CERN 61-11

Data Handling Division

11th April, 1961

ORGANISATION EUROPÉENNE POUR LA RECHERCHE NUCLÉAIRE
CERN EUROPEAN ORGANIZATION FOR NUCLEAR RESEARCH

THE SCATTERING OF POSITIVE 120 MEV PIONS ON PROTONS

Part I

A. Loria, P. Mittner, R. Santangelo, I. Scotoni and G. Zago

Part II

B. Aubert, A. Brenner, Y. Goldschmidt-Clermont, F. Grard
G.R. Macleod, A. Minguzzi-Ranzi and L. Montanet

G E N E V E

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FOREWORD

The present report describes the first bubble chamber experiment performed with a beam of particles produced by a CERN accelerator. Apart from its intrinsic contents, it has at CERN some historical value. The work reported here goes to the early days when a way to establish an effective collaboration between inside and outside physicists was a major problem. This first trial proved useful in several respects. The analysis of the photographs was carried out as a collaboration between a University group and CERN, thus being a forerunner of the fruitful pattern later established for the other bubble chamber exposures. It also offered an early and valuable opportunity of systematic tests for the methods of measurements developed by the CERN Iep group.

G. Bernardini

THE SCATTERING OF POSITIVE 120 MEV PIONS ON PROTONS

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CERN - Genève

SUMMARY. An investigation of the elastic scattering of 120 MeV positive pions by protons is described. The experiment was done by exposing a liquid propane chamber to the CERN 600 MeV synchro-cyclotron. The results refer to 5405 selected events in which the contamination from scattering on carbon nuclei is shown to be negligible.

The values obtained for the phase-shifts are:

$$\alpha_{31} = -2.60^\circ \pm 0.69^\circ$$

$$\alpha_3 = -11.05^\circ \pm 1.32^\circ$$

$$\alpha_{33} = +31.67^\circ \pm 1.01^\circ$$

The value of α_3 differs significantly from that expected if the linear dependence of α_3 on the momentum, which has been proposed by some authors, is assumed. The present result is discussed, particularly in connection with some recent theoretical development.

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Introduction

Because of the current interest in the behaviour of the s wave phase-shifts, the differential cross section for the elastic scattering of positive 120 MeV kinetic energy pions by protons was measured by exposing the Padua propane chamber, without magnetic field, to the CERN 600 MeV synchro-cyclotron. The chamber and the auxiliary equipment used, including the optics and timing circuits, have already been described⁽¹⁾.

Pions internally produced in the synchro-cyclotron were used. The method of extraction is explained in detail by J. Ashkin et al⁽²⁾. The general layout of the beam is shown in Fig.1. The pions were first analysed and focussed by two bending magnets and then, after having been filtered by 2 cm of polythene in order to stop the protons, were collimated by means of a hole in a thick lead block. Monitoring was effected by two counters at either ends of the hole. The bubble chamber was located at 80 cm distance from the second counter.

The energy was measured in the conventional manner by tracing differential range curves. One of these is reproduced in Fig. 2, and indicates a mean energy of 142 MeV, which was requested in order to have pions of 120 MeV in the middle of the chamber. The spread in beam energy was ± 2.5 MeV.

Two exposures were effected. A suitable number of satisfactory tracks per picture, four on the average in the first and six in the second, was recorded by using the beam at the maximum available intensity. A photograph of some typical events, which consist of two prong stars produced by beam tracks, is reproduced in Fig.3.

During the first exposure, which took place in September 1958, 10,000 pictures were taken and the analysis was carried out at Padua. During the second, in April 1959, another 30,000 pictures were taken; 10,000 of these were also analysed by the Padua group, and 20,000 at Geneva by the CERN group. Preliminary results referring to the first exposure were quoted at the 1959 Kiev Conference⁽³⁾.

Part I of this paper is devoted to the description of the methods employed and to the results obtained by the Padua group; the relevant data were presented to the 1960 Rochester Conference⁽⁴⁾. Part II refers to the CERN contribution. The two parts are practically independent reports. Repetitions have been sometimes preferred to the lack of precision and of clarity which would have inevitably followed from fusing the two reports into one.

The phase-shifts calculated by using the two different procedures are compatible with each other. The combined values are quoted and discussed as the final result of this work in Part III.

A list of abbreviations used throughout follows.

List of abbreviations

X	- coordinate perpendicular to Y and Z.
Y	- coordinate in the direction of the beam.
Z	- coordinate in the direction of the optical axes.
The origin of the coordinates is the centre of the bubble chamber.	
c.m.s.	- the centre of mass system of reference; parameters referring to the c.m.s. are indicated by a star, e.g. θ^* .
π_i	- the incident π .
γ	- the angle of π_i with the average direction of the beam.
π_s	- the scattered π .
p	- the scattered proton, or momentum in general, according to the context.
hydrogen event	- the scattering of π_i by a free proton.
carbon event	- the scattering of π_i by a proton of a carbon nucleus.
i, j, k	- the unit vectors respectively of the π_i , π_s and p momenta.
θ_π	- scattering angle of π_i : $\sin \theta_\pi = \mathbf{i} \times \mathbf{j} $.
θ_p	- scattering angle of p : $\sin \theta_p = \mathbf{j} \times \mathbf{k} $.
c and d	- deviation from coplanarity: $c = \cos \beta = \frac{\mathbf{i} \cdot \mathbf{j} \times \mathbf{k}}{ \mathbf{i} \times \mathbf{k} }$ CERN $d = \cos \alpha = \frac{\mathbf{i} \times \mathbf{j} \cdot \mathbf{k}}{ \mathbf{i} \times \mathbf{j} }$ Padua
φ	- angle between the plane of π_i and π_s , and the glass windows.
θ'_p	- angle between π_i and p, expected for the measured value of θ_π in the hypothesis of a 120 MeV hydrogen event.
Δ	- $\theta'_p - \theta_p$.
σ_d	- r.m.s. value of d.
σ_Δ	- r.m.s. value of Δ .
σ	- cross section.
Ω	- solid angle.

Part I. Padua contribution

1. Scanning of the pictures

1.1 Reprojection and spatial reconstruction.

The two stereoscopic pictures were reprojected onto a screen and a drawing was made where, by shifting the paper, the images of the front fiducial marks given by the two lenses were brought into coincidence. Every track was approximated by eye with the "best" straight line, taking into account multiple scattering.

