of a \(N\) plus a \(\overline{K}\) in a \(p_\frac{1}{2}\)-orbit. Lacking the necessary information for


**) Christy assumes \(H(K+N) = -H(\overline{K}+N)\), which is perhaps an over simple expression of the observation that all of the hyperons have \(S \leq 0\).
choosing, let us consider the simpler of the two cases and assume (arbitrarily) that the $N-K$ interaction is in the $1=0, s_1$ state $^*$. 

With this choice, and making the further assumption of negligible $\bar{K}-\bar{K}$ interaction, both $\bar{K}$'s may be put into $s$-orbits with, however, the condition that they are in the charge state with $T_{\bar{K}}=1$ ($T_{\bar{K}} = t_{\bar{K}} + t_{\bar{K}}$), since the wave function of the compound system must be symmetrical with respect to interchange of the two $K$-mesons (bosons). (The charge wave function corresponding to $T_{\bar{K}}=0$ is antisymmetrical!) 

Our model gives, for the mass of the $\Xi$ under these assumptions

\[ M_{\Xi} = M_{\Xi}^0 = \sum_{\Xi} = 2\lambda_a + B_{1N}^a (t_1 + t_2) = 2\lambda_a + B_{1N}^a T_{\bar{K}} = 2\lambda_a \]

\[ +(1/2)B \left[ T(T+1) - t_N (T_N + 1) - T_{\bar{K}} (T_{\bar{K}} + 1) \right] \tag{5.12c} \]

Using the constants derived above, Eq. (5.12c), with $T_{1/2}, t_{1/2}, T_{1/1}$, we obtain

\[ \delta_{\Xi} = 2\lambda_a - B \approx 391 \text{ MeV} \]

\[ M_{\Xi} \approx 1330 \text{ MeV} \tag{5.12f} \]

in too excellent agreement with the observed value ($M_{\Xi} \approx 1320 \text{ MeV}$, Table 2.1). This triumph is short-lived, however, when we observe that the same considerations predict another $N-2\kappa$ state with $T=3/2$, of mass

\[ \delta_{3/2} = 2\lambda_a + (1/2)B = 506 \text{ MeV} \]

\[ M_{3/2} \approx 1445 \text{ MeV}. \tag{5.12g} \]

---

Such a particle would have four charge states, of which one \( T_3 = -3/2 \) would be multiply charged \( (q = -2) \) since
\[
q = T_3^+ \bar{N}/2 + S/2 = T_3 - 1/2.
\]
(5.14)

The decay of this particle would be slow, since the simplest decay permitted to it by conservation of nucleons, charge, and energy,
\[
Y^\pm \longrightarrow \Xi^- + \pi^-, \quad \Xi^\pm \longrightarrow \Xi^- + \gamma; \quad \Xi^+ \text{ could decay via } \Lambda^0 + \bar{\pi}^+, \Sigma^+, \bar{\pi}^0 + \pi^0, \text{ or } p + K^0, \text{ all with } \Delta S = 1. \text{ Of course, the } T = 3/2 \text{ cascade particle would be unstable against rapid (} \Delta S = 0 \text{) decays}
\]
\[
Y_{3/2} \longrightarrow \Xi + \pi
\]
(5.15b)

if its mass were only \( \sim 25 \text{ MeV} \) greater than that computed above. In this case, the "particle" would be analogous to the \((3/3)\) nucleon isobar, and would not be directly observable.

Further difficulties are encountered in considering the possibilities of bound states of \( \Xi \bar{N} \) and more than two \( \Xi \)’s, but the one example suffices to demonstrate the type of problems which are encountered in attempting too literal an application of models such as Goldhaber’s. These difficulties do not, in principle, exclude the model, since it is possible to conceive of \( N - \Xi \) interactions where the \( \Xi \) is not necessarily in an \( s_1 \) state, or to invoke appropriate \( \Xi - \bar{\Xi} \) interactions, which may eliminate the unwanted states; but in so doing the model is deprived of some of the basic simplicity which represents one of its most attractive features.
Many other compound models have been suggested. One of these *) describes the baryons in a rather more symmetrical fashion. Thus, as may be noted by referring to Fig. 5.1, the charge states of the baryons exhibit a symmetry with respect to the \( S = -1 \) axis. Accordingly, one may devise a compound model which, in essence, reflects this symmetry, by considering the baryons as bound states of a fundamental singlet \( B_0 \) (essentially the \( \Lambda^0 \)) plus a \( K \)-meson \((N)\), or a \( \bar{K}(\Xi) \) or a \( \Lambda(\Sigma) \). One must still invoke a difference between the \( K-B_0 \) and \( \bar{K}-B_0 \) interactions to account for the \( N-\Xi \) mass difference; in view of the approximate equality of \( M_\Xi - M_\Lambda \sim 200 \text{ MeV} \) and \( M_\Lambda - M_N = 176 \text{ MeV} \), this may be achieved by addition of a term to the interaction Hamiltonian of the form \( CS \). On this model, the basic associated production reactions, c.f. Eqs. (6.15), are described as relatively simple processes of disintegration of a compound particle; on the other hand, the \( K-N \) and \( \bar{K}-N \) interactions are less direct.

Of considerable interest from the field-theoretic point of view is the model of Sakata. This is a natural extension of the Fermi-Yang model (in which the pions are treated as bound \( N-N \) systems) to include the strange particles. This is done by adding the \( \Lambda^0 \)-hyperon to the nucleon as a fundamental constituent. Table 5.1 lists the particles, their structure and properties on the basis of the Sakata model. Also listed are some other possible particles which would be predicted on this model. The main problem seems to be associated with the \( z \) \((S = +1 \text{ baryon})\). Note also the parity predictions for the compound baryons.

*) D.H. Frisch, private communication.
# Table 5.1

<table>
<thead>
<tr>
<th>Particle</th>
<th>Structure</th>
<th>State</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Pi$-meson</td>
<td>$N\bar{N}$</td>
<td>$^1S_0^-(0^+), t=1$</td>
<td>Most strongly bound $N-\bar{N}$ state</td>
</tr>
<tr>
<td>K-meson</td>
<td>$N\bar{\Lambda}$</td>
<td>$^1S_0^-(0^-), t=\frac{1}{2}$</td>
<td>Most strongly bound $N-\bar{\Lambda}$ state, but less strongly than $\Pi$</td>
</tr>
<tr>
<td>?-meson</td>
<td>$\Lambda\bar{\Lambda}$</td>
<td>$^1S_0^-(0^-), t=0$</td>
<td>Also $N\bar{\Lambda}$ in $t=0$ state less bound than $\Pi$, if at all</td>
</tr>
<tr>
<td>$\Sigma$-hyperon</td>
<td>$N\bar{N}\Lambda$</td>
<td>$^1S_0^-(S=1)$</td>
<td>Assume $B^{\Sigma}_{2\Sigma}$ pair try to bind in $^1S_0$ states</td>
</tr>
<tr>
<td>$\Xi$ (cascade)</td>
<td>$\Lambda\bar{N}\Lambda$</td>
<td>$^1S_0^-(S=2)$</td>
<td>Heavier than $\Sigma$ due to weaker $\bar{\Pi}\Lambda$ binding than $N\bar{N}$</td>
</tr>
<tr>
<td>$N'$</td>
<td>$N\Lambda\Lambda$</td>
<td>$^1S_0^-(S=0)$</td>
<td>Could be very heavy due to weak $\Lambda\Lambda$ binding - rapid decay to $\bar{\Pi}\Pi$</td>
</tr>
<tr>
<td>$Z^+$</td>
<td>$N\bar{\Lambda}\bar{N}$</td>
<td>$^1S_0^-(S=1)$</td>
<td>Expect mass $\sim \Xi$ which would make it stable against strong decay to $N\bar{K}$</td>
</tr>
</tbody>
</table>

- **N**: Neutron
- **N**: Proton
- **\Lambda**: Lambda
- **\bar{N}**: Antinucleon
- **\bar{\Lambda}**: Antilambda
- **\bar{\Lambda}**: Antilambda
- **\Pi**: Pion
- **\Sigma**:Sigma
- **\Xi**:Xi
- **\bar{\Pi}**: Antipion
- **B^{\Sigma}_{2\Sigma}**: Binding energy
C. Schwinger’s scheme: symmetry properties of the Yukawa reactions

A field theoretical description of the strong interactions between mesons and baryons aims at an understanding of the Yukawa reactions responsible for these interactions and of the selection rules which govern the choice among the various possibilities. The list of Yukawa reactions of interest includes

\[ \begin{align*}
B & \xrightarrow{\gamma} B^+ \pi^- \\
\Sigma & \xrightarrow{\gamma} \Lambda^+ \pi^- \\
N & \xrightarrow{\gamma} (\Lambda \text{ or } \Sigma) + K \\
\Xi & \xrightarrow{\gamma} (\Lambda \text{ or } \Sigma) + K^* \\
N & \xrightarrow{\gamma} \Xi^- + 2K \\
\Lambda & \xrightarrow{\gamma} \Xi^- + 2\pi^- \\
K & \xrightarrow{\gamma} \Xi^- + 2\pi^- \tag{5.16a} \end{align*} \]

The \( B \) in Eq. (5.16a) stands for \( N \) or \( \Lambda \) or \( \Sigma \) or \( \Xi \) (any baryon). The above reactions (5.16) imply, in addition, all permutations resulting from a shift of any one of the particles from one side to the other (but, in so doing, the particle must be changed to its antiparticle; e.g. \( N \xrightarrow{\gamma} \Xi^- + K \) becomes \( N^+ \xrightarrow{\gamma} \Xi^- \) or \( \Sigma \xrightarrow{\gamma} N^+ \bar{K} \), etc.), or from the addition, on either side of any of the reactions (5.16), of a particle – antiparticle pair or/and any number of pions.

