ORGANISATION EUROPÉENNE POUR LA RECHERCHE NUCLÉAIRE
CERN EUROPEAN ORGANIZATION FOR NUCLEAR RESEARCH

SCATTERED-OUT EXTERNAL PROTON BEAM
AT THE CERN PROTON SYNCHROTRON

by

B. Dayton and H. Winzeler
Propriété littéraire et scientifique réservée pour tous les pays du monde. Ce document ne peut être reproduit ou traduit en tout ou en partie sans l'autorisation écrite du Directeur général du CERN, titulaire du droit d'auteur. Dans les cas appropriés, et s'il s'agit d'utiliser le document à des fins non commerciales, cette autorisation sera volontiers accordée.

Le CERN ne revendique pas la propriété des inventions brevetables et dessins ou modèles susceptibles de dépôt qui pourraient être décrits dans le présent document; ceux-ci peuvent être librement utilisés par les instituts de recherche, les industriels et autres intéressés. Cependant, le CERN se réserve le droit de s'opposer à toute revendication qu'un usager pourrait faire de la propriété scientifique ou industrielle de toute invention et tout dessin ou modèle décrits dans le présent document.

© Copyright CERN, Genève, 1961

Literary and scientific copyrights reserved in all countries of the world. This report, or any part of it, may not be reprinted or translated without written permission of the copyright holder, the Director-General of CERN. However, permission will be freely granted for appropriate non-commercial use.

If any patentable invention or registrable design is described in the report, CERN makes no claim to property rights in it but offers it for the free use of research institutions, manufacturers and others. CERN, however, may oppose any attempt by a user to claim any proprietary or patent rights in such inventions or designs as may be described in the present document.
SCATTERED-OUT EXTERNAL PROTON BEAM
AT THE CERN PROTON SYNCHROTRON

by

B. Dayton and H. Winzeler

GENEVA
SCATTERED-OUT EXTERNAL PROTON BEAM
AT THE CERN PROTON SYNCHROTRON

by

B. Dayton* and H. Winzeler†.

****

INTRODUCTION

Protons elastically scattered through more than 14 milliradians to the left in target No.1 of the proton synchrotron are able to penetrate the vacuum chamber and emerge from the fringing field of the magnet with nearly their full orbit energy. These protons, together with other positive particles of lower momentum, come out in a band that stretches several metres along the main shielding wall. At each point in the median plane all of the particles have nearly the same direction so that a collimating slit, properly oriented, permits a beam of highly monodirectional particles to pass through the shielding wall into the experimental area of the South Hall. A subsequent magnetic analysis results in a very pure beam of monoenergetic protons since all secondaries from the target have momenta lower than that of the elastically scattered primaries.

The fringing field not only deflects all positive particles but defocuses them horizontally and focuses them vertically towards the median plane to an extent that depends upon their initial angles of emission and upon their momenta. By making use of the vertical focusing property, it has been possible to choose a location for the collimator through the shielding wall such that the emerging beam of

*) Physikalisches Institut der Universität Bern, and Universitetets Institut for teoretisk fysik, Copenhagen.
†) Physikalisches Institut der Universität Bern, and CERN, Geneva.
elastically scattered protons reaches its narrowest vertical dimension in the middle of the experimental area (see Fig. 1). At this point protons with full momenta are therefore relatively intense compared to particles with lower momentum which are already out of focus.

Soon after the proton synchrotron commenced operation, a number of emulsion exposures were carried out to explore the scattered proton beam. Since then various groups have made use of and further developed the beam for emulsion and bubble chamber exposures and for a number of studies with counters using a magnetically analysed beam, the intensity of which, in the middle of the South Hall, is of the order of $10^4$ protons/cm$^2$ per pulse for a circulating current of $2 \times 10^9$ protons/pulse. In the meantime it has been possible to increase the intensity of the scattered proton beam by an order of magnitude, using quadrupole focusing magnets.

**COMPUTED TRAJECTORIES IN THE HORIZONTAL PLANE**

Particle trajectories in the horizontal plane have been calculated by the CERN Mercury computer$^1$ for positive particles emitted from target No.1 at various angles $\theta_0$ up to 60 milliradians and having various momenta $p$ from $p_0$ to 0.15 $p_0$, where $p_0$ is the momentum of the circulating protons when hitting the internal target.