It can be shown that, in the approximation in which straight lines in the chamber transform into straight lines in the pictures⁽⁵⁾, the coordinates of points in space may be calculated, using simplified formulae, from measurements on the images⁽⁶⁾. The geometric quantities useful in the description and in the analysis of the events were calculated by means of an electronic computer.

1.2 Fiducial criteria.

On the basis of some exploratory work on distributions of samples of events, a fiducial region was fixed in order to reduce as much as possible the geometrical biases. A parallelepiped was chosen, centrally located in the chamber, and whose dimensions were 12 cm in the direction of the beam, 10 cm in width and 4 cm in the direction of the optical axes. The dimensions of the chamber were, in the same order, 21 x 14 x 7 cm, while the beam window was 10 x 4 cm. We also excluded the events for which $\gamma > 6^\circ$, as the r.m.s. value of γ was measured to be 2.7° , and those for which $\theta_\pi < 43^\circ$, as the proton range appeared too short to allow satisfactory measurements to be made.

1.3 Criteria of selection of the hydrogen events.

It is practically certain that a two prong star produced by a beam track is not a hydrogen event if any one of the following criteria is not satisfied within the limits of error of the measurements.

- a) π_i , π_s and p must be coplanar. We measured the coplanarity by means of d .
- b) The correlation between the angles θ_{π} and θ_p must be consistent with the kinematics of the π - p collision at 120 MeV. We measured this correlation by means of Δ .
- c) The range of any particle stopping in the liquid must be consistent with the kinematics of the π - p collision at 120 MeV. In applying this criterion we took account of the fact that the proton range may be shorter than expected owing to interactions with carbon nuclei, e.g. the $C^{12}(p,pn)C^{11}$ (7). Four percent of the hydrogen events in our second exposure have in fact a significantly short proton range.

Criterion a) does not involve the identification of the tracks, which is necessary before applying criteria b) and c). On the basis of the direction of the track, of the bubble density and of the delta rays,

π_i can generally be distinguished from π_s . In 70% of the cases the scattered proton was recognizable from the range; if this was not available the bubble density was almost always sufficient for a decision. In some instances it was impossible to identify the particles and, in a very few cases of stars lying in planes almost normal to the windows, even the association of the tracks in the two stereoscopic pictures was uncertain.

All possibilities were always examined and if one of them was consistent with a hydrogen event, the star was considered as such.

1.4 Scanning procedure.

Scanning was done by looking along all incident tracks for two prong stars. Some of them could be excluded at the very first inspection on the basis of the fiducial criteria either because their apex was located, in at least one of the reprojections, outside a well defined fiducial area, or because at least one of the reprojections of δ was greater than 6° . Other stars were excluded at once from being hydrogen events on the bases of the selection criteria because the coplanarity test mentioned by Bassi et al⁽⁵⁾ clearly failed, or because the transverse momentum was not conserved in at least one of the reprojections, or because the proton was scattered backwards in the laboratory system of reference. In connection with criterion c) we also took advantage of the fact that the kinetic energy of π_1 , calculated from Θ_π and from the proton range, should be smaller than the upper limit of the π_1 energy. A quick estimate was sometimes sufficient to allow exclusion of the star.

The number of two prong stars meeting the requirements for acceptance at the scanning was 1595 for the first, and 2046 for the second exposure.

Apart from the clear cut cases mentioned above, the selection procedure described below was adopted.

2. Selection procedure for the hydrogen events. Contamination of carbon events.

2.1 Selection procedure for the hydrogen events.

When the second exposure at the synchro-cyclotron was made, the analysis of the first was practically completed. Each set of events was dealt with in a rather different manner and some results of the first exposure were even used as a basis for the analysis of the second exposure.

This point of view was for instance adopted in fixing the basic feature of the selection procedure, according to which we rejected an event if

$$\rho = \left[\left(\frac{d}{\sigma_d} \right)^2 + \left(\frac{\Delta}{\sigma_\Delta} \right)^2 \right]^{\frac{1}{2}}$$

was too great. In fact instead of using σ_d and σ_Δ as they result from the distribution of d and Δ of all the events, we took the values referring to the first exposure only, as they were certainly satisfactory.

As the precision of the measurements is affected by the geometry, the events were divided into three intervals of θ_π : 43° - 90° , 90° - 135° and 135° - 180° . The distributions of d are reported in Fig.I.1 and those of Δ in Fig.I.2.

It is apparent that the distributions, apart from a tail which is clearly due to carbon events, are very nearly gaussian. The r.m.s. values are .048, .024, .025 for d , and 1.8° , 1.3° , 1.4° for Δ , respectively in the above intervals.

The distribution of ρ for all the events of both exposures is shown in Fig.I.3 which we used to selected the hydrogen events. If

$\rho > 7$ the event was rejected.

If $2 < p < 7$ the event was remeasured and reanalysed with great care by taking into account all available information; on this basis a reasonable decision could always be reached.

If $p < 2$ the event was as a rule accepted; a few events in this category were also rejected because the proton range was appreciably greater than was consistent with the upper energy limit of the beam.

There were 1141 events from the first exposure and 1624 from the second exposure selected by the procedure we have described. All results are based on these 2765 events. If we ignore for the first exposure the condition $\theta < 6^\circ$ the number of events increases to 1392.

2.2 The energy of the primary pions.

We have described how a suitable beam was used so as to have pions of 120 MeV energy in the middle of the chamber. In order to feel quite sure about the behaviour of the beam during the experiment and also in order to acquire more detailed information about the primary pions of the events, it is of course desirable to infer their characteristics from the events themselves. This was done separately, but with the same procedure, for the two exposures.

For the second exposure we took into account the 1211 events in which the proton range could be measured, excluding temporarily the 390 in which the proton left the illuminated region of the chamber; 23 events were not considered because the proton range was too small for a reliable calculation of the energy.

In Fig.I.4 the number of events with stopping protons is plotted vs the momentum of the primary pions at the point of interaction. The spread is evidently due both to the beam spread and to the energy loss in the fiducial region of the chamber. As the tail towards the low values

of the momentum is interpreted as due to nuclear collisions of the scattered protons, a cut-off was made at 204 MeV/c so that 49 more events were temporarily excluded. For the remaining 1162 events we calculated the total energy of the primary pions.