The problem with which we are confronted is to derive, from observations on the particle properties and their interactions, the nature of the fields which give rise to these reactions. While it is possible that the field equations associated with reactions (5.16) are entirely
unconnected with each other, this is a rather sterile point of view to adopt at the outset; it would appear to be more fruitful to aim at uncovering the relations between the Yukawa reactions and to arrive, if possible, at a unified theory of the fundamental particles. Towards this end, there have been many attempts to understand the conservation laws governing the strong (and weak) interactions in terms of the properties of the basic field equations and, in particular, of the symmetry or invariance principles which could lead to the observed constants of motion (quantum numbers) and selection rules.

Among these, one of the most attractive is that of Schwinger*) in which the quantum numbers associated with the fundamental particles are derived from symmetry properties inherent in assumed form of the description of the particle fields. Schwinger's representation leads to a classification of mesons and baryons in terms of two quantum numbers: the nucleonic (baryonic) charge $\mathcal{N}$ and the "hyperonic charge" $\mathcal{Y}$, each of which may assume, independently, the values 1, 0, −1. The classification of particles according to these quantum numbers is summarized in Table 5.2.

In this scheme, the relation between the particle's charge and the other quantum numbers is

$$Q = t\frac{3}{2} + \frac{Y}{2}$$  \hspace{1cm} (5.17)

from which follows the connection with the "strangeness" classification scheme of Gell-Mann and Nishijima

$$S = Y - \mathcal{N}$$  \hspace{1cm} (5.18)

**Table 5.2**

Schwinger's scheme for classifying mesons and baryons

<table>
<thead>
<tr>
<th>( N )</th>
<th>( \gamma )</th>
<th>1</th>
<th>0</th>
<th>-1</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>N</td>
<td>( \Lambda )</td>
<td>( \Sigma ) (^*)</td>
<td>( \Xi )</td>
</tr>
<tr>
<td>0</td>
<td>K</td>
<td>( \pi )</td>
<td></td>
<td>( \bar{K} )</td>
</tr>
<tr>
<td>-1</td>
<td>( \Xi )</td>
<td>( \bar{\Lambda} )</td>
<td>( \bar{\Sigma} )</td>
<td>( \bar{N} )</td>
</tr>
</tbody>
</table>

*) \( \Lambda \) and \( \Sigma \) stands for either the \( \Lambda \) or \( \Sigma \)-hyperon, or an appropriate combination of the two, as described in the text.

So far, what we have presented is merely an alternative classification which substitutes the hypercharge for the strangeness quantum number, but describes the same (observed) set of particles. However, by associating the quantum numbers with the invariance properties of the field equations, the approach of Schwinger provides a framework within which one may attempt to understand the dynamical features of the Yukawa reactions and their effects on the particle properties. Thus, it is the essence of the scheme that all the baryons have their common origin in a single baryonic type which, owing to the symmetries inherent in the field equations, comprises a number of degenerate substates characterized by the quantum numbers defined above. The effect of the meson fields is to remove this degeneracy, through the (self) energies associated with their Yukawa interactions, and to split the primordial baryon into the observed multiplets. Thus, one may regard the mass spectrum of the hyperons as representing a "fine structure"
of the baryon. In this sense, the small mass differences of particles within a given multiplet represent a "hyperfine structure" arising out of the weaker electromagnetic interactions.

For example, in an early version of his theory, Schwinger attempted to account for the baryonic spectrum by assuming two Yukawa reactions of the form (5.16a) each associated with one of the quantum numbers, \( \mathcal{N} \) and \( Y \), and having strengths (coupling constants), respectively \( \mathcal{N} G_n \) and \( \mathcal{N} G_Y \). Accordingly, the pion couplings associated with the nucleons \( \Lambda \) and \( \Sigma \) would be, respectively: \( (G_n + G_Y) \), \( G_n \), and \( (G_n - G_Y) \). The corresponding self-energies, proportional to the squares of these coupling constants (and negative for an attractive interaction), would account for the major mass splittings. The further splitting of the \( \Lambda \Sigma \) state into \( \Lambda \) and \( \Sigma \) is provided by the isotopic spin dependent Yukawa reactions involving the kaon field

\[
\Lambda \Sigma \rightarrow N + \bar{K} \quad (5.16c')
\]

\[
\Lambda \Sigma \rightarrow N + \bar{K} \quad (5.16d')
\]

Such an interpretation of the baryon mass spectrum implies very large difference in the strengths of the pion Yukawa reactions for the different type of baryons, and its adoption would necessitate the relinquishment of a possible symmetry among the strong interactions - the so-called "global" symmetry - which requires a universal coupling of pions to all baryons. For this reason, it has not been looked upon with great favour, even by Schwinger. Nevertheless, it indicates the kind of relations which might follow from a dynamical theory of the meson fields, and it does follow from at least one of the schemes suggested \(^*\).

Other possible symmetries among the Yukawa interactions have been suggested with consequences for the strong interactions which may be confronted with the observations. Thus, one might assume, following Schwinger and Gell-Mann *, that the coupling of pions to all baryons, \( \mathcal{F} \) reactions (5.16a) is of the same form and the same strength (global symmetry). As a consequence, if we disregard the effects of the \( K \)-meson interactions, the baryons can be classified into four isotopic doublets:

\[
\begin{align*}
B_1 &= \left( \frac{P}{n} \right) \\
B_2 &= \left( \frac{Z^0}{Y^0} \right) \\
B_3 &= \left( \frac{Z^0}{\Sigma^-} \right) \\
B_4 &= \left( \frac{\bar{Z}^0}{\bar{\Sigma}^-} \right)
\end{align*}
\]

(5.19a) (5.19b) (5.19c) (5.19d)

with

\[
\begin{align*}
Y^0 &= \sqrt{1/2}(Z^0 - \Lambda^0) \\
Z^0 &= \sqrt{1/2}(Z^0 + \Lambda^0).
\end{align*}
\]

(5.20a) (5.20b)

In order to see how this representation of the baryons follows from the assumption of a global pion interaction, it is convenient to write the Yukawa reaction in terms of the isotopic spin operators and charge eigenvectors defined in Appendix 1. Thus, for the pion-nucleon interaction ** we have


**) Aside from an irrelevant normalization factor.
$$\begin{pmatrix} p \\ n \end{pmatrix} \leftrightarrow \begin{pmatrix} n^0 & -\Sigma^+ \\ \sqrt{2} n^- & n^0 \end{pmatrix} \begin{pmatrix} p \\ n \end{pmatrix}$$  \hspace{1cm} (5.21a)

which may be abbreviated

$$N \leftrightarrow \zeta \Pi N,$$  \hspace{1cm} (5.21b)

where $\zeta$ is the isotopic spin "vector" for $t=\frac{1}{2}$ and the components of the vector $\Pi$ represent the creation of pions according to the scheme

$$\Pi^\pm = \sqrt{\frac{1}{2}} (\Pi_x \pm i \Pi_y)$$  \hspace{1cm} (5.21c)

$$\Pi^0 = \Pi_z$$  \hspace{1cm} (5.21d)

Since the cascade baryon $\Xi$ is also an isotopic doublet, the global pion interaction requires

$$\Xi = \begin{pmatrix} \Xi^0 \\ \Xi^- \end{pmatrix} \leftrightarrow \zeta \Pi \Xi$$  \hspace{1cm} (5.21e)

For the $\Lambda$ and $\Sigma$ baryons, the Yukawa pion reactions look different, since there are, respectively, an isotopic singlet and triplet. Using charge independence and the properties of the isotopic spin operator $\Sigma$ corresponding to $t=1$, we may write these as

$$\Sigma^- \leftrightarrow \Pi^- \Sigma^-$$  \hspace{1cm} (5.22a)

$$\Sigma^0 \leftrightarrow \Pi^0 \Lambda^0$$  \hspace{1cm} (5.22b)

$$\Lambda^0 \leftrightarrow \Pi^0 \Sigma^-$$  \hspace{1cm} (5.22c)
which are abbreviations for

\[
\begin{pmatrix}
\Sigma^+ \\
\Xi^0 \\
\Xi^-
\end{pmatrix}
\Leftrightarrow
\begin{pmatrix}
\Lambda^0 & -\pi^+ & 0 \\
\pi^- & 0 & -\pi^+ \\
0 & \pi^- & -\pi^0
\end{pmatrix}
\begin{pmatrix}
\Sigma^+ \\
\Xi^0 \\
\Xi^-
\end{pmatrix}
\]

(5.22a')

\[
\begin{pmatrix}
\Xi^+ \\
\Lambda^0 \\
\Lambda^-
\end{pmatrix}
\Leftrightarrow
\begin{pmatrix}
\Lambda^0 \\
\pi^- \\
\pi^-
\end{pmatrix}
\]

(5.22b')

\[
\Lambda^0 \Leftrightarrow
\begin{pmatrix}
-\pi^- & \pi^0 & \pi^+
\end{pmatrix}
\begin{pmatrix}
\Sigma^+ \\
\Xi^0 \\
\Xi^-
\end{pmatrix}
\]

(5.22c')

Global symmetry assumes equal strength for all of the reactions, Eqs. (5.22),
and accordingly, after addition of (5.22a) and (5.22b), leads to four Yukawa
reactions (\(\Sigma^+, \Xi^0, \Xi^-, \Lambda^0\)), of which a typical one is

\[
\Sigma^- \Leftrightarrow -\Sigma^- \pi^0 + \Sigma^0 \pi^- + \Lambda^0 \pi^-
\]

