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COULOMB BLOCKADE IN TWO-DIMENSIONAL ELECTRON SYSTEMS IN A STRONG MAGNETIC FIELD

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Coulomb blockade in two-dimensional electron systems in a strong magnetic field

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Abstract

We consider tunneling in a system of two parallel two-dimensional layers of electrons in the presence of a strong perpendicular magnetic field. The tunneling current between the two layers is blocked at low voltages due to strong correlations in each subsystem. We calculate the current-voltage characteristics by modeling the two subsystems as Wigner crystals, and find a current peak at a finite voltage. The peak is broadened by the excitation of collective modes of the Wigner crystals. Our results are in a good quantitative agreement with recent experiments.

Keywords: Coulomb blockade, quantum Hall effect, Wigner lattice

Two areas of condensed matter physics which have been studied intensively in the last years are small, man-made structures (quantum dots), [1] and two-dimensional electron systems in a strong magnetic field, [2] which are also the topics of this meeting. One of the main focuses of the first subfield are phenomena classified under the heading Coulomb blockade. [3] Coulomb blockade is a consequence of the granularity of the electric charge which becomes particularly important in small systems: the motion of a single electron can change the electrostatic energy of the quantum dot by an amount comparable to the thermal energy $k_B T$. Under suitable circumstances this can result in a energy barrier blocking the movement of electrons through the device. Therefore, in a transport measurement the current flow can be suppressed at small bias voltages, and the current-voltage dependence may be strongly nonlinear. A key requirement for observing Coulomb blockade is that the charge of an electron is confined to a sufficiently small region so that the effective capacitance C associated with the electron motion is small and consequently the charging energy $\frac{e^2}{2C}$ is large compared to $k_B T$. In quantum dots the confinement is usually achieved by means of gate electrodes connected to external voltage sources. By this technique electrons can be confined to areas that are less than one micrometer in size, resulting in an effective capacitance C of less than 1 fF, which leads to a temperature requirement of the order of 1 K.

The study of the properties of an extended two-dimensional electron system in a strong magnetic field was greatly intensified after the discoveries of the integer [4] and fractional [5] quantum Hall effects. Much of the interest in these systems stems from the fact that a strong magnetic field quenches the kinetic energy of electrons, thereby effectively amplifying the effects of electron-electron interactions. Consequently, novel phases of matter have been discovered, all of which owe their existence to the strong Coulomb interaction. A common property of the strongly interacting phases is that the motion of electrons is more correlated than in the two-dimensional electron *gas* (in the absence of the magnetic field), the electron system may form a fractional quantum Hall *liquid*, or even a Wigner *solid*. As the experimental techniques have been developed further, more complicated structures like

double-layer systems have been studied both experimentally and theoretically, which have also revealed a great deal more structure than initially expected.

The two systems described above appear to be very different, in particular in size, and it was therefore quite surprising when experiments on double-layer quantum Hall systems showed current-voltage behavior quite reminiscent of that seen in Coulomb-blockading nanostructures. Ashoori *et al.* [6] observed that the tunneling conductance from a three-dimensional to a two-dimensional electron system was strongly suppressed at low voltages in the presence of a perpendicular magnetic field, and Eisenstein *et al.* [7–9] observed a similar suppression of the tunneling conductance between two two-dimensional electron layers. In the experiments by Eisenstein the current was found to exhibit a peak at a voltage of about 7 mV, and the structure was seen to persist up to about 10K. It was suggested in Ref. [7] that the effect may be due to correlations within the two layers, i.e. the “small scale” necessary for Coulomb blockade is due to the correlations introduced by electron-electron interactions, a view that was later confirmed by our work. [10–12] In the following we concentrate on the experimental setup used by Eisenstein *et al.*, and take all relevant parameters (electron density, device structure) from Ref. [7].