Some of the trajectories have been computed for the case where the target is on the centreline of the equilibrium orbit ($y_0 = 0$) and others for the case when the target is located 2 cm radially inward ($y_0 = -0.02$) from the equilibrium orbit, since both modes of operation have been employed.

In describing the horizontal trajectories it is convenient to express positions about the proton synchrotron in polar co-ordinates where the distance $r$ is measured from target No.1 and the angle $\alpha$ is measured from the line tangent to the orbit at the target. $\beta$ is the angle between the trajectory at some point P outside of the fringing field and the line connecting this point P with the target T.
All symbols are illustrated in Fig. 2, in which angles have been magnified for the sake of clarity.

The computer output data which characterize a trajectory are the distance $d$ between the target and the asymptotic trajectory, measured in the direction normal to the proton orbit, and $\tan \Theta$, where $\Theta$ is the angle between the asymptotic trajectory and the zero-degree neutral beam. The computer output data for a wide range of trajectories are tabulated in Table I.

Particles of different momenta reaching the same point $F(r_0, a)$ outside of the fringing field have very nearly the same direction. This is illustrated in Fig. 3, where the computer output data are plotted in the form: $r \Theta(\approx d)$ vs. $\alpha + \beta (= \Theta)$. The full range of momenta $(0, 3 p_0 \leq p \leq p_0)$ shown on Fig. 3 is transmitted through the collimator that was placed in the shielding wall with entrance at $(37, 0.031)$, the acceptance of which is shown by the cross-hatched area. The unwanted lower momentum particles must therefore be removed by magnetic analysis following the collimator. Alternatively, the low momentum components may be eliminated by careful collimation in the fringing field region, where the trajectories are still momentum dependent, but this would make the momentum and intensity of the beam depend rather critically on the effective target position. Figure 3 is discussed further in the appendix.

The horizontal defocusing (or demagnification) is expressed by the ratio $\Delta\Theta_0/\Delta \Theta$, where $\Delta \Theta$ is the angular interval in the median plane containing the asymptotic trajectories of those particles of a given momentum which are emitted from the target within the angular interval $\Delta \Theta_0$. The demagnification $\Delta\Theta_0/\Delta \Theta$ is shown in Fig. 4 as a function of the initial angle of emission $\Theta_0$ for particles of momentum $p = p_0$. The same graph shows also the distance $r_0$ of the real target from the virtual target defined by the diverging beam. The angular position of the hole in the shielding wall through which the scattered proton beam entered the experimental hall is also shown in Fig. 4.
Horizontal trajectories have been computed for protons having $p_0$ of both 23 and 29 GeV/c, but they do not differ significantly. The calculations are valid therefore for all lower energies where the magnetic field is well below saturation.

**COMPUTED TRAJECTORIES IN THE VERTICAL PLANE**

A few such trajectories have been computed for particles emitted at a finite angle to the median plane by graphical methods, using measured values$^2)$ for the magnetic field gradient along paths already evaluated in the median plane. Vertical trajectories are shown in Fig. 5 for 24 GeV/c protons, elastically scattered through horizontal angles $\Theta_0$ of 14, 17.4, 22 and 26 milliradians, and making an arbitrary small angle with the median plane. [The maximum accepted initial vertical angles of emission $\phi_0$ are limited to a little over ± 5 milliradians by the pole faces of the magnet.] For a given value of $\Theta_0$ all particles whose vertical angle $\phi_0 \leq \phi_0 (\text{max})$ come to a focus as shown in the figure. Focal points corresponding to different horizontal angles of emission lie approximately on a straight line; this is shown in Fig. 6. These vertical trajectories apply only to positive particles of essentially full orbit momentum. Vertical trajectories have also been computed for three different values of momenta; they are discussed further in connection with the momentum distribution in the beam.