(5.22d)

\[
= -\Sigma^- \pi^0 + \sqrt{2} \left( \frac{\Sigma^0 + \Lambda^0}{\sqrt{2}} \right) \pi^-
\]

It is a matter of simple book-keeping to show that the four Yukawa reactions
may be written
\[ Y = \begin{pmatrix} \bar{X}^+ \\ X^0 \end{pmatrix} \xrightarrow{\Xi} \bar{\Xi} \Xi Y \]  
\[ Z = \begin{pmatrix} Z^+ \\ Z^- \end{pmatrix} \xrightarrow{\Xi} \bar{\Xi} \Xi Z \]  

when \( Y^0 \) and \( Z^0 \) are defined by Eqs. (5.20). Thus, global symmetry permits all the reactions to be written in terms of the four doublets, Eqs. (5.19), as

\[ \bar{B} \xrightarrow{\Xi} \bar{\Xi} \Xi \Xi B. \]  

The effect of the K-meson interactions, Eqs. (5.16c, d), is to render a description in terms of doublet hyperon states, Eqs. (5.19), inappropriate, at least as it concerns the \( \Lambda \) and \( \Sigma \). Nevertheless, it has been suggested that effects of the global symmetry persist to the extent that the K-B interaction strength may turn out to be relatively weaker than that of the \( \bar{\eta} \)-B interaction. Pais *) has examined the consequences of an extreme form of this suggestion, in which the K-interaction is assumed to have a form such as to maintain the appropriateness of the doublet representation, Eqs. (5.19). It is clear that this type of symmetry can be maintained only if the K-B couplings each involve just one member of K-meson multiplet i.e., reactions (5.16c' and d') mix \( B_2 \) and \( B_3 \) if \( K \) and \( \bar{K} \) are treated as isotopic doublets. For the maintenance of global symmetry, then, the Yukawa reactions with K-mesons must be

\[ B_1 \leftrightarrow B_2 + K^0 \quad (5.2a) \]
\[ B_1 \leftrightarrow B_3 + K^+ \quad (5.2b) \]
\[ B_4 \leftrightarrow B_2 + K^- \quad (5.2c) \]
\[ B_4 \leftrightarrow B_3 + \bar{K}^0 \quad (5.2d) \]

with correspondingly drastic limitations on the possible strong interactions. Thus, reactions like

\[ \bar{\Xi}^+ + p \rightarrow \Sigma^+ + K^+ \quad (5.25a) \]

and

\[ K^+ + n \rightarrow p + K^0 \quad (5.25b) \]

would be forbidden, since Eqs. (5.24) contain no strong interactions connecting the initial and final states \(^*)\). But such reactions, especially (5.25a), are observed, and with appreciable strength, which is not at all unexpected in view of the obvious effects of the isotopic doublet nature of the \(K\)-mesons in separating the \(\Lambda\Sigma\) hyperonic state into a distinct singlet (\(\Lambda^0\)) and triplet (\(\Sigma^+\)). If, further, the large mass difference between the nucleons and the \(\nu\)-cascade-hyperons is connected with the

\(^*)\) Since such a global symmetry completely uncouples the \(K^\pm\) from the \(K^0-K^\circ\), it is no longer necessary for the charged \(K\)'s even to have the same intrinsic parity as the neutrals if this symmetry prevails \(**\). But the global symmetry demands that the members of the \(\Lambda-\Sigma\) and the \(N-\Xi\) pairs have the same intrinsic parity.

B-K interactions \( \overline{\text{Eqs. (5.16c', d')}} \) extended global symmetry schemes of the type discussed by Pais could have very little physical justification.

According to the point of view which we have been discussing, the validity of a global pion coupling scheme, Eqs. (5.23), implies a breakdown of the universal kaon coupling scheme, Eqs. (5.24). It is, of course, possible to start from the opposite end, assuming a universal kaon coupling, and to depend on a breakdown of the global character of the pion coupling for the splitting of the baryon masses (Sakurai, Phys. Rev. 112, 1679 (1959) refers to this as "cosmic" symmetry). On this scheme, the Yukawa reactions (5.24) are all of the same strength. In discussing this scheme, however, it is more appropriate to define the baryon doublet in such a way as to preserve, explicitly, the isotopic multiplet nature of the \( K \) and \( \bar{K} \). Thus, we may write for the cosmic interactions

\[
N \rightarrow (\Lambda^0 + \Xi^0) K
\]

\[
\Xi \rightarrow (\Lambda^0 + \Xi^0) \bar{K}
\]

with

\[
K = \begin{pmatrix} K^+ \\ K^0 \end{pmatrix}, \quad \bar{K} = \begin{pmatrix} K^0 \\ K^- \end{pmatrix}
\]

Eqs. (5.26) again imply four relations which may now be abbreviated as

\[
B_1^1 \rightarrow \Xi^0 \bar{K} \quad B_2^1
\]

\[
B_4^1 \rightarrow \Xi^- K \quad B_3^1
\]
Thus, the cosmic kaon symmetry is one which mixes baryons of different strangeness. Since the observed mass spectrum is such that the strangeness is the main determining feature, the pion interactions, on this scheme, must clearly be strangeness dependent and such as to destroy most effects of such a cosmic symmetry. Some effects, such as the prohibition of reaction (5.25b), would, however, still persist, even with a cosmic-destroying pion interaction if the kaon interactions are universal, so that experimental tests are possible.

On the whole, however, the evidence concerning universal symmetries is rather discouraging, due mainly to the large magnitude of the mass splitting among the baryon. Nevertheless, since the purpose of an analysis of the observed interactions, both strong and weak, among the mesons and baryons is to uncover the existence and effects of possible symmetries and correlations, such schemes will provide a useful framework for the discussion of the experimental information.
Relation between the masses of mesons and baryons and the "strangeness" quantum number of Gell-Mann and Nishijima. The expression connecting the particle's charge, isotopic spin, nucleonic (baryonic) number, and \( S \) is

\[ Q = t_3 + \eta/2 + S/2. \]

The antibaryons are not shown; they are obtained by reversing the signs of all of the above quantum numbers (i.e., by reflection about the \( S=0 \) axis).
Is the list of fundamental particles complete?

All of the particles listed in Table 2.1 have been verified by direct observation, with the exception of the heaviest antibaryons \( \Xi \). Most (indeed, all) of the unstable charged particles were first detected in the cosmic radiation, a reliable source of projectiles of energies up to \( \sim 10^{18-19} \) eV; but it was only with the advent of high-energy particle accelerators, with their high intensities and the attendant possibilities for controlled experimentation, that the properties of the particles could be elucidated in sufficient detail to permit the formulation of classification schemes such as those outlined in Chapter 5.

As the number of particle accelerators and their energies become greater and as available experimental techniques are further refined, it is not unlikely that more particles will be discovered. Certainly, as new accelerators become available in new energy ranges, considerable effort will be expended in searching for new particles or for new states of excitation of known particles. It is accordingly of interest to consider, in as systematic a fashion as possible, what are the characteristics which enable the identification and classification of particles, and how these may be applied in new experimental situations. We have already developed, in the foregoing, the basis for such exploration; in this chapter we illustrate the application by a number of examples.

We shall assume that the hierarchy of interactions — strong, electromagnetic, and weak — required for the classification of all known particles and their interactions, will continue to suffice for the understanding of future observations. Then we may generally assume that a particle
which is permitted to decay through a strong or electromagnetic interaction, with reasonable energy release, will not be directly observable in the parent state. This point is, of course, amply illustrated by, e.g., the decays \( \pi^0 \rightarrow 2\gamma \) and \( \Sigma^0 \rightarrow \Lambda^0 + \gamma \) in both cases the lifetime is sufficiently long, however, so that the production of either of these particles in a given reaction requires a unique energy. Thus in the reaction

\[
\pi^- + p \rightarrow n + \pi^0
\]  \hspace{1cm} (6.1)

the c.m. momentum of the product neutron is unique, although any observation of the reaction products will reveal three \( n + 2\gamma \).

For "particles" which decay through strong (rather than electromagnetic) interactions, on the other hand, lifetimes may be sufficiently short so that the particles do not behave in reactions as though they had a unique mass, owing to the uncertainty relation

\[
\Delta E \Delta t \sim \hbar.
\]  \hspace{1cm} (6.2)

A case in point is the so-called "nucleon isobar", of spin and isotopic spin \( 3/2 \), which decays

\[
N^* \rightarrow N + \pi^0
\]  \hspace{1cm} (6.3)

with a meanlife \( \tau \sim 10^{-23} \text{ sec} \) \( \Delta E \sim 50 \text{ MeV} \). Nevertheless, the formation of the "isobar state" is manifested in many pion reactions, through the observation of strong cross-section maxima (resonances) at energies corresponding to possible isobar formation, as well as by kinematical correlations among the reaction products which are characteristic of the production and subsequent decay of the isobar.

We have already considered briefly one example of a possible new particle among the hyperons. In Chapter 5, in developing the Goldhaber model of the strange particles, we noted the possibility of a "cascade-like"
hyeron \( (\Pi) \) with \( S = -2, \ t = 3/2 \), of which the four members would be \( \Pi^-, \Pi^-, \Pi^0, \Pi^+ \). If the members of this quartet should have mass greater than that of a \( \Xi \) plus a pion \( (M_{\Xi} + m_\pi \approx 1450 \text{ MeV}) \), the decays

\[
\Pi \rightarrow \Xi + \pi \quad (6.4a)
\]

would be so rapid as to preclude the direct observation of any of the \( \Pi \)'s; it would then be necessary to detect them through kinematical effects on reactions in which \( \Xi \)'s are the observed reaction products.

