THE BEAM GUIDE, A DEVICE FOR THE TRANSPORT OF CHARGED PARTICLES

by

S. van der Meer

Geneva
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Summary

A coaxial transmission line, fed with direct current or pulsed, may be used as a focusing channel for charged particles. The particles will follow a screw-type trajectory around the inner conductor.

The most important properties of the trajectories are described, and normalized diagrams are given for computing all important parameters for high-energy applications.

A method of calculating the acceptance is outlined and some examples are given. It is shown that both angular acceptance and phase acceptance may be an order of magnitude higher than that of a quadrupole channel.

Some remarks are made about the method of excitation and the direction in which applications might be found.
1. Introduction

In fig. 1 a coaxial transmission line is shown, carrying a current. A rotationally symmetric magnetic field exists between the inner and outer conductor. Charged particles, introduced into this field and not coplanar with the axis, will follow a screw-type trajectory around and along the inner conductor. The trajectory is in general not a pure helix, but it is a periodic function of the axial coordinate $z$. Within a range of initial conditions the trajectories are such that they do not touch the conductors.

The "beam guide", formed in this way, may be used to transport particles over great distances. It may also be bent slightly, thus changing the general direction of the particles.

As an illustration of the type of movement obtained, fig. 3 a shows the projection of a trajectory on a plane perpendicular to the axis, and fig. 3 b the distance from the axis as a function of $z$. Another type of trajectory is shown in fig. 3 c and 3 d. With high-energy particles and reasonable excitation currents, the type of fig. 3 a and 3 b will always be found.

The angle between the trajectory and the axis will then always be small, and the focusing action of the beam guide may be understood by considering the moving particle as a current that is attracted by the parallel current in the central conductor. The continuous focusing (no defocusing, as in AG systems) results in a relatively high acceptance \(^{3}\).

Trajectories of charged particles in the field of a linear current have been studied in connection with plasma physics (ref. 1 and 2). Part of par. 3 is an extract from ref. 1 with a slightly different notation.

\(^{3}\) The magnetic horn (ref. 3) belongs to the same family as the beam guide. Its field obeys the same law, and the method of computing trajectories, explained in the following, may be applied to it.
2. Scaling law

If all linear dimensions of an electromagnetic device are increased by the same factor, currents being kept constant, particle trajectories will scale up by the same factor, but will not change their shape.

This general principle may be applied to a beam guide. In this case, a simplification arises from the fact that scaling up the diameters of a beam guide does not change the field between the conductors at a fixed distance from the axis. We may therefore scale up all trajectories in one and the same beam guide linearly, provided they do not touch the conductors.

3. Movement of particles in a straight beam guide

We define the coordinate system of the particle as shown in fig 2. The MESA unit system will be used throughout ($\mu_0 = 4 \pi \times 10^{-7}$). The particle momentum $p$ will be expressed in eV/c.

The field between inner and outer conductor with current $i$ is

$$ B = \frac{\mu_0 i}{2\pi r} \quad (1) $$

The equations of motion of a particle are

$$ \ddot{r} - \frac{1}{r} \dot{r}^2 = - \eta \frac{z}{r} B \quad (2a) $$
$$ z = \eta \frac{r}{B} \quad (2b) $$
$$ \frac{d}{dt} (r^2 \dot{r}) = 0 \quad (2c) $$

in which

$$ \eta = \frac{v_0}{p} \quad (v = \text{velocity of particle}) $$

No solution in $r$, $z$ and $\dot{r}$ can be found for these equations, but it may be shown by substitution that they are satisfied by

$$ \dot{r} = \frac{1}{r} \sqrt{f(r)} \quad (3a) $$
$$ \dot{z} = \varphi (r) \quad (3b) $$

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in which
\[ f(r) = v^2 - \left( \frac{\eta \mu_i}{2\pi} \ln \frac{r}{r_o} + \hat{z}_o \right)^2 - \frac{\hat{z}_o^2}{r^2} \] \hfill (4a)

\[ \varphi(r) = \frac{\eta \mu_i}{2\pi} \ln \frac{r}{r_o} + \hat{z}_o \] \hfill (4b)

where \( r_o, \hat{z}_o \) and \( \hat{\varphi}_o \) are the initial values of \( r, \hat{z} \) and \( \hat{\varphi} \).

