AC

RUB-MEP-23/94
September 1994

to be published in
Phys. Rev. D 1995

Simple Model Amplitude of the Nucleon: A Hint for Important Higher-Twist Effects?

Robert Eckardt, Jörg Hansper, and Manfred F. Gari

Ruhr-Universität Bochum

SW 9512

Institut für Theoretische Physik

Arbeitsgruppe Mittelenenergiephysik

A Simple Model Amplitude of the Nucleon: a Hint for Important Higher-Twist Effects ?

Robert Eckardt, Jörg Hansper, and Manfred F. Gari

*Ruhr-Universität Bochum, Institut für Theoretische Physik, Agr. Mittelerneuerphysik,
D-44780 Bochum, Germany. e-mail: QCD-MEP@gari.MEP.Ruhr-Uni-Bochum.DE*

(September 1994)

The convergence properties of a simple model for the nucleon quark distribution amplitude into Appell polynomials are studied and the model is used to calculate nucleon form factors in perturbative QCD. Treating the gluon propagator and the strong coupling α_S in various ways, we demonstrate that the freedom left in the unknown $\vec{k}_\perp$ -dependence of the full wave function is large enough to account for a wide range of form factor results. This is interpreted as a hint that higher-twist effects may give significant contributions to hadronic form factors in PQCD. Additionally, we find a ratio of $|\frac{F_1^n}{F_1^p}| \approx \frac{1}{2}$ at large momentum transfers.

The problem of constructing realistic hadronic wave functions in a reliable way is of outstanding theoretical interest because they are *process-independent*. This task, however, is far from being completed. On the contrary, a large number of different models for hadronic quark distributions has been put forward in the past by various authors. However, it has been shown recently [1] that it is not possible to find polynomial models for hadronic distribution amplitudes from their corresponding moments calculated by means of QCD sum-rules. The reason for this is the extreme instability of the polynomial expansion with respect to small variations in the moments.

Using the specific example of the nucleon, the nucleon quark distribution amplitude ϕ_N is defined by:

$$\phi_N(x, \mu^2) := \int_{\vec{k}_\perp < \mu^2} [d^2 k_\perp] \psi_N(x_i, \vec{k}_\perp) \quad , \quad (1)$$

where ϕ_N can be interpreted as the probability amplitude for the hadron to consist of quarks collinear up to the scale μ^2 with longitudinal momentum fractions x_i . The dependence on μ^2 is completely determined by the evolution equation [2]. Note that ϕ_N has lost its dependence on the transverse momenta of the quarks, in contrast to the full light-cone wave function $\psi_N(x, \vec{k}_\perp)$.

The full light-cone wave function $\psi_N(x, \vec{k}_\perp)$ is usually constructed by multiplying ϕ_N with with an additional function that also has a dependence on the transverse momenta, e.g.

$$\psi_N(x, \vec{k}_\perp) = \phi_N(x) \cdot f(x, \vec{k}_\perp) \quad . \quad (2)$$

Using this kind of ansatz for the wave function, the condition of Eq. (1) has to be fulfilled for sake of consistency. In some calculations [3,4], a Gaussian model [3] for $f(x, \vec{k}_\perp)$ is used in the following form (with normalization factor N_f):

$$f(x, \vec{k}_\perp) = N_f \cdot \frac{1}{x_1 x_2 x_3} \exp \left(-a^2 \sum_{i=1}^3 \frac{(\vec{k}_{i\perp})^2}{x_i} \right) \quad . \quad (3)$$

As a consequence of the indeterminacy of a polynomial expansion, one has to formulate, *in addition to QCD sum-rule moments*, other strong criteria that any candidate for a realistic model wave function must fulfill, so as to rule out the pathological cases [5–8]. We summarize the criteria for the model distribution of the nucleon that are essential in our opinion:

(i) functional simplicity, (ii) minimum number of free parameters, (iii) smoothness, (iv) absence of oscillations, (v) strict positivity, (vi) substantially non-polynomial form (e.g. exponential) and (vii) process-independent construction.

Following these criteria, we propose a model wave function of simpler (“haplusterotic”) type by merely modelling the completely symmetric asymptotic distribution amplitude, mainly by shifting the position of the maximum.