On the other hand, suppose \( M_{\Xi} + m_\pi > M_{\Pi} > M_{\Xi} \), as predicted by the rough estimate given in Chapter 5. In this case, two of the members could still decay rapidly through electromagnetic processes

\[
\Pi^0 \rightarrow \Xi^0 + \gamma \quad (6.4b)
\]

but the other two could only decay through (weak) interactions requiring \( \Delta S = 1 \)

\[
\Pi^+ \rightarrow \Lambda^0 + \pi^+ \\
\rightarrow \Sigma^{\pm} + \pi^0 \text{(if } M_\pi > 1325 \text{ MeV}) \\
\rightarrow p + \Xi^0 \text{ (if } M_\pi > 1435 \text{ MeV}) \quad (6.4c)
\]

and

\[
\Pi^- \rightarrow \Sigma^+ + \pi^- \quad (6.4d)
\]

or leptonic decay modes (also weak). Such decays have long meanlives \( (\tau \gg 10^{-10} \text{ sec}) \) and would be easily observable by most visual techniques if the particles were produced in numbers in any way comparable to the other unstable baryons. It is especially difficult to believe that the \( \Pi^- \) would until now have escaped detection, since it is doubly charged, a feature which, together with its unique decay mode, Eq. (6.4d), would render it difficult to overlook.
From such observations, we may conclude that it is extremely unlikely that a \( \Gamma \)-multiplet \((S = -2, \ t = 3/2)\) exists with \( M \lesssim 1450\) MeV. However, the evidence is at present insufficient, as regards the dynamical features of \( \Xi \)-hyperon production, to draw any conclusions concerning a possible \( \Gamma \)-hyperon of greater mass.

It would, in fact, appear possible to draw a more general conclusion: on the basis of available evidence, both from the cosmic radiation and observations at high-energy accelerators, the existence of any multiply charged particles with lifetimes characteristic of the weak interactions is extremely unlikely. We shall use this "observation" as one of the bases for the considerations to follow.

Let us now consider, in a somewhat more systematic fashion, some possibilities which may exist for additional particles, with the qualification that we confine our attention mainly to singly charged (or uncharged) possibilities which, if unstable, decay through weak interactions.

**Mesons:** two types of mesons are known, the \( \overline{\tau} \) and the \( K \). Both have spin 0 and are believed to be pseudoscalar. We know of no a priori reasons why spin 1 (vector or pseudovector) mesons should not exist. Nor can we present any convincing theoretical argument for the non-existence of an ordinary (\( \tau \)-like) meson of isotopic spin zero (the so-called \( \tau^0 \)).

Recent observations indicate a number of unstable mesons: the \( K' \) (\( t=\frac{1}{2}, S=0^+ \) or \( 1^- \), not yet settled) of mass \( \sim 850\) MeV, which decays into \( \pi^+ + K \). The \( \omega (t=1, S=1^-) \) decaying into 2\( \pi \); a possible \( t=0, S=1^+ \) decaying into \( \pi^0 + \chi (or \ 3 \pi) \).

There has also been a number of theoretical proposals for additional mesons, based mainly on arguments of utility. Thus, a spin and I-spin \((J=1, I=1)\) meson has been suggested as the common intermediary for the universal
β-decay interaction \textsuperscript{*)}; and the \[\Gamma^0_0\] has been invoked to explain various experimental discrepancies in pion physics \textsuperscript{**)}. In our previous discussion of the "structure" of the pion, in Chapter 5, it was pointed out that the static properties and decay of the pions could be qualitatively understood in terms of the Yukawa reaction

\[\pi \rightarrow N + N.\] \hspace{1cm} (6.5)

In particular, the nucleon-antinucleon state associated with the pion has isotopic spin \(t=1\) and the \(^1S_0\) spin combination. As mentioned in that discussion, other spin and isotopic spin combinations are possible, and these could lead to different mesons, including those mentioned above. Of course, only charged mesons lighter than the pion would be relatively stable; other mesons would decay very rapidly into pions and/or photons, but one would nevertheless hope to detect them either through peculiar features of their decays or kinematical aspects of their production.

In Table 6.1 we have summarized those properties of the simplest \(N-N\) combinations which determine the decays (or annihilation) into various combinations of pions, photons, or pions and photons. The first five columns list the properties of the \(N-N\) states under consideration. In addition to the usual quantum numbers describing the spin, parity, and isotopic spin, we have shown, in columns 4 and 5, the symmetry of the \(N-N\) wave function with respect to the charge conjugation (particle \(\rightarrow\) anti-particle) operation \(C\) and the "isotopic parity" operation \(\text{***} G \text{****}\).

\textsuperscript{***}) Defined as simultaneous charge conjugation and rotation through \(180^0\) about the \(\gamma\)-axis in isotopic spin (charge) space, \(G = 0 e^{i \frac{\pi}{2} \gamma 2}\).
\textsuperscript{****}) G.C. Wick, Annual Rev. of Nuclear Science 2, 1 (1958).
It must be borne in mind that the symmetry under $C$ is conserved (and definite) only for neutral systems; on the other hand, the symmetry under $G$, and $G$ conservation, while applicable to charged as well as to neutral systems, implies charge-independence, or conservation of $t$, and is accordingly only approximately valid for the strong interactions and certainly invalid for decays involving photons. Finally, the weak decays ($\beta$-decay and the decays of the strange particles) violate both $P$ and $C$, but conserve the product $CP$, and replace $G$ conservation by the $\Delta T = \frac{1}{2}$ selection rule.

The remaining columns (6–10) in Table 6.1 indicate the results of application of the selection rules governing the decays into various possible end products. Decays into pions only are assumed to conserve all the quantum numbers; decays involving photons do not conserve $t$ or $G$ but, for neutral parents, also conserve $C$. For forbidden decays, indicated by "no", the symmetry property which prevents the decay is indicated in parentheses in each case. Of course, the possibility of any decay requires an initial parent energy (mass) greater than the sum of the masses of the products. In comparing the probabilities of various alternative decay modes, we may recall that the emission of each photon normally introduces a factor $\alpha' = 1/137$ into the decay rate, and that the "phase space" generally favours the minimum number of decay products.

In determining the quantum numbers of the $N-N$ systems, we use the following relationships:

$C = \text{spin exchange (S) } \times \text{ parity (P)}$

$= (-1)^S \times (-1)^I = (-1)^{S+I}$

$G = \text{particle exchange (C) } \times \text{ isotopic parity (T)}$

$= (-1)^{S+I} \times (-1)^T = (-1)^{S+I+T}$. 

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For the pions:
\[ C \eta^0 = +\bar{\eta}^0, \quad T \eta^0 = -\eta^0 \]
\[ G \bar{n} = -\bar{n} \]
\[ G(n\bar{n}) = (-1)^n \]
\[ P(n\bar{n}) = (-1)^n p_\perp \]

For the photon (\( C \) is the only good quantum number)
\[ C \gamma = -\gamma \]
\[ C(n\gamma) = (-1)^n \]

For \( K^-\bar{K} \) systems:
\[ C = P = (-1)^l = (-1)^S \]
\[ T = (-1)^t \]
\[ G = CT = (-1)^{S+t} \]

For \( K^-'\bar{K} \) systems (\( P \) is the only good quantum number)
\[ P = -(-1)^S \quad \text{(assuming} \ S_{K'}=0) \]

In arriving at the conclusions shown in the table, a number of considerations should be borne in mind:

1) for the \( 2\eta \)-decays, the final \( t=0 \) state is symmetrical with respect to interchange of the pions; the state of \( t=1 \) is antisymmetrical (and the neutral member contains no \( 2\eta^0 \)). Since the spin \( j \) of a \( 2\eta^- \)-state is equal to its orbital angular momentum, the parity is \((-1)^j\).
2) for the $3\pi$-states, the only possible combinations which are symmetrical with respect to the interchange of any two of the pions have $t \geq 1$. The $t=0$ state is antisymmetrical (no $3\pi^0$). In general, for $n$ pions $P = (-1)^n P_{\text{space}}$ and $C = (-1)^n$.

3) the charge conjugation quantum number of $n$ photons is $C = (-1)^n$; when combined with $C_{\pi^0} = 1$ and the charge-conjugation properties of the $2\pi$-states as discussed in 1) above, this leads to some of the selection rules shown in the last two columns. In addition, conservation of angular momentum and statistics prevents the decay of a spin 0 system into $\pi^+ + \gamma$ and the decay of a spin 1 system into $2\gamma$ (note the decay of the $3\pi^1_1$ state).

4) $\beta$-decay processes, provided they follow the pattern of the known decays – conserving CP, lepton number, and "chirality" – will prefer decay of a charged system into $\mu^\pm + \nu(\bar{\nu})$ for spin 0, and into $e^\pm + \nu(\bar{\nu})$ for higher spins. The reasons have been discussed in the case of the $\pi^- $-meson decay.

5) for the decays involving pions only, the spin and space properties are related as mentioned above. Owing to the conservation rules and the $\Delta T = \pm 1$ selection rule, the K-meson decays must be into final $0^+$ and $0^-$ states for the $2\pi(\pi^-)$ and $3\pi(\pi^-)$ decays, respectively. This is the origin of the famous $\Theta - \pi$ paradox. In the $3\pi$-decay, in particular, different initial spin values are manifested in different final state angular momenta, which lead to the types of angular correlations among the products as have been derived by Dalitz [**] in his analysis of the $\pi^-$-decay. These


will be discussed in greater detail.