Since the initial angles of emission $\Theta_0$ in the horizontal plane are several times larger than the largest acceptable angles of emission $\phi_0$ in the vertical plane, the space angle of emission and therefore the source intensity of scattered protons are practically independent of the vertical angle of emission.
BEAM MEASUREMENTS IN NUCLEAR EMULSIONS

The first observations about the focusing properties of the fringing field were made in nuclear emulsion test plates which had been exposed while lying flat in the median plane and arranged in several radial rows. At successive distances from the target these rows intersected the calculated focal line in places corresponding to successively larger scattering angles $\Theta_0$. The angular distribution and flux density of relativistic particle tracks were measured in these emulsions. The angular distribution in the median plane was found to be sharply peaked about the direction computed for elastically scattered protons. The peak stood out clearly against a low intensity background of tracks spread over a large range of angles. In each row of emulsions the flux density of these collimated tracks, as a function of distance from the orbit, was found to pass through a maximum at a point close to the calculated focal line. The positions of these experimental foci are shown in the plan view of Fig. 6.

Along each row of emulsions the position of any point is related to the initial angle $\Theta_0$ of the computed trajectory (for $p=p_0$) that intersects the row at the given point. The measured particle-intensity profiles, plotted as a function of the common abscissa $\Theta_0$, are shown in Fig. 7 for three different distances from the target.

In the case of the intensity profile at $r=19\text{ m}$, some useful information has been obtained from the ratio of height to width of the peak. Without making any strong assumptions about the angular distribution of elastically scattered protons at the target, the effective thickness at focus of the layer containing all particles admitted by the magnet gap may be obtained from the intensity distribution and the computed vertical foci by simply requiring the conservation of particles. At $r=19\text{ m}$, the layer containing half of the protons of nearly full orbit momentum is approximately 1 cm thick at its thinnest point. This is consistent with the observation that in an emulsion exposed 1 cm above the median plane at this point only background tracks were found.
The sharpness of the focus is limited at least by multiple Coulomb scattering in the wall of the vacuum chamber, the momentum spread of the emerging particles, and imperfections in the focusing properties of the fringing field. Physically these are quite different things but their combined effect on the final direction of the trajectory may be represented most simply by ascribing to the target an effective size that depends upon the angle of emission $\Theta_c$ and the momentum $p$. In the appendix it is shown that multiple scattering in the vacuum chamber wall can account for a substantial part, if not all, of the effective target height. The effective target width, on the other hand, probably comes mainly from the distribution over momentum of the particles.

The thickness of the sheet of particles at focus is then simply the height of the image of the apparent target. In the case considered above, the target is effectively distributed vertically with a standard deviation of 1 cm.

At this same point, where the particles are most concentrated in space, their vertical angle distribution should reach its greatest spread. The full vertical aperture, as seen from the target, is limited to about 10 milliradians by the magnet pole faces; the corresponding computed vertical angular spread of the tracks at focus is 14 milliradians. The measured dip distribution extends over about 16 milliradians and is shown in Fig. 8. It indicates that the entire vertical aperture transmits particles fairly uniformly.

At a point far from focus, the vertical angular distribution is limited by the effective target height rather than by the available aperture. In general, the span of angles may be obtained by drawing all rays through the given point that intersect both the image of the effective target and the image of the limiting aperture of the magnet. This is equally true for the horizontal trajectories, though in this case the target image is virtual and the limiting aperture, if any, is the actual horizontal gap of any collimator through which the
particles must pass to reach the given point. The image formation is illustrated by the ray diagrams of Fig. 9 and is discussed further in the appendix.

Figure 10 is a target diagram of the directions of particle tracks measured in an emulsion which was exposed at the point in the median plane \( r = 37 \, \text{m}, \alpha = 31 \, \text{mr} \); i.e., at the point marking the entrance to the collimator which subsequently was built through the shielding wall. The measured angular distribution, when corrected for the multiple Coulomb scattering in the emulsion, is characterized by standard deviations \( \delta_H' = \delta_V' = \pm 1.0 \) milliradians for the horizontal and vertical directions.