As a specific ansatz for our haplusterotic (“ Ha^+ ”) model of the quark distribution amplitude of the nucleon, we chose the following form:

$$\phi_N^{Ha^+}(x) = N \exp \left(-\left(\frac{b_1^{(r)}}{x_1^r} + \frac{b_2^{(r)}}{x_2^r} + \frac{b_3^{(r)}}{x_3^r} \right) \right) \quad . \quad (4)$$

$r = (1, \frac{1}{2})$ for model $(\phi_N^{Ha^+ -1}, \phi_N^{Ha^+ -\frac{1}{2}})$, and N is a normalization factor. The three parameters $b_i^{(r)}$ are adjusted in order to change the position of the maximum according to the requirements of QCD sum-rule moments, i.e. the $b_i^{(r)}$ are chosen so as to yield optimal agreement between the moments of the model distribution amplitude and the corresponding values of the same moments as obtained from QCD sum rules.

Model $\phi_N^{Ha^+ -1}$ and $\phi_N^{Ha^+ -\frac{1}{2}}$ reproduce the 19 moments of Ref. [9] with sufficient accuracy with only *three* parameters! This can be seen in Table I, where also the moments of Ref. [10] and those of the asymptotic distribution amplitude are given. As an example, we show plots of an $r = 1$ model in Fig. 1 in comparison with the distribution amplitude of Ref. [9].

TABLE I. Two different sets of sum-rule moments of King and Sachrajda (KS) [10] and of Chernyak, Ogloblin and Zhitnitsky [9], in comparison with the moments of the following wave functions: $\phi_{N,as}$, ϕ_{COZ} [9] and our new model $\phi_N^{Ha^+}$. For $r = 1$ ($\phi_N^{Ha^+} - 1$) we have $b_1 = 0.196$, $b_2 = 2.91 \cdot 10^{-13}$, $b_3 = 0.00804$ and for $r = \frac{1}{2}$ ($\phi_N^{Ha^+} - \frac{1}{2}$) we have $b_1 = 0.652$, $b_2 = 6.17 \cdot 10^{-11}$, $b_3 = 0.0548$, see Eq. (4).

$n_1 n_2 n_3$	Moments $\langle n_1 n_2 n_3 \rangle$					
	KS	COZ	$\phi_{N,as}$	ϕ_{COZ}	$\phi_N^{Ha^+} - 1$	$\phi_N^{Ha^+} - \frac{1}{2}$
0 0 0	1	1	1	1	1	1
1 0 0	0.46 ... 0.59	0.560	$^{+0.06}_{-0.02}$	0.333	0.531	0.534
0 1 0	0.18 ... 0.21	0.192	$^{+0.008}_{-0.012}$	0.333	0.211	0.209
0 0 1	0.22 ... 0.26	0.229	$^{+0.021}_{-0.029}$	0.333	0.257	0.257
2 0 0	0.27 ... 0.37	0.350	$^{+0.07}_{-0.03}$	0.142857	0.316	0.323
0 2 0	0.08 ... 0.09	0.084	$^{+0.004}_{-0.019}$	0.142857	0.0709	0.0704
0 0 2	0.10 ... 0.12	0.109	$^{+0.011}_{-0.019}$	0.142857	0.0939	0.0956
1 1 0	0.08 ... 0.10	0.090	$^{+0.01}_{-0.01}$	0.095238	0.0960	0.0942
1 0 1	0.09 ... 0.11	0.102	$^{+0.008}_{-0.012}$	0.095238	0.119	0.117
0 1 1	—	-0.03 ... 0.03	0.095238	0.0270	0.0444	0.0441
3 0 0	—	0.236	$^{+0.014}_{-0.026}$	0.0714285	0.205	0.212
0 3 0	—	0.032	$^{+0.008}_{-0.004}$	0.0714285	0.0297	0.0300
0 0 3	—	0.052	$^{+0.004}_{-0.004}$	0.0714285	0.0417	0.0435
2 1 0	—	0.045	$^{+0.004}_{-0.004}$	0.035714285	0.0494	0.0489
2 0 1	—	0.050	$^{+0.005}_{-0.006}$	0.035714285	0.0624	0.0617
1 2 0	—	0.035	$^{+0.002}_{-0.008}$	0.035714285	0.0285	0.0277
1 0 2	—	0.041	$^{+0.002}_{-0.004}$	0.035714285	0.0386	0.0382
0 2 1	—	-0.004 ... 0.007	0.035714285	0.0056	0.0127	0.0127
0 1 2	—	-0.005 ... 0.008	0.035714285	0.0073	0.0136	0.0138
1 1 1	—	—	0.0238095	0.0141	0.0181	0.0176