All decays into pions only manifest the property of "isotopic invariance" - i.e., an initial combination of states of net charge zero will decay into equal numbers of $\Pi^+$, $\Pi^-$, and $\Pi^0$ mesons. This last point has a special bearing on the question of the possible existence of mesons heavier than the pion. Thus, from the observations on $p-\bar{p}$ annihilation *) that equal numbers of pions of all charges are produced, to an accuracy of $\sim 3-4\%$ (which is, roughly, also the relative probability of $K-\bar{K}$ production in the annihilation reaction), it may be concluded that any heavier mesons, produced in the annihilation with probability $\gamma$, a few per cent, exhibit isotopic invariance in their decay. Since decays with photon emission will almost certainly violate isotopic invariance, we may eliminate a number of the possibilities contained in Table 6.1.

With regard to the possibility of an isotopic singlet ($t=0$) meson of mass comparable to the pion, if its spin were $0$ or $2$ its decay would be easily confused with that of the $\Pi^0$; for spin $1$, the $3\gamma$ (or $4\gamma'$) decay is, in principle, distinguishable from that of the $\Pi^0$, but this might be difficult in practice if the meson were accompanied by a large background of $\Pi^0$'s, however, if its mass were sufficiently less than the $\Pi^0$ it could be produced below the $\Pi^0$-threshold. There is no definite indication of such a meson. On the other hand, it has been argued that a neutral meson of mass not too different from that of the $\Pi^0$ should have been detected through the observation of deviations from the predictions of "charge independence" in reactions like

$$\Pi^- + p \rightarrow n + \Pi^0$$  \hspace{1cm} (6.6a)

$$\Downarrow n + \Pi^0$$

$$\downarrow n + \Pi^0$$  \hspace{1cm} (6.6b)

*) E. Segrè, Annual Rev. of Nuclear Science 2, 127 (1955).
This is certainly true, but the difficulty in applying this argument to
the available observations arises from the abnormally strong (enhanced)
pion-nucleon interaction in the $T=3/2$ state. The final state in reaction
(6.6b) must have $T=1/2$ and, therefore, cannot be produced in the $(3,3)$
resonance. Thus even a reasonably "normal" cross-section for reaction
(6.6b) would result in a relatively small change in the observed reactions
in the region of pion energies which has been most carefully investigated.
Therefore, although there is no definite evidence in favour of an iso-
topic-spin $0$ meson, it remains difficult to exclude it.

Turning to mesons of $S\neq 0$, the same comments as were made con-
cerning the prospects for additional ordinary mesons of different spin
and/or parity could be applied to the mesons of $S=\pm 1$, of which the
only known example is the $K$-meson. However, in this case, the only possible
value of the isotopic spin is $1/2$, provided we exclude multiply charged par-
ticles. This follows from the connection between charge and strangeness
(or hypercharge) which is, for mesons,

$$Q = t_3 + S/2; \quad (6.7)$$

thus, for $S = \pm 1$, the condition $|Q| \leq 1$ requires $|t_3| \leq 1/2$.

If, however, we consider the possibility of $|S| > 1$, but still
maintain the restriction on the charge, there is one more meson type
available: a meson with $S = \pm 2$ which, according to Eq. (6.7) must have
t_3 = 0 = t; this is a charged isotopic singlet of $(Q = \pm 1)$, which was labelled
$\omega$ by Gell-Mann (*). ($\omega^+$ for $S=2$; $\omega^- = \bar{\omega}^+$ for $S=-2$). For such a
meson to be directly observable, it must have $M_\omega < 2m_K (=992$ MeV); other-
wise, the decay $\omega \rightarrow 2k$, resulting from a strong interaction, would be
too rapid. **


(**) These remarks are correct for scalar or pseudovector $\omega$.

However, for a pseudoscalar or vector $\omega$, the decay would be
$\omega \rightarrow 2K+\gamma$ or $\pi$, which would be somewhat slower, but probably still
too fast to observe the $\omega$ as a definite particle.
On the other hand, for \( m_\omega < m_K \), the decay could only be into pions, photons, and/or leptons. Since such decays require \( \Delta S = \pm 2 \), the meanlife would be exceedingly long, \( \tau_\omega \gg 10^3 \) sec. It appears unlikely that a particle of such great stability would have escaped detection until now, even if it were produced in small numbers *).

For an intermediate mass, \( m_K < m < 2m_K \), however, the decay would be

\[
(\omega^\pm \rightarrow K^\pm \gamma) \quad (6.8a)
\]

or

\[
(\omega^\pm \rightarrow K^0 + \mu^\pm (\mu^\pm) + \gamma (\overline{\nu})) \quad (6.8b)
\]

or

\[
(\omega^\pm \rightarrow K^\pm \pi^0 \quad \rightarrow K^0 (K^0) + \pi^\pm) \quad (6.8c)
\]

All of these decays, with \( \Delta S = \pm 1 \), would exhibit typical weak interaction lifetime (\( \sim 10^{-10} - 10^{-8} \) sec), which would render the \( \omega^\pm \) directly observable, provided it could be produced in sufficient numbers. For the pionic decay modes (6.8c) the \( \Delta T = \pm 1 \) selection rule predicts a branching ratio of 2/1 in favour of the charged pion mode.

We may consider a number of possible reactions for the production of such mesons.

*) Thus, for example, if the \( \sim 280 \) MeV particle, discussed in the previous section, were the \( \omega^- \)-meson, the cross-section for its production by photons, or by some of the reactions to be discussed in the following, would be large enough for it to be present in appreciable numbers with existing accelerators.
1) Pair production by photons

\[ \gamma + p \rightarrow p + \omega^+ + \omega^- \]  \hspace{1cm} (6.9a)

The threshold for this reaction is in the range \( 1.5 \leq E_\gamma < 4.0 \ \text{GeV} \), which lies above the photon energies available in accelerators.*

2) Associated production via

\[ \eta^- + p \rightarrow \Xi^- + \omega^+ \]  \hspace{1cm} (6.9b)

requires pions of (threshold)kinetic energy \( 1.2 \leq E_\pi < 2.3 \ \text{GeV} \).

Pion beams of energies in the region of the lower limit have been available for some time. So we may assume that reaction (6.9b) would have been detected if the \( \omega^- \) mass were close to \( m_K^- \). But there have been very few extensive studies of reactions for pions of \( E_\pi \gtrsim 1.5 \ \text{GeV} \).

3) Strange particle interactions, e.g.,

\[ K^+ + n \rightarrow \Lambda^0 + (n^+) \]  \hspace{1cm} (6.9c)

the range of threshold kaon kinetic energies is \( 0.29 \leq E_{K^+} \leq 1.3 \ \text{GeV} \). In this case, again, it is unlikely that reaction (6.9c) would have been missed if \( m_\omega \simeq m_K^- \). However, as in the previous case, an \( \omega^- \)-meson with mass nearer \( 2m_K^- \) might have escaped detection this far.

One further comment is pertinent concerning the possible production and detection of \( \omega^- \)-mesons with existing accelerators. If this meson exists, in the mass range under consideration, it should be produced relatively

\[ \text{For } m_\omega \simeq m_K^- \text{ and for production in the Coulomb field of heavy nucleus,} \]  
\[ E_\gamma \simeq 1.0 \ \text{GeV}, \text{ which is only slightly lower than the maximum available photon energies. But the cross-section will be small close to threshold, if only because of the phase-space factor.} \]
copiously in the internal targets of existing accelerators. However, in the absence of inhibiting factors, decays such as (6.8) would exhibit a mean-life of $\sim 10^{-10}$ sec, which would be too short to permit such mesons to survive in the external beams extracted from such accelerators $^*$; they would have to be observed close to the target, a situation which has not been so extensively investigated.

**Baryons:** all measured baryon spins are $1/2$. Lacking evidence to the contrary, we assume this to be a general rule for baryons (also for leptons). Whether or not we expect to observe any baryons beyond the $N$, $\Lambda$, $\Sigma$, and $\Xi$ depends on our view, if any, of the origin of the known baryonic states. Thus, for example, the scheme of Schwinger discussed in Chapter 5 C, permits of no additional baryons beyond the ones already observed $^{**}$. However, this restriction arises from the finite dimensionality (four) of the geometry assumed by Schwinger to determine the symmetry among the fundamental particles; his scheme gives rise to only one new quantum number (the hypercharge) with values $-1$, $0$, $1$. If it were necessary to account for any additional particle types, the Schwinger scheme would have to be enlarged by increasing the dimensionality of the basic geometry. But this would lead to a whole new set of particles. The present situation favours the more restrictive Schwinger scheme, but it does so only on grounds of economy, which would no longer be applicable if another type of strange baryon or meson were discovered.

$^*$) In this respect, it is fortunate that the specific form of the weak interactions inhibits the $K^{\pm}$-meson decay, and results in a mean-life of $\sim 10^{-8}$ sec, sufficiently long to permit relatively long flight paths.

$^{**}$) Nor would it permit the "strangeness" $\pm 2$ boson, $\omega$.
On the other hand, the strangeness classification scheme of Gell-Mann and Nishijima contains no such restriction. Baryons are only required to satisfy the relation

\[ Q = t_3 + S/2 + \frac{\eta}{2}. \]  
(6.10)

If we restrict our consideration of new possibilities to nucleonic types \((\eta = 1, \text{ or } -1 \text{ for antimucleons})\) and to charge multiplets in which there are no multiply charged members, then, in addition to the known baryons, there are two possibilities:

1) \(S=1, t=0\), a positively charged isotopic singlet, labelled \(Z^+\) by Gell-Mann;

2) \(S=-3, t=0\), a negative isotopic singlet, \(\Sigma^-\).