Several other theories have addressed the same problem that we are studying. By means of a numerical diagonalization of a finite-size system Hatsugai, Bares, and Wen [13] calculated the single-electron spectral function and discussed their results in terms of selection rules. He, Platzman, and Halperin [14] also presented analytic results in the low-voltage regime based on the Chern-Simons theory for filling factor $\nu = 1/2$. They found an “activated” behavior at small voltages, $I \sim e^{-V_0/V}$, with $V_0 = \frac{2\pi e^2}{\epsilon \ell_c}$ (ϵ is the dielectric constant and ℓ_c the magnetic length). Yang and MacDonald [15] considered the effects of impurities, but did not find a pseudogap at small voltages. Efros and Pikus [16] used a lattice gas model which they studied by Monte Carlo techniques. Finally, Varma, Larkin, and Abrahams [17], and also Renn and Roberts, [18] studied the role of interlayer correlations in the experiments by Eisenstein *et al.*.

The individual electron layers in Eisenstein’s experiment were sufficiently clean to exhibit

the fractional quantum Hall effect at low enough temperatures. The tunneling experiments, however, were performed at higher temperatures so that the FQHE structure was washed away. Consequently, the correlations in the individual layers in the actual experiment are most likely liquid-like. However, properties of correlated electron liquids are difficult to calculate. Therefore we make the simplifying assumption that individual layers are described by classical Wigner crystals. [19] This clearly overestimates the long range correlations, but may be sufficient for the short correlations, and is certainly a convenient first approximation. If the two Wigner crystals were purely static, transferring an electron from one layer to another would require an external voltage corresponding to the Madelung energy of the triangular lattices, and the current-voltage curve would show a sharp peak at the corresponding voltage. In reality the peak is quite broad, suggesting that we must include some dynamic effects. The dynamics of Wigner crystals is due to lattice vibrations, which in the presence of a magnetic field are divided into magnetophonons, which are gapless, and magnetoplasmons, which have a gap equal to the cyclotron frequency. Since we are primarily interested in the low-energy properties of the system, we ignore the magnetoplasmon modes and only consider the gapless magnetophonons. The inclusion of magnetoplasmons is straightforward and has been discussed in Ref. [11].

To write down a model Hamiltonian we consider an area of the size $2\pi\ell_c^2$, which accommodates one electron state per Landau level. It is convenient to treat the electron state within this small area separately as a fermion, and describe all the other electrons in a layer collectively as a Wigner crystal. The individually treated electron is allowed to tunnel between the two layers, which we call left (L) and right (R). We assume that all tunneling events take place between lattice sites and interstitial sites of the Wigner crystals, as depicted in Figure 1. Since the tunneling electron is charged, it must couple to the dynamic modes of the Wigner crystals. The coupling can be evaluated in exactly the same way as the electron-phonon coupling in ordinary solids. Hence, we arrive at the model Hamiltonian

$$H = H_0 + H_T^+ + H_T^- = [\varepsilon_L + \sum_{\alpha} (M_{\alpha L} a_{\alpha} + M_{\alpha L}^* a_{\alpha}^{\dagger})] c_L^{\dagger} c_L + [\varepsilon_R + \sum_{\alpha} (M_{\alpha R} a_{\alpha} + M_{\alpha R}^* a_{\alpha}^{\dagger})] c_R^{\dagger} c_R + \sum_{\alpha} \hbar \omega_{\alpha} a_{\alpha}^{\dagger} a_{\alpha} + T_{LR} c_R^{\dagger} c_L + T_{LR}^* c_L^{\dagger} c_R. \quad (1)$$

Here $c_{L(R)}$ destroys an electron in the left (right) quantum well, and $\varepsilon_{L(R)}$ is the corresponding site (Madelung) energy. The operator a_{α} destroys a magnetophonon whose wavevector and quantum well are collectively given by α , $M_{\alpha L(R)}$ is the coupling between the magnetophonon and the tunneling electron in the left (right) layer, and ω_{α} is the magnetophonon frequency. The tunneling matrix elements T_{LR} can be calculated using the method of Bardeen. [20]

The Hamiltonian (1) has two great virtues: first, it contains no adjustable parameters, and second, H_0 is exactly solvable. The exact solvability is due to the fact that the state of the tunneling electron is not affected by the emission and absorption of magnetophonons, and there is no interaction between the magnetophonons. Therefore, magnetophonons with different α are independent, and the number of phonons in each mode obeys a Poisson distribution. [21]