The *exact* functional form for our model distribution amplitude of the nucleon is now known by definition, see Eq. (4). Consequently, the expansion coefficients c_i with respect to the Appell polynomials $A_i(x)$ of Ref. [2] can be determined *directly* via the corresponding orthonormality relations so that our model can be expanded into an infinite series of Appell polynomials according to:

$$\phi_N^{Ha^+}(x, Q^2) = \phi_{as}(x) \sum_{i=0}^{\infty} c_i \left(\frac{\alpha_S(Q^2)}{\alpha_S(\mu_0^2)} \right)^{\gamma_i} A_i(x) \quad (5)$$

The polynomials used in the expansion are taken from Refs. [11] and [12].

It is demonstrated in Fig. 2 that the convergence of the whole series is quite slow: a very large number of polynomials (up to tenth order) has to be summed up until the shape of the (exponential) model function is sufficiently reproduced. It is emphasized that we are dealing with the *exact* polynomial expansion that yields the *optimal* approximation to the corresponding function in every order. We are *not* dealing with a polynomial expansion that has coefficients determined from moments with uncertainties (e.g. QCD sum-rule moments), which would lead to “neurotic” behaviour (cf. Refs. [1]).

FIG. 1. The well-known nucleon quark distribution amplitude ϕ^{COZ} of Chernyak, Ogloblin and Zhitnitsky and our model amplitude $\phi_N^{Ha^+}$. Also shown are the shapes of the amplitudes as modified by the additional effective $k_{\perp}$ -dependence f .

FIG. 2. Appell polynomial expansions of different degree of our model amplitude $\phi_N^{Ha^+}$ and the exact function itself. The model shown needs an expansion up to 10th degree to sufficiently resemble the exact function. When the maximum approaches the edge the convergence becomes worse and even higher degrees of expansion are necessary.

Obviously, we have here a clear counterexample to the optimistic assumption [13] that higher polynomials are negligible and that a polynomial of second degree, that is in reasonable agreement with existing QCD sum-rule moments, can sufficiently describe the “true” quark distribution amplitude: Although the *exact* moments of $\phi_N^{Ha^+}(x)$ are known e.g. up to fifth order, this does not allow to reconstruct a polynomial model for the distribution amplitude with sufficient accuracy!

Calculating form factors in perturbative QCD by means of such a polynomial expansion shows that the form factors converge slowly and indeed grow with increasing order in the model expansion. For the exact distribution amplitude we find very large form factor values if the conventional approximations in PQCD calculations are used [13].

However, in the end-point regions, which give significant contributions to the form factors, various problems appear. Although the end-point regions give the main contributions to the form factors, PQCD is not applicable because the exchanged gluon momenta are too small, as first pointed out by Isgur and Llewellyn-Smith [14]. Another difficulty arises when the collinear approximation of Lepage and Brodsky [2] is discarded and transverse degrees of freedom in the particle momenta are allowed. It is expected that such $k_\perp$ - (or higher-twist) effects are important just in the end-point regions [11]. A further point is the indeterminacy of (polynomial) models of hadronic quark distribution amplitudes [1] that is particularly large in the end-point regions [11], where the precise form of the distribution amplitude is mainly determined by higher moments that have not yet been calculated.

Furthermore, Botts, Li and Sterman [15,16] have shown that gluonic radiative corrections in terms of transverse separations, i.e. Sudakov form factors, have to be taken into account in PQCD calculations of hadronic form factors. Using these ideas, Kroll and collaborators [17] have chosen a Gaussian model for the transverse momentum dependence of the nucleon wave function according to Eq. (3) and incorporated Sudakov form factors into the hard-scattering formalism. Simplifying

the difficult problem of an eleven-dimensional integration over the transverse momenta to a seven-dimensional one, they found that the conventional quark distribution amplitudes, e.g. [9,13], yield a magnetic form factor G_M^p that is much smaller than the experimental value.