We may dispose of the \(Z^+\) possibility with relative ease. It could not have a mass less than that of the nucleons; for if it did, the nucleons would decay into \(Z^+\) + a pion or photon or leptons, with typical \(\beta\) -decay lifetimes \((\Delta S=1)\). But the proton is extraordinarily stable \(^\ast\). On the other hand, for \(M_Z > M_N + m_K = 1433 \text{ MeV}\), the decay of \(Z^+ \rightarrow N + K\) would be very fast \((\Delta S=0)\) and the \(Z\)-baryon observable only through kinematical effects. We are left with the possibility \(M_N < M_Z < M_N + m_K\). In this case, the decay into a nucleon plus a pion or photon or leptons would be slow \((\Delta S=1)\), but it could be confused with the decay of the \(\Sigma^+\) if \(M_Z \sim 1200 \text{ MeV}\). However, the associated production reactions

\[ (\pi \text{ or } \gamma) + N \rightarrow Z^+ + K \]  
(6.11a)

would result in the direct production of \( \bar{K} \)-mesons with thresholds comparable to those for \( K \)-meson production and lower than those for the production of \( K-\bar{K} \) pairs; this is contrary to the experimental evidence. Of course, if \( M_\pi \) were close to the upper limit, the difference between \( Z^+-\bar{K} \) and the \( K-\bar{K} \) thresholds would be difficult to observe. But, in any case, the existence of a \( Z^+ \) in the mass range under consideration would give rise to the allowed exoergic reaction

\[
K^+ + n \rightarrow Z^+(\gamma \text{ or } \Pi^0),
\]

(6.11b)

with the consequence that the \( K^+ - n \) cross-section would exhibit a \( 1/\nu \) absorption at low energies and other properties associated with a "capture resonance" at negative or low \( K^+ \) energies. No such effects are indicated by the observations.

Similar considerations, as applied to the \( S = -3 \) possibility (\( Q^- \)), lead to rather different conclusions: if this particle exists, it surely has \( M_\pi \gg M_\pi \); otherwise it would be extremely stable, its decay requiring \( \Delta S \gg 2 \). But a very long-lived, negative particle could hardly have escaped detection, even if it were produced only infrequently in the cosmic radiation. On the other hand, if \( M_\pi \gg M_\pi + m_K \) (1814 MeV), the decay would be very fast, and the particle would be extremely difficult to establish. For a mass between these limits, \( M_\pi \lesssim M_\pi + m_K \), a variety of weak decay possibilities (\( \Delta S = 1 \)) exist. Possible decay modes of such a \( Q^- \) are summarized in Table 6.2, column 1. In column 2 are shown the minimum values of \( M_\pi \) at which each decay mode becomes energetically possible. The branching ratios for the alternative charge states of a given decay mode, based on a \( \Delta T = \pm \frac{1}{2} \) selection rule, are given in columns 3 and 4.
The production of $\Omega^-$ would involve reactions of considerable complexity, e.g.,

$$\pi^+ N \rightarrow \Omega^- + 3K$$  \hspace{1cm} (6.12a)

$$K^- N \rightarrow \Omega^- + 2K$$  \hspace{1cm} (6.12b)

owing to the $\Delta S=0$ selection rule in the strong interactions. Such reactions require very high energy projectiles (reaction (6.12a) needs a threshold pion energy of 4.5 GeV). In addition, the small cross-section, expected as a result of the small phase-space factors corresponding to the large number of reaction products, will render such reactions relatively improbable.

An interesting event, observed by Eisenberg *) in the cosmic radiation, could be interpreted as the $\Omega^-$. He noted, among the products of a high energy reaction, a particle of apparent mass (determined by multiple scattering and ionization density) $1650 \pm 360$ MeV; it appears to decay into a $K^-$-meson and one or more neutral particles; assuming a 2-body decay, the energy release ($Q$-value) would be $\sim 5$ MeV. These observations are consistent with $\Omega^-$ decaying by the $\Lambda^0 K^-$ or $\Sigma^0 K^-$ decay mode (see Table 6.2); but the observation could also be interpreted as a $\Xi^-$ or $\Sigma^-$ interaction in which the $K^-$ is the only charged product **).

Although we have noted at least two possible particles ($\omega^\pm, \Omega^-$) which cannot be definitely excluded on the basis of existing observations, this does not imply that we know of any good reasons why they should be expected. On the other hand, in the absence of a convincing theory of the elementary particles, we can neither predict nor eliminate the possibility


**) Or, for a single event, one can never completely exclude the possibility of an accident.
of entirely new types of particles, whose characteristics cannot be derived by extrapolation of existing classification schemes. It is precisely on account of such ignorance on our part, and in the hope of narrowing the boundaries of this ignorance, that the extension of accelerators into higher energy ranges, and the more careful exploration of available energies, is a field of such intense interest and vast promise. Nevertheless, notwithstanding the unquestioned depth of our ignorance, considerations such as those presented in this chapter provide the essential basis for the interpretation of new observations and for the guidance of investigations into the unknown.
### Decay properties of N-N combinations

<table>
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<th>N-N state</th>
<th>spin and parity</th>
<th>t</th>
<th>C</th>
<th>G</th>
<th>possible decays</th>
<th>(ny) min</th>
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<th>K'→K</th>
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<td></td>
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<td>+</td>
<td>-</td>
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<td>no (P) yes</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1</td>
<td>-</td>
<td>+</td>
<td>no (P) no (G)</td>
<td>π&lt;sup&gt;-&lt;/sup&gt;+γ</td>
<td>no (P) yes</td>
<td></td>
</tr>
<tr>
<td>3P&lt;sub&gt;0&lt;/sub&gt;</td>
<td>0&lt;sup&gt;+&lt;/sup&gt;</td>
<td>0</td>
<td>+</td>
<td>+</td>
<td>yes (π&lt;sup&gt;-&lt;/sup&gt;-π&lt;sup&gt;-&lt;/sup&gt;-2π&lt;sup&gt;0&lt;/sup&gt;) no (P,G)</td>
<td>π&lt;sup&gt;0&lt;/sup&gt;+2γ</td>
<td>2</td>
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<td></td>
<td>1</td>
<td>+</td>
<td>-</td>
<td>no (G) no (P)</td>
<td>π&lt;sup&gt;-&lt;/sup&gt;+2γ, 2π&lt;sup&gt;-&lt;/sup&gt;+γ</td>
<td>yes * no</td>
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</tr>
<tr>
<td>3P&lt;sub&gt;1&lt;/sub&gt;</td>
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<td>0</td>
<td>+</td>
<td>+</td>
<td>no (P) no (G)</td>
<td>π&lt;sup&gt;0&lt;/sup&gt;+2γ</td>
<td>4</td>
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<td>1</td>
<td>+</td>
<td>-</td>
<td>no (P,G) yes (see 1S&lt;sub&gt;0&lt;/sub&gt;)</td>
<td>π&lt;sup&gt;-&lt;/sup&gt;+2γ, π&lt;sup&gt;-&lt;/sup&gt;+γ, 2π&lt;sup&gt;-&lt;/sup&gt;+γ (no 2π&lt;sup&gt;0&lt;/sup&gt;)</td>
<td>no (P,G) yes</td>
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<tr>
<td>3P&lt;sub&gt;2&lt;/sub&gt;</td>
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<td>+</td>
<td>+</td>
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<td>yes</td>
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<td></td>
<td>1</td>
<td>+</td>
<td>-</td>
<td>no (G) yes (see 1S&lt;sub&gt;0&lt;/sub&gt;)</td>
<td>π&lt;sup&gt;-&lt;/sup&gt;+2γ, π&lt;sup&gt;-&lt;/sup&gt;+γ, π&lt;sup&gt;-&lt;/sup&gt;-π&lt;sup&gt;-&lt;/sup&gt;+γ</td>
<td>yes</td>
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# Table 6.2

$\Delta S=1$ decay modes of an $S=-3$ hyperon, $\Omega^-$

<table>
<thead>
<tr>
<th>decay</th>
<th>$M_{\Omega_{min}}$(MeV)</th>
<th>charge state</th>
<th>branching</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Xi^- + \gamma$</td>
<td>1319</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>$\Xi^0 + e^- + \bar{\nu}$</td>
<td>1320</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>$\Xi^0 + \mu^- + \bar{\nu}$</td>
<td>1425</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>$\Xi + \pi^-$</td>
<td>1455</td>
<td>$\Xi^- + \pi^0$</td>
<td>1/3</td>
</tr>
<tr>
<td>$\Lambda^0 + K^-$</td>
<td>1609</td>
<td>$\Xi^0 + \pi^-$</td>
<td>2/3</td>
</tr>
<tr>
<td>$\Sigma + \bar{\pi}$</td>
<td>1686</td>
<td>$\Sigma^0 + K^-$</td>
<td>1/3</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\Sigma^- + \bar{K}^0$</td>
<td>2/3</td>
</tr>
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</table>
CHAPTER 7

The Fermi-Watson theorem

We consider, in this chapter, a number of general properties of the reaction amplitudes (including those which describe decays) which follow from the "S-matrix" formulation of reaction theory. These results are based on an approach due to Heisenberg \(^*\); applications to reactions and to decays are discussed in detail by a number of authors \(^**\). Our approach follows that of Fermi \(^***\).

The S-matrix is a formal device for describing the connections between all possible reactions, consistent with known conservation laws, which can be associated with a given group of particles. A given set of possible reactions, e.g.,

\[
\begin{align*}
A + B &\rightarrow A + B \quad \text{(elastic scattering)} \\
&\rightarrow A' + B' \quad \text{(inelastic scattering) (7.1)} \\
&\rightarrow C + D \quad \text{(transmutations)} \\
&\rightarrow A + B + E, \text{ etc.},
\end{align*}
\]

is represented by the matrix

\[
S = \begin{pmatrix} S_{ij} \end{pmatrix} \quad (7.2)
\]

\(^*)\) W. Heisenberg (S-matrix formulation of reaction theory).