The tunneling current can be obtained, to first order in $|T_{LR}|^2$, from the interaction picture correlation functions $I^{-+}(t) = \langle H_T^{-}(t) H_T^{+}(0) \rangle$ and $I^{+-}(t) = \langle H_T^{+}(t) H_T^{-}(0) \rangle$ as

$$I(V) = \frac{S}{2\pi \ell_c^2} \frac{e}{\hbar^2} \int_{-\infty}^{\infty} dt \left[e^{ieVt/\hbar} I^{-+}(t) - e^{-ieVt/\hbar} I^{+-}(t) \right], \quad (2)$$

where the angular brackets indicate thermal average of the magnetophonon modes and S is the sample area. By following the procedure outlined in standard text books, [21] we find an expression for the current between two mutually uncorrelated Wigner crystals as

$$I(V) = \frac{S}{2\pi \ell_c^2} \frac{e}{\hbar^2} \nu(1 - \nu) |T_{LR}|^2 [C(eV/\hbar) - C(-eV/\hbar)]. \quad (3)$$

Here the function $C(\omega)$ is the Fourier transform of

$$C(t) = e^{i(\varepsilon_L - \Delta_L - \varepsilon_R + \Delta_R)t/\hbar} \exp \left(- \sum_{\alpha} \frac{|M_{\alpha R} - M_{\alpha L}|^2}{(\hbar \omega_{\alpha})^2} [(1 + N_{\alpha})(1 - e^{-i\omega_{\alpha} t}) + N_{\alpha}(1 - e^{i\omega_{\alpha} t})] \right), \quad (4)$$

where the polaron shifts $\Delta_{L(R)}$ are given by

$$\Delta_{L(R)} = \sum_{\alpha} \frac{|M_{\alpha L(R)}|^2}{\hbar \omega_{\alpha}}. \quad (5)$$

The exponential prefactor in (4) tells how much energy is lost to lattice distortions that remain after the Wigner crystal has come to rest after a tunneling event and the subsequent magnetophonon emissions. We neglect this shift, mainly because such distortions most likely do not exist in the experimental system, but even in a true Wigner crystal this distortion energy is small compared with the peak energy. This means that the threshold voltage for tunneling is zero.

While we find an explicit expression for $C(t)$ in time space, it is difficult to calculate its Fourier transform by means of direct integration. The method of choice is an integral-equation method due to Minnhagen, [22] which at zero temperature reads

$$\omega C(\omega) = \int_0^{\infty} \frac{d\Omega}{2\pi} g(\Omega) C(\omega - \Omega), \quad (6)$$

where the coupling constant is given by

$$g(\Omega) = 2\pi \sum_{\alpha} \frac{|M_{\alpha R} - M_{\alpha L}|^2}{\hbar^2 \Omega} \delta(\Omega - \omega_{\alpha}). \quad (7)$$

This, in connection with the conditions $C(\omega) = 0$ for $\omega < 0$ and

$$\int_0^{\infty} \frac{d\Omega}{2\pi} C(\Omega) = 1, \quad (8)$$

can be used to effectively calculate $C(\omega)$. At a finite temperature it is necessary factorize $C(t)$ into emission and absorption parts, derive integral equations for both of them, and calculate the Fourier transform through a convolution. [11]

The current-voltage characteristics of our model are displayed in Fig. 2. The current at small voltages is seen to be strongly suppressed, and it exhibits a broad peak centered around 9 mV. Compared to the experimental data, the theoretical peak occurs at a slightly higher voltage (9 mV vs. 7 mV) and the theoretical current is somewhat higher (4...12 nA vs. 2...3 nA), but the overall agreement is quite impressive, considering the lack of adjustable parameters and the crudeness of the model. The large variation in the theoretical peak

current as a function of the magnetic field is primarily due to the assumption that the electron lattices in the two layers are *mutually* uncorrelated which yields the statistical prefactor $(1 - \nu)$; including inter-layer correlations would reduce the variation. The model predicts that the peak position varies as inverse distance between electrons, and the peak width scales as $1/\sqrt{B}$, at least qualitatively in agreement with experiments (however, see also Ref. [23]). Experimentally, Eisenstein *et al.* found that over the voltage range $\frac{1}{3}V_{\text{peak}} < V < \frac{2}{3}V_{\text{peak}}$ the current very accurately followed an “activated” curve $I \sim e^{-V_0/V}$, where $V_0^{\text{exp}} \approx 13.4$ mV at $B = 13.5$ T; our model yields an approximately “activated” behavior over the same voltage range with $V_0^{\text{th}} \approx 15$ mV, as shown in Fig. 3.