We introduce the initial angles \( \alpha_o \) and \( \beta_o \), as illustrated in fig. 2, and define

\[ a_o = \tan \alpha_o \] \hfill (5a)

\[ b_o = \tan \beta_o \] \hfill (5b)

\[ A = \frac{\eta \mu_i}{\pi p} \] \hfill (5c)

Equations (4) may then be written as follows

\[ f(r) = v^2 \left[ \frac{a_o^2 + b_o^2}{1 + a_o^2 + b_o^2} \right] - \frac{r_o^2}{r^2} \cdot \frac{a_o^2}{1 + a_o^2 + b_o^2} - \frac{A^2}{4} \left( \ln \frac{r}{r_o} \right)^2 - \frac{\hat{\varphi}_o^2}{\sqrt{1 + a_o^2 + b_o^2}} \] \hfill (6a)

\[ \varphi(r) = v \left[ \frac{A}{2} \ln \frac{r}{r_o} + \frac{1}{\sqrt{1 + a_o^2 + b_o^2}} \right] \] \hfill (6b)

The minimum and maximum distance between the particle and the axis, \( r_{\text{min}} \) and \( r_{\text{max}} \), are the zeros of \( f(r) \). This is evident by substitution into (3a). If \( A \neq 0 \), the right hand side of (6a) will always have two positive zeros, whatever the values of \( a_o \) and \( b_o \). Mathematically speaking, therefore, a charged particle will always describe periodical trajectories of the type of fig. 3, around a current-carrying wire, whatever its initial direction or momentum. With some initial conditions, however, \( r_{\text{max}} \) may become very large and \( r_{\text{min}} \) very small. Obviously, the particle will only be accepted if

\[ \begin{align*}
    r_{\text{max}} &< R_o \\
    r_{\text{min}} &> R_i
\end{align*} \] \hfill (7)

where \( R_o \) and \( R_i \) are the radii of outer and inner conductor respectively.
4. Approximate solutions and graphical representation

If the conditions

\[ |A \ln \frac{r}{r_0}| \ll 4 \]  
\[ |a_o| \ll 1 \]  
\[ |b_o| \ll 1 \]  

are satisfied (as will be the case in high-energy applications), we may write in good approximation

\[ f(r) \approx \sqrt{a_o^2 \left( 1 - \frac{r^2}{r_0^2} \right) + b_o^2 - A \ln \frac{r}{r_0}} \]  
\[ \varphi(r) \approx v \]

\( r_{\text{max}} \) and \( r_{\text{min}} \) are now the solutions of

\[ a_o^2 \left( 1 - \frac{r_0^2}{r^2} \right) + b_o^2 - A \ln \frac{r}{r_0} = 0 \]  

By introducing the normalized quantities

\[ x_o = \sqrt{\frac{E}{I}} a_o \]  
\[ y_o = \sqrt{\frac{E}{I}} b_o \]  

we may write

\[ x_o^2 \left( 1 - \frac{r_0^2}{r^2} \right) + y_o^2 - \frac{e \mu_o}{\pi} \ln \frac{r}{r_0} = 0 \]  

The values of \( r_{\text{max}} \) and \( r_{\text{min}} \) may be found from fig. 4, which is a graphical representation of eq. (12) \( \star \).

\( \star \) Since the curves are symmetrical with respect to the \( x_o \) and \( y_o \) axes, all information is contained in one quadrant. Two are shown in order to make the construction of par. 5 b possible.

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For $r_{\text{max}} / r_0 = 1$ and $r_{\text{min}} / r_0 = 1$ the curves degenerate into the parts of the $x_0$ axis to the left and to the right of point H, respectively. This point represents the case of a helical trajectory around the inner conductor ($r_{\text{max}} / r_0 = r_{\text{min}} / r_0 = 1$; $x_0 \approx \sqrt{\frac{cp_0}{2}}$).