\(^**\) J.M. Blatt and V.F. Weisskopf, "Theoretical Nuclear Physics" (John Wiley and Sons, N.Y. 1952).


in terms of which a given incident wave

\[ \psi_{1}^{(\text{in})} = (4\pi v_{1})^{\frac{1}{2}} e^{-ik_{1}r} / r \]  \hspace{1cm} (7.3a)

is converted into the various possible outgoing waves

\[ S\psi_{1}^{(\text{in})} = \sum_{j} S_{i j} \psi_{j}^{(\text{out})} \]  \hspace{1cm} (7.3b)

where

\[ \psi_{j}^{(\text{out})} = (4\pi v_{j})^{\frac{1}{2}} e^{ik_{j}r} / r \]  \hspace{1cm} (7.3c)

(The normalizations in Eqs. (7.3) are chosen to correspond to unit flux.)

Pictorially, as indicated in Fig. 7.1, we may think of the S-matrix as summarizing the effects of a 'black box' which distributes the outgoing states among the various exit channels according to the requirements of the conservation laws and the strengths of the interactions concerned.

The form of the S-matrix is primarily determined by two physical requirements:

1) **Conservation of flux.** For unit (steady state) incident flux the total reaction rate must be unity. The mathematical expression of this condition is that the S-matrix must be unitary

\[ S \tilde{S}^{*} = 1 \]  \hspace{1cm} (7.4a)

where

\[ (\tilde{S}^{*})_{i j} = S_{j i}^{*} \]  \hspace{1cm} (7.4b)
2) **time-reversal invariance.** The condition that the interactions are invariant with respect to inversion of the time-axis \((\psi \to \psi^*)\) gives rise to the requirement that the \(S\)-matrix must be symmetrical

\[
S_{ij} = S_{ji}. \tag{7.5}
\]

Combining Eqs. (7.4a) and (7.5), we have

\[
(S^*)_{ij} = S^*_{ji} = S^*_{ij} \tag{7.6a}
\]

\[
SS^* = 1. \tag{7.6b}
\]

Now, as noted, the elements \(S_{ij}\) are non-zero only when the transitions between channels are permitted by the conservation laws. In particular, if we are careful to choose our channels as states with definite values of the 'good' quantum numbers, the corresponding \(S\)-matrix may be nearly diagonal — i.e., the off-diagonal elements small as compared to the diagonal elements — at least for reactions at low energies. As an example, if we consider the \(S\)-matrix corresponding to the possible reactions of \(\pi + N\) at relatively low energies, and choose as our initial \((A+B)\) state a \(\pi + N\) state of definite angular momentum, parity, and definite isotopic spin, then the possible exit channels will include, in addition to elastic scattering, \(N + \gamma\), \(N + \pi\) (with a different isotopic spin, to the extent that charge independence is violated), \(N + \pi^* + \gamma\), \(\Lambda\) or \(\Sigma\) plus \(K\) (for initial states of appropriate energy and spin), etc. All of these are only relatively weakly connected to the initial state; accordingly, in such situations, to a good approximation, the \(S\)-matrix may be written

\[
S = S_0 + i \varepsilon, \tag{7.7a}
\]

where \(S_0\) is diagonal and the elements of \(\varepsilon\) are all small.
We consider, first, some properties of $S_0$ in lowest order. From Eq. (7.6) we have

$$|S_{0j}|^2 = 1$$  \hspace{1cm} (7.7b)

$$S_{0j} = e^{2i \alpha_j}.$$  \hspace{1cm} (7.7c)

That $\alpha_j$ is the conventional scattering phase shift, follows from the form of the elastic scattering cross-section corresponding to an incident plane wave

$$\psi^{\text{(in)}} = v_j e^{ikz},$$  \hspace{1cm} (7.8)

from which we must first extract the portion corresponding to the incident spherical wave of the required angular momentum, Eq. (7.3a), and then subtract that portion corresponding to an outgoing spherical wave; the net result of these manipulations is

$$\frac{d\sigma}{d\Omega} = \frac{1}{4k_j^2} |S_{0j}^{-1}|^2$$  \hspace{1cm} (7.9)

which defines $S_{0j}$ in terms of the conventional phase shift, Eq. (7.7c). Now, applying Eqs. (7.6) to the S-matrix described by Eq. (7.7a)

$$(S_0 + i\xi)(S_0^* - i\xi^*) = 1$$  \hspace{1cm} (7.10a)

$$S_0 S_0^* + i(\xi S_0^* - S_0 \xi^*) + \xi \xi^* = 1$$  \hspace{1cm} (7.10b)

$$1 + i(\xi S_0^* - S_0 \xi^*) = 1$$  \hspace{1cm} (7.10c)

845
we obtain the condition

$$\xi S_0^* = S_0 \xi^*.$$  \hfill (7.10d)

We write, for the elements of $\xi$

$$\xi_{ij} = s_{ij}^{\gamma} e^{i(\alpha_i - \delta_j)}$$  \hfill (7.11a)

with $s_{ij}^{\gamma}$ real and $|s_{ij}^{\gamma}| \ll 1$. Combining Eq. (7.11a) and Eq. (7.7c),
we obtain by straightforward matrix algebra

$$\xi_{ij} = s_{ij}^{\gamma} e^{i(\alpha_i + \alpha_j)}.$$  \hfill (7.11b)

Equation (7.11b) contains the essence of the Fermi-Watson theorem. Thus, if we are interested in the weak decay

$$Y^+(x) \rightarrow N^+ \pi^-$$  \hfill (7.12a)

($Y$ is a hyperon, $x$ a 'spurion'), we have $\alpha_i \approx 0$ (in the reaction

$$x + Y \rightarrow Y + x$$; $\alpha_j$ is the $N^- \pi$ scattering phase shift in the state of
appropriate angular momentum and isotopic spin, and the decay amplitude may
be written

$$A_i = \sum_j \xi_{ij} = \sum_j a_j e^{i\alpha_j}.$$  \hfill (7.12b)

Another application is to photoproduction reactions, say

$$\gamma + N \rightarrow N^+ \pi^-.$$  \hfill (7.13)
Here, the amplitude corresponding to a definite final state is again given by an equation of the form of (7.12b), since the phase shift \( \alpha_i \) for elastic photon scattering is \( \ll 1 \).

It is of interest to derive the lowest order correction to the elastic scattering phase shift arising from the small non-diagonal contribution \( \xi \), to the S-matrix. Thus, from Eqs. (7.10a)

\[
S_0 S^*_0 \simeq 1 - \xi \xi^*
\]

(7.14a)

and

\[
(S_0 S^*)_ii = 1 - \sum_j \rho_{ij}^2 \approx \xi_i^2 \ll 1.
\]

(7.14b)

Thus, we may write

\[
S_{0 ii} = \xi_i e^{2i_\alpha_i} = e^{2i(\alpha_i + i\eta_i)}
\]

(7.14c)

with

\[
e^{-2\eta_i} = (1 - \sum_j \rho_{ij}^2)^{1/2} \approx 1 - \frac{1}{2} \sum_j \rho_{ij}
\]

(7.14d)

\[
\eta_i = \frac{1}{4} \sum_j \rho_{ij}.
\]

(7.14e)

The effect of the weak reaction channels is to introduce a small imaginary phase shift.

Returning to the general form of the S-matrix, Eqs. (7.1-6), we may note another important general property of reactions. Consider the reaction \( i \to j \); from the preceding we have, for given incident and outgoing channels
\[
\frac{d\sigma}{d\Omega} (i \rightarrow j) = \frac{1}{4k_i^2} |S_{ij}|^2 .
\] (7.15a)

If the incident particles, say \((A+B) \equiv i\), have spins \(s_A\) and \(s_B\) and the outgoing particles, \((C+D) \equiv j\), have spins \(s_C\) and \(s_D\), the cross-section for an incident unpolarized plane wave is obtained by averaging over initial and summing over final spin states

\[
\frac{d\sigma}{d\Omega} (A+B \rightarrow C+D) = \frac{(2k_i)^{-2}}{(2s_A+1)(2s_B+1)} \sum_{m_i} \sum_{m_j} |S_{ij}|^2
\] (7.15b)

The cross-section for the inverse reaction is

\[
\frac{d\sigma}{d\Omega} (C+D \rightarrow A+B) = \frac{(2k_j)^{-2}}{(2s_C+1)(2s_D+1)} \sum_{m_j} \sum_{m_i} |S_{ji}|^2
\] (7.15c)

But time-reversal invariance requires

\[
S_{ij} = S_{ji} ,
\] (7.16)

which results in the principle of detailed balancing

\[
k_i^2(2s_A+1)(2s_B+1) \frac{d\sigma}{d\Omega} (A+B \rightarrow C+D)
\]

\[
= k_j^2(2s_C+1)(2s_D+1) \frac{d\sigma}{d\Omega} (C+D \rightarrow A+B)
\] (7.17)
"Cusps" and associated phenomena

A number of connections between reaction amplitudes for inelastic, elastic, and production processes were exhibited in this chapter, in which we discussed some general properties of the $S$-matrix.

Other interesting effects of this nature may occur in the behaviour of reaction cross-sections at energies corresponding to the inception (threshold) of new, competing reactions. Such effects, manifested as discontinuities in the curves of cross-section vs. energy, were first discussed by Wigner *; more recently, their possible importance with respect to reactions involving strange particles has been pointed out by Adair ** and by Baz' and Okun' ***.