We know that the energy spread of the beam is comparatively small, so we may safely assume that the visibility of the whole proton track is related only to the geometry of the event and not to the momentum of the primary pion. The mean momentum $\bar{p}'$ of all particles of the beam, normalized to the central plane of the chamber, may therefore be calculated by taking into account only the events with stopping protons, i.e. the only ones for which we know the momentum of the primary pions at the point of interaction.

In Fig.I.5 the number of the 1162 remaining events is plotted versus the momentum p' ; the spread in momentum is now much smaller and a new cut-off was made at 207 MeV/c. We obtained:

$$\bar{p}' = 220.5 \pm 4.3 \frac{\text{MeV}}{c}, \text{ to which corresponds the energy}$$

$$\bar{E}' = 121.4 \pm 3.6 \text{ MeV}$$

The energy spread may be compared with the value ± 2.5 MeV deduced from Fig.2.

In Fig.I.6 the number of all the 439 excluded events is plotted vs the coordinate Y in the direction of the beam. From $\bar{p}'$ we have calculated the energy which can be attributed to each event in order to estimate the total energy of the primary pions for the 1601 events of the second exposure which we have considered.

In conclusion we have for the energy of the primary pions of the second exposure:

$$\bar{E}_2 = 121,7 \pm 5.6 \text{ MeV},$$

where the spread, deduced from Fig.I.4, includes the energy loss in the fiducial region.

For the first exposure we obtained similarly:

$$\bar{E}_1 = 119.9 \pm 5.4 \text{ MeV.}$$

2.3 Contamination of carbon events in the selected hydrogen events.

It is interesting to evaluate how many carbon events were misinterpreted as hydrogen events on the basis of the analysis we have just outlined.

For this purpose, using the method given by Mr. Chrétien et al⁽⁸⁾ we have plotten in Fig.I.7 the angular correlation of all the carbon events of both exposures for which $d < 2\sigma_d$. In the same figure five lines have been drawn: the central one is the expected angular correlation for a π -p collision at 120 MeV and the others define two regions of equal area, a central one B, and a peripheral one consisting of two strips A and C. On the two regions we have, respectively, $\Delta < \sigma_\Delta$ and $\sigma_\Delta < \Delta < 2\sigma_\Delta$, where we adopted for σ_Δ a mean value of 1.5° .

It appears that the carbon events do not show any tendency to group themselves anywhere on the θ_p, θ_π -plane, so that the number of carbon events which have been included in the hydrogen events can be estimated by subtracting the carbon events in the region B from those in the region A+C. The result is, for the number c of the erroneously included events:

$$c = 1 \pm \sqrt{13 + 12} = -1 \pm 5.$$

The contamination thus appears to be completely negligible. This conclusion has been confirmed as follows.

We may assume that the carbon events are uniformly distributed both with respect to $\frac{\Delta}{\sigma_{\Delta}}$ and $\frac{d}{\sigma_d}$. The distribution vs ρ is therefore given by the equation $dN_c = k \rho d\rho$ where k is a constant, and the histogram of the carbon events should be approximated, up to a certain value of ρ , by a straight line passing through the origin of the coordinate system, and having a slope k .

For practically all the hydrogen events $\rho < 4.5$.

From the carbon events distributed between $\rho = 4.5$ and $\rho = 12$ we have first calculated k by means of a weighted mean squares procedure, and then the number of carbon events expected to lie between $\rho = 0$ and $\rho = 4.5$. The number of events misinterpreted as hydrogen events is obviously given by the difference between expected and recognized carbon events in the same interval. Using this method we obtained for the contamination the value of 0.8 ± 0.3 percent, where the error is purely statistical.

3. Angular distributions and phase shift analysis

3.1 Detection efficiency and discussion of experimental biases.

The detection efficiency was satisfactorily uniform throughout the fiducial volume.

For the first exposure this is demonstrated by Fig.I.8 and Fig.I.9. These show that the 1392 pions colliding inside the fiducial region and a generic sample of 900 beam tracks are equally distributed, within the limits of statistical error, both if they are plotted vs Z as in Fig.I.8 or vs X as in Fig.I.9. The same holds for the second exposure.

It is clear however that even within the fiducial region some events get lost because the visibility is in some cases reduced. In fact when φ is near 90° or θ_π near 180° , the projected angles are small and the tracks difficult to distinguish from the beam, while when θ_π approaches its lower limit of 43° the recoil proton may be difficult to see.

In order to investigate this problem we have plotted the number of selected events vs the azimuthal angle φ for various intervals of θ_{π}^* . Fig.I.10 refers to the first and Fig.I.11 to the second exposure. Note that 53° in the c.m.s. are equivalent to 43° in the laboratory system of reference.

It may be seen that, within the limits of the statistical error, Fig.I.10 b) and c) do not show any bias, while a) and d) indicate a possible small bias for big values of φ . A similar and bigger bias is noticeable in Fig.I.11 b), c), and d), but a new feature of Fig.I.11 in comparison with Fig.I.10 is the systematic lack of events for small values of φ .

There are two strong arguments in favour of the conclusion that the events concerned either represent merely a statistical fluctuation, or have been shifted and not lost.

First, as it is absurd that the visibility should be affected by small values of φ , the only possibility is that the missing events are included in the carbon group; nevertheless if we plot the number of carbon events vs φ we obtain the lower histogram of Fig.I.11 e), which also shows a lack of events for small values of φ .

Secondly, by comparison of several independent scanings the absolute scanning efficiency for events with $\varphi < 45^\circ$ was found to be 99.0 ± 0.8 percent for the first and 98.0 ± 1.0 percent for the second exposure, with no limitation on θ_{π} . The number of events lost in independent scanings is plotted respectively vs φ and vs θ_{π} in Fig.I.12 a) and b) for 101 events of the first exposure, and in Fig.I.13 a) and b) for 117 events of the second exposure. It appears clearly from Figs.I.12 a) and I.13 a) that there was no bias for losing events at small values of φ and it is worthwhile noticing that the histograms for the two exposures have very nearly the same shape.

We may add that the direction of the beam in the second exposure had a dip of about 2° . This certainly caused a shift of events from small values of φ into greater ones.