The period of movement is given by

$$\lambda = 2 \int_{r_{\text{min}}}^{r_{\text{max}}} \frac{dz}{dr} dr = 2 \int_{r_{\text{min}}}^{r_{\text{max}}} \frac{\frac{d}{dr} \frac{r}{z}}{r} dr = 2 \int_{r_{\text{min}}}^{r_{\text{max}}} \frac{q(r)}{\sqrt{f(r)}} dr$$

Using the approximations (8) and definition (11), we may write

$$\frac{\lambda}{r_0} \sqrt{\frac{1}{p}} = 2 \int_{r_{\text{min}}}^{r_{\text{max}}} \frac{d(r/r_0)}{\sqrt{x_0^2 \left(1 - \frac{r_0^2}{r^2}\right) + y_0^2 \frac{\omega}{\pi} \ln \frac{r}{r_0}}}$$

This quantity was numerically computed and is shown in fig. 5 as a function of $x_0$ and $y_0$.

The rotation around the axis per period (see fig. 3) is equal to

$$\varphi = 2 \int_{r_{\text{min}}}^{r_{\text{max}}} \frac{v}{r} dr$$

Since, according to (2c),

$$r^2 \varphi = r_0^2 \varphi_0,$$

we may write

$$\varphi = 2 \int_{r_{\text{min}}}^{r_{\text{max}}} \frac{v}{\sqrt{r^2 f(r)}} dr = \frac{2 v \omega}{a_0} \frac{r_0}{\sqrt{1 + a_0^2 + b_0^2}} \int_{r_{\text{min}}}^{r_{\text{max}}} \frac{dr}{r^2 \sqrt{f(r)}}$$

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or, using (3) and normalizing,

$$\psi = 2 \frac{x_o}{r_o} \int_{r_{\min}}^{r_{\max}} \frac{d(r/r_o)}{r^2/r_o^2 \sqrt{x_o^2 \left(1 - \frac{r_o^2}{r^2}\right) + y_o^2 - \frac{c \mu_o}{\pi} \ln \frac{r}{r_o}}}$$  \hspace{1cm} (14)$$

This angle is also shown in fig. 5 as a function of $x_o$ and $y_o$.

The angle $\psi$ is a constant for a given particle trajectory. Therefore, if $x_o$ and $y_o$ are considered as variables (instead of initial conditions), the particle will follow its $\psi$ - curve, which is closed in the second quadrant. In one period it will travel around this curve.

It follows from this that all trajectories with the same $\psi$ have the same shape. It is therefore possible to determine the shape of trajectories in a beam guide from a set of standard curves with $\psi$ as parameter.

These are shown in fig. 6 and 7. Represented are $\frac{r}{r_o}$ and $\mathcal{U}$ respectively, both as a function of $\frac{x_o}{\lambda}$. These curves were computed numerically from the equations

$$\frac{d \left( \frac{r}{r_o} \right)}{d \left( \frac{2}{\lambda} \right)} = \frac{\lambda}{r_o} \frac{\sqrt{f(r)}}{\varphi(r)} = \frac{\lambda}{r_o} \frac{\sqrt{\frac{1}{p} \left(1 - \frac{r_o^2}{r^2}\right) + y_o^2 - \frac{c \mu_o}{\pi} \ln \frac{r}{r_o}}}$$

and

$$\mathcal{U} = x_o \int_{r_{\min}}^{r_{\max}} \frac{d \left( \frac{r}{r_o} \right)}{r^2/r_o^2 \sqrt{x_o^2 \left(1 - \frac{r_o^2}{r^2}\right) + y_o^2 - \frac{c \mu_o}{\pi} \ln \frac{r}{r_o}}}$$

Finally, the relation between $\psi$ and $\frac{r_{\max}}{r_{\min}}$ is shown in fig. 8. This relation follows from (14) by substituting $r_o = r_{\min}$, $y_o = 0$, and the corresponding value of $x_o$ from eq (12).