The model is also quite successful in explaining the thermal properties of the system. The zero-bias conductance is experimentally observed to be activated, $I \sim e^{-T_A/T}$, with an activation temperature that increases with magnetic field. The model yields approximately activated behavior with activation temperatures $T_A^{\text{th}} \approx 8$ K at $B = 8$ T and $T_A^{\text{th}} \approx 13$ K at $B = 13$ T, which are approximately 20% larger than the experimental values. It is worth noticing that the energy scale associated with V_0 is about ten times larger than the scale T_A ; V_0 is primarily controlled by the maximum of $C(\omega)$ (the peak voltage), whereas T_A is most sensitive to the tails of the emission and absorption parts of $C(\omega)$.

From a theoretical point of view one of the most interesting questions concerns the current-voltage relation at a small voltage. This is the regime in which the low-energy, small- q processes are most important and our model is expected to be least accurate. At the very smallest voltages, we find a non-analytic behavior $I \sim e^{-\sqrt{V_0/V}}$. At voltages in excess of a few percent of the maximum magnetophonon frequency (i.e. of the order of few tens of microvolts), we find a powerlaw $I \sim V^\gamma$, where the exponent increases with the magnetic field ranging from 4.8 at $B = 8$ T to 7.0 at $B = 13$ T. This is to be contrasted with the behavior $I \sim e^{-V_0/V}$ found by He *et al.* [14] Unfortunately, at low voltages the current signal is so weak that no direct comparison with the experiments has been possible.

There is one striking difference between the current-voltage characteristics seen in a standard Coulomb blockade experiment and in the tunneling setup that we have considered:

in the former the current is a *monotonically increasing* function of the voltage, whereas in the latter it exhibits a *peak* at a finite voltage. The monotonically increasing current in the former case is essentially due to single-particle excitations, which have a by and large continuous spectrum. In our case, however, the single-particle spectrum (Landau levels) is discrete, and, moreover, the magnetophonon spectrum is bounded from above, so that transferring a large amount of energy to the magnetophonon system requires the excitation of a large number of phonons. Since the number of magnetophonons in each mode is Poisson distributed, multi-phonon effects are suppressed by a combinatorial factor, and the tunneling current is consequently small.

In conclusion, we have analyzed a simple model of tunneling between two two-dimensional electron systems in a strong perpendicular magnetic field. The model was found to reproduce surprisingly accurately the experimentally observed Coulomb blockade -like behavior. In many ways the result is discouraging as it suggests that the phenomenon is not very sensitive to the detailed structure of the two electron systems. On the other hand, it provides a simple reference point for more sophisticated calculations. It would be very interesting to identify the points where our simple model clearly fails, as they might serve as starting points for the refinements of the model.

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FIGURES

FIG. 1. Schematic picture of a tunneling event in our model. An electron starts from a lattice position in one of the triangular Wigner crystals and ends up in an interstitial position on the other side of the barrier. The surrounding electrons experience a sudden change in the potential and as a result relaxation processes take place in the electron systems on both sides of the barrier. In our model the energy released in these relaxation processes is carried away mainly in the form of magnetophonons.

FIG. 2. Calculated tunnel current as a function of voltage for four different magnetic fields. The parameter values used in the calculations have been taken directly from the experiment. [7] The lattice parameter of the Wigner crystals $a_0=270$ Å corresponding to an electron density $n \approx 1.6 \times 10^{11} \text{ cm}^{-2}$. The quantum well width was $L=200$ Å, the barrier thickness $d=175$ Å, and the total sample area was $S=0.0625 \text{ mm}^2$. The barrier height was $V_b=250$ meV.

FIG. 3. The current on a logarithmic scale vs. $1/V$. The thick curve shows our calculated results for $B=13$ T, while the straight line corresponds to an “activated” I - V curve, $I \sim e^{-V_0/V}$ with $V_0=15$ mV. In the experiment [8] this kind of activated behavior was found over a wide range of voltages, at $B=13.5$ T the measured V_0 was ≈ 13.4 mV.

Figure 1

Figure 2

Figure 3