In regard to θ_π , the curves in Figs. I.12 b) and I.13 b) indicate what the distribution of the events just considered would have been in the absence of biases, on the basis of the final result of this work. As the curves fit the histograms rather well, we can conclude that the biases against the detection of events with either great or small θ_π were not very important.

For the first exposure we decided that a correction for scanning inefficiency at small values of φ should be applied to the histograms reported in Fig. I.10 a) and d). Carbon contamination was also considered. For this purpose three histograms referring to the first exposure only, similar to that reported in Fig. I.3, were drawn for the usual intervals of θ_π . The results for the contamination, in order of increasing θ_π , were respectively:

$$.8 \pm .6, .6 \pm .5 \text{ and } 1.2 \pm .4 \text{ percent.}$$

As customary at this energy of the primary pions, the angular distribution was approximated by means of the expression:

$$A + 2 B \cos \theta^* + 3 C \cos^2 \theta^* .$$

A calculation of the coefficients A, B and C showed, as appears from Table I.1, that the geometrical biases and the carbon contamination were negligible in comparison with the statistical errors.

For the second exposure however, the histograms of Fig. I.11 were first accepted without modification, and then successively cut at $\varphi = 75^\circ$ and $\varphi = 60^\circ$. It also appears from Table I.1 that the cut-offs did not appear to have any considerable effect on the values of the coefficients

A, B and C. Nevertheless we finally adopted the cut-off $\varphi = 60^\circ$, taking into account that the decrease in statistical precision was small. Carbon contamination was neglected as it was shown to be practically negligible in the first exposure.

3.2 Corrected angular distribution and phase shift analysis

As the difference of 1 MeV energy is irrelevant for the phase - shift analysis, and in any case small in comparison with the energy spread of the primary pions, we have considered the events as if they were all due to positive pions of 120 MeV.

The calculation of the phase-shifts from our experimental data was made by the Department of Physics of Rochester University. The formulae used and the method by which the computation was performed has been explained in detail by Barnes et al⁽⁹⁾.

In our case the program had to be modified in some places, since we could not obtain the experimental differential cross section $\frac{d\sigma}{d\Omega}$ but only quantities which are proportional to it; if N is the number of events in an interval of θ^* we may write

$$\frac{N}{\sin \theta^*} = K \frac{d\sigma}{d\Omega}$$

where K is a normalization factor. K is determined by the total cross section, for which we took 95 ± 6 mb, interpolated from experimental values at energies in the neighbourhood of 120 MeV⁽¹⁰⁾. The modification was done so that the best values of the phase shifts α_{31} , α_3 , α_{33} and of the normalization factor K , i.e. those which gave the minimum value of χ^2 , could be found simultaneously. $\frac{\sqrt{N}}{\sin \theta^*}$ was taken as the statistical error of $\frac{N}{\sin \theta^*}$.

Five degrees has been taken as the interval of θ^* , and χ^2 has the following expression :

$$\chi^2 = \sum_{i=1}^{25} \left(\frac{N}{\sin \theta^*} - K \frac{d\sigma}{d\Omega} \right)^2 \times \left(\frac{\sin \theta^*}{\sqrt{N}} \right)^2 + (\sigma_{\text{exp}} - \sigma_{\text{calc}})^2 \times \left(\frac{1}{\Delta \sigma_{\text{exp}}} \right)^2$$

In Table I.2 are reported the essential steps of the calculations referring to the first exposure. The meaning of the headings is obvious and the only thing we should perhaps note is that the errors were estimated as simply the square roots of the number of the events recorded, since the corrections due to carbon contamination and geometrical biases are small.

In Table I.3 the same steps are reported for a total distribution obtained by considering together the events of the first distribution corrected, and of the second distribution cut at $\varphi = 60^\circ$.

Table 4 contains the phase shifts, the error matrices and the χ^2 values for the first corrected, for the second and for the total distributions. We may note that the error matrix does not involve K for the first distribution; the reason is that in this case the error matrix was computed by means of the original, and not the modified program.

No rounding off of the computer data has been done in the tables.

We consider the compatibility of the phase-shifts obtained for the three distributions as a demonstration of the internal consistency of our measurements, but assume of course as the final result of the work done in Padua the values, here reported in degrees:

$$\alpha_{31} = -3.15^\circ \pm 0.95^\circ \quad \alpha_3 = -11.12^\circ \pm 2.10^\circ \quad \alpha_{33} = 31.63^\circ \pm 1.15^\circ$$

$$\chi^2 = 17.6$$

referring to the total distribution.

We reproduce in Fig.I.14 the experimental normalized differential cross section of Table I.3, third and fourth columns, together with the curve showing the calculated distribution corresponding to the above phase-shifts.

Part II. CERN contribution

1. Analysis of the pictures

1.1 Scanning criteria

The pictures were examined using two projectors allowing the two stereoscopic views to be scanned side-by-side; the orientation of the view was such as to allow the observer to scan easily along the direction of the beam tracks. Pictures with more than 15 incident particles were disregarded, so as not to miss scattering close to 0 or 180 degrees. In general the quality of the pictures was such as to allow the identification of the recoil proton by ionisation; but as this was not always possible (particularly for events in planes orthogonal to the chamber glasses) this information was not used systematically in the analysis. Only in those cases where the kinematic analysis of the events lead to ambiguities was ionisation information used to identify the proton track. All two prong stars where the conservation of transversal momentum was not obviously violated were included in the list of events to be measured; those events where the proton was ejected at more than 90° to the incident particle were eliminated from this list.

In addition those interactions where the projected angle χ was greater than 10° were not included for measurement, as well as any events where the incident particle underwent a scattering $> 2^\circ$ without visible recoil.

We have not fixed any limits to the useful volume of the chamber in order to be able to determine the influence of such geometrical effects on the final angular distribution. Similarly we have not excluded, a priori, the interactions with a very short proton track, in so far it was possible to define the range of such protons by a minimum of two points.

The films were scanned twice independently using the above criteria. From the results we were able to estimate the scanning efficiency: it was 95% for all the events. For those events where $\phi > 60^\circ$ it was 97% and for events where $\phi > 60^\circ$ and having a scattering angle $\theta > 40^\circ$ (proton range > 5 mm) it was 99%.

1.2 Measurement

Measurements were made using two digitised instruments: the Digitised Protractor⁽¹¹⁾ and the IEP⁽¹²⁾.