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Angular acceptance of beam guides

a) Point source inside beam guide

If \( r_o, R_i, R_o \) and \( p/i \) are given, the conditions (7) applied to fig. 4 define a range of accepted initial angles. In fig. 9 this is illustrated. Particles with initial conditions inside the crosshatched areas are accepted. The solid angle accepted is equal to this area, multiplied by \( i/p \).

b) Point source outside beam guide

For many applications (e.g. transport of secondary particles from an "internal target") it will not be feasible to locate the source inside the beam guide. In this case the analysis of the angular acceptance becomes much more complicated. One possible method is the graphical construction outlined below.

We shall suppose that the source is located at a distance \( L_1 \) before the beginning of the beam guide, and at a distance \( d \) from its axis (see fig. 10). It is immaterial whether \( d \) is greater or smaller than \( R_o \), but it should be greater than \( R_i \).

The complication arises from the fact that the angles with the reference frame at the source (\( \gamma \) and \( \delta \)) are not the same as the initial angles \( \alpha_o \) and \( \beta_o \) at the point of entry into the beam guide.

By elementary but tedious reasoning it may be shown that

\[
\left(\frac{d}{L_1}\right)^2 = \tan^2 \alpha_o + \left(\tan \beta_o - \frac{r_o}{L_1}\right)^2
\]  

(15)

By substituting (5) and (11) we find

\[
\left(\frac{d}{L_1}\right)^2 \cdot \frac{p}{i} = x_o^2 + \left(y_o - \frac{r_o}{L_1} \sqrt{\frac{p}{i}}\right)^2
\]  

(16)
This equation may be represented in fig. 4 as a set of circles with radius \( \frac{d}{L_1} \sqrt{\frac{p}{i}} \) and with their centre on the \( y \) axis at \( y_0 = \frac{r_0}{L_1} \sqrt{\frac{p}{i}} \).

For several values of \( r_0 \) such circles may be drawn. The intersections of each circle with the corresponding curves for \( \frac{r_{\max}}{r_0} = \frac{R}{r_0} \) and \( \frac{r_{\min}}{r_0} = \frac{R_1}{r_0} \) represent trajectories that pass through the source and will just touch the outer and inner conductor respectively.

In order to translate the values of \( x_0 \) and \( y_0 \) of these trajectories into the angles \( \gamma \) and \( \delta \) at the source (fig. 10), we may remark that

\[
\tan \gamma = \frac{r_0}{d} \tan \alpha_0 \tag{17a}
\]

\[
\tan \delta = \frac{r_0}{L_1} \left( r_0 - L_1 \tan \beta_0 \right) - \frac{d}{L_1} \tag{17b}
\]

With (11) and (16), this is equivalent to

\[
\tan \gamma = \frac{1}{d} \sqrt{\frac{i}{p}} \ r_0 \ x_0 \tag{18a}
\]

\[
\tan \delta = \frac{r_0^2}{L_1 d} - \frac{d}{L_1} - \frac{1}{d} \sqrt{\frac{i}{p}} \ r_0 \ y_0 \tag{18b}
\]

The values of \( \tan \gamma \) and \( \tan \delta \) corresponding to \( x_0 \), \( y_0 \) and \( r_0 \) in the intersection points mentioned before may now be found from equations (18) and plotted in a \( (\tan \gamma, \tan \delta) \) plane. The enclosed surfaces so obtained are equal to the accepted solid angle.

Since this procedure is too slow if the angular acceptances of many different geometries have to be compared, a Mercury programme has been written that performs essentially the same operations. It computes the boundaries of the accepted area in the \( (\tan \gamma, \tan \delta) \) plane, as well as the solid angle accepted (ref. 4).
c) **Source of finite width**

This case may be evaluated as above by dividing the source into parts, each of which is treated separately as a point source.

d) **Example**

As an example, the following case is considered:

\[
R_\perp = 2 \text{ cm} \\
R_u = 10 \text{ cm} \\
p/i = 2 \times 10^4 \text{ eVc}^{-1} \text{ A}^{-1} \quad (\text{e.g. } p = 2 \text{ GeV/c, } i = 100 \text{ KA})
\]

Fig. 11 shows the angular acceptance as a function of \( d \) and \( L_\perp \). Obviously, the maximum solid angle is accepted for \( L_\perp = 0 \) (source in beam guide). With a distance of 1 m between source and beam guide, the solid angle has decreased by about 40 o/o.