In the appendix it is shown that such an angular distribution is equivalent to the effective target being distributed vertically with a standard deviation \( h = \pm 5 \) mm and horizontally with a standard deviation \( w = \pm 30 \) mm. If all of the particles were elastically scattered protons, multiple Coulomb scattering in the vacuum chamber wall alone would give rise to an effective target size \( h = w = \pm 4 \) mm; evidently this is the source of a large part of the vertical spread but contributes very little to the horizontal spread.

MOMENTUM DISTRIBUTION IN THE BEAM

In order to investigate the relative importance of lower momentum particles in the beam, vertical trajectories have been computed for three different momenta; for each momentum \( p \), a value of \( \Theta_c \) is chosen such that the three groups of particles have a common asymptotic trajectory in the median plane. Figure 10 gives the relevant numbers and illustrates the extent to which particles of different momenta are spread out vertically at that location where the first emulsion exposures were made in a momentum analysed proton beam, the point \( P(75 \, \text{m}, 36 \, \text{mr}) \). In the appendix there is calculated for each of the three momenta the solid angle \( \Delta \Omega \) subtended at the target containing those particles emitted from the target within the solid angle
$\Delta \Omega_2$. The momentum distribution of particles emitted from the target (with the appropriate $\Theta_0$) should be multiplied by the weighting factor $\Delta \Omega_0/\Delta \Omega$ to obtain the expected momentum distribution at this particular point of observation. This weighting factor, normalized to unity at $p = p_0$, is also multiplied by the transparency of the vacuum chamber wall and the resulting function, $f(p)$, is shown in Fig. 11.

The first experimental determination of the momentum spectrum of the scattered beam was made, using a counter telescope $S_1$, $S_2$ to define the beam incident on an analysing magnet and a second telescope $S_3$, $S_4$ to determine the angle of deflection.

A similar experimental set-up and the resulting measured momentum distribution for the target position $y_0 = -0.02$ m are shown in Fig. 12. The spectrum shows that approximately 30% of the particles counted by this apparatus have lower momenta than that of the elastically scattered protons.

Figure 13 shows a comparable arrangement in which the beam-defining counters $S_1$, $S_2$ have been replaced by a 1 cm wide slit in a lead collimator and emulsions are used to detect the directions and lateral distribution in intensity of the analysed particles. These measurements again indicate that about 70% of the particles entering the emulsions belong to the group of nearly full orbit momentum. The target diagram of Fig. 14 shows the measured direction of the tracks at the high momentum end of the plate. This angular distribution, when corrected for the multiple Coulomb scattering in the emulsion, agrees well with a distribution that is limited in horizontal angle by the lead collimator and in vertical angle by the proton synchrotron magnet gap; just as expected for particles near their vertical focus.

The vertical angular spread of ±0.8 milliradians is greater than the vertical acceptance angle of the counter telescope (less than 0.5 milliradians). Particles of lower momentum, being less well focused, have flatter trajectories. The counter telescope might thus discriminate slightly against counting the elastic components.
The peak intensity of the analysed protons observed in the emulsion is approximately $10^4/\text{cm}^2$ for a pulse of $2 \times 10^{11}$ protons in orbit.

**A HIGH INTENSITY SCATTERED PROTON BEAM**

For some experiments employing a small external target it is desirable to have as intense a beam as possible of monochromatic protons in the experimental hall.

The beam at the location discussed above could be increased in intensity by means of a magnetic quadrupole lens on the machine side of the shielding wall to increase the horizontal angle subtended, together with a pair of quadrupole lenses, following the analyser magnet, to form an image on the external target.

By using protons elastically scattered through a smaller horizontal angle, one gains intensity by virtue of the larger cross-section for elastic scattering; however, the amount of this increase is not known and is at least partially offset by the greater horizontal demagnification for those particles that travel a longer distance through the fringing field. At any rate, for practical reasons of avoiding interference between collimators used for other experiments, the scattered proton beam has been moved to approximately $\theta = 20$ milliradians for counter experiments and emulsion exposures. Another scattered proton beam of approximately $\theta_c = 40$ milliradians was used for bubble chamber exposures.