Reconstruction and kinematic analysis of the events were made using programs written for the CERN Mercury Computer^(13, 14, 15). The reconstruction programs calculated, for each coordinate of a measured point, an error determined by the method of least squares. Unfortunately this procedure could not be applied to the determination of track directions, as only two stereoscopic views were available for the reconstruction. Nevertheless, a set of repeated measurements on a series of selected tracks were used to estimate the precision of the digitised protractor. The standard deviations thus obtained were

X, Y coordinates	: 0.3 mm
Z coordinate	: 2.5 mm
Direction of tracks	: 1.2°
Coplanarity	: 0.03

The precision of IEP is superior to that of the digitised protractor, and we therefore used the IEP to measure those events which were intrinsically difficult to measure, and which therefore required the maximum available precision. These events were of two types: those where the recoil proton

track was very short i.e. > 5 mm, and those where the pion was scattered at about 90° so that the image of its track lay nearly parallel to the X axis. For this second type we tried to measure on IEP the coordinate of corresponding points on the two views; some 90 cases were left at the end of experiment as un-measurable since the quality of these photographs did not allow corresponding bubbles to be identified on the two views.

The complementary use of IEP and of the digitised protractor allowed us to measure all the events satisfying the scanning criteria, with the exception of the 90 cases mentioned above. The quality of the measurements depends naturally on the apparatus used, and we have analysed the two groups of measurements as independently, as possibly by weighting each measurement by a function not only of the geometry of the event, but also of the precision of its measurement.

2. Selection of elastic events

2.1 Selection Criteria

Since the selection of elastic events by measurement of the proton range is too restrictive, we preferred to use the coplanarity C and the angular correlation Δ . Each event is characterised by two values of Δ , one for each of the two possible mass assignments for the secondary particles. Since the best angular correlation for a given measurement of a given event, does not necessarily correspond to the best interpretation for an elastic scatter due to the errors of measurement, we have always chosen the correct interpretation by a systematic comparison of the computed quantities with the original photographs, using ionisation and proton range data to resolve any ambiguities.

For the systematic selection of the hydrogen events we used practically the same parameter ρ defined in Part I; however, the normalisation factors σ_c and σ_Δ were based on all the events including the carbon contamination satisfying the following criteria.

- 1) $-51 \leq x \leq + 51$ mm
 - 2) $-61 \leq y \leq + 70$ mm
 - 3) $-33 \leq z \leq + 30$ mm
 - 4) $c < 0.4$
 - 5) $\Delta < 15^\circ$
 - 6) $\gamma < 6^\circ$
 - 7) $\theta_p > 40^\circ$ ($\theta_{cm} = 50^\circ$)
 - 8) $\varphi \leq 80^\circ$
- } coordinates of the point of interaction

In fact the events not satisfying criteria 1 2 3 7 and 8 can be excluded at the scanning table; events excluded by 4 or 5 are due to collisions on carbon; criteria 6 is intended to exclude low energy interactions which may still give a good value of ρ .

2.2 Histograms of C and Δ

From the application of the various tests described above, each event is allocated on index V, whose value depends upon

- 1) the position of the point of interaction in the chamber,
- 2) the angle ϕ ,
- 3) the angle γ ,
- 4) whether or not the proton stops in the chamber.

To define more closely the useful volume of the chamber and the angle ϕ we have plotted histograms of events as functions of these quantities.

To determine σ_c and σ_Δ as a function of θ_π we made histograms of C and Δ for events satisfying the conditions described in 2.1 and for 20° intervals of θ_π from 40° to 180° .

All these operations were carried out by a program written for the CERN Mercury Computer. In addition to the histograms this program gives, for each event, the data necessary for the first analysis: the index V, the coordinate of the point of interaction x y and z, the deviation from coplanarity C, the dip angle φ , the measured scattering angle θ_p and θ_π the calculated proton angle θ_p , the centre of mass pion angle θ_π^* , and the proton energy derived from the observed range in the chamber.

2.3 Results

After excluding events which do not satisfy the conditions described in 2.1, the histograms mentioned above represented a total of 2640 events, of which 2100 were measured with the digitised protractor. The final analysis corresponds to this total of events.

Fig.II.1 and Fig.II.2 show the distributions of the measured events respectively as a function of Y and φ ; table II.1 reports the results obtained for σ_c and σ_Δ as a function of θ_π . Fig.II.1 shows a loss of event for $|Y| > 60$ mm. Fig.II.2 shows that the loss of events is noticeable above 60° . These two effects result from the reduction in scanning efficiency, but we need not take account of the loss in the following analysis since we limit ourselves to the angular distribution of events satisfying the condition

$$\varphi < 60^\circ \quad |Y| < 60 \text{ mm}$$

One notice in table II.1 that the parameter σ_c shows a maximum for value of θ_π near 90° ; this effect was foreseen in 1.2.

3. Final Analysis: angular distribution and phase analysis

3.1 Determination of the incident energy

To check the incident beam energy we have taken 350 events for which the proton range was measurable. The incident energy was determined from the proton kinetic energy and the pion scattering angle, with a precision better than 3% on each event. (The angle θ_{π} is known to $\pm 1.2^\circ$ and the proton range to ± 0.5 mm). From these 350 events we have found, with the same method used in Part I, a primary energy of 121.5 ± 3 MeV.

3.2 Angular distribution

To see whether the measured angular distribution depends in any way upon the geometrical selection criteria used, we made histograms of the distribution under the following conditions:

1. Useful volume

1.1	$-6 \leq y \leq 6$	$-5.1 \leq x \leq 5.1$	$-3.3 \leq z \leq +3.0$
1.2	$-4.5 \leq y \leq 6$	$-4.5 \leq x \leq 4.5$	$-2.8 \leq z \leq +2.5$
1.3	$-4.5 \leq y \leq 5.5$	$-4 \leq x \leq 4.5$	$-2.8 \leq z \leq +2.5$

2. Limits to φ

2.1	$\varphi \leq 80^\circ$
2.2	$\varphi \leq 60^\circ$

3. Limits to γ

3.1	$\cos \gamma \geq 0.9986$
3.2	$\cos \gamma \geq 0.9945$

4. Limits to ρ

$$4.1 \quad \rho \leq 2$$

$$4.2 \quad \rho \leq 1.414$$

Thus a total of 24 histograms were made.