The phase acceptance may be found by integrating over the cross section. In this example it is about 1.5 cm\(^2\) steradians.

The tolerance on \( d \) is quite large in terms of FS targets. If, for instance, a surface of 1 cm\(^2\) would be used, only a small fraction of the intrinsic phase acceptance of this beam guide would be exploited. Matching the beam guide dimensions to the source size by reducing its diameters would not only necessitate a proportional reduction of \( L_\perp \), but would also be limited by the increasing current density in the inner conductor. In par. 7 this will be considered in more detail.

For \( L_\perp = 0 \), the accepted solid angle is proportional to \( i/p \), as long as the approximations (8) are valid. This proportionality breaks down for finite values of \( L_\perp \), as fig. 12 shows.

6. **Curved beam guides**

If the axis of a beam guide is not straight, particles will still tend to be kept together by it. The bending action is not very efficient, however, due to its alternating character as the particles screw around the inner conductor.
The simple equations of par. 3 and 4 are no longer valid and the movement is no longer periodical with \( z \). It seems that numerical computation of separate trajectories is necessary in order to understand such a system. A Mercury programme (described in ref. 4) has been written that performs these computations.

As an example, the same beam guide as described above will be taken. It is supposed that the first part with a length of 10 m is straight (see fig. 13). Then follows a bend of \( 3^\circ \) with a bending radius of 100 m. The third part is straight again, and is supposed to be very long.

A point source is located inside the beam guide, as shown, at \( r_o = 3.5 \) cm.

In fig. 14 the range of angles accepted is shown in full lines for the case described, and in dashed lines for a straight beam guide of the same characteristics. The accepted solid angle has decreased by a factor 6 in this example due to the bending.

7. Power supply - Practical limitations

As is seen from the examples given, high currents are necessary if high energy particles have to be transported. The current may be supplied continuously, or if the particles come in bursts, a pulsed supply may also be used.

a) d.c. excitation

It may be possible in future to construct superconducting beam guides. This possibility is examined in Appendix 1.

In all other cases, the maximum excitation current may be limited either by the available power or by the difficulty of cooling the inner conductor. The obvious way of doing this is to make it hollow and pump water through it. It is shown in Appendix 2 that, if the maximum current is determined by the cooling problem, and if the outer diameter of the inner conductor (\( D \)) is given, the current limit is maximum if the diameter of the water hole is \( D/\sqrt{3} \). This gives the optimum balance between conductor section and cooling surface.
The actually obtainable maximum current is then dependent on the rate of heat transport per unit cooling surface. Supposing a reasonable value for this parameter, it is shown in Appendix 2 that the beam guide of the example used before could safely carry a direct current of about 40 kA. This is not a very high value, for applications in high energy physics; however, for particles of 750 MeV/c, the angular acceptance would still be as shown in fig. 11. A length of 25 m would dissipate 1 MW at this current.

For particles of still lower momentum the d.c. excitation becomes very cheap, since the power decreases with the square of the current.

b) Pulsed excitation

If the particles come in bursts, as they do with most accelerators, power may be saved and/or the peak current increased by pulsing. During each pulse not only the energy that is lost in the resistance must be supplied, but also the (sometimes much higher) inductive energy. The latter may, of course, be recuperated. The peak power will usually be such that an energy storage device between the mains supply and the beam guide is necessary.

An additional complication is the skin effect, which may increase the power dissipation if the pulse is fast. Also, the sudden heating during the pulse and the subsequent cooling may produce thermal stresses in the material, giving rise to fatigue problems. All these factors must be examined from case to case; a general treatment seems to be difficult.