A significant improvement can be expected by providing a thin window in the vacuum chamber for the scattered protons, as well as by making sure that they do not have to pass through any other large amount of material (such as the valve housing following magnet unit No.1). A factor 2 is obtained simply by eliminating the 50% loss due to inelastic nuclear collisions in the vacuum tube wall. At the same time this will eliminate an important source of random scattering.
and thereby permit a better focus and higher final intensity to be obtained.

The purity of the beam can probably be improved by interposing a lead absorber somewhere on the line connecting the target and the beam-defining aperture of the first lens, in order to prevent neutrons from reaching the proton beam transport system and so avoid that source of background.

A pure beam of intensity $10^6$ proton/cm$^2$ per pulse should be obtainable as a very conservative estimate.

ACKNOWLEDGEMENTS


Dr. G. Plass, CERN, provided the machine-computed trajectories.

Professor G. Cocconi did much to facilitate the emulsion exposures and has, with his group, continued to develop and make use of the scattered-out beam for counter and emulsion experiments.

We are grateful to Professor F.G. Houtermans for his continued interest and one of us (B.D.) for the hospitality extended at his institute. Professor B. Peters contributed with many helpful discussions.

Financial assistance for the study was provided, in part, by the Schweizerische Kommission für Atomwissenschaften.
TABLE I

Computer Output Data - Horizontal Trajectories

<table>
<thead>
<tr>
<th>tan ( \theta ) d</th>
<th>1.0 po</th>
<th>0.9 po</th>
<th>0.8 po</th>
<th>0.7 po</th>
<th>0.6 po</th>
<th>0.5 po</th>
<th>0.4 po</th>
<th>0.3 po</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \theta_c = 15 )</td>
<td>0.0809</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>.687</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>0.0575</td>
<td>0.0837</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>.467</td>
<td>.732</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>25</td>
<td>0.0439</td>
<td>0.0597</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>.358</td>
<td>.501</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>30</td>
<td>0.0329</td>
<td>0.0450</td>
<td>0.0664</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>.311</td>
<td>.381</td>
<td>.576</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>35</td>
<td>0.0210</td>
<td>0.0307</td>
<td>0.0461</td>
<td>0.0781</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>.248</td>
<td>.309</td>
<td>.405</td>
<td>.705</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>40</td>
<td>0.0069</td>
<td>0.0144</td>
<td>0.0253</td>
<td>0.0435</td>
<td>0.0939</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>.190</td>
<td>.229</td>
<td>.292</td>
<td>.403</td>
<td>.893</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>45</td>
<td>-0.0075</td>
<td>-0.0020</td>
<td>0.0055</td>
<td>0.0165</td>
<td>0.0365</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>.140</td>
<td>.165</td>
<td>.202</td>
<td>.260</td>
<td>.383</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>50</td>
<td>-0.0209</td>
<td>-0.0169</td>
<td>-0.0116</td>
<td>-0.0043</td>
<td>0.0071</td>
<td>0.0277</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>.100</td>
<td>.117</td>
<td>.140</td>
<td>.174</td>
<td>.229</td>
<td>.345</td>
<td></td>
<td></td>
</tr>
<tr>
<td>55</td>
<td>-0.0327</td>
<td>-0.0297</td>
<td>-0.0259</td>
<td>-0.0208</td>
<td>-0.0132</td>
<td>-0.0013</td>
<td>0.0224</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>.063</td>
<td>.075</td>
<td>.092</td>
<td>.120</td>
<td>.153</td>
<td>.206</td>
<td>.332</td>
<td></td>
</tr>
<tr>
<td>60</td>
<td>-0.0428</td>
<td>-0.0406</td>
<td>-0.0378</td>
<td>-0.0341</td>
<td>-0.0287</td>
<td>-0.0206</td>
<td>-0.0065</td>
<td>0.0274</td>
</tr>
<tr>
<td></td>
<td>.041</td>
<td>.050</td>
<td>.061</td>
<td>.076</td>
<td>.099</td>
<td>.134</td>
<td>.203</td>
<td>.383</td>
</tr>
</tbody>
</table>

(The above trajectories are computed for \( y_c = -0.02 \) m. The tan \( \theta \) and \( d \) appear in PS/Int.EA 59-14 with negative signs. There is an unexplained 2.5 milliradian discrepancy between these computed trajectories and the direction of the observed maximum in the unanalysed beam. The observed direction has the smaller values of tan\( \theta \).)
REFERENCES

1) The computer programme has been developed by E. Kuiper, D. Lake and G. Plass - PS/Int. EA 59-14.