The study of these histograms shows that the angular distribution is not affected by the following choice of criteria:

$$-5.1 \leq x \leq +5.1 \text{ cm} \quad -6 \leq y \leq 6 \text{ cm} \quad -3.3 \leq z \leq +3.0 \text{ cm}$$

$$\psi \leq 60^\circ \quad \cos \chi \geq 0.9945 \quad \rho \leq 2$$

The distribution is not sensitive to the choice of χ and of the volume; on the other hand, the choice of ψ and ρ is indicated by inspection of figures II.3a, II.3b, II.3c.

In addition the histograms show a loss of events in the regions

$$\cos \theta_{\pi^+} > 0.55$$

$$\cos \theta_{\pi^-} < -0.95$$

It is to be noted that the angular distribution shows also an irregularity for

$$-0.15 > \cos \theta_{\pi^+} > -0.35$$

This region corresponds to angles θ_{π^+} between 85° and 95° ; in the analysis of the angular distribution we have treated this anomaly in the two following ways:

1) by neglecting the zone defined by

$$-0.15 > \cos \theta_{\pi^+} > -0.35$$

- 2) by correcting the distribution by a special analysis of the 90 unmeasurable events which correspond to this region (see § 1.2). We estimate that one third of these events can be interpreted as elastic scatters, and we have completed the angular distribution by taking account of the additional 30 unmeasurable events. By introducing larger errors on the points in this region, we find that the two methods give compatible results.

- 3) To determine the coefficients ABC of the distribution

$$A + 2B \cos \theta^* + 3C \cos^2 \theta^*$$

we have varied the limits of $\cos \theta^*$ from 0.55 to -0.95 in intervals of 0.05 from 0.55 to 0.30, and from -0.95 to -0.75. The table II.2 shows the results obtained: from these results we take the following values :

$$A = 0.340 \pm 0.010 \quad B = -0.140 \pm 0.035 \quad C = 0.247 \pm 0.053$$

after normalisation to a total cross section of $95 \pm 6\text{mb}$, (10).

- 4) The distribution of events as a function of ρ (Fig.II.4) allows an estimate to be made of the contamination of carbon scatters. Extrapolating the curve in the region $0 < \rho < 2$ we find that an upper limit for the carbon contamination is of the order of 2.5% for the events for which $\rho < 2$

3.3 Phase-Shift analysis

Each solution in table II.2 has been analysed by a computer program to calculate the phase-shifts α_3 , α_{31} and α_{33} with their errors.

The inspection of this table leads to the choice :

$$\alpha_3 = -11.0 \pm 1.7$$

$$\alpha_{31} = -2.0 \pm 1.0$$

$$\alpha_{33} = 31.8 \pm 2.1$$

$$\chi^2 = 19.5$$

which correspond to the limits :

$$\cos \theta_{\pi}^x > -0.95 \quad \cos \theta_{\pi}^x < 0.50$$

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Part III. Conclusion

The results obtained at Padua and Geneva are consistent with each other. By combining them we have for the phase shifts :

$$\alpha_{31} = -2.60^\circ \pm 0.69^\circ$$

$$\alpha_3 = -11.05^\circ \pm 1.32^\circ$$

$$\alpha_{33} = +31.67^\circ \pm 1.01^\circ$$

The values of α_{31} and α_{33} agree well with previous determinations and will not be discussed here. However, α_3 differs by more than two standard deviations from the value -8.2° , expected on the basis of the linear dependence on the momentum, which has been proposed by some authors, e.g. J. Orear⁽¹⁶⁾ and more recently S.W. Barnes et al⁽⁹⁾. This dependence previously thought to be valid at least up to a pion energy of about 200 MeV in the laboratory can now be excluded.

Our result has in fact confirmed with markedly better precision the measurements with nuclear plates in the region around 100 MeV⁽¹⁹⁾ which gave for α_3 values significantly smaller than indicated by that linear dependence. More evidence from other sources has accumulated in the meantime both at higher and at lower energies. We have plotted some representative points in Fig.III.1, where three lines are drawn.

The straight line represents the linear dependence.

By neglecting the re-scattering one can easily obtain from the dispersion relations given by G.F. Chew et al⁽¹⁷⁾ the following relations:

$$\sin \alpha_3 \cos \alpha_3 = \eta \frac{M}{\omega + M} \left(-2 \lambda^+ - \frac{\omega}{M} \lambda^- \right) \quad (1)$$

$$\sin \alpha_1 \cos \alpha_1 = \eta \frac{M}{\omega + M} \left(-2 \lambda^+ + \frac{2 \omega}{M} \lambda^- \right),$$

where η is the momentum in the c.m.s. in units of $m_\pi c$, M is the mass of the proton and ω is the total energy in the c.m.s., minus the mass of the proton.

The dashed curve in Fig.III.1 corresponds to the above relations with :

$$\lambda^+ = + 0.007 \quad \text{and} \quad \lambda^- = + 0.562$$

The values of λ^+ and λ^- have been obtained by normalizing the relations (1) to the phase shifts.

$$\alpha_3 = -4.01^\circ \quad \text{and} \quad \alpha_1 = + 6.65^\circ$$

measured by S.W. Bernes et al⁽⁹⁾ in an experiment of π^+p and π^-p elastic scattering at 41.5 MeV.

Neglecting the re-scattering though could hardly be justified, as the phase shifts are already too big in the energy region which we consider; in fact the curve appears to be too low, especially towards the higher energies.

It is certainly more correct to neglect the re-scattering for the expression

$$\frac{1}{2} \left[\sin 2 \alpha_1 - \sin 2 \alpha_3 \right] \quad (2)$$

as the value of $\sin^2 \alpha_1 - \sin^2 \alpha_3$, which appears in the integral representing the re-scattering, is experimentally known to be very small.

In fact J. Bowcock et al⁽¹⁸⁾ have recently calculated the expression (2), neglecting the re-scattering and taking instead into account a possible $\pi\pi$ interaction in the $J = 1, T = 1$ state.

Taking now for α_3 our experimental value and for α_1 the value calculated at 120 MeV and assuming that

$$\alpha_1 = \eta \times [0.1668 \pm 0.0230] \quad (9)$$

we obtain for the above expression the value 0.390 ± 0.034 .