FIG. 3. Proton and neutron form factor values for the Ha^+ -model and its Appell polynomial expansions of up to 5th degree using different prescriptions in the form factor calculations. All calculations were performed at 30 GeV² without taking the Q^2 -evolution of the quark distribution amplitude into account and using an effective gluon mass of $m_g = 300$ MeV in the gluon propagator in the form $1/(xyQ^2 + m_g^2)$. For * a constant $\alpha_S \equiv 0.3$, for • and o a running $\alpha_S(xyQ^2 + m_g^2)$ was used. Moreover, for o an additional effective $k_\perp$ -dependence of the wave function was assumed that suppresses the end-point region in the distribution amplitude. Note that for all calculation procedures we have a convergence on $|F_1^n/F_1^p| \approx 0.5$, as already found in Ref. [9]!

In contradistinction to Ref. [17], we follow a slightly different way: we choose the values of $a = 0.9939$ GeV⁻¹

and $\sqrt{\langle \vec{k}_\perp^2 \rangle} \approx 271 \text{ MeV}$ [17] and supplement the Gaussian Ansatz of Ref. [17], see Eq. (3), with a delta function:

$$\psi_N(x, \vec{k}_\perp) := \phi_N(x) \cdot f(x, \vec{k}_\perp) \cdot \prod_i \frac{1}{\pi} \delta(\vec{k}_{i\perp}^2 - \langle \vec{k}_\perp^2 \rangle) \quad (6)$$

FIG. 4. Form factors at 30 GeV^2 for the Ha^+ -model and its Appell polynomial expansions of up to 5th degree using different prescriptions in the form factor calculations. For the polynomial approximations the Q^2 -evolution of the QDA was taken into account. Since the BL-evolution equation of the exact model amplitude $\phi_N^{Ha^+}$ can only be solved numerically, estimated values, based on the polynomial approximations, are given here. An effective gluon mass of $m_g = 300 \text{ MeV}$ in the gluon propagator in the form $1/(xyQ^2 + m_g^2)$ and a running $\alpha_S(xyQ^2 + m_g^2)$ was used. For $\circ$ an additional effective $k_\perp$ -dependence of the wave function was assumed. Note that for all calculation procedures we have a convergence on $|F_1^n/F_1^p| \approx 0.5$, as already found in Ref. [9]!

This means that we use an *effective* mean value for the transverse momenta instead of performing the integrations explicitly. Therefore, we can replace the transverse momenta by a simple “gluon mass” m_g : $\vec{k}_{i\perp}^2 \rightarrow \langle \vec{k}_\perp^2 \rangle =: m_g^2$ [11,18,19]. The effect of this modification is shown in Fig. 1, where the distribution amplitude of Ref. [9] and the new model $\phi_N^{Ha^+}$ are multiplied by $f(x, \langle \vec{k}_\perp^2 \rangle)$. We obtain results very close to those of Ref. [17]: including the $k_\perp$ -dependence reduces the form factors of conventional distribution amplitudes by a factor of 1.5 ... 6 and the Q^2 -behaviour of the form factor becomes much flatter. Therefore we believe that the Ansatz of Eq. (6) is an excellent approximation, and indeed a useful one because the true $k_\perp$ -dependence of the wave function is still unknown.

Calculating the form factor F_1^p by means of the model $\phi_N^{Ha^+}$, we find a large variety of possible values, as demonstrated in Figs. 3 and 4, according to the calculation procedure used. Furthermore, form factors calculated by means of the *exact* polynomial expansion of the model $\phi_N^{Ha^+}$ are given for various orders of expansion. We note that for all types of form factor calculations the convergence properties of the form factors are similar: up to fourth order of expansion we have strongly growing contributions that may even begin to oscillate around the “true” value for increasing order.

Obviously, the calculations are very sensitive to the specific choice and the magnitude of α_S . An effective constant $\alpha_S = 0.3$ gives much smaller results than a dynamical coupling that depends on the internal gluon momenta. Taking into account also the Q^2 -evolution of the distribution amplitude, which has an α_S -dependence as well, this effect is even amplified.

We furthermore observe an interesting phenomenon: the ratio $|F_1^p/F_1^n|$ is very insensitive to the order of expansion as well as to the calculation procedure used. We have in practically all cases: $|F_1^p/F_1^n| \approx 2 \dots 2.2$, cf. Figs. 3 and 4. This seems to be caused by the QCD sum rules alone, as already stated in Ref. [9], and can be regarded as valuable information when data for nucleon form factors have to be analyzed.