2) PS/Int. MM 59-5.

3) G. Cocconi, A. Diddens and A. Wetherell - 2 August 1960

7 September 1960.

* * *
APPENDIX

a) The Universal Graph for Horizontal Trajectories

In Fig. 3 the computer output data for the particle trajectories in the median plane are plotted directly: $d(\approx r \beta)$ vs. $\tan \theta(\approx \alpha + \beta)$. The values for different momenta $p$ lie along closely spaced curves. Numbers identifying the values of $\Theta_0$ (in milliradians) are placed alongside the corresponding points for the $p = p_0$ curve. This latter curve may be used to find the appropriate direction for a collimator to be placed with entrance at the point $(r_1, \alpha_1)$.

In this diagram any point $(r, \alpha)$ in the median plane is represented by a straight line of horizontal intercept $\alpha$ and slope $r$. The intersection of two such straight lines therefore represents the direction in the median plane of the line connecting the given points, and a family of lines on the graph having slopes between $r_1$ and $r_2$, intersecting in a single point, corresponds to a line of finite length in the median plane. Thus any collimator may be defined by the almond-shaped area that is bounded by two such families.

In Fig. 3 the cross-hatched area represents the acceptance of an 11 m long tapered collimator, 10 cm wide at the entrance and 20 cm wide at the exit that was placed in the shielding wall with entrance at $(37, .031)$.

b) Interactions in the Vacuum Chamber Wall

The elastically scattered protons penetrate the vacuum chamber wall at a very small angle so that the actual thickness of steel traversed corresponds approximately to a mean free path for nuclear interaction. The resulting loss of particles has been calculated using the computed horizontal trajectories and a wall thickness of 2 mm. The absorption mean free path for high-energy protons in steel was assumed to be $\lambda = 136 \text{ g/cm}^2$. The transmission through the wall is
plotted against the initial scattering angle $\Theta_0$ in the upper graph of Fig. 14.

The r.m.s. projected angle $\epsilon$ of multiple Coulomb scattering in the iron has been computed for 25 GeV/c protons. Scattering out from the beam is, in first approximation, equal to scattering into the beam; the resulting angular distribution of particles is equivalent to that from an extended target whose distribution is characterized by a standard deviation $h = w = \pm \epsilon r_w$ in both vertical and horizontal directions, where $r_w$ is the distance from the target to the point where the average trajectory intersects the wall of the vacuum chamber. At the effective centre of the fringing field ($\sim 10.5$ m for both projections) this effective target subtends the angle $\delta$. Thus $\delta$ measures the incoherent angular spread due to the multiple scattering. This quantity is plotted as a function of $\Theta_0$ in the lower graph of Fig. 15.

c) **Effective Target Size and Angular Distribution at a Point**

The fringing field of the proton synchrotron may be approximated by a highly astigmatic lens and a prism when describing its effect on the positive particles which we are considering. The net deflection is of no concern in regard to the image formation, so we ignore the prismatic component. The path length of the particles in the fringing field is only a few metres, a distance which is a moderately small fraction of the total trajectory. It is therefore a reasonably good approximation to describe the geometrical optics of the beam in terms of a diverging lens for the projection of the trajectories on to the median plane and of a converging lens for the vertical projection of trajectories (see Fig. 9).

In the median plane a target of effective half width $w$ appears to be at $r_0$, demagnified by the factor $\delta_0/\delta$. This virtual target subtends, at the point of observation $P$ an angle:

$$\delta' = \frac{w \Delta \Theta_0}{r-r_0}.$$
the quantities \( r_0 \) and \( \Delta \Theta / \Theta \) can be calculated from the Mercury computer output data describing the horizontal trajectories. They are plotted in Fig. 4.