As an example, the beam guide of pur. 5d will be taken again. The following two cases could, for instance, be considered: x)

---

x) Some details about the calculation of these parameters are given in Appendix 3.
<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>length</td>
<td>100 m</td>
<td>20 m</td>
</tr>
<tr>
<td>pulse duration (1/2 cycle of damped oscillation)</td>
<td>40 ms</td>
<td>0.3 ms</td>
</tr>
<tr>
<td>repetition frequency</td>
<td>0.5 c/s</td>
<td>55 c/s</td>
</tr>
<tr>
<td>peak current</td>
<td>100 kA</td>
<td>100 kA</td>
</tr>
<tr>
<td>peak magnetic energy</td>
<td>185 kJ</td>
<td>32 kJ</td>
</tr>
<tr>
<td>resistive loss per pulse</td>
<td>440 kJ</td>
<td>3.8 kJ</td>
</tr>
<tr>
<td>energy in storage capacitor</td>
<td>610 kJ</td>
<td>35 kJ</td>
</tr>
<tr>
<td>temperature rise on surface of inner conductor during pulse</td>
<td>1.7 °C</td>
<td>0.4 °C</td>
</tr>
<tr>
<td>skin depth</td>
<td>20 mm</td>
<td>1.7 mm</td>
</tr>
<tr>
<td>average power consumption</td>
<td>220 kW</td>
<td>210 kW</td>
</tr>
<tr>
<td>max. voltage on beam guide</td>
<td>600 V</td>
<td>6700 V</td>
</tr>
</tbody>
</table>

In both cases, the angular acceptance would be as shown in fig. 11 for 2 GeV/c particles and proportionally less for higher momenta.

The power supply for each of these examples would not present major problems. In case A the energy storage device would be the main part, in case B the switching device.

At the ends of a beam guide, water and current connections must be made to the inner conductor. This will cause some scattering and loss of particles, but it seems to be possible to keep 80 - 90 c/o of the aperture free from obstructions.

The inner conductor must be supported at regular distances. However, the forces exerted by the magnetic field on this conductor are always zero with d.c excitation and, with good alignment, nearly zero with pulsed operation. The supports may therefore be light and need not influence high energy particles greatly.

The outer conductor may serve as vacuum pipe at the same time, thus avoiding air scattering.

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8. Possible applications

The main desirable properties of beam guides are low price, simplicity of construction and high acceptance, for a wide momentum range.

They can, however, not be used for experiments that require small images (good focusing), since their nonlinear properties will distribute the beam of particles over the whole cross-section after a few wavelengths. Moreover, all particle trajectories will be at appreciable angles with the axis. Therefore the particles will spread out after leaving the beam guide.

However, for some applications this might not be too objectionable. Experiments with counters, for instance, do not always require a sharp focus. Directing many particles upon a large target for the production of other particles is another application where high intensity might be more important than good optics.

If used for focusing a secondary beam from an internal target in an accelerator like the CERN PS, a beam guide may be brought nearer to this target than conventional magnets or quadrupoles. Secondary beams might thus be made with an intensity that could not be obtained by other means.

S. van der Meer

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Appendix 1

Superconducting beam guides

For some applications, where high currents and d.c. operation are required, superconducting beam guides might be a proposition. The magnetic fields encountered are not particularly high (in the examples given only 10 kGauss), so that the critical field strength of the superconducting material need not be a serious limitation.

The current density obtained at present in Nb-Zr wire is already sufficient to make this application possible. In short wire samples, 3000 A/cm² has been reached. One should perhaps allow for a safety factor of 3 in constructing a large device. With 500 wires of 0.5 mm diameter, distributed in two layers around an inner conductor of 4 cm diameter, a total current of 100 KA could be obtained. Of course, at each end 500 joints would be needed between the wires on inner and outer conductor. These joints, if made near the outer conductor, would be in a weak magnetic field and should not give trouble.

In spite of the complicated construction, the expensive material and the necessity of providing the low temperature, the cost of such a construction might not be much higher than that of a nonsuperconducting device, because the power supply is very much simplified. Moreover, the d.c. excitation may be necessary for some applications.

If superconducting materials with a higher maximum current density would become available, their use in beam guides might become even more attractive.