We have inserted the resulting point in Fig.III.2, which is a copy from J. Bowcock et al; the agreement with the theoretical predictions is very good. Unfortunately the energy at which our experiment was planned, when the problem had not yet arisen, is too low in order to choose among the various curves, and therefore no indication on the $\pi\pi$ interaction can be given. A measurement on π^+p scattering at 310 MeV is in progress.

It might perhaps be of some interest to note that, if we replace empirically $\sin \alpha \cos \alpha$ by $\tan \alpha$ in the relations (1) we obtain from $\tan \alpha_3 = \eta \frac{M}{\omega + M} (-2\lambda^+ - \frac{\omega}{M}\lambda^-)$ the solid curve, which fits rather well the experimental data up to 600 MeV.

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	First distribution	First distribution corrected	Second distribution	Second distribution cut at $\varphi=75^\circ$	Second distribution cut at $\varphi=60^\circ$
A	1.000 ± 0.059	1.000 ± 0.060	1.000 ± 0.049	1.000 ± 0.054	1.000 ± 0.060
B	-0.445 ± 0.060	-0.399 ± 0.062	-0.376 ± 0.047	-0.389 ± 0.056	-0.389 ± 0.062
C	0.532 ± 0.082	0.582 ± 0.083	0.603 ± 0.064	0.617 ± 0.075	0.582 ± 0.083
χ^2	30.6	33.1	50	47.8	33.8
$\exp.\chi^2$	40	40	40	40	40

TABLE I.1
Values of the coefficients A, B and C for various distributions

θ^*	N_1	$N_{1, \text{corr.}}$	$(\frac{d\sigma}{d\Omega})_{\text{exp}} = \frac{N_{1, \text{corr.}}}{K \sin \theta^*}$	$\frac{\sqrt{N_1}}{K \sin \theta^*}$	$(\frac{d\sigma}{d\Omega})_{\text{calc.}}$
55.5°	39	41.64	6.054 $\frac{\text{mb}}{\text{sterad}}$	0,9381 $\frac{\text{mb}}{\text{sterad}}$	5.426 $\frac{\text{mb}}{\text{sterad}}$
60.5°	30	32.31	4.448	0.7825	5.009
65.5°	27	29.20	3.845	0.7112	4.663
70.5°	36	38.19	4.854	0.7855	4.407
75.5°	41	42.21	5.219	0.8032	4.255
80.5°	35	35.00	4.252	0.7189	4.231
85.5°	35	35.00	4.206	0.7112	4.311
90.5°	39	39.00	4.673	0.7485	4.564
95.5°	47	47.00	5.657	0.8251	4.930
100.5°	49	49.00	5.971	0.8532	5.424
105.5°	39	38.64	4.804	0.7728	6.040
110.5°	43	42.48	5.434	0.8333	6.764
115.5°	62	61.26	8.132	1.0390	7.580
120.5°	72	71.14	9.892	1.1727	8.471
125.5°	70	69.16	10.178	1.2238	9.414
130.5°	68	67.18	10.585	1.2913	10.387
135.5°	71	70.15	11.991	1.4316	11.365
140.5°	57	56.32	10.608	1.4136	12.324
145.5°	61	60.84	12.869	1.6499	13.237
150.5°	56	56.00	13.625	1.8208	14.082
155.5°	52	52.00	15.023	2.0833	14.837
160.5°	41	41.00	14.716	2.2983	15.480
165.5°	36	37.73	18.054	2.9394	15.995
170.5°	25	27.99	20.318	3.8402	16.369
175.5°	10	11.30	17.256	5.1335	16.590
Tot.	1141				

$$\chi^2 = 16.010$$

TABLE I.2

Essential steps of the phase shift analysis
for the first corrected distribution

θ^*	$N_{2,\psi < 60^\circ}$	N_T	$(\frac{d\sigma}{d\Omega})_{\text{exp.}} = \frac{N_T}{K \sin \theta^*}$	$\frac{\sqrt{N_T}}{K \sin \theta^*}$	$(\frac{d\sigma}{d\Omega})_{\text{calc.}}$
55.5°	35	76.64	5.654 $\frac{\text{mb}}{\text{sterad}}$	0.6460 $\frac{\text{mb}}{\text{sterad}}$	5.307 $\frac{\text{mb}}{\text{sterad}}$
60.5°	30	62.31	4.353	0.5513	4.921
65.5°	31	60.20	4.022	0.5184	4.605
70.5°	38	76.19	4.914	0.5631	4.379
75.5°	34	76.21	4.782	0.5476	4.257
80.5°	36	71.00	4.377	0.5195	4.256
85.5°	45	80.00	4.879	0.5456	4.380
90.5°	38	77.00	4.682	0.5336	4.633
95.5°	35	82.00	5.009	0.5531	5.016
100.5°	44	93.00	5.751	0.5963	5.525
105.5°	41	79.64	5.025	0.5631	6.151
110.5°	50	92.484	6.003	0.6242	6.883
115.5°	51	112.256	7.562	0.7138	7.704
120.5°	59	130.136	9.183	0.8052	8.597
125.5°	63	132.216	9.870	0.8584	9.540
130.5°	78	145.184	11.608	0.9634	10.511
135.5°	75	145.148	12.591	1.0450	11.486
140.5°	81	137.316	13.125	1.1201	12.439
145.5°	68	128.844	13.830	1.2185	13.347
150.5°	46	102.00	12.594	1.2470	14.186
155.5°	48	100.00	14.661	1.4661	14.935
160.5°	37	78.00	14.207	1.6085	15.573
165.5°	30	67.73	16.447	1.9984	16.084
170.5°	18	45.99	16.941	2.4981	16.454
175.5°	11	22.30	17.281	3.5804	16.674
Tot.	1122				