In the vertical direction the target of half height \( h \) has a real image \( h' \) at the distance \( r_F \). This image subtends at the point of observation, the angle

\[
\delta'_V = \frac{h \varphi_0 / \varphi_1}{r_F - r};
\]

the focal distance \( r_F \) and the corresponding value of \( \varphi_0 / \varphi_1 \), having been computed graphically for a few cases only. See Figs. 5 and 11. Particles fill the entire angular interval only as long as \( \delta'_V > A' \), where \( A' \) is the angle subtended by the image of the limiting gap between the pole faces of the magnets.

The ray diagrams shown apply to the angular distribution at the point (37, 0.31) appropriate to the target diagram of Fig. 10. In this latter case the image of the gap subtends an angle \( A' = 1.7 \) milliradians, while \( \delta'_V = 1.0 \) milliradians; at this point, therefore, the size of the gap does not affect the angular distribution of particle tracks appreciably.

d) The Distribution of Momenta in the Beam

The particles emitted from the target within the solid angle \( \Delta \Omega_0 = \Delta \Theta_0 \cdot 2 \varphi_0 \) are spread cut over a height \( 2Z = 2(r_F - r) \varphi_1 \), assuming that there is a perfect focus and that the gap of the magnet transmits particles uniformly; the corresponding spread in the horizontal direction is \( \Delta Y = (r - r_0) \Delta \Theta_1 \).

Thus the area illuminated by the point target subtends a solid angle \( \Delta \Omega = 2 \Delta Y Z / r^2 \). One obtains

\[
\frac{\Delta \Omega}{\Delta \Omega_0} = \left( 1 - \frac{r_F}{r} \right) \frac{\Delta \Theta_1}{\Delta \Theta_0} \left( \frac{r_F - 1}{r} \right) \frac{\varphi_1}{\varphi_0}.
\]

NP/1081/nc
This expression is valid far from the focus where the effective target height may be neglected. At the focus, however, we must use for Z the expression:

$$Z = \delta_v \left( r - r_1 \right).$$

The distance $r_1$ from the target to the effective centre of the fringing field is given by:

$$r_1 = \left( \frac{\varphi_1}{\varphi_1 + \varphi_0} \right) r_2.$$

In the neighbourhood of the focus, one should properly form the convolution of the two distributions: the rectangular distribution given by the ray optics, and the spread that comes from those sources which are included under the effective target height. In the case we are considering, this makes little difference for out-of-focus trajectories.

The relevant numbers for the three trajectories which have been calculated graphically are given in Fig. 11.
The Computer Program yields values
of $-d$ and $-\tan \Theta$ for asymptotic
trajectories of positive particles in the median plane.

Fig. 2
TRAJECTORIES IN MEDIAN PLANE

"Mercury" Computer Output Data

$0.3 \rho_0 \leq \rho \leq 1.0 \rho_0$ ; $15 \leq \theta_0 \leq 60$ milliradians

Target at: $y_0 = -0.02$ m

$d (\approx \beta_0)$ [meters]

$\tan \theta \approx d + \beta$

Numbers attached to $\beta_0$ curve are values of $\theta_0$ in milliradians.

Fig. 3
HORIZONTAL TRAJECTORY PARAMETERS (COMPUTED)

VIRTUAL TARGET DISTANCE:

DEMAGNIFICATION:

Initial Horizontal Angle: $\theta_0$ (milliradians)

Fig.4
VERTICAL FOCUSING OF 24 GeV PROTONS

Calculated for 4 Angles of Elastic Scattering

Vertical trajectories are given for an arbitrary initial vertical angle of 5 milliradians. The actual limiting aperture is slightly different for different trajectories as shown at right:

Fig. 5
(a) Plan View Showing Emulsion Rave A, B, C:

(b) Flux Density of Relativistic Particle Tracks:

Fig. 7
Dip Distribution of Minimum Ionizing Tracks
at 19 m. Focal Point

Expected span of Dip Angles

Dip Angle (milliradians)

Fig. 8
TARGET DIAGRAM SHOWING ANGULAR DISTRIBUTION IN SPACE OF SCATTERED 24 GeV PROTONS IN PROPOSED BEAM

(Measured at entrance to new collimator through the shielding wall. Corresponds to elastic scattering in target of 25.7 mr)

67 Events
Test Plate 102
Exposure II

Approximately One-Half of the Particles have a Space Angle that is ≤ 1 Milliradian from the Average Direction.

Fig. 10
INTENSITY OF DIFFERENT MOMENTA - WEIGHTING FACTOR
(at r = 75 m)

Vertical Trajectories:

- Shielding Wall

Relative Intensity at P:

<table>
<thead>
<tr>
<th>$\varphi/\varphi_0$</th>
<th>1.0</th>
<th>.9</th>
<th>.7</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Theta_0$ (mrad)</td>
<td>27</td>
<td>92</td>
<td>40</td>
</tr>
<tr>
<td>$r_F$ (m)</td>
<td>64</td>
<td>35</td>
<td>12</td>
</tr>
<tr>
<td>$\varphi_i/\varphi_0$</td>
<td>.19</td>
<td>.33</td>
<td>.77</td>
</tr>
<tr>
<td>$2Z$ (cm)</td>
<td>2</td>
<td>13</td>
<td>50</td>
</tr>
<tr>
<td>$\frac{1}{2}E(z/\delta)$</td>
<td>.31</td>
<td>.14</td>
<td>.04</td>
</tr>
<tr>
<td>$\Delta \Theta/\Delta \Theta_0$</td>
<td>2.5</td>
<td>3.1</td>
<td>6.2</td>
</tr>
<tr>
<td>$T$</td>
<td>.62</td>
<td>.68</td>
<td>.74</td>
</tr>
<tr>
<td>$T \Delta \Theta_0/\Delta \Theta$</td>
<td>3.14</td>
<td>1.26</td>
<td>.18</td>
</tr>
<tr>
<td>$f(p)$</td>
<td>.100</td>
<td>.40</td>
<td>.06</td>
</tr>
</tbody>
</table>

Fig.11
APERTURE = 1 cm WIDE, 7.5 cm HIGH

\[ \frac{3+4}{1+2} \]

COUNTS
GeV/c

A. WETHERELL
A. DIDDENS
G. COCONI
6.9.60

Fig. 12
ANALYZED SCATTERED PROTON BEAM

Particle Flux: \((\text{cm}^{-2} \text{cm}^{-2} \text{ proton/yr})\)

- Elastic Scattered Protons: \(\sim 70\%\)
- Lower Momentum Secondaries: \(\sim 30\%\)

Distance Across Emulsion Plate (cm.)
TARGET DIAGRAM FOR ANALYZED PROTON BEAM
(Exposure of 6, 9, 60)

$r = 7.5 \text{ m.}$

Measured Standard Deviations:

- Horizontal: $0.5\text{ milliradians}$
- Vertical: $0.9 \text{ milliradians}$

Multiple Scattering in Emulsion: $0.75 \text{ milliradians}$

Net Angular Distribution:

\[
\delta_H' = 0.3 \text{ milliradians}
\]
\[
\delta_V' = 0.8 \text{ milliradians}
\]

Fig. 14
INTERACTIONS OF 24 GEV PROTONS
IN VACUUM CHAMBER WALL

Inelastic Nuclear Collisions:
(Using $\lambda_{abs} = 136 \text{ g/cm}^2$)

$$T = 100 e^{-\frac{t_0(\text{g/cm}^2)}{\lambda \cdot \theta}}$$

Multiple Coulomb Scattering:

INCOHERENT ANGULAR SPREAD $\delta$ DUE TO EFFECTIVE TARGET SIZE ARISING FROM MULTIPLE SCATTERING

$$\delta = \frac{f_m}{t_i} \cdot \varepsilon$$
$$\varepsilon = \frac{21}{\sqrt{\theta}} \cdot \frac{\text{p (mev)}}{t_0 \text{ (rad. length)}}$$

Projected r.m.s. angle of multiple scattering = $\varepsilon$:

$\text{Effective Target Size}$

Vacuum Chamber

$\text{Orbit}$

$\text{(Effective center of Fringe Field Lens)}$

Fig. 15