For particles with higher momentum than available at present (e.g. 300 GeV/c), superconducting beam guides might be the only reasonable choice, because of the potentially high excitation current they allow.

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Appendix 2

Cooling of inner conductor

If the inner conductor is a tube with outer diameter \( D \), inner diameter \( d \) and resistivity \( \rho \), the power dissipated per unit length is

\[
P = \frac{4 i^2 \rho}{\pi (D^2 - d^2)} \tag{19}
\]

If \( Q \) is the heat carried away by the water per unit time and unit cooling surface, we have also

\[
P = \pi d Q \tag{20}
\]

From (19) and (20) we find

\[
i^2 = \frac{\pi^2 Q d (D^2 - d^2)}{4 \rho} \tag{21}
\]

If \( D, \rho \) and \( Q \) are given, this can be made maximum by making

\[
d = \frac{D}{\sqrt{3}} \tag{22}
\]

The power per unit length is then

\[
P = \frac{6 i^2 \rho}{\pi D^2} = \frac{\pi D Q}{\sqrt{3}} \tag{23}
\]

The maximum current is

\[
i = \pi \sqrt{\frac{D^3 Q}{6 \sqrt{3} \rho}} \tag{24}
\]
If we take the same example as in par. 5 d, supposing that the tube is made of copper and that the maximum safely obtainable $Q$ is 50 W/cm$^2$, we find $i_{\text{max}} \approx 40$ kA.

Higher values of $Q$ (up to 2000 W/cm$^2$) may be obtained by the "nucleate boiling" principle (ref. 5). This would bring the maximum d.c. current in the example up to 250 kA. However, apart from other disadvantages, the power dissipation at such a current (1.5 MW/m) would be excessive, as well as the water pressure needed.
Appendix 3

Calculation of parameters given on page 13

The following parameters for the conductors are assumed:

<table>
<thead>
<tr>
<th></th>
<th>material</th>
<th>resistivity ($\Omega \cdot m$)</th>
<th>outer diam (mm)</th>
<th>inner diam (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>inner conductor</td>
<td>copper</td>
<td>$2 \times 10^{-3}$</td>
<td>40</td>
<td>23</td>
</tr>
<tr>
<td>outer conductor</td>
<td>aluminium</td>
<td>$3.1 \times 10^{-8}$</td>
<td>210</td>
<td>200</td>
</tr>
</tbody>
</table>

It is further supposed that a storage capacitor is used with a capacitance such that the first half-cycle of the discharge will have the duration as indicated. Of course, the useful time is shorter by a factor 2 or 3.

The effective resistance of the inner and outer conductor (skin effect included) was supposed to be equal to the value for the fundamental frequency. This is not quite right for a pulse of one half-cycle, but the difference is not very important.

In computing the necessary energy storage, the damping of the oscillation by the resistance of the beam guide was taken into account. This is an important factor in example A, but not in example B.

The skin effect is negligible in example A, but considerable in case B.

After the first half-cycle the current is supposed to be cut off by the ignitron switch. This is thought to be possible without danger of reverse ignition in both examples.

In case A, the beam guide could be fed through a pulse transformer, in order to obtain more suitable parameters for the capacitor bank and the (single) ignitron.

Case B might require 10 ignitrons, but only a small capacitor bank would be needed.

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\[ x = \sqrt{\frac{F}{c}} \tan \alpha \]
\[ y = \sqrt{\frac{F}{c}} \tan \beta \]

\( F \) in eV, \( c \) in \( m/s \)
\( \alpha \) in degrees
\( \beta \) in degrees

Fig. 1
$\frac{\lambda}{\pi_0} \sqrt{\frac{E}{p}}$ ($i$ in $\text{mm}$, $p$ in $\text{eV/c}$)

\[
\psi \text{ (radians)}
\]
Fig. 8
\[ \frac{i}{\rho} = 5 \times 10^{-5} \frac{A}{(eV)^{-1} \text{C}} \]

**Fig. 11**

**Fig. 12**

\[ d = 7 \text{ cm} \]
\[ L = 80 \text{ cm} \]