$$\chi^2 = 17.617$$

TABLE I.3

Essential steps of the phase shift analysis
for the total distribution

	First distribution corrected	Second distribution					Total distribution		
α_{31}	$-.0520 \pm .0234$	$-.0595 \pm .0232$					$-.0550 \pm .0165$		
α_3	$-.1780 \pm .0421$	$-.2075 \pm .0540$					$-.1940 \pm .0367$		
α_{33}	$-.5559 \pm .0190$	$.549 \pm .022$					$.552 \pm .020$		
$K \times 10^{-28}$	$.8347 \pm .0622$	$.799 \pm .060$					$1.645 \pm .114$		
			α_{31}	α_3	$K \times 10^{-28}$	α_{33}	α_{31}	α_3	$K \times 10^{-28}$
α_{31}	$.54275 \times 10^{-3}$	$.39060 \times 10^{-3}$	$.5388 \times 10^{-3}$	$.4639 \times 10^{-3}$	$.3177 \times 10^{-3}$	$.1130 \times 10^{-3}$	$.2732 \times 10^{-3}$	$.2460 \times 10^{-3}$	$.4205 \times 10^{-3}$
α_3	1.77290×10^{-3}		$.65133 \times 10^{-3}$	2.919×10^{-3}	1.7280×10^{-3}	$.4070 \times 10^{-3}$		1.3483×10^{-3}	2.2766×10^{-3}
Error Matrix									
$K \times 10^{-28}$									
α_{33}			$.36193 \times 10^{-3}$	3.5695×10^{-3}	$-.6530 \times 10^{-3}$	$.4726 \times 10^{-3}$		13.0028×10^{-3}	-1.6787×10^{-3}
χ^2	16.010	18.444							17.617

TABLE I.4
Results of the phase shift analysis for various distributions

θ°		40-60	60-80	80-100	100-120	120-140	140-160	160-180
ϕ	A	0.102	0.134	0.136	0.120	0.109	0.098	0.086
	B	0.13	0.104	0.13	0.105	0.101	0.102	0.074
Δ	A	3.15	3.05	3.25	3.16	3.07	3.15	2.70
	B	3.33	3.24	3.49	4.19	3.44	3.39	2.26

TABLE II.1

A : events measured on IEP

B : events measured on Baby-IEP

Cos θ^* > Cos θ^* <		- 0.95	- 0.90	- 0.85	- 0.80	- 0.75
0.55	A	48.79	49.37	49.60	48.60	49.45
	B	- 27.32	- 27.25	- 27.16	- 27.82	- 26.90
	C	27.14	25.47	24.72	28.42	24.69
0.50	A	48.83	49.47	49.74	48.61	49.62
	B	- 27.44	- 27.54	- 27.53	- 27.85	- 27.23
	C	27.00	25.10	24.24	28.38	24.13
0.45	A	48.37	49.03	49.29	47.91	49.04
	B	- 25.55	- 25.90	- 26.00	- 25.72	- 25.67
	C	29.05	27.10	26.15	31.32	26.52
0.40	A	47.65	48.24	48.39	46.61	47.83
	B	- 21.02	- 21.52	- 21.71	- 20.21	- 21.05
	C	33.72	32.90	31.12	38.21	33.05
0.35	A	47.84	48.54	48.79	46.95	48.35
	B	- 23.12	- 24.15	- 24.73	- 22.12	- 23.90
	C	31.50	28.90	28.00	35.95	25.43
0.30	A	47.76	48.29	48.34	46.02	47.36
	B	- 15.75	- 17.12	- 17.17	- 12.05	- 14.72
	C	38.72	36.02	35.95	47.81	40.07

TABLE II.2

Fig. 1 - Layout of the pion beam.

Fig. 2 - Differential range curve for a 142 MeV pion beam.

Fig. 3 Photograph showing three typical scattering events.

Fig. I. 1 - First exposure. Plot of the number of events accepted at the scanning, vs the deviation from coplanarity d .

Fig. I. 2 - First exposure. Plot of the number of events accepted at the scanning, vs $\Delta = \theta_p - \theta_\pi$

Fig. I. 3 - Plot of the number of events accepted at the scanning of both exposures, vs $\left[\left(\frac{d}{\sigma_d} \right)^2 + \left(\frac{\Delta}{\sigma_\Delta} \right)^2 \right]^{\frac{1}{2}}$.

Fig. I. 4 - Second exposure. Plot of the number of selected events with stopping protons, vs the momentum of the primary pions at the point of interaction.

Fig. I. 5 - Second exposure. Plot of the number of selected events with stopping protons, vs the momentum of the primary pions, normalized to the central plane of the chamber.

Fig. I. 6 - Second exposure. Plot of the number of selected events without stopping protons plus selected events with significantly short R, vs a coordinate Y in the direction of the beam.

Fig. I. 7 - Angular correlation of the coplanar carbon events ($d < 2\sigma_d$) of both exposures. Dashed lines include the acceptance region for hydrogen events (B). Region A + C and region B have equal area.

Fig. I. 8 - First exposure. Depth distribution of 1392 events found in the fiducial region, and of 900 beam tracks (shaded area).

Fig. I. 9 - First exposure. Lateral distribution of 1392 events found in the fiducial region, and of 900 beam tracks (shaded area).

Fig. I.10 - First exposure. Plots of the number of selected events vs the azimuthal angle ϑ , for various intervals of the pion scattering angle ϑ_π^* .

Fig. I.11 - Second exposure. Plots of the number of selected events and of the number of carbon events (N_c) vs the azimuthal angle φ , for various intervals of the pion scattering angle ϑ_π^* .

Fig. I.12 - First exposure. Plots of the number of events lost in three independent scannings, vs ϕ and vs $90^\circ - \phi$.

a)

b)

Fig. I.13 - Second exposure. Plots of the number of events lost in two independent scannings, vs ϕ and vs $\phi\pi$.

Fig. I.14 - Experimental normalized differential cross section deduced from the total distribution. The curve shows the cross section calculated with the phase shifts α_{31} , α_3 and α_{33} extracted from the total distribution.

Fig II .1

*Distribution of the measured events
along the direction of the beam.*

Fig II.2

*Distribution of the measured events
vs. the azimuthal angle φ .*

Fig: II.3a.

Angular distribution

$$\varphi \leq 60^\circ \quad \rho \leq 2$$

Fig. II.3b.

Angular distribution

$$\varphi \leq 80^\circ \quad p \leq 2.$$

Fig. II.3c.

Angular distribution

$\varphi \leq 60^\circ$ $\rho < 1,414$

Fig. II.4

ρ distribution

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- (*)- D waves are also considered.

Fig. III. 1 - α_3 versus momentum.

Fig. III. 2 - Theoretical curves for S - waves, from J. Bowcock et al.
See text for reference.