In summary, we have shown that the *non-polynomial* model $\phi_N^{Ha^+}$ is a good starting point for investigating the role of higher-twist effects in PQCD calculations of hadronic form factors. It has a simple structure because it has only *three* parameters determined solely by QCD sum-rule moments and is stable under variations in its functional form (e.g. $r = \frac{1}{2}, 1$). Of course, the quality of $\phi_N^{Ha^+}$ depends on that of the sum-rule moments and can only be improved if a set of significantly more accurate moments up to higher orders were calculated. With fixed $\phi_N^{Ha^+}$, we found a large variety of possible form factor values according to different parameters and calculation procedures. This shows that the higher-twist effects are essential and deserve the main attention in

future analyses — in contrast to the distribution amplitude $\phi_N^{Ha^+}$ that seems to be sufficiently fixed by existing QCD sum-rule moments. Furthermore, we have given evidence that it is sufficient to use an *effective* model for the $k_\perp$ -dependence instead of integrating over the full set of transverse momentum variables. Using the example of $\phi_N^{Ha^+}$, we have demonstrated the insufficiency of a polynomial limited to small orders of expansion and we have emphasized the sensitivity of the whole procedure to the specific form and magnitude of α_s . In addition, we made the observation that the ratio $|\frac{F_1^a}{F_1^p}|$ is nearly independent of the approximation used in the calculations, which might have a deeper, physical meaning and can be used for an interesting prediction for the Dirac form factor of the neutron in terms of F_1^p .

ACKNOWLEDGMENTS

We are grateful to J.A. Eden for useful discussions.

-
- [1] R. Eckardt, J. Hansper and M.F. Gari, Phys. Rev. **D50**, R26 (1994); R. Eckardt, J. Hansper and M.F. Gari, Phys. Lett. **B 329**, 345 (1994).
 - [2] G.P. Lepage and S.J. Brodsky, Phys. Rev. **D22**, 2157 (1980).
 - [3] S. Brodsky, T. Huang and P. Lepage, Report No. SLAC-PUB-2540, 1980 (unpublished); G.P. Lepage, S. Brodsky, T. Huang, P.B. Mackenzie, Banff Summer Institute, Particles and Fields 2, A.Z. Capri, A.N. Kamal (eds.), 1983.
 - [4] C.E. Carlson and F. Gross, Phys. Rev. **D36**, 2060 (1987).
 - [5] V.L. Chernyak and A.R. Zhitnitskiĭ, Nucl. Phys. **B201**, 492 (1982).
 - [6] A.R. Zhitnitskiĭ, Phys. Lett. **B 329**, 493 (1994).
 - [7] A. Mueller, private communication.
 - [8] V.A. Radyushkin, private communication.
 - [9] V.L. Chernyak, A.A. Ogloblin and A.R. Zhitnitsky, Z. Phys. **C 42**, 569 (1989).
 - [10] I.D. King and C.T. Sachrajda, Nucl. Phys. **B279**, 785 (1987).
 - [11] J. Hansper, R. Eckardt and M.F. Gari, Z. Phys. **A341**, 339 (1992); R. Eckardt, J. Hansper and M.F. Gari, Z. Phys. **A343**, 443 (1992).
 - [12] M. Bergmann, Dissertation, Ruhr-Universität Bochum 1993.
 - [13] N.G. Stefanis and M. Bergmann, Phys. Rev. **D47**, R3685 (1993).
 - [14] N. Isgur and C.H. Llewellyn-Smith, Nucl. Phys. **B317**, 526 (1989).
 - [15] J. Botts and G. Sterman, Nucl. Phys. **B325**, 62 (1989).
 - [16] H.-N. Li and G. Sterman, Nucl. Phys. **B381**, 129 (1992).
 - [17] J. Bolz, R. Jakob, P. Kroll, M. Bergmann, and N.G. Stefanis, Universität Wuppertal, Report No. WU-B-94-06, submitted to Z. Phys. **C**.
 - [18] C.R. Ji, A.F. Sill and R.M. Lombard-Nelsen, Phys. Rev. **D36**, 165 (1987).
 - [19] C. Bernard, Phys. Lett. **108 B**, 431 (1982); J.M. Cornwall, Phys. Rev. **D26**, 1453 (1982).