To illustrate the form and to understand better the origin of such effects, we consider a simple system having only two possible reaction channels (say, $a+A$ and $b+B$). The scattering matrix

---

*) E. Wigner, Phys. Rev. 72, 1002 (1948).


\[
S = \begin{pmatrix}
    2i \alpha_a & i \varphi_{ab} \\
    i \varphi_{ae} & 2i \alpha_b \\
    i \varphi_{be} & i \varphi_{ab}
\end{pmatrix}
\] (7.18a)

describes elastic scatterings

\[
a + A \rightarrow A + a
\] (7.18b)

\[
b + B \rightarrow B + b
\] (7.18c)

through the diagonal elements, and the reaction

\[
a + A \rightarrow b + B,
\] (7.18d)

and its inverse, through the off-diagonal elements, as discussed.

Eq. (7.18a) already contains the symmetry requirement on the S-matrix; to satisfy the unitarity requirement \( S^2 = 1 \),

\[
\xi_a^2 = \xi_b^2 = 1 - \bar{\varphi}^2
\] (7.19a)

and

\[
\alpha_{ab} = \alpha_a + \alpha_b.
\] (7.19b)

We now consider the effect of the onset of reaction (7.18d) on the cross-section for elastic scattering (7.18b). Near threshold, the dominant term in the amplitude for reaction (7.18d) is that corresponding to S-wave production of the products, \( b + B \), for which we may write

\[
\bar{\varphi}^2 \approx 2k\eta
\] (7.20)
For the cross-sections

\[ \sigma_{ab} = \frac{\pi \varphi^2}{k^2} = \frac{2\pi \eta k}{k^2} \quad (7.21a) \]

\[ \sigma_a(E_a > E_t) = \frac{4\pi}{k^2} \left| \frac{(1-\eta k)e^{-\alpha_a}}{2i} \right|^2 \approx \frac{4\pi}{k^2} (1-k\eta)\sin^2 \alpha_a \quad (7.21b) \]

where \( \eta \) is a constant (length), \( K \) and \( k \) are the wave-numbers of \( a \) and \( b \), respectively.

\[ \hbar^2 k^2 = 2\mu_a E_a \quad (7.22a) \]

\[ \hbar^2 k^2 = 2\mu_b (E_a - E_t) \quad (7.22b) \]

the \( \mu \)'s are appropriate reduced masses, and \( E_t \) is the threshold (kinetic) energy for \( (7.18d) \).

Note that as the threshold energy is approached from above \( (k \to 0) \), \( \sigma_{ab} \to 0 \), \( \sigma_{aa} \to \sigma_0 = (4\pi/k^2)\sin^2 \alpha_{ao} \); but the derivative \( d\sigma_{aa}/dE_a \to -\infty \), indicating that there is a discontinuity in the scattering cross-section at \( E_a = E_t \). To specify fully the nature of this discontinuity, it is necessary to consider the behaviour of \( \sigma_{aa} \) as \( E_a \to E_t \) from below. Of course, for \( E_a \ll E_t \), we require \( \varphi = 0 \). But in the immediate vicinity of the threshold, albeit below effects of reaction \( (7.18d) \) are still present; these may be derived by observing, from \( (7.22b) \), that \( k \) becomes imaginary for \( E_a < E_t \).
\[ k \rightarrow i \varphi = i (2 \mu_b / \hbar^2)^{1/2} (E_t - E_a)^{1/2} \]  \hspace{2cm} (7.23a)

in which case

\[ \alpha_b \rightarrow ia_b = i \frac{1}{2} \varphi \]  \hspace{2cm} (7.23b)

and

\[ \delta^2 \approx 2i \varphi \gamma e^{-2a_b} \]  \hspace{2cm} (7.23c)

Substitution into (7.21b) yields

\[ \sigma_{aa} (E_a < E_t) \propto \frac{4 \pi}{k^2} \left\{ \sin^2 \alpha_a - \varphi \gamma \right\} e^{-2 \frac{i}{2} \sin \alpha_a \cos \alpha_a} \]  \hspace{2cm} (7.21b')

Eq. (7.21b') joins onto Eq. (7.21b) at \( E_a = E_t \), but not smoothly; rather, approaching the threshold from below, \( d \sigma_{aa} / dE_a \rightarrow \infty \) for \( \alpha_a \) lying in the first or third quadrant, while \( d \sigma_{aa} / dE_a \rightarrow -\infty \) for \( \alpha_a \) in the second or fourth quadrant. The possible forms of this scattering cross-section anomaly are illustrated in Fig. 7.2.

In the more general case, where the reaction of interest (scattering, in the cases considered above) is accessible through more than one channel, the threshold anomaly can have other forms, in addition to those illustrated in Fig. 7.2. Below the threshold for a new reaction, we may write the cross-section *) referring to the following equation.

*) We assume, for simplicity, no spin dependence. The generalization to include spin dependence does not introduce any new physics.
\[
\frac{d\sigma}{d\Omega} = \frac{1}{4k^2} \sum_\ell \left| s_\ell P_\ell (\theta) \right|^2 = \left| F(\theta) \right|^2 = \left| f(\theta)e^{i\delta} \right|^2 \quad (7.24a)
\]

(the subscript, \( \ell \), symbolizes all the quantum numbers characterizing a given combination of initial and final channels). Near the threshold for a new reaction, say \( \text{a}+\text{A} \to \text{b}+\text{B} \), the only amplitude whose alteration we need to consider is that corresponding to the set of channels \( L \) which leads to s-wave production of the products \( \text{b}+\text{B} \). For this amplitude we may write

\[
S_L = s_{0L} e^{-i\eta} e^{i\alpha} \quad (7.24b)
\]

in which the symbols have the same meaning as in the preceding and, in particular, \( k \to 1 \) below the threshold. Accordingly, near the threshold, Eq. (7.24a) becomes

\[
\frac{d\sigma}{d\Omega} = \left| f(\theta)e^{i\delta} - \frac{k \eta e^{i\alpha} P_L(\theta)}{2ik} \right|^2 \quad (7.24c)
\]

\[
\propto \frac{d\sigma}{d\Omega} - \frac{k \eta}{k} f(\theta)P_L(\theta)\sin(\alpha - \delta), \quad \text{(for } E_a > E_t \text{)}, \quad (7.24d)
\]

\[
\propto \frac{d\sigma}{d\Omega} - \frac{k \eta}{k} f(\theta)P_L(\theta)\cos(\alpha - \delta), \quad \text{(for } E_a < E_t \text{)} \quad (7.24e)
\]

The threshold anomalies have the forms of cases (a) and (b) of Fig. 7.2 if \( 0 < (\alpha - \delta) < \pi/2 \) or \( \pi/2 < (\alpha - \delta) < \pi \), respectively. For \( (\alpha - \delta) \) in the third and fourth quadrants, the shapes of the anomalies are reversed, as illustrated in Fig. 7.3.
A few comments concerning formal S-matrix theory: consider a reacting system with \( n-1 \) channels, below the threshold for an \( n \)th channel. Let the S-matrix elements be \( s_{ij} = s_{ji}' \),

\[
\sum_{\ell=1}^{n-1} a_{i\ell} s^*_{\ell j} = \delta_{ij},
\]

(7.25a)

Now, above the threshold, we have a new row and a new column added to the S-matrix

\[
s_{i'n} = s_{ni}' = m_i \frac{1}{2} k^2
\]

(7.25b)

while the old elements become

\[
s_{ij}' = s_{ij} + a_{ij} k.
\]

(7.25c)

Application of \( SS^* = 1 \) both above and below the threshold, yields

\[
\sum_{\ell=1}^{n-1} s_{j\ell} s^*_{\ell n} = -m_j e^{-2i \alpha_n}
\]

(7.25d)

which is a set of \( n-1 \) simultaneous equations for determining, essentially, the phases of the \( m_j \)'s and

\[
a_{ij} = \frac{1}{2} m_i m_j e^{-2i \alpha_n}.
\]

(7.25e)

It is the form of this last expression which permits us to write Eq. (7.24b) and to derive the forms of the anomaly shown in Fig. 7.3.
There is a number of additional features of these "cusp" anomalies which are more or less directly derivable from the $S$-matrix formalism as outlined above. Thus, while a given channel (say $a + A \rightarrow C + c$) may exhibit anyone of the four forms of the anomaly at the threshold for a new reaction, only those anomalies shown in Figs. 7.2/3 in which the cross-section below threshold is less than it would have been in the absence of the new channel, appear if we consider the sum of all processes but the new one

$$\sigma = \sum_{j=1}^{n-1} \sigma_{aj}.$$ 

Finally, a possible use of these phenomena is to determine the intrinsic parities of the products in the new channel ($b + B$) relative to those in the reaction in which the anomaly is observed. This possibility arises from the determination of the orbital angular momentum of the product channel which exhibits the anomaly, by the condition that it follow from the same initial channel as gives rise to $s$-state production of the new products $P_L$ in Eqs. (7.24). This application has been suggested *) as a means of determining the relative $\Lambda - \Sigma$ parity by observing the behaviour of the reactions $\pi + N \rightarrow \Lambda + K$ at the threshold for $\pi + N \rightarrow \Sigma + K$.

PICTURE 7.1

Pictorial representation of relations among reactions which are described by the S-matrix

\[ S \psi_i^{(in)} = \sum_j S_{ij} \psi_j^{(out)} \cdot \]

The ingoing arrow represents a unit flux of projectiles \((A+B)\)

\[ \psi_i^{(in)} = (4\pi v_i)^{-\frac{1}{2}} e^{-ik_i r}/r \]

and the outgoing arrows represent appropriate fluxes in the exit channels of form

\[ \psi_j^{(out)} = (4\pi v_j)^{-\frac{1}{2}} e^{ik_j r}/r \cdot \]

The S-matrix elements are the appropriate reaction amplitudes

\[ \frac{d\sigma}{d\Omega}(i \rightarrow f) = \frac{1}{4k_i^2} |S_{if}|^2 \cdot \]
Fig. 7.2

Fig